

MAT257 PSET 10—Question 3

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Let $g(u, v) = (u/v, uv)$, and $A = \{(u, v) : 1 < u < \sqrt{2}, 1 < v < 2\}$. Then, $g(A) = B$ and g has continuous partial derivatives except for $v = 0$. Also, using mathematica $\det(g') = 2u/v$, which is nonzero except when $u = 0$. There are no points in A where either u or v is zero, hence, using the change of variables theorem,

$$\int_B x^2 y^3 = \int_A (u/v)^2 (uv)^3 = \int_A u^5 v.$$

As A 's boundary is a set of content zero, we can integrate over the closure of A , $\bar{A} = [1, \sqrt{2}] \times [1, 2]$ and get the same result. Then, using Fubini's theorem,

$$\int_B x^2 y^3 = \int_{\bar{A}} u^5 v = \int_1^{\sqrt{2}} u^5 du \int_1^2 v dv = \frac{7}{4}$$