Report for project 3

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In this project we analysed some cooking data and did fairly well.

1 Introduction

The concept of machine learning have gained a hughe popularity boost over the last couple of years. And this is no wonder: the techniques have a wide range of applications and can be a major asset if you know when to use what.

When to use what is exactly what we're going to have a brief peek into in this project. We will evaluate the performance of three different methods for classification on a data set consisting of recipies labeled with the cuisines they belongs to. That is we are looking at a classification problem with multiple classes. The methods we will look at are logistic regression, support vector machines and random forests.

All the code we have implemented can be found at https://github.com/???. A print of the jupyter notebook is also attached at the end of this report.

2 Methods

First we need to establish some notation. In the following we assume that we are given $N \in \mathbb{N}$ samples consisting of $p \in \mathbb{N}$ features and one target. Further we let $K \in \mathbb{N}$ be the number of classes in our classification problem. We then denote by $x_{i,j}$ the j-th feature of the i-th sample and by $t_i \in \{1, 2, ..., K\}$ the target of the i-th sample.

2.1 Logistic Regression

Let $\mathbf{x}_i := (1, x_{i,1}, x_{i,2}, \dots, x_{i,p})$ be the row-vector in \mathbb{R}^{p+1} consisting of a one followed by the features of sample i. Also let $\boldsymbol{\beta}_k$ be a column-vector in \mathbb{R}^{p+1} for $k \in \{1, 2, \dots, K-1\}$ and let $\boldsymbol{\theta} := [\boldsymbol{\beta}_1 \ \boldsymbol{\beta}_2 \ \dots \ \boldsymbol{\beta}_{K-1}]$ be the matrix with the $\boldsymbol{\beta}_k$'s as it's columns. In the case of multinomial logistic regression our model is then given by

$$P_{k}(\boldsymbol{x};\boldsymbol{\theta}) = \frac{\exp(\boldsymbol{x}\boldsymbol{\beta}_{k})}{1 + \sum_{l=1}^{K-1} \exp(\boldsymbol{x}\boldsymbol{\beta}_{l})} \qquad k \in \{1, 2, \dots, K-1\}$$

$$P_{K}(\boldsymbol{x};\boldsymbol{\theta}) = \frac{1}{1 + \sum_{l=1}^{K-1} \exp(\boldsymbol{x}\boldsymbol{\beta}_{l})}$$

$$(1)$$

where the β_k 's are the coefficients we wish to estimate. $P_k(x; \theta)$ is then the probability that a sample with features x belongs to category k.

The way we fit our model in the case of logistic regression deviates slightly from the usual proceedure with the introduction of a loss function. We now instead define a function we wish to maximize, namely the log-likelihood function given by

$$L(\boldsymbol{\theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \chi_{\{k\}}(t_i) \log(P_k(\boldsymbol{x}_i; \boldsymbol{\theta}))$$
(2)

However the difference doesn't go further than the fact that this function shouldn't be interpreted exactly as a loss function. Other than that we proceed as usual by minimizing the negative of this function in order to maximize the original function. To do this we first need to compute the derivative of this function with respect to θ . Some computations gives us

$$\frac{\partial L(\boldsymbol{\theta})}{\partial \beta_{k,j}} = \sum_{i=1}^{N} \chi_{\{k\}}(t_i) x_{i,j} (1 - P_k(x_i; \boldsymbol{\theta}))$$
(3)

We are now all set to use the gradient method of your choice to minimize -L as a function of θ .



2.2

To call a Support Vector Machine classifier with no kernel we use the following Python code:

- 2.3 Decision trees
- 3 Model Selection and Verification
- 4 Results
- **5** Conclusion

We did good.