

# MAT292—ODE

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## Contents

### 1 Newton's Law of Cooling

1

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Newtons law of cooling states that

$$\frac{du}{dt} = -k(u - T_0) \quad (1.1)$$

Note that there is one trivial solution, the equilibrium solution is  $u(t) = T_0$ . The meaning of this solution is the temperature of an object doesn't change when it is already at the equilibrium temperature.

$$\frac{\frac{du}{dt}}{u - T_0} = -k \quad (1.2)$$

$$\frac{d}{dt} \log(u - T_0) = -k \quad (1.3)$$

$$\log(u - T_0) = -kt + c_1 \quad (1.4)$$

$$u = T_0 + \exp(c_1) \exp(-kt) = T_0 + c_2 \exp(-kt) \quad (1.5)$$

Note that  $c_2 = \exp(c_1) > 0$ . However, this is not a complete solution as it cannot describe the solutions with  $u < T_0$ .

**Warning:** note that the integral of  $\frac{1}{x}$  is  $\log|x|$ , **NOT**  $\log(x)$ . This is what caused the solution to be incomplete.

Hence,  $c_2 = \pm \exp(c_1)$ . Note that  $c_1 = \pm\infty$  is allowed thus so  $c_2$  can take any value.

Below are the integral curves.