# CHE260—Thermodynamics & Heat Transfer

Jonah Chen

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## Contents



## 1 Conduction

### 1.1 Fourier's Law (relates heat flux to temperature)

$$\dot{q} = -k\frac{\partial T}{\partial x} \tag{1.1}$$

### 1.2 Heat Equation

The heat equation is a statement of the first law (conservation of energy), which describes conduction.

$$(\partial_t - \alpha\nabla^2)T = 0 \tag{1.2}$$

In steady state, $\partial_t T = 0$, so the heat equation reduces to Laplace's equation for temperature $\nabla^2 T = 0$. Energy Balance for something:

$$\rho c_p \partial_t T = -\frac{1}{A}\partial_x(\dot{q}A) \tag{1.3}$$

1-dimensional Heat Equation

$$(\partial_t - \alpha\partial_x^2)T = 0 \tag{1.4}$$

where $\alpha := \frac{k}{\rho c_p}$, with units of $\mathrm{m}^2/\mathrm{s}$.

- High $k$ — Material conducts heat well.
- High $\rho c_p$ — Material stores energy well.

In cylinderical coordinates (in the r direction),

$$(\partial_t - \frac{\alpha}{r}\partial_r(r\partial_r))T = 0 \tag{1.5}$$

In spherical coordinates (in the r direction),

$$(\partial_t - \frac{\alpha}{r^2}\partial_r(r^2\partial_r))T = 0 \tag{1.6}$$

Common Solutions: Steady state solution for cartesian coordinates is

$$T = mx + b \tag{1.7}$$

## 1.3 Thermal Resistance

When dealing with steady state behavior for multiple connected mediums, it is useful to define a quantity called thermal resistance with the property that

$$\dot{Q} = \frac{T_1 - T_2}{R} \tag{1.8}$$

The calculated values for resistances are

- For conduction, $R = \frac{L}{kA}$.
- For convection, $R = \frac{1}{hA}$.
- For radiation, $R = \frac{1}{\varepsilon\sigma A(T^2 - T_\infty^2)(T - T_\infty)}$.

For two mediums connected in series,

$$R_T = R_1 + R_2 \tag{1.9}$$

It is also useful to define an overall heat transfer coefficient $U = \frac{1}{R_T A}$

$$\dot{Q} = UA(T_1 - T_2) \tag{1.10}$$

# 2 Convection

Newton's Law of Cooling:

$$\dot{q} = h(T - T_\infty) \tag{2.1}$$