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Homework - ICS 2020 Problem Sheet #7

Problem 7.1

A Boolean function ϕ is defined using the following sum of minterms:

$\phi(A, B, C, D, E) = m_0+m_2+m_4+m_6+m_9+m_{10}+m_{13}+m_{14}+m_{15}+m_{16}+m_{17}+m_{21}+m_{26}+m_{28}+m_{30}+m_{31}$

literals	minterm	pattern	used	minterms	pattern	used	minterms	pattern	used
1L	m_0	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0	✓
	m_2	00010	✓	$m_{0,4}$	00-00	✓	$m_{0,4,2,6}$	00--0	
	m_4	00100	✓	$m_{0,16}$	-0000				✓
	m_{16}	10000	✓						
2L	m_6	00110	✓	$m_{2,6}$	00-10	✓	$m_{2,6,10,14}$	0--10	✓
	m_9	01001	✓	$m_{2,10}$	0-010	✓			
	m_{10}	01010	✓	$m_{4,6}$	001-0	✓	$m_{2,10,6,14}$	0--10	
	m_{17}	10001	✓	$m_{16,17}$	1000-				✓
3L	m_{13}	01101	✓	$m_{6,14}$	0-110	✓	$m_{10,26,14,30}$	-1-10	✓
	m_{14}	01110	✓	$m_{9,13}$	01-01				✓
	m_{21}	10101	✓	$m_{10,14}$	01-10	✓	$m_{10,14,26,30}$	-1-10	
	m_{26}	11010	✓	$m_{10,26}$	-1010	✓			
	m_{28}	11100	✓	$m_{21,17}$	10-01				✓
4L	m_{15}	01111	✓	$m_{13,15}$	011-1				✓
	m_{30}	11110	✓	$m_{14,15}$	0111-	✓	$m_{14,15,30,31}$	-111-	✓
				$m_{14,30}$	-1110	✓	$m_{14,30,15,31}$	-111-	
				$m_{26,30}$	11-10	✓			
				$m_{28,30}$	111-0				✓
5L	m_{31}	11111	✓	$m_{15,31}$	-1111	✓			
				$m_{30,31}$	1111-	✓			

a) Calculate the prime implicants of ϕ.

- $m_{0,2,4,6}$ = 00--0 = ¬A ∧ ¬B ∧ ¬E
- $m_{2,6,10,14}$ = 0--10 = ¬A ∧ D ∧ ¬E
- $m_{10,26,14,30}$ = -1-10 = B ∧ D ∧ ¬E
- $m_{14,15,30,31}$ = -111- = B ∧ C ∧ D
- $m_{0,16}$ = -0000 = ¬B ∧ ¬C ∧ ¬D ∧ ¬E
- $m_{16,17}$ = 1000- = A ∧ ¬B ∧ ¬C ∧ ¬D ∧ ¬E
- $m_{9,13}$ = 01-01 = ¬A ∧ B ∧ ¬D ∧ E
- $m_{21,17}$ = 10-01 = A ∧ ¬B ∧ ¬D ∧ E
- $m_{13,15}$ = 011-1 = ¬A ∧ B ∧ C ∧ E
- $m_{28,30}$ = 111-0 = A ∧ B ∧ C ∧ ¬E

b) Construct the prime implicant chart and identify the essential prime implicants.

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}
$m_{0,2,4,6}$	✓	✓	✓	✓												
$m_{2,6,10,14}$		✓	✓			✓		✓								
$m_{10,26,14,30}$						✓		✓					✓		✓	
$m_{14,15,30,31}$								✓	✓						✓	✓
$m_{0,16}$	✓									✓						
$m_{16,17}$										✓	✓					
$m_{9,13}$					✓		✓									
$m_{21,17}$											✓	✓				
$m_{13,15}$							✓		✓							
$m_{28,30}$														✓	✓	
E.P.I.C	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

From the table, we can see there are 6 essential prime implicants:

- $m_{0,2,4,6}$ = 00--0 = $\neg A \wedge \neg B \wedge \neg E$ (only one that covers m_6)
- $m_{10,26,14,30}$ = -1-10 = $B \wedge D \wedge \neg E$ (only one that covers m_{26})
- $m_{14,15,30,31}$ = -111- = $B \wedge C \wedge D$ (only one that covers m_{31})
- $m_{9,13}$ = 01-01 = $\neg A \wedge B \wedge \neg D \wedge E$ (only one that covers m_9)
- $m_{21,17}$ = 10-01 = $A \wedge \neg B \wedge \neg D \wedge E$ (only one that covers m_{21})
- $m_{28,30}$ = 111-0 = $A \wedge B \wedge C \wedge \neg E$ (only one that covers m_{28})

We also take $m_{0,16}$ or $m_{16,17}$ as essential prime implicants as they're the only ones covering m_{16}.

- $m_{0,16}$ = -0000 = $\neg B \wedge \neg C \wedge \neg D \wedge \neg E$ (let's take this one for the expression)
- $m_{16,17}$ = 1000- = $A \wedge \neg B \wedge \neg C \wedge \neg D \wedge \neg E$

c) Write out all minimal boolean expressions defining ϕ using the mathematical logic notation.

The resulting minimal expressions defining $\phi(A, B, C, D)$ linked in mathematical logic notation:

$$\phi(A, B, C, D, E) = (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$$

or

$$\phi(A, B, C, D, E) = (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D \wedge \neg E)$$