MIE407 Nuclear Reactor Theory & Design

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Nuclear Stability:

- Stability of nucleus is the result of the balance between the strong nuclear force and the electromagnetic force.
- Range of the strong nuclear force is about $1\,\mathrm{fm}$ (femtometre, $10^{-13}\,\mathrm{cm}$)
- The strong nuclear force acts equally between protons and neutrons, but neutrons also reduce repulsions between protons by pushing them further apart from each other.
- Fermi approx: $r = R_0 A^{1/3}$ where $R_0 = 1.2\,\mathrm{fm}$
- The strong nuclear force is repulsive at very small distances, which contributes to the incompressibility of the nucleus.
- The strong nuclear force is insignificant at larger than around 4 proton diameters.

Nuclear Binding Energy:

- The binding energy of a nucleus can be found by calculating the mass defect of the nucleus compared to the mass of the unbound nucleons.

$$E_B = [Zm_p + (A - Z)m_n - m]c^2 \equiv \Delta M \times c^2 \tag{1}$$

- The binding energy is usually expressed as the average binding energy per nucleon $\varepsilon = E_B/A$.
- $1\,\mathrm{u}{=}931.5\,\mathrm{MeV/c^2}$

Compound nucleus decay modes:

- Neutron capture (n, γ): Nucleus decays to a lower energy state by emitting some gamma rays. $n +^A X \rightarrow^{A+1} X^* \rightarrow^{A+1} X + \gamma$
- Elastic scattering (n, n'): Neutron is re-emitted after leaving the nucleus in the ground state.
- Inelastic scattering $(n, n'\gamma)$: Neutron (usually high energy) is re-emitted at a lower energy, leaving the nucleus in an excited state. Then, the nucleus emits some gamma rays to return to the ground state.
- Fission (n, f): Nucleus splits into two fragments with approx. $2:3$ mass ratio.
- Particle emission $(n, \alpha), (n, p), (n, kn)$: A particle other than a neutron, or multiple neutrons are emitted. Only occurs with very high energy neutrons. Hence, only a small number of $(n, 2n)$ reactions occur in nuclear reactors.

Nuclear Fission

- When a thermal neutron ($\leq 1\,\mathrm{eV}$) is absorbed by a U-235 nucleus, it forms a compound nucleus that fissions with $\approx 84\%$ probability.
- Fission is a threshold reaction. In cases for U-235, the threshold energy is lower than the energy gained by binding the extra neutron. Hence, fissions may occur by neutrons with any energy. However, for U-238, the threshold energy is about 1 MeV greater than the energy gained by binding the extra neutron. Hence, fissions only occur by neutrons with kinetic energy greater than 1 MeV, which rarely occurs in even a fast reactor.
- Under thermal conditions ($0.0253\,\mathrm{eV}$ or $2200\,\mathrm{m/s}$) the distribution of fission products is a strong bimodal distribution (Peaks at $A = 96, 135$).
- Most neutrons (prompt neutrons) emitted in fission are released at the instant of fission.
- A small number of fission fragments also emit neutrons (delayed neutrons). These compose of $< 1\%$ of the total neutrons, but they are significant in the transient behavior of the reactor.
- Delayed neutrons result from high nuclear excitation of the daughter (when the excess energy exceed the nuclear binding energy of the neutron)
- Between 0 to 7 prompt neutrons may be emitted by fission, on average around 2.4. This is very important.
- The fission neutron spectrum is approximately $\chi(E) = 0.484 \sinh\left(\sqrt{2E}\right) e^{-E} \mathrm{MeV}^{-1}$. Integrate this over a interval of E to obtain the probability of energy in that interval. (Average $2\,\mathrm{MeV}$)
- Delayed neutrons are much lower energy ($400\,\mathrm{keV}$)
- Reaction Rate stuff
  - Intensity $I = nv$ $(1/\mathrm{cm}^2\mathrm{s})$
  - Flux ϕ $(1/\mathrm{cm}^2\mathrm{s})$
  - Number density $N = \frac{\rho N_A}{M}$ $(1/\mathrm{cm}^3)$
  - Microscopic cross section σ $(1/\mathrm{cm}^2)$
  - Macroscopic cross section $\Sigma = N\sigma$ $(1/\mathrm{cm})$
  - Reaction rate $R = \Sigma\phi$ $(1/\mathrm{cm}^3\mathrm{s})$
- *Passive* or non-multiplying media are characterized by the scattering (σ_s) and capture (σ_c) cross sections.
- *Multiplying* media contain at least one fissile or fissionable (fissile at high energy only) isotope and are further characterized by fission cross-section σ_f and ν.

1 Propogation of Neutrons in a Passive Medium

- Due to interactions with the medium, the initial intensity I_0 decreases to $I(x)$ at depth x.
- The rate of decrease is the reaction rate $I' = -R = \Sigma I$, with solution $I(x) = I_0 e^{-\Sigma x}$. Σ is the total cross-section.
- Understood in probability terms, $e^{-\Sigma x}$ is the probability that a neutron survive to depth x. Also, for very small Δx, $\Sigma \Delta x$ is the probability that a neutron will interact in Δx.
- Finally, $\Sigma e^{-\Sigma x} \Delta x$ is the probability that a neutron will interact between distance x and $x + \Delta x$.
- The **mean free path** is defined as the average distance a neutron travels before interacting

  $$\lambda = \int_0^\infty x p(x) \mathrm{d}x = \int_0^\infty x \Sigma e^{-\Sigma x} \mathrm{d}x = \frac{1}{\Sigma} \tag{2}$$

  Note that the mean free path can be used for a single reaction type also, e.g. $\lambda_s, \lambda_c, \lambda_f$.
- Point neutron source. The intensity of the source is $\dot{S}$.

- In empty space, $\phi(r) = \frac{\dot{S}}{4\pi r^2}$.
- In medium, $\phi(r) = \frac{\dot{S}}{4\pi r^2} e^{-\Sigma r}$
- If there is scattering, these equations will underestimate the flux.

2 Neutron Flux and Current

- In analyzing a nuclear reactor, we are interested in the neutron population at any position/time, and the rates of all nuclear reaction at any position/time.
- The neutron density is $n(\mathbf{r}, \mathbf{v}, t)$. It is useful to express the velocity vector as the speed v and the unit direction $\mathbf{\Omega}$. We also use kinetic energy (which contains the same information as the speed $E = \frac{1}{2}mv^2$) i,e, $n(\mathbf{r}, E, \mathbf{\Omega}, t)$.
- $n(\mathbf{r}, E, \mathbf{\Omega}, t)$ is a density function with units $1/\mathrm{cm}^3\mathrm{eVsr}$. $n(\mathbf{r}, E, \mathbf{\Omega}, t)\mathrm{d}V\mathrm{d}E\mathrm{d}\Omega$.