Introduction to Computer Science
Jacobs University Bremen
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Course: CH-232-A
Date: 2019-10-18
Time: 75 min.

ICS 2019 Practice Midterm Exam (Version 1)

The Jacobs University's Code of Academic Integrity applies to this examination. Please fill in your name (please write readable) and sign below.

Name (last, first):	
Signature:	

This exam is **closed book**. You are also not allowed to use any electronic equipment such as computers, smart phones, or calculators. However, you are allowed to use an A4 (single-sided) or A5 (double-sided) handwritten cheat sheet. Please answer the questions on the problem sheets. If you need more space, feel free to write on the back of the pages. Please keep the papers stapled.

Problem	Max. Points	Points	Grader
1.1: mixed questions	10		
1.2: utopia number system	10		
1.3: proof by induction	16		
1.4: equivalence & order relations	18		
1.5: number representations	16		
1.6: collatz in haskell	10		
Total	80		

70 points will earn you 100%. You cannot obtain more than 100% in the exam.

Problem 1.1: *mixed questions* (10 points)

Indicate which of the following statements are correct or incorrect by marking the appropriate boxes. For every correctly marked box, you will earn two points. For every incorrectly marked box, you will loose one point. Statements which are not marked or which are marked as both true and false will be ignored. The minimum number of points you can achieve is zero.

true	false	
☐	☐	A computer program is a collection of instructions that perform a specific calculation when executed by a computer. The underlying method used for the calculation is known as an algorithm.
☐	☐	The time complexity of an algorithm describes its computational effort while the space complexity of an algorithm describes the memory space needed to store the instructions of an algorithm.
☐	☐	A deterministic algorithm always terminates and hence a deterministic algorithm is totally correct if it is partially correct.
☐	☐	A spanning tree T of a graph G is a cycle-free subgraph of G that includes all vertices (nodes) of G and a subset of all edges of G.
☐	☐	The Boyer-Moore string search algorithm uses the good character rule to skip efficiently over matching alignments and the bad suffix rule to remove suffixes from the pattern that can never be found in the text.

Problem 1.2: *utopia number system* (3+5+2 = 10 points)

You have found a calculator from the planet Utopia. You do not know the number system used on planet utopia, Utopians likely do not use decimal numbers. After playing with the calculator for a while, you have observed the following:

$$\begin{aligned} \alpha^2 &= \alpha \\ \alpha + \alpha &= \beta \\ \gamma^2 &= \gamma \\ \gamma + \gamma &= \gamma \\ \delta^2 &= \beta\beta \\ \delta + \delta &= \alpha\alpha \end{aligned}$$

a) What is the decimal value of α, β, and γ?

b) What is the decimal value of δ? What is the base used by the number system?

c) How is the decimal number 99 written on planet Utopia?

Problem 1.3: *proof by induction* (16 points)

Prove that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

for $n \in \mathbb{N}$ and $n > 0$.

Problem 1.4: *equivalence and partial order relations* (7+2+7+2 = 18 points)

Let $\mathbb{N}_0 = (\mathbb{N} \cup \{0\})$ be the set of natural numbers including 0. Let $M = \mathbb{N}_0^3 = (\mathbb{N}_0 \times \mathbb{N}_0 \times \mathbb{N}_0)$. The relation $\sim$ on $M \times M$ is defined as

$$x \sim y \iff (a, b, c) \sim (d, e, f) \iff a + b + c = d + e + f$$

a) Prove that $\sim$ is an equivalence relation.

b) Determine all elements of the equivalence class $(1, 0, 1)$.

c) Let $x = (a, b, c) \in M$ and $y = (d, e, f) \in M$. the relation $\preceq$ on $M \times M$ is defined as follows:

$$x \preceq y \iff (a, b, c) \preceq (d, e, f) \iff a \leq d \wedge b \leq e \wedge c \leq f$$

Prove the $\preceq$ is a partial order.

d) Is $\preceq$ also a linear order? Explain why or why not.

Problem 1.5: *number representations* (2+2+2+10 = 16 points)

a) Represent the hexadecimal number 0xf0 as an octal number.

b) Consider a fixed size integer numeral system with the base $b = 6$ and $n = 3$ digits using the b-complement notation for negative integers. What is the representation of -20_{10} in this numeral system?

c) Calculate $12 + (-6)$ using 8-bit binary numbers and the b-complement representation of negative numbers. (The numbers in the expression are decimal numbers.)

d) Convert the decimal fraction -35.75 into IEEE 754 single precision floating point format:

```
 0                   1                   2                   3
 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|S|    exponent   |         mantissa (23 bits)                    |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

What is the hexadecimal representation of the IEEE 754 single precision floating point number representation? (Recall that the bias for the exponent is 127.)

Problem 1.6: *collatz sequence in haskell* (1+1+2+4+2 = 10 points)

Consider the following Haskell definition of the function `collatz`.

```haskell
collatz :: Int -> [Int]
collatz 1 = [1]
collatz n | even n = n : collatz (n `div` 2)
          | odd  n = n : collatz (3 * n + 1)
```

a) Where is pattern maching used in the function definition?

b) Where are guards used in the function definition?

c) Provide a mathematical definition of the sequence that the Haskell function `collatz` returns.

d) What is the result of the expression `collatz 5`? Explain how the result is calculated.

e) A function is tail recursive if the last computational action is a recursive call of the function. Is the function `collatz` tail recursive? Explain why or why not.