AER210—Fluids

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Contents

1 Tutorial 1

- 15.1 #15

$$\int_1^4 \int_0^2 (6x^2y - 2x)\mathrm{d}y\mathrm{d}x \tag{1.1}$$
$$= \int_1^4 (12x^2 - 4x)\mathrm{d}x \tag{1.2}$$
$$= (256 - 32) - (4 - 2) \tag{1.3}$$
$$= 222 \tag{1.4}$$

- 15.1 #29

$$\iint_R \frac{xy^2}{x^2+1}\mathrm{d}A, R = \{(x,y)|0 \le x \le 1, -3 \le y \le 3\} \tag{1.5}$$
$$= \int_0^1 \int_{-3}^3 \frac{xy^2}{x^2+1}\mathrm{d}y\mathrm{d}x \tag{1.6}$$
$$= 18\int_0^1 \frac{x}{x^2+1}\mathrm{d}x \tag{1.7}$$
$$= 18 \times \frac{1}{2}\log\left(x^2+1\right)\Big|_0^1 \tag{1.8}$$
$$= 9\log 2 \tag{1.9}$$

- 15.1 #31

$$\iint_R x\sin(x+y)\mathrm{d}A, R=[0,\frac{\pi}{6}]\times[0,\frac{\pi}{3}] \tag{1.10}$$
$$=\int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{3}} x\sin(x+y)\mathrm{d}y\mathrm{d}x \tag{1.11}$$
$$=\int_0^{\frac{\pi}{6}} x\cos(x)-x\cos\Big(x+\frac{\pi}{3}\Big) \tag{1.12}$$
$$=x\left(\sin x-\sin\Big(x+\frac{\pi}{3}\Big)_0^{\frac{\pi}{6}}\right)-\int_0^{\frac{\pi}{6}}\sin x-\sin\Big(x+\frac{\pi}{3}\Big)\mathrm{d}x \tag{1.13}$$
$$=\frac{\pi}{6}(\frac{1}{2}-1)-\Big(-\cos x+\cos\Big(x+\frac{\pi}{3}\Big)\Big)_0^{\frac{\pi}{6}} \tag{1.14}$$
$$=\frac{\sqrt{3}-1}{2}-\frac{\pi}{12} \tag{1.15}$$

2 Intro to Fluid Flow

2.1 Description of Mechanics

Definition: [Lagrangian Description] Describe fluids as a small "fluid particle", describe fluids like particles in solid mechanics. i.e. obey euler-lagrange equations.

$$\mathbf{r}(t)=(x,y,z) \tag{2.1}$$
$$\mathbf{v}(t)=(\dot{x},\dot{y},\dot{z})=(u,v,w) \tag{2.2}$$

Definition: [Eulerian Description] Uses density at each points of the fluid in a flow field.

$$\mathbf{v}(x,y,z,t) \tag{2.3}$$

More suitible for analysis of a continum.

2.2 Flow visualization

There are three common concepts used in flow visualization

- Streamlines
  - A line that is tangent to the local velocity vector at each point at a given instant.
  - No flow across a streamline.
  - Can use particle image velocimetry(PIV) to find streamlines experimentally.
    - Streamtubes are streamline
- Pathlines
  - A path a particle takes as it moves.
  - Experimentally, particle tagged and captured using large exposure.
- Streaklines
  - A line that connects all the fluid particles that have passed through the same point in space at a previous time
  - Experimentally, use smoke o

3 Flow Distinction

- A steady flow is when velocity, pressure, temperature, and density is time-independent.
- A unsteady flow is when these are time-dependent.
- The streamline, streakline, and pathline passing through a particular location will be identical in a steady flow.
- Viscous flow regions are regions in which frictional effects are significant.
- Invicid flow regions are regions where viscous forces are negligibly small compared to other forces.
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