PHY356: Quantum Mechanics

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1 Introduction

Schrodinger's Equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(\frac{p^2}{2m} + V(x,t)\right)\psi(x,t) \tag{1}$$

Conservation of Probability:

$$\frac{\partial(\psi^*\psi)}{\partial t} = \frac{\partial\psi^*}{\partial t}\psi + \psi^*\frac{\partial\psi}{\partial t} = -\mathrm{div}J \tag{2}$$

We can use the Schrodinger's equation and its complex conjugate to get the following:

$$\frac{\partial\psi^*}{\partial t}\psi = \tag{3}$$
$$\psi^*\frac{\partial\psi}{\partial t} = \tag{4}$$

Probability Current is then

$$J = -\frac{i\hbar}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) \tag{5}$$

This requires the wave function ψ to be continuously differentiable (for non-singular potentials), unlike the electric field which can be discontinuous on surfaces.

Question: How does the wave mechanics recapture the notion of a classical particle?

Superposition Principle: If ψ_1 and ψ_2 are two solutions to the Schrodinger equation, then $\alpha\psi_1 + \beta\psi_2$ is also a solution for $\alpha, \beta \in \mathbb{C}$.

Simple Example:

$$\cos k_1 x + \cos k_2 x = 2\cos\left(\frac{k_1 + k_2}{2}x\right)\cos\left(\frac{k_1 - k_2}{2}x\right) \tag{6}$$

Let $k_1 = k_0 + \Delta k$ and $k_2 = k_0 - \Delta k$, assume $\Delta k \ll k_0$.

Fourier Decomposition: General solution of the free-particle schrodinger equation

$$\psi(\mathbf{r}, t) = (2\pi)^{-3/2} \int \mathrm{d}^3\mathbf{k}\, g(\mathbf{k}) \exp\left(i(\mathbf{k}\cdot\mathbf{r} - \omega t)\right) \psi_k \tag{7}$$

For simplicity, we will use $t = 0$ and 1 dimension, with a normal distribution of frequencies $g(k) = \exp\left\{-\frac{(k-k_0)^2}{2(\Delta k)^2}\right\}$. We can perform the fourier transform to get $\psi(x) = \exp\{i\Delta k^2 x^2\}$