CHE260—Thermodynamics & Heat Transfer

Jonah Chen

November 15, 2021

Contents

1 Conduction

1.1 Fourier's Law (relates heat flux to temperature)

$$\dot{q} = -k\nabla T \tag{1.1}$$

1.2 Heat Equation

The heat equation is a statement of the first law (conservation of energy), which describes conduction.

$$(\partial_t - \alpha\nabla^2)T = 0 \tag{1.2}$$

In steady state, $\partial_t T = 0$, so the heat equation reduces to Laplace's equation for temperature $\nabla^2 T = 0$. Energy Balance for something:

$$\rho c_p \partial_t T = -\frac{1}{A}\partial_x(\dot{q}A) \tag{1.3}$$

1-dimensional Heat Equation

$$(\partial_t - \alpha\partial_x^2)T = 0 \tag{1.4}$$

where $\alpha := \frac{k}{\rho c_p}$, with units of m^2/s.

- High k — Material conducts heat well.
- High ρc_p — Material stores energy well.

In cylinderical coordinates (in the r direction),

$$(\partial_t - \frac{\alpha}{r}\partial_r(r\partial_r))T = 0 \tag{1.5}$$

In spherical coordinates (in the r direction),

$$(\partial_t - \frac{\alpha}{r^2}\partial_r(r^2\partial_r))T = 0 \tag{1.6}$$

Common Solutions: Steady state solution for cartesian coordinates is

$$T = mx + b \tag{1.7}$$

1.3 Thermal Resistance

When dealing with steady state behavior for multiple connected mediums, it is useful to define a quantity called thermal resistance with the property that

$$\dot{Q} = \frac{T_1 - T_2}{R} \tag{1.8}$$

The calculated values for resistances are

- For conduction, $R = \frac{L}{kA}$.
- For convection, $R = \frac{1}{hA}$.
- For radiation, $R = \frac{1}{\varepsilon\sigma A(T^2 - T_\infty^2)(T - T_\infty)}$.

For two mediums connected in series,

$$R_T = R_1 + R_2 \tag{1.9}$$

For two mediums connected in parallel,

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \tag{1.10}$$

It is also useful to define an overall heat transfer coefficient $U = \frac{1}{R_T A}$

$$\dot{Q} = UA(T_1 - T_2) \tag{1.11}$$

The rvalue is L/k, hence

$$\dot{Q} = \frac{\Delta T}{R_{value}} \times A \tag{1.12}$$

which is in english units. L is in feet and k is in BTU per h square feet Fahernhit.

The critical radius of insulation is when the resistance is stationary. When increasing the radius, we are increacing conductive resistance but decreasing convective resistance.

$$r_{crit} = \frac{k}{h} \tag{1.13}$$

2 Convection

Newton's Law of Cooling:

$$\dot{q} = h(T - T_\infty) \tag{2.1}$$

For a very long fin with perimeter P, conduction and convection coefficients k and h in an enviroment at T_∞, (we are also assuming steady state somehow, but not sure how)

$$(k\partial_x(A\partial_x) - hP)T = -hPT_\infty \tag{2.2}$$

Assume A, k, P are constant. Define $\Theta = T - T_\infty$ and $a^2 = \frac{hP}{kA}$. Then,

$$(\partial_x^2 - a^2)\Theta = 0 \tag{2.3}$$

Using boundary conditions of $\Theta(0) = \Theta_b$ and $\Theta(\infty) = 0$ (from the definition of T_∞), the unique solution is

$$\Theta = \Theta_b e^{-ax} \tag{2.4}$$

The rate of heat loss is just equal to the heat entering the base of the fin. Using fourier's law,

$$\dot{Q} = \sqrt{hPkA}(T_b - T\infty) \tag{2.5}$$

For a finitely long fin, assume the heat transfer to the air at the end is negligiable (adiabatic). Hence, $\partial_x \Theta|_{x=L} = 0$.

$$\Theta = \Theta_b \frac{\cosh(a(L-x))}{\cosh(aL)} \tag{2.6}$$

By similar means, the rate of heat loss for this case is

$$\dot{Q} = \dot{Q}_\infty \tanh(aL) \tag{2.7}$$

If there is some heat lost to the air, we can correct the length $L_C = L + \frac{A}{P}$