MAT257 PSET 3—Question 3

Jonah Chen

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As $|f(x)| \leq |x|^2$ for all x, $|f(0)| \leq 0$ hence $f(0) = 0$. Then,

$$\lim_{h \to 0} \frac{|f(h) - f(0) - 0h|}{|h|} = \lim_{h \to 0} \frac{|f(h)|}{|h|} \leq \lim_{h \to 0} \frac{|h|^2}{|h|} = 0$$

Thus, f is differentiable at 0 and the derivative is 0.