

# MIE407 Nuclear Reactor Theory & Design

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## Nuclear Stability:

- Stability of nucleus is the result of the balance between the strong nuclear force and the electromagnetic force.
- Range of the strong nuclear force is about 1 fm (femtometre,  $10^{-13}$  cm)
- The strong nuclear force acts equally between protons and neutrons, but neutrons also reduce repulsions between protons by pushing them further apart from each other.
- Fermi approx:  $r = R_0 A^{1/3}$  where  $R_0 = 1.2$  fm
- The strong nuclear force is repulsive at very small distances, which contributes to the incompressibility of the nucleus.
- The strong nuclear force is insignificant at larger than around 4 proton diameters.

## Nuclear Binding Energy:

- The binding energy of a nucleus can be found by calculating the mass defect of the nucleus compared to the mass of the unbound nucleons.

$$E_B = [Zm_p + (A - Z)m_n - m]c^2 \equiv \Delta M \times c^2 \quad (1)$$

- The binding energy is usually expressed as the average binding energy per nucleon  $\varepsilon = E_B/A$ .
- $1 \text{ u} = 931.5 \text{ MeV}/c^2$

## Compound nucleus decay modes:

- Neutron capture ( $n, \gamma$ ): Nucleus decays to a lower energy state by emitting some gamma rays.  $n + {}^A_Z X \rightarrow {}^{A+1}_Z X^* \rightarrow {}^{A+1}_Z X + \gamma$
- Elastic scattering ( $n, n'$ ): Neutron is re-emitted after leaving the nucleus in the ground state.
- Inelastic scattering ( $n, n'\gamma$ ): Neutron (usually high energy) is re-emitted at a lower energy, leaving the nucleus in an excited state. Then, the nucleus emits some gamma rays to return to the ground state.
- Fission ( $n, f$ ): Nucleus splits into two fragments with approx. 2 : 3 mass ratio.
- Particle emission ( $n, \alpha$ ), ( $n, p$ ), ( $n, kn$ ): A particle other than a neutron, or multiple neutrons are emitted. Only occurs with very high energy neutrons. Hence, only a small number of ( $n, 2n$ ) reactions occur in nuclear reactors.

## Nuclear Fission

- When a thermal neutron ( $\leq 1$  eV) is absorbed by a U-235 nucleus, it forms a compound nucleus that fissions with  $\approx 84\%$  probability.
- Fission is a threshold reaction. In cases for U-235, the threshold energy is lower than the energy gained by binding the extra neutron. Hence, fissions may occur by neutrons with any energy. However, for U-238, the threshold energy is about 1 MeV greater than the energy gained by binding the extra neutron. Hence, fissions only occur by neutrons with kinetic energy greater than 1 MeV, which rarely occurs in even a fast reactor.
- Under thermal conditions (0.0253 eV or 2200 m/s) the distribution of fission products is a strong bimodal distribution (Peaks at  $A = 96, 135$ ).
- Most neutrons (prompt neutrons) emitted in fission are released at the instant of fission.
- A small number of fission fragments also emit neutrons (delayed neutrons). These compose of  $< 1\%$  of the total neutrons, but they are significant in the transient behavior of the reactor.
- Delayed neutrons result from high nuclear excitation of the daughter (when the excess energy exceed the nuclear binding energy of the neutron)
- Between 0 to 7 prompt neutrons may be emitted by fission, on average around 2.4. This is very important.
- The fission neutron spectrum is approximately  $\chi(E) = 0.484 \sinh(\sqrt{2E}) e^{-E} \text{MeV}^{-1}$ . Integrate this over a interval of  $E$  to obtain the probability of energy in that interval. (Average 2 MeV)
- Delayed neutrons are much lower energy (400 keV)
- Reaction Rate stuff
  - Intensity  $I = nv$  ( $1/\text{cm}^2\text{s}$ )
  - Flux  $\phi$  ( $1/\text{cm}^2\text{s}$ )
  - Number density  $N = \frac{\rho N_A}{M}$  ( $1/\text{cm}^3$ )
  - Microscopic cross section  $\sigma$  ( $1/\text{cm}^2$ )
  - Macroscopic cross section  $\Sigma = N\sigma$  ( $1/\text{cm}$ )
  - Reaction rate  $R = \Sigma\phi$  ( $1/\text{cm}^3\text{s}$ )
- *Passive* or non-multiplying media are characterized by the scattering ( $\sigma_s$ ) and capture ( $\sigma_c$ ) cross sections.
- *Multiplying* media contain at least one fissile or fissionable (fissile at high energy only) isotope and are further characterized by fission cross-section  $\sigma_f$  and  $\nu$ .

## 1 PROPOGATION OF NEUTRONS IN A PASSIVE MEDIUM

- Due to interactions with the medium, the initial intensity  $I_0$  decreases to  $I(x)$  at depth  $x$ .
- The rate of decrease is the reaction rate  $I' = -R = \Sigma I$ , with solution  $I(x) = I_0 e^{-\Sigma x}$ .  $\Sigma$  is the total cross-section.
- Understood in probability terms,  $e^{-\Sigma x}$  is the probability that a neutron survive to depth  $x$ . Also, for very small  $\Delta x$ ,  $\Sigma \Delta x$  is the probability that a neutron will interact in  $\Delta x$ .
- Finally,  $\Sigma e^{-\Sigma x} \Delta x$  is the probability that a neutron will interact between distance  $x$  and  $x + \Delta x$ .
- The **mean free path** is defined as the average distance a neutron travels before interacting

$$\lambda = \int_0^\infty x p(x) dx = \int_0^\infty x \Sigma e^{-\Sigma x} dx = \frac{1}{\Sigma} \quad (2)$$

Note that the mean free path can be used for a single reaction type also, e.g.  $\lambda_s, \lambda_c, \lambda_f$ .

- Point neutron source. The intensity of the source is  $\dot{S}$ .

- In empty space,  $\phi(r) = \frac{\dot{S}}{4\pi r^2}$ .
- In medium,  $\phi(r) = \frac{\dot{S}}{4\pi r^2} e^{-\Sigma r}$
- If there is scattering, these equations will underestimate the flux.

## 2 NEUTRON FLUX AND CURRENT

- In analyzing a nuclear reactor, we are interested in the neutron population at any position/time, and the rates of all nuclear reaction at any position/time.
- The neutron density is  $n(\mathbf{r}, \mathbf{v}, t)$ . It is useful to express the velocity vector as the speed  $v$  and the unit direction  $\Omega$ . We also use kinetic energy (which contains the same information as the speed  $E = \frac{1}{2}mv^2$ ) i.e,  $n(\mathbf{r}, E, \Omega, t)$ .
- $n(\mathbf{r}, E, \Omega, t)$  is a density function with units  $1/\text{cm}^3\text{eVsr}$ .  $n(\mathbf{r}, E, \Omega, t)dVdEd\Omega$ .
- With  $\mu := \cos\theta$ , we have  $d\Omega = -d\varphi d\mu/4\pi$ , when we integrate over the all solid angles, we can integrate  $\varphi$  from 0 to  $2\pi$  and  $\mu$  from 1 to  $-1$ .
- Here, we define  $d\Omega = dS/4\pi = -d\varphi d(\cos\theta)/4\pi$ . Note that  $\int_{4\pi} d\Omega = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_{-1}^1 d\mu = 1$
- The **angular neutron density**  $n(\mathbf{r}, E, \Omega)$  is an important reactor parameter used in the definition of neutron flux and current.
- The integral of the angular neutron density over all solid angles is the **neutron density** in  $\text{cm}^{-3}\text{eV}^{-1}$

$$n(\mathbf{r}, E) = \int_{4\pi} n(\mathbf{r}, E, \Omega) d\Omega. \quad (3)$$

- If some quantity  $q$  with density  $\rho$  can flow by a velocity field  $\mathbf{v}$ , we can define the **vector flux**

$$\mathbf{j} = \rho \mathbf{v}. \quad (4)$$

We can define the **neutron flux density** as

$$\phi(\mathbf{r}, E, \Omega) = n(\mathbf{r}, E, \Omega) \mathbf{v}(\mathbf{r}, E, \Omega). \quad (5)$$

- The magnitude of vector flux density is the combined distance traveled by these neutrons. The **scalar neutron flux**,

$$\Phi(\mathbf{r}, E) = \int_{4\pi} \phi(\mathbf{r}, E, \Omega) d\Omega = v \int_{4\pi} n(\mathbf{r}, E, \Omega) d\Omega. \quad (6)$$

- The **neutron current density** is

$$\mathbf{J}(\mathbf{r}, E) = \int_{4\pi} \mathbf{v} n(\mathbf{r}, E, \Omega) d\Omega = v \int_{4\pi} \hat{\Omega} n(\mathbf{r}, E, \Omega) d\Omega \quad (7)$$

- Since the integral in (7) is performed over all direction, which yields a vector that points in a particular direction. The direction of  $\mathbf{J}$  represents the direction of **net** neutron flow.
- We can specify a unit vector  $\hat{n}$  that is normal to  $dA$ , then  $\mathbf{J} \cdot \hat{n} dA$  is the net number of neutrons crossing the surface  $dA$  per unit energy and unit time.