MAT351 PDE

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Transport Equation (simplest form)

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{1}$$

where $u : (x,t) \in \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}, c \in \mathbb{R}$

Diffusion Equation (simplest form)

$$\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0 \tag{2}$$

where $k > 0$

Wave Equation

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0 \tag{3}$$

where c is the speed of the wave.

In the 19th century, the research focused on finding explicit solutions. Most notably:

1. Method of characteristics (Hamilton)
2. Fourier analysis
3. Green's functions

In the 20th century, numerical methods were developed for equations without explicit solutions. This is used for modelling, weather prediction, finance, etc.

Theoretical questions are posed like uniqueness, behavior, stability, and other qualitative properties of PDEs.

> **Definition**: [Well-posedness] A PDE (problem) is well posed if
>
> 1. A solution exists.
> 2. The solution is unique.
> 3. The solution is stable. (i.e. small perturbations of initial conditions or boundary conditions leads to small perturbations of the solution)

Classification:

1. Order of PDE (highest order of derivative)

2. Linearity/non-linearity

$$L(u) = f(x) \tag{4}$$

where L is a linear operator and f is a function.

$$\frac{\partial u}{\partial t} - k\frac{\partial}{\partial 2ux^2 + u^3 = x^2 + 1(5)}$$