HW 1

- Derivatives by definition:

$$\frac{df}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h} \qquad \frac{\partial f}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

- $\frac{d}{dx}\sin(x) = \cos x$ $\qquad \frac{d}{dx}\cos(x) = -\sin x$
- $\frac{d}{dx}\tan(x) = \frac{1}{\cos^2 x}$ $\qquad \frac{d}{dx}\cot(x) = -\frac{1}{\sin^2 x}$
- $\frac{d}{dx}\sec(x) = \tan x \sec x$ $\qquad \frac{d}{dx}\csc(x) = -\cot x \csc x$
- $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$ $\qquad \frac{d}{dx}\mathrm{arccot}(x) = -\frac{1}{1+x^2}$
- $\frac{d}{dx}\mathrm{arcsec}(x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad \frac{d}{dx}\mathrm{arccsc}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}a^x = a^x\ln(a)$ $\qquad \frac{d}{dx}\ln(x) = \frac{d}{dx}\log(x) = \frac{1}{x}$
- $\frac{d}{dx}e^x = e^x \rightarrow \frac{d}{dx}e^{ax} = ae^{ax}$ $\qquad \frac{d}{dx}x^x = x^x(\ln x + 1)$

- Equation of the tangent plane to the surface given by $f(x,y,z) = k$, at $A(x_0, y_0, z_0)$:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$a = \frac{\partial f}{\partial x}(x_0, y_0, z_0)$$

$$b = \frac{\partial f}{\partial y}(x_0, y_0, z_0)$$

$$c = \frac{\partial f}{\partial z}(x_0, y_0, z_0)$$

- The tangent plane at (x_0, y_0, z_0) is parallel to the xy– plane if $\nabla f = k\begin{bmatrix}0\\0\\1\end{bmatrix}$ for some k.

HW 2

- $\ln e^x = x, \log\left(a^b\right) = b\log(a), \log_x\left(\frac{1}{a^x}\right) = -a, \log_a b = \frac{\ln b}{\ln a}, \ \log_a b = \frac{1}{\log_b a}, \ \log_a \frac{1}{b} = \log_{\frac{1}{a}} b = -\log_a b$
- $\frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} + \frac{dz}{dy}\frac{dy}{dt}$
- $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$
- $\nabla f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$
- $D_u f(x,y) = \nabla f \cdot u$
- $\hat{u}(\text{unit vector with the maximum rate of change}) = \frac{1}{|\nabla f(x,y)|} \times \nabla f(x,y)$

HW 3

- For a function $f(x, y)$:

$$\text{Find} \rightarrow \frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y} \rightarrow \text{then solve} \begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases} \rightarrow \text{find critical point } (a, b)$$

$$\text{Find} \rightarrow \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\frac{\partial f}{\partial x} \ \mid \ \frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}\frac{\partial f}{\partial y} \ \mid \ \frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial f}{\partial x}\frac{\partial f}{\partial y} \ \mid$$

$$\mid \quad D = \frac{\partial^2 f}{\partial x^2}(a, b)\frac{\partial^2 f}{\partial y^2}(a, b) - \left(\frac{\partial^2 f}{\partial x\,\partial y}(a, b)\right)^2 \quad \mid$$

$$\text{Classify critical point } (a, b) \rightarrow \text{if} \begin{cases} D > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} > 0 \rightarrow (a, b) \rightarrow \text{minimum} \\ D > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} < 0 \rightarrow (a, b) \rightarrow \text{maximum} \\ D < 0 \rightarrow (a, b) \rightarrow \text{saddle point} \end{cases}$$

- Shortest distance from a point (a, b, c) to a plane defined by $z = f(x, y)$:

$$\text{distance} = \sqrt{(x-a)^2 + (y-b)^2 + (f(x, y) - c)^2}$$

$$\Delta(x, y) = (x-a)^2 + (y-b)^2 + (f(x, y) - c)^2 \rightarrow \begin{cases} \frac{\partial \Delta}{\partial x} = 0 \\ \frac{\partial \Delta}{\partial y} = 0 \end{cases} \rightarrow \text{find closest point } (x, y, z)$$

$$\text{susbstitute point in } \sqrt{(x-a)^2 + (y-b)^2 + (f(x, y) - c)^2}$$

- Absolute maximums and minimus in polygons:

1) Find critical points → substitute → find value of f → no need to determine min or max.

2) Find equations for all sides (triangle, square, rectangle) →

→ substitute them in f → set borders → substitute borders → find min & max.

3) Compare all final values, to set absolute minimum and maximum

- Absolute maximum and minimum of function $f(x, y)$, subject to the constraint $g(x, y)$:

$$1)\ \text{Find critical points of f} \rightarrow \text{substitute in f}$$

$$2)\ *\ \text{Use method of Langrage}'\text{s multipliers}\ *$$

$$\left\{\begin{matrix} \dfrac{\partial f}{\partial x} = \lambda \dfrac{\partial g}{\partial x} \\ \dfrac{\partial f}{\partial y} = \lambda \dfrac{\partial g}{\partial y} \\ g(x, y) \end{matrix}\right\} \rightarrow \text{solve system} \rightarrow \text{find points} \rightarrow \text{substitue in f}$$

$$3)\ \text{Compare all final values, to set absolute minimum and maximum}$$

HW 4

- Lagrange multipliers to find the max and min of the function $f(x, y)$ subject to the given constraint $g(x, y)$:

$$\left\{\begin{matrix} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \\ g(x, y) \end{matrix}\right\} \rightarrow \text{separate } \lambda \text{ on both equations} \rightarrow \text{equalize} \rightarrow \text{ find relation } x - y \rightarrow \text{substitute in } g(x, y)$$

$$\rightarrow \text{find points} \rightarrow \text{substitute in } f(x, y) \rightarrow \text{evaluate maximum and minimum values}$$

- The plane $f(x, y, z) = k$ intersects the paraboloid $z = f(x, y)$ in an ellipse. Find the points on this ellipse that are nearest and farthest from the origin.

$$g_1 \rightarrow f(x, y, z) = k \quad | \quad g_2 \rightarrow f(x, y) - z = 0$$

$$(x, y, z) \rightarrow \text{a point that satisfies both of the constraints}$$

$$\text{distance from origin} = \sqrt{(x - 0)^2 + (y - 0)^2 + (z - 0)^2}$$

$$\rightarrow \text{by minimizing } f(x, y, z) = x^2 + y^2 + z^2$$

$$\nabla f = (2x, 2y, 2z) \mid \nabla g_1 = (\ , \ , \ ) \mid \nabla g_2 = (\ , \ , \ )$$

$$\left\{\begin{matrix} 2x = \lambda_1 \frac{\partial g_1}{\partial x} + \lambda_2 \frac{\partial g_2}{\partial x} \\ 2y = \lambda_1 \frac{\partial g_1}{\partial y} + \lambda_2 \frac{\partial g_2}{\partial y} \\ 2z = \lambda_1 \frac{\partial g_1}{\partial z} + \lambda_2 \frac{\partial g_2}{\partial z} \\ g_1 \rightarrow f(x, y, z) = k \\ g_2 \rightarrow f(x, y) - z = 0 \end{matrix}\right\} \rightarrow \text{solve system} \rightarrow \text{find points} \rightarrow \text{substitue in } f \rightarrow \begin{matrix}\text{deterine nearest} \\ \text{\& farthest}\end{matrix}$$

- Differential equations, initial value problems – solution method:

  a) linear:

$$1) \text{ transform to form: } y' + ky = f(x)$$

$$2) \text{ calculate } \mu = e^{\int (k) dx}$$

$$3) \text{ solve } y = \frac{1}{\mu} \int (f(x)\mu) dx$$

  b) separable:

$$1) \text{ transform to } y' = \text{sth} \rightarrow y' = \frac{dy}{dx} \rightarrow \text{separate x, and y on each side}$$

$$2) \text{ integrate both sides and separate the y on the left side}$$

$$3) \text{ this is the general solution, although it can become a unique one, if a point is given}$$

- Integration $\rightarrow \left\{ \begin{matrix} \text{a) By parts} \rightarrow \int u dv = uv - \int v du \\ \text{b) By substitution} \rightarrow \text{set part as u, find du, substitute and integrate} \end{matrix} \right.$

$$\text{c) Special cases} \rightarrow \int a^x = \frac{a^x}{\ln a} + C \ \Big| \int \ln x = x \ln x - x + C \Big| \int \sec^2 x = \tan x + C$$

$$\int \sin^n(x) dx = -\frac{1}{n} \sin^{(n-1)}(x) \cos(x) + \frac{(n-1)}{n} \int \sin^{(n-2)} dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

HW 5

- Projection of vector v on subspace W spanned by another vector w:

$$\text{proj}_w v = \frac{\langle v, w\rangle}{\|w\|^2} w$$

- Orthogonal projection of vector v on subspace W spanned by another vector w:

$$\text{orth}_w v = v - \text{proj}_w v = v - \frac{\langle v, w\rangle}{\|w\|^2} w$$

- Inner product of polynomials f(x) and g(x) viewed as elements of the space of continuous functions on $[b, a]$ equipped with the inner product:

$$\langle f, g\rangle = \int_b^a f(x)g(x)\, dx$$

- The length of a vector v is defined by

$$\|v\| = \sqrt{\langle v, v\rangle}$$

- Two vectors v, w are orthogonal if $\langle v, w\rangle = 0$.
- For vectors $v, w \in R^n$, the angle between v and w is defined by

$$\ltimes (v, w) = \cos^{-1} \frac{\langle v, w\rangle}{\|v\|\|w\|}$$

- $\{L_0, L_1, L_2\}$is an orthonormal set, if

$$\text{i.e.} \langle L_i , L_j \rangle = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

- Projection of function g(x) on the space spanned by 2 functions L_0, L_1(should be orthogonal)

$$\text{1) Normalize the orthogonal functions to orthonormal bases: } v_0 = \frac{L_0}{\|L_0\|}, v_1 = \frac{L_1}{\|L_1\|}$$

$$\text{2) Use formula to find projection} \rightarrow \text{proj}_W u = \sum_{i=0}^{k} \langle u, v_i\rangle\, v_i$$

- Apply Gram − Schmidt process to the vectors v_1, v_2, v_3 below to find an orthonormal basis e_1, e_2, e_3 for the space W they span

$$e_1 = \frac{v_1}{\|v_1\|} \quad | \quad \tilde{e_2} = v_2 - \langle v_2, e_1\rangle e_1 \quad | \quad \tilde{e_3} = v_3 - \langle v_3, e_1\rangle e_1 - \langle v_3, e_2\rangle e_2$$

HW 6

- QR decomposition for $\mathrm{n} \times \mathrm{n}$ matrix, i. e.

$$\mathrm{M} = \begin{pmatrix} \mathrm{a} & \mathrm{d} & \mathrm{g} \\ \mathrm{b} & \mathrm{e} & \mathrm{h} \\ \mathrm{c} & \mathrm{f} & \mathrm{i} \end{pmatrix} \rightarrow \mathrm{u}_1 = \begin{pmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{pmatrix}, \mathrm{u}_2 = \begin{pmatrix} \mathrm{d} \\ \mathrm{e} \\ \mathrm{f} \end{pmatrix}, \mathrm{u}_3 = \begin{pmatrix} \mathrm{g} \\ \mathrm{h} \\ \mathrm{i} \end{pmatrix}$$

1) find orthonormal bases $\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3$

$$\widetilde{\mathrm{e}_1} = \mathrm{u}_1 \rightarrow \mathrm{e}_1 = \frac{\mathrm{u}_1}{\|\mathrm{u}_1\|}$$

$$\widetilde{\mathrm{e}_2} = \mathrm{u}_2 - \langle \mathrm{u}_2, \mathrm{e}_1 \rangle \mathrm{e}_1 \rightarrow \ \mathrm{e}_2 = \frac{\widetilde{\mathrm{e}_2}}{\|\widetilde{\mathrm{e}_2}\|}$$

$$\widetilde{\mathrm{e}_3} = \mathrm{u}_3 - \langle \mathrm{u}_3, \mathrm{e}_1 \rangle \mathrm{e}_1 - \langle \mathrm{u}_3, \mathrm{e}_2 \rangle \mathrm{e}_2 \rightarrow \mathrm{e}_3 = \frac{\widetilde{\mathrm{e}_3}}{\|\widetilde{\mathrm{e}_3}\|}$$

2) find $\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3$ (leave $\mathrm{e}_1 . \mathrm{e}_2, \mathrm{e}_3$ at the end of each inner product as it is)

$$\mathrm{v}_1 = \langle \mathrm{u}_1, \mathrm{e}_1 \rangle \mathrm{e}_1 + \langle \mathrm{u}_1, \mathrm{e}_2 \rangle \mathrm{e}_2 + \langle \mathrm{u}_1, \mathrm{e}_3 \rangle \mathrm{e}_3$$

$$\mathrm{v}_2 = \langle \mathrm{u}_2, \mathrm{e}_1 \rangle \mathrm{e}_1 + \langle \mathrm{u}_2, \mathrm{e}_2 \rangle \mathrm{e}_2 + \langle \mathrm{u}_2, \mathrm{e}_3 \rangle \mathrm{e}_3$$

$$\mathrm{v}_2 = \langle \mathrm{u}_3, \mathrm{e}_1 \rangle \mathrm{e}_1 + \langle \mathrm{u}_3, \mathrm{e}_2 \rangle \mathrm{e}_2 + \langle \mathrm{u}_3, \mathrm{e}_3 \rangle \mathrm{e}_3$$

$$\mathbf{M} = \mathbf{QR} \rightarrow \begin{pmatrix} \mathbf{a} & \mathbf{d} & \mathbf{g} \\ \mathbf{b} & \mathbf{e} & \mathbf{h} \\ \mathbf{c} & \mathbf{f} & \mathbf{i} \end{pmatrix} = (\mathbf{u_1} \quad \mathbf{u_2} \quad \mathbf{u_3})(\mathbf{v_1} \quad \mathbf{v_2} \quad \mathbf{v_3})$$

HW 7

- Fourier Series for i. e.

$$f(x)=\begin{cases}0 & \text{if } -\pi<x<0\\ \sin x & \text{if } \quad 0<x<\pi\end{cases}$$

- $\mathbf{a_0}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{f(x)\ dx}\rightarrow\frac{1}{2\pi}\left(\int_{-\pi}^{0}0\ dx+\int_{0}^{\pi}\sin x\ dx\right)=\frac{1}{\pi}$
- $\mathbf{a_n}=\frac{1}{\pi}\int_{-\pi}^{\pi}\mathbf{f(x)\cos nx\ dx}\rightarrow\frac{1}{\pi}\int_{0}^{\pi}\sin x\cos nx\ dx=-\frac{\cos n\pi+1}{\pi(n^2-1)}$
- $\mathbf{b_n}=\frac{1}{\pi}\int_{-\pi}^{\pi}\mathbf{f(x)\sin nx\ dx}\rightarrow\frac{1}{\pi}\int_{0}^{\pi}\sin x\sin nx\ dx=-\frac{-\sin n\pi}{\pi(n^2-1)}$

$$\mathbf{f(x)}=\mathbf{a_0}+\sum_{\mathbf{n=1}}^{\infty}\mathbf{a_n\cos nx}+\sum_{\mathbf{n=1}}^{\infty}\mathbf{b_n\sin nx}\rightarrow f(x)=\frac{1}{\pi}+\sum_{n=1}^{\infty}\left(-\frac{\cos n\pi\ +\ 1}{\pi(n^2-1)}\right)\cos nx+\sum_{n=1}^{\infty}\left(-\frac{-\sin n\pi}{\pi(n^2-1)}\right)\sin nx$$

- Sum and Difference + Product to Sum − trigonometric identities

$$\sin(a+b)=\sin a\cos b+\cos a\sin b$$
$$\sin(a-b)=\sin a\cos b-\cos a\sin b$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$
$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$
$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$
$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

$$\cos(a)\cos(b)=\frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right)$$
$$\sin(a)\sin(b)=\frac{1}{2}\left(\cos(a-b)-\cos(a+b)\right)$$
$$\sin(a)\cos(b)=\frac{1}{2}\left(\sin(a+b)+\sin(a-b)\right)$$
$$\cos(a)\sin(b)=\frac{1}{2}\left(\sin(a+b)-\sin(a-b)\right)$$

- Determine the sign of each one of the following permutations i. e.

$$\sigma_1=\begin{pmatrix}1&2&3&4&5\\3&1&5&4&2\end{pmatrix}\rightarrow\underset{\text{2 flips}}{\begin{pmatrix}3&1&2&4&5\\3&1&5&4&2\end{pmatrix}}\rightarrow\underset{\text{2 flips}}{\begin{pmatrix}3&1&5&2&4\\3&1&5&4&2\end{pmatrix}}\rightarrow\underset{\text{1 flip}}{\begin{pmatrix}3&1&5&4&2\\3&1&5&4&2\end{pmatrix}}\rightarrow\begin{matrix}\text{total}=5\text{ flips}\\ \operatorname{sgn}\sigma_1=-1\text{ (odd)}\end{matrix}$$

HW 8

- A square matrix is invertible if and only if the determinant is not equal to zero.
- Adjoint of a matrix:

$$A=\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} \to \mathrm{adj}A=\begin{pmatrix} \begin{vmatrix} e & h \\ f & i \end{vmatrix} & -\begin{vmatrix} b & h \\ c & i \end{vmatrix} & \begin{vmatrix} b & e \\ c & f \end{vmatrix} \\ -\begin{vmatrix} d & g \\ f & i \end{vmatrix} & \begin{vmatrix} a & g \\ c & i \end{vmatrix} & -\begin{vmatrix} a & d \\ c & f \end{vmatrix} \\ \begin{vmatrix} d & g \\ e & h \end{vmatrix} & -\begin{vmatrix} a & g \\ b & h \end{vmatrix} & \begin{vmatrix} a & d \\ b & e \end{vmatrix} \end{pmatrix}^{T}$$

- Inverse of a matrix:

$$A^{-1}=\frac{1}{\det A}\mathrm{adj}A$$

- Find the characteristic polynomial of matrix, its eigenvalues and the corresponding eigenvectors:

$$A=\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} \to \text{characteristic polynomial } f(t)=\begin{vmatrix} t-a & -d & -g \\ -b & t-e & -h \\ -c & -f & t-i \end{vmatrix}$$

$$eigenvalues\ \lambda_{1,2,3}=t \to \begin{vmatrix} t-a & -d & -g \\ -b & t-e & -h \\ -c & -f & t-i \end{vmatrix}=0$$

$$\text{eigenvectors } v_{1,2,3}=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \to \begin{pmatrix} a-\lambda_{1,2,3} & d & g \\ b & e-\lambda_{1,2,3} & h \\ c & f & i-\lambda_{1,2,3} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \to solve\ system$$

HW 9

- An $n \times n$ diagonalizable matrix should have n distinct eigenvalues.
- How to diagonalize matrix $A \to$ find P, P^{-1} and D:

$$P = (v_1 \quad v_2 \quad v_3) \to P^{-1} = \frac{1}{\det P} \operatorname{adj} P \to D = P^{-1}AP$$

- Find a formula for A^n:

$$A^n = PD^nP^{-1}$$

- Find the general solution for the initial value problem:

$$x' = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Find the characteristic polynomial of matrix, its eigenvalues and the corresponding eigenvectors $v_1 . v_2, v_3$

$$x(t) = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2 + c_3 e^{\lambda_3 t} v_3$$

$$x(0) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c_1 v_1 + c_2 v_2 + c_3 v_3 \to \text{solve system to get general solution}$$

- The exponential function and its power series:

$$\text{a) } e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$

$$\text{b) } e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!} = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$\text{Steps to find } e^A \to \text{find P, find } P^{-1} \to e^D = \begin{pmatrix} e^{\lambda_1} & 0 \\ 0 & e^{\lambda_2} \end{pmatrix} \to e^A = Pe^DP^{-1}$$

- Dynamics of growth with immigration:

Suppose that A_1, A_2, and A_3 are three populations, each with growth rates of r_1, r_2, r_3 respectively. Suppose, further that there is a population emigration from A_1 to A_2 at rate s_1 and from A_2 to A_3 at rate s_2. Let $x_1(t), x_2(t), x_3(t)$ denote the population of these communities at time t. Hence:

$$\begin{cases} x'_1(t) = (r_1 - s_1)x_1(t) \\ x'_2(t) = s_1x_1(t) + (r_2 - s_2)x_2(t) \\ x'_3(t) = s_2x_2(t) + r_3x_3(t) \end{cases} \to A = \begin{pmatrix} (r_1 - s_1) & 0 & 0 \\ s_1 & (r_2 - s_2) & 0 \\ 0 & s_2 & r_3 \end{pmatrix}$$

Find the characteristic polynomial of matrix, its eigenvalues and the corresponding eigenvectors $v_1 . v_2, v_3$

$$x_i(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2 + c_3 e^{\lambda_3 t} v_3$$

$$x_i(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = c_1 v_1 + c_2 v_2 + c_3 v_3 \to \text{solve system to get general solution}$$

HW 10

- An $n \times n$ matrix A is called symmetric if it is equal to its transpose and $A = A^t$
- An $n \times n$ matrix A is called Hermitian if it is equal to its complex conjugate transpose and $A = A^*$
- A Hermitian matrix is positive definite, if its eigenvalues are positive.
- Orthonormal basis $\rightarrow$ matrix of eigenvectors
- A matrix is called unitary if $AA^* = I$
- Compute the singular values of A:

  Calculate matrix $A^TA \rightarrow$ find the characteristic polynomial of matrix and its eigenvalues $\lambda_{1,2,3}$

  The singular values in S are square roots of eigenvalues A^TA $\sigma_{1,2,3} = \sqrt{\lambda_{1,2,3}}$