MAT257 PSET 13—Question 2

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a) Using the standard basis for $\mathbb{R}^2$, L_1 can be represented as a matrix $M_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, and $\det M_1 = -1$ so L_1 is orientation reversing.

b) L_2 is represented as a matrix $M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and $\det M_2 = -1$ so L_2 is orientation reversing.

c) Rotations have determinent 1, so L_3 is orientation preserving.

d) Same as c, L_4 is orientation preserving.

e) Using $a + bi \to (a, b)$, the complex conjugation map can be represented as a matrix $M_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and $\det M_5 = -1$ so L_5 is orientation reversing.

f) Using the standard basis for $\mathbb{R}^3$, L_6 can be represented as a matrix $M_6 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, and $\det M_6 = 1$ so L_6 is orientation preserving.

g) Using the standard basis for $\mathbb{R}^n$, the matrix representing L_7 is $M_7 = (-1)\mathbb{1}_{n\times n}$, so $\det M_7 = (-1)^n \det \mathbb{1} = (-1)^n$, so L_6 is orientation preserving if n is even, and orientation reversing if n is odd.

h) Using the standard basis for $\mathbb{R}^{m+n}$, the matrix describing L_8 is $M_8 = \begin{pmatrix} 0_{m\times n} & \mathbb{1}_{n\times n} \\ \mathbb{1}_{m\times m} & 0_{n\times m} \end{pmatrix}$

We can define the matrix P_j which swap the j-th and $j+1$-th rows of a matrix. This matrix has determinant -1. Then,

$$(P_n P_{n-1} \dots P_1)(P_{n+1} P_n \dots P_2) \dots (P_{n+m-1} \dots P_m) M_8 = \mathbb{1}.$$

There are a total of nm of these P_j matrices, so $(-1)^{nm} \det M_8 = \det \mathbb{1} = 1, \det M_8 = (-1)^{nm}$, so L_8 is orientation reversing if both m and n are odd, and orientation preserving otherwise.