MAT257 PSET 15—Question 3

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As $x = r\cos\theta, y = r\sin\theta$ are zero forms, their exterior derivatives are

$$\begin{aligned}\mathrm{d}x &= \frac{\partial x}{\partial r}\mathrm{d}r + \frac{\partial x}{\partial \theta}\mathrm{d}\theta = \cos\theta \mathrm{d}r - r\sin\theta\mathrm{d}\theta \\ \mathrm{d}y &= \frac{\partial y}{\partial r}\mathrm{d}r + \frac{\partial y}{\partial \theta}\mathrm{d}\theta = \sin\theta \mathrm{d}r + r\cos\theta\mathrm{d}\theta\end{aligned}$$

We also know that $x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$. Then,

$$\begin{aligned}\frac{-y}{x^2+y^2}\mathrm{d}x + \frac{x}{x^2+y^2}\mathrm{d}y &= \frac{-r\sin\theta}{r^2}(\cos\theta\mathrm{d}r - r\sin\theta\mathrm{d}\theta) + \frac{r\cos\theta}{r^2}(\sin\theta\mathrm{d}r + r\cos\theta\mathrm{d}\theta) \\ &= \frac{-\cos\theta\sin\theta\mathrm{d}r}{r} + \sin^2\theta\mathrm{d}\theta + \frac{\sin\theta\cos\theta\mathrm{d}r}{r} + \cos^2\theta\mathrm{d}\theta \\ &= \mathrm{d}\theta,\end{aligned}$$

as desired.