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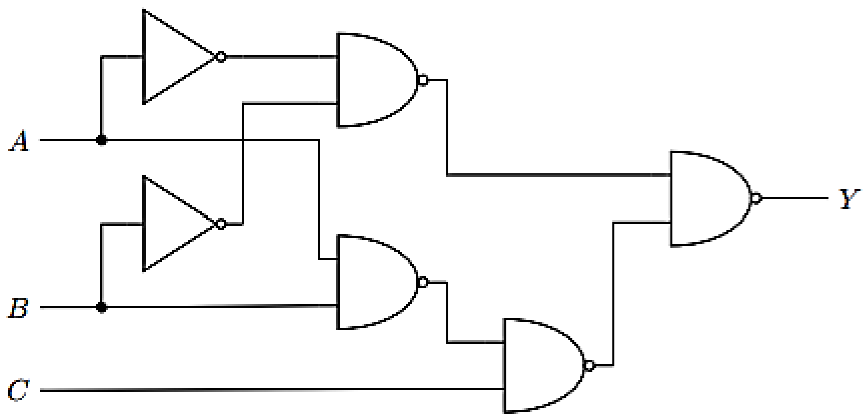
Homework - ICS 2020 Problem Sheet #8

Problem 8.1

You are given the following digital circuit.



a) What is the Boolean expression implemented by the digital circuit?

$Y = (\neg A \uparrow \neg B) \uparrow ((A \uparrow B) \uparrow C)$

Convert to using logical operators ($\neg$) and ($\wedge$):

$Y = \neg[(\neg(\neg A \wedge \neg B)) \wedge (\neg(\neg(A \wedge B) \wedge C))]$

b) Derive algebraically step-by-step the disjunction (sum) of minterms from the Boolean expression implemented by the digital circuit.

$Y = \neg[(\neg(\neg A \wedge \neg B)) \wedge (\neg(\neg(A \wedge B) \wedge C))]$

Apply de Morgan’s laws:

$Y = \neg[(A \vee B) \wedge (\neg(\neg A \vee \neg B) \vee \neg C)]$
$Y = \neg[(A \vee B) \wedge ((A \wedge B) \vee \neg C)]$
$Y = \neg(A \vee B) \vee \neg((A \wedge B) \vee \neg C)$
$Y = (\neg A \wedge \neg B) \vee (\neg(A \wedge B) \wedge C)$
$Y = (\neg A \wedge \neg B) \vee ((\neg A \vee \neg B) \wedge C)$

Apply distributivity law:

$Y = (\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C)$ -> DNF form

Problem 8.2

A full adder digital circuit was introduced in class. It is defined by the following two boolean functions:

$$S = A \dot{\vee} B \dot{\vee} C_{in}$$
$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a) Write both functions as a disjunction of product terms.

$S = (\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$
$C_{out} = (\neg A \wedge B \wedge C_{in} \wedge \neg S) \vee (A \wedge \neg B \wedge C_{in} \wedge \neg S) \vee (A \wedge B \wedge \neg C_{in} \wedge \neg S) \vee (A \wedge B \wedge C_{in} \wedge S)$

b) Write both functions as a conjunction of sum terms.

$S = (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in}) \wedge (\neg A \vee \neg B \vee C_{in})$
$C_{out} = (A \vee B \vee C_{in} \vee S) \wedge (A \vee B \vee \neg C_{in} \vee \neg S) \wedge (A \vee \neg B \vee C_{in} \vee \neg S) \wedge (\neg A \vee B \vee C_{in} \vee \neg S)$

c) Write both functions using only not ($\neg$) and not-and ($\uparrow$) operations.

$S = A \dot{\vee} B \dot{\vee} C_{in}$

Convert to using ($\neg$) and ($\uparrow$) operators:

$S = ((A \vee B) \wedge \neg(A \wedge B)) \dot{\vee} C_{in}$

$S = [(\neg(\neg A \wedge \neg B) \wedge \neg(A \wedge B))] \dot{\vee} C_{in}$

$S = [\neg((\neg(\neg A \wedge \neg B) \wedge \neg(A \wedge B))) \uparrow \neg C_{in}] \wedge [((\neg(\neg A \wedge \neg B) \wedge \neg(A \wedge B))) \uparrow C_{in}]$

$S = [\neg((\neg A \uparrow \neg B) \wedge (A \uparrow B)) \uparrow \neg C_{in}] \wedge [((\neg A \uparrow \neg B) \wedge (A \uparrow B)) \uparrow C_{in}]$

$S = [((\neg A \uparrow \neg B) \uparrow (A \uparrow B)) \uparrow \neg C_{in}] \wedge [((\neg A \uparrow \neg B) \wedge (A \uparrow B)) \uparrow C_{in}]$

$S = \neg[((\neg A \uparrow \neg B) \uparrow (A \uparrow B)) \uparrow \neg C_{in}] \uparrow [(\neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B))) \uparrow C_{in}]$

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$

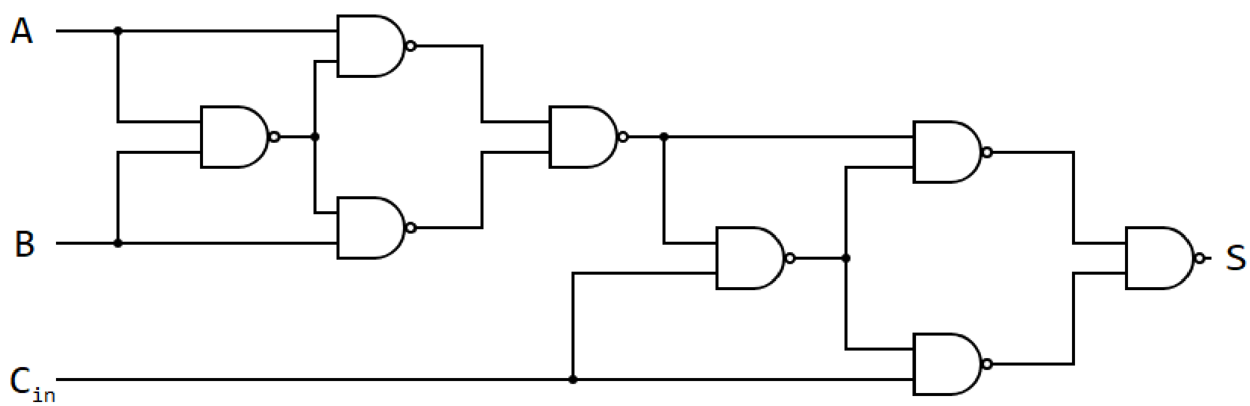
Now A ∨ B is represented by ¬A ↑ ¬B and A ∧ B is represented by ¬(A ↑ B)

Convert to using (¬) and (↑) operators:

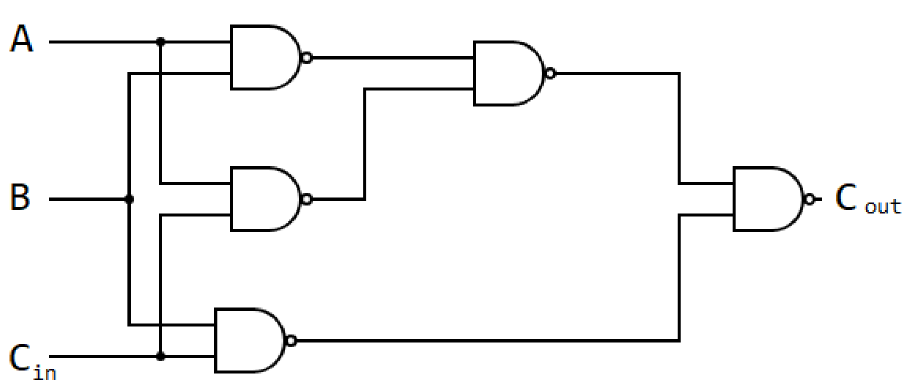
$C_{out} =$ (A ∧ B) ∨ (C_{in} ∧ (A $\dot{\vee}$ B)) = (A ∧ B) ∨ (C_{in} ∧ ¬((A↑B) ↑(¬A↑¬B)))
$C_{out} =$ (A ∧ B) ∨ ¬ (C_{in} ↑¬((A ↑ B) ↑ (¬A ↑ ¬B)))
$C_{out} =$ ¬(A ↑ B) ∨ ¬(C_{in} ↑¬((A ↑ B) ↑ (¬A ↑ ¬B)))
$C_{out} =$ ¬(¬(A ↑ B)) ↑ ¬(¬C_{in} ↑¬((A ↑ B) ↑ (¬A ↑ ¬B))))
$C_{out} =$ (A ↑ B) ↑ (C_{in} ↑ ¬((A ↑ B) ↑ (¬A ↑ ¬B)))

d) In a digital circuit, we can easily reuse common terms. Draw a small digital circuit implementing S and C_{out} using NAND gates only.

Digital circuit for S :



Digital circuit for C_{out} :



Integrated full adder digital circuit using only NAND gates: