FISH 621 Laboratory #9: Spatial Mark-recapture and CWT

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Instructions

The purpose of this lab is to:

- Estimate hatchery contribution to catches based on coded wire tagging (CWT) of hatchery releases.
- Explore matrix algebra operations in R.
- Implement the Darroch abundance estimation method for stratified mark-recapture experiments.

If you have a question during the lab, please un-mute yourself and ask, or type it into the chat box. There is a high likelihood that someone else has the same question. It is more fun if we all learn together in our distance-learning world.

I have posted the lecture slides to the ***Canvas site***, so you can reference this material as you work through the lab.

This and all other labs will be graded based on your attendance and participation.

Lab Contents

- **621_Lab 8_Spatial MR.pdf** (this file)
- **621_Lab 8_Spatial MR.R** R script with exercises

Exercise 1: CWT Data

In this exercise we will explore how the hatchery contribution to fishery catches can be estimated based on CWT tagging of hatchery fish at release. Specifically, we will explore estimation of the total number of hatchery fish in the catch, the proportional hatchery contribution to catches, and associated estimates of uncertainty.

CWT Estimators

P = tag proportion
N = total catch
C = hatchery catch
$M = CP$ = total # of marked fish

- Number of hatchery fish in the catch
  - $\hat{C} = \frac{\hat{M}}{P} = x\left(\frac{N}{sP}\right)$
- Variance estimator from Clark and Bernard (1987)
  - For hatchery catch
    - $\widehat{var}(\hat{C}) = x\left(\frac{N}{sP}\right)^2\left[1 - x + \left(\frac{s-1}{N-1}\right)\left(\frac{xN}{s} - P\right)\right]$

Exercise 2: Matrix Algebra in R

In preparation for the Darroch analysis to follow and further expand our R skillset we will practice implementing matrix algebra operations in R, including:

- Matrix multiplication
- Matrix transposition
- Calculating the inverse of a matrix
- Calculating the matrix determinant

Review: Matrix Algebra

- Matrix multiplication
  - Matrix X Matrix = Matrix
    - $\mathbf{M} * \mathbf{M}$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \times \begin{bmatrix} g & j \\ h & k \\ i & l \end{bmatrix} = \begin{bmatrix} ag + bh + ci & aj + bk + cl \\ dg + eh + fi & dj + ek + fl \end{bmatrix}$$

Review: Matrix Algebra

- Matrix transpose
  - **Switch rows and columns**
    - A' or A^T
    - $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Review: Matrix Algebra

- Determinant
  - **A single numeric value derived from a square matrix**
    - $|A|$ or $det(A)$
    - $det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \cdot \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$= a(ei - hf) - b(di - gf) + c(dh - ge)$$

Review: Matrix Algebra

- Matrix inverse
  - Definition of the inverse
    - $A^{-1}A = AA^{-1} = I$
    - Where I is an identity matrix: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

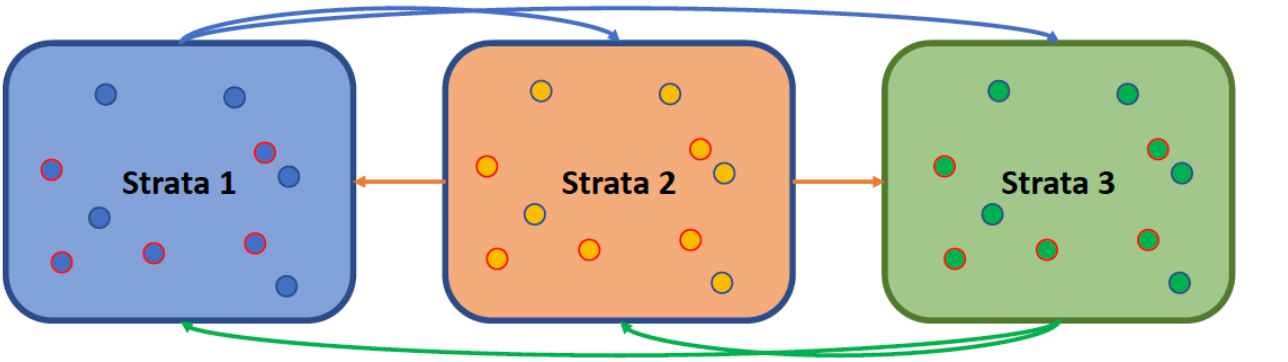
Exercise 3: Darroch Method

In this exercise we will explore Darroch's method for estimating abundance based on mark-recapture data. In this case we have three spatial strata (locations) in which we mark and release individuals at time t1, using a separate tag color for marking individuals in each stratum. This allows us to know the stratum origin of each recaptured individual.

At time t2 then collect a sample of individuals in each of these three strata and record:

1) The number of unmarked individuals in the sample from each stratum
2) The number of recaptures, and their stratum of origin

- Animals in different strata receive different tags
  - To distinguish stratum of marking
    - Different color, number, character ID



Notation

$\theta_{i,j}$
$\theta_{from,to}$

- Probability of ***moving*** from stratum i to stratum j
  - $\boldsymbol{\Theta} = \{\theta_{ij}\}$
    - Square matrix: $s \times s$
- Probability of moving from stratum i to stratum j
  - And then being recaptured
    - $\Psi = \{\psi_{ij}\}$
- n_j = sample size in recapture stratum j
- u_j = unmarked individuals ***captured*** in stratum j
- m_{ij} = marked animals in sample from stratum j
  - Which originated from stratum i
    - $m_i = \sum_j m_{ij}$ - Total marks from stratum i recaptured

Exercise 4: Comparison of Darroch Estimates with Petersen Estimates

It is important to note that with this experimental design we might be tempted to ignore our strata, and by extension variability in capture probability at time $t2$ and use our basic Petersen estimator. However, based on theory we should expect this estimate to be ***biased***. But, here we will compare the simple the simple mark-recapture estimates (Petersen, Chapman, Bailey) we get if we ignore stratification and movement.