

MAT257 PSET 4—Question 1

Jonah Chen

October 15, 2021

(a) By 1-variable Fundamental Theorem of Calculus,

$$\partial_x f(x, y) = g(x + y)$$

$$\partial_y f(x, y) = g(x + y)$$

(b) $\partial_x f(x, y) = g(x)$. Note that $\int_y^x g = -\int_x^y g$. Hence, $\partial_y f(x, y) = -g(y)$.

(c) Define $h(u) = \int_a^u g$ and $k(x, y) = xy$. Then, $f(x, y) = h(k(x, y))$ By chain rule,

$$\partial_x f(x, y) = h'(k(x, y)) \cdot \partial_x k = g(xy) \cdot y$$

$$\partial_y f(x, y) = h'(k(x, y)) \cdot \partial_y k = g(xy) \cdot x$$

(d) As f does not depend on x , $\partial_x f = 0$. For the y partial derivative, note that $f(x, y) = h(h(y))$ so $\partial_y f(x, y) = (h \circ h)'(y)$. By chain rule (single variable),

$$\partial_y f(x, y) = (h \circ h)'(y) = h'(h(y)) \cdot h'(y) = g(h(y)) \cdot g(y) = g\left(\int_b^y g\right) \cdot g(y)$$