MAT257 PSET 4—Question 5

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Consider the function $g : \mathbb{R} \to \mathbb{R}$ where $g(t) = f(tx) = t^m f(x)$. Then, $g'(t) = mt^{m-1} f(x)$.

Consider the function $h : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ where $h(x,t) = tx$. Then, $g(t) = f(h(x,t))$. By chain rule,

$$g'(t) = [f'(h(x,t))][\partial_t h(x,t)] = Df(tx) \cdot x$$

$$g'(1) = Df(x) \cdot x = mt^{1-1} f(x) = mf(x)$$

As f is differentiable,

$$Df(x) = \begin{pmatrix} D_1 f(1) & D_2 f(1) & \dots & D_n f(1) \end{pmatrix}$$

Thus,

$$g'(1) = Df(x) \cdot x = \sum_{i=1}^{n} x_i D_i f(x) = mf(x)$$