Problem 1.1

a) True
b) True
c) False
d) True
e) False

Problem 1.2

a) $\alpha = 1$, $\beta = 2$, $\gamma = 0$
b) $4^2 = 22_{(7)} = 16_{(10)}$
$4 + 4 = 11_{(7)} = 8_{(10)}$

(base 7)	0	1	2	3	4	5	6	10	**11**	12	13	14	15	16	20	21	**22**
(base 10)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Decimal value of $\delta = 4$ in a number system of base 7.

c) $99_{(10)} = 201_{(7)} = \beta\gamma\alpha$

Problem 1.3

1) Base Case:

$$\frac{1}{1(1+1)} = \frac{1}{1+1}$$

$$\frac{1}{2} = \frac{1}{2}$$

2) Induction step: n = k

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{k}{k+1} => \mathit{Induction\ Hypothesis}$$

3) Assuming the induction hypothesis is true, prove for k+1:

LHS RHS

$$\left(\sum_{k=1}^{n} \frac{1}{k(k+1)}\right) + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

Expand LHS:

$$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} =$$

$$\frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} =$$

$$\frac{k^2 + 2k + 1}{(k+1)(k+2)} =$$

$$\frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2} = RHS$$

Problem 1.4

Let $N_0 = (N \cup \{0\})$ be the set of natural numbers including 0. Let $M = N_0^3 = (N_0 \times N_0 \times N_0)$.

The relation $\sim$ on $M \times M$ is defined as:

$$x \sim y \Leftrightarrow (a, b, c) \sim (d, e, f) \Leftrightarrow a + b + c = d + e + f$$

a) Prove that $\sim$ is an equivalence relation.

Reflexive: Suppose (a, b, c) is an ordered pair in M.

[We must show that $(a, b, c) \sim (a, b, c)$.]

We have $a + b + c = a + b + c$. Thus, by definition of $\sim$, $(a, b, c) \sim (a, b, c)$. ✓

Symmetric: Suppose (a, b, c) and (d, e, f) are two ordered pairs in M and $(a, b, c) \sim (d, e, f)$.

[We must show that $(d, e, f) \sim (a, b, c)$.]

Since $(a, b, c) \sim (d, e, f)$, $a + b + c = d + e + f = S$ (a common sum). Therefore, even if we switch the sides of the equality, the relation holds: $d + e + f = a + b + c = S \Rightarrow (a, b, c) \sim (d, e, f)$. ✓

Transitive: Suppose (a, b, c), (d, e, f), and (g, h, i) are elements of M, (a, b, c) R (d, e, f), and (d, e, f) R (g, h, i).

[We must show that $(a, b, c) \sim (g, h, i)$.]

Since $(a, b, c) \sim (d, e, f)$ means $a + b + c = d + e + f = S$, and since $(d, e, f) \sim (g, h, i)$ means $d + e + f = g + h + i$, then $S = g + h + i$. Thus\ $a + b + c = g + h + i$, by definition of $\sim \Rightarrow (a, b, c) \sim (g, h, i)$. ✓

b) Determine all elements of the equivalence class (1, 0, 1).
$[(1,0,1)] = \{a, b, c \in N \mid (a, b, c) \sim (1,0,1)\} = \{(0, 1, 1), (1, 0, 1), (1, 1, 0), (0, 0, 2), (0, 2, 0), (2, 0, 0)\}$

c) Let $x = (a, b, c) \in M$ and $y = (d, e, f) \in M$. the relation $\preccurlyeq$ on $M \times M$ is defined as follows:

$$x \preccurlyeq y \Leftrightarrow (a, b, c) \preccurlyeq (d, e, f) \Leftrightarrow a \le d \wedge b \le e \wedge c \le f$$

Prove the $\preccurlyeq$ is a partial order.

Reflexive: Suppose (a, b, c) is an ordered pair in M.

[We must show that $(a, b, c) \preccurlyeq (a, b, c)$.]

We have $(a \le a) \wedge (b \le b) \wedge (c \le c)$. The comparison holds, thus $(a, b, c) \preccurlyeq (a, b, c)$. ✓

Antisymmetric: Suppose (a, b, c) and (d, e, f) are two ordered pairs in M.

[We must show that if $(a, b, c) \preccurlyeq (d, e, f) \wedge (d, e, f) \preccurlyeq (a, b, c)$, $(a, b, c) = (d, e, f)$.]

Since $(a, b, c) \preccurlyeq (d, e, f)$, $(a \le d) \wedge (b \le e) \wedge (c \le f)$ and since $(a, b, c) \preccurlyeq (d, e, f)$, $(d \le a) \wedge (e \le b) \wedge (f \le c)$.

For the relation $\le$ (smaller or equal than) to hold, both sides need to be equal (as smaller then doesn't hold when switched). Therefore, $a = d$, $b = e$ and $c = f$, thus $(a, b, c) = (d, e, f)$. ✓

Transitive: Suppose (a, b, c), (d, e, f), and (g, h, i) are elements of M, $(a, b, c) \preccurlyeq (d, e, f)$, and $(d, e, f) \preccurlyeq (g, h, i)$.

[We must show that $(a, b, c) \preccurlyeq (g, h, i)$.]

Since $(a, b, c) \preccurlyeq (d, e, f)$, $(a \le d) \wedge (b \le e) \wedge (c \le f)$

Since $(d, e, f) \preccurlyeq (g, h, i)$, $(d \le g) \wedge (e \le h) \wedge (f \le i)$

⇨ $(a \le d \le g) \wedge (b \le e \le h) \wedge (c \le f \le i)$

⇨ $(a \le g) \wedge (b \le h) \wedge (c \le i)$, thus $(a, b, c) \preccurlyeq (g, h, i)$

d) Is $\preccurlyeq$ also a linear order? Explain why or why not.
Yes, it is linear order, as 1) it is partial order and 2) every two elements of the poset are comparable, meaning $\forall (a, b, c)\ (d, e, f) \in M$, $[((a, b, c), (d, e, f)) \in \preccurlyeq] \vee [((d, e, f), (a, b, c)) \in \preccurlyeq]$.

Problem 1.5

a) $0xf0 = 240_{(10)} = 360_{(8)}$

b) $|-20_{(10)}| = 20_{(10)} = 032_{(6)} \rightarrow 523_{(6)} + 1 = 524_{(6)}$

$a'_i = (6-1) - a_i$

$a'_2 = (6-1) - 0 = 5$

$a'_1 = (6-1) - 3 = 2$

$a'_0 = (6-1) - 2 = 3$

c) $12_{(10)} = 00001100_{(2)}$

$-8_{(10)} = 8_{(10)} = 10001000_{(2)} \rightarrow 11110111_{(2)} + 1 = 11111000_{(2)}$

	00001100	–>		12
+	11111000	–>	-	8
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	~~1~~00000100	–>		4

(We drop the first 1 as it is an overflow // n = 8)

d) $-35.75_{(10)}$

```
 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|S|   exponent    |             mantissa (23 bits)              |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Convert integral part and fraction part:

$35_{(10)} = 100011_{(2)}$

$0.75_{(10)} = 110..._{(2)}$

Find exponent:

$1\underline{00011}.110... = 1.00011110... \cdot 2^5$ Exp = 5

Add bias: $5 + 127 = 132_{(10)} = 10000100_{(2)}$

Convert into IEEE 754 single precision floating point format:
S = 1 Exponent = 10000100 Mantissa = 00011110000000000000000

$$\underbrace{1}_{\text{S}}\ \underbrace{10000100}_{\text{exponent}}\ \underbrace{00011110000000000000000}_{\text{mantissa}}$$

Hexadecimal representation: 0xc20e0000

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1100 0010 0000 1111 0000 0000 0000 0000

 12    2    0   15    0    0    0    0

  C    2    0    F    0    0    0    0
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