MAT257 PSET 6—Question 1

Jonah Chen

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(a) If f satisfies the hypotheses of the implicit function theorem at $x = 3$ (and $y = (-1, 2)$ because $f(x, y) = 0$ for this choice of x and y. Then as a consequence, there exists g where $f(x, g(x)) = f(x, g_1(x), g_2(x)) = 0$ in a neighborhood about $x = 3$.

f is C^1 everywhere so it is continuously differentiable at $(3, -1, 2)$. Secondly, $\dfrac{\partial f}{\partial y} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ which has a nonzero determinent of 3.

(b) Consider the function $h(x) = (x, g(x))$. Then $f(x, g(x)) = (f \circ h)(x) = 0$ inside B. By chain rule,

$$0 = f'(3, -1, 2)h'(3) = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ g'(3) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} g'(3)$$

$$g'(3) = -\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

(c) Consider solving this equation for the unknowns x and y_2 in terms of y_1. Then, let $z = (x, y_2)$ so $\det\left(\dfrac{\partial f}{\partial z}\right) = 0$ so the implicit function theorem does not apply.