MAT257 PSET 8—Question 3

Jonah Chen

As $f : [a,b] \times [c,d] \to \mathbb{R}$ is continuous and $D_2 f$ is continuous and hence integrable, by the 1D fundamental theorem of calculus,

$$f(x,y) = f(x,c) + \int_c^y D_2 f(x,t) \mathrm{d}t$$

Define $\alpha = \int_a^b f(x,c)\mathrm{d}x$ and $g(t) = \int_a^b D_2 f(x,t)\mathrm{d}x$. Then, by Fubini's theorem,

$$F(y) = \alpha + \int_a^b \int_c^y D_2 f(x,t)\mathrm{d}t\mathrm{d}x = \alpha + \int_c^y \int_a^b D_2 f(x,t)\mathrm{d}x\mathrm{d}t = \alpha + \int_c^y g(t)\mathrm{d}t$$

By the 1D fundamental theorem of calculus,

$$F'(y) = g(y) = \int_a^b D_2 f(x,y)\mathrm{d}x$$