MAT292—ODE

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1 Newton's Law of Cooling

Newtons law of cooling states that

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -k(u - T_0) \tag{1.1}$$

Note that there is one trivial solution, the equilibrium solution is $u(t) = T_0$. The meaning of this solution is the temperature of an object doesn't change when it is already at the equilibrium temperature.

$$\frac{\frac{\mathrm{d}u}{\mathrm{d}t}}{u - T_0} = -k \tag{1.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\log(u - T_0) = -k \tag{1.3}$$

$$\log(u - T_0) = -kt + c_1 \tag{1.4}$$

$$u = T_0 + \exp(c_1)\exp(-kt) = T_0 + c_2\exp(-kt) \tag{1.5}$$

Note that $c_2 = \exp(c_1) > 0$. However, this is not a complete solution as it cannot describe the solutions with $u < T_0$.

Warning: note that the integral of $\frac{1}{x}$ is $\log|x|$, **NOT** $\log(x)$. This is what caused the solution to be incomplete.

Hence, $c_2 = \pm\exp(c_1)$. Note that $c_1 = \pm\infty$ is allowed thus so c_2 can take any value.

Below are the integral curves.