

(a)  $A$  is closed  $\implies A^C$  is open. Then,  $x \in A^C \implies \exists$  open ball with radius  $d > 0$ ,  $B = \{x_0 \in \mathbb{R}^n : |x_0 - x| < d\} \subset A^C$ .

Take any point  $y \in A$ . Then,  $y \notin B$  as  $B \subset A^C$ . Then,  $|y - x| \geq d$ .

(b) B

(c) Consider increasing sequences  $a_n = \binom{n + \frac{1}{n}}{0}$  and  $b_n = \binom{n}{0}$  for  $n \in \mathbb{Z}_+$ . Let  $A = \{a_n\}_{n=2}^\infty$  and  $B = \{b_n\}_{n=2}^\infty$ .

$A$  is closed because  $A^C$  is the union of open sets  $((-\infty, a_2) \times \mathbb{R}) \cup \left( \bigcup_{n=2}^\infty (a_n, a_{n+1}) \times \mathbb{R} \right) \cup (\mathbb{R} \times (0, \infty)) \cup (\mathbb{R} \times (-\infty, 0))$ .

Similarly,  $B$  is closed because  $B^C = ((-\infty, b_2) \times \mathbb{R}) \cup \left( \bigcup_{n=2}^\infty (b_n, b_{n+1}) \times \mathbb{R} \right) \cup (\mathbb{R} \times (0, \infty)) \cup (\mathbb{R} \times (-\infty, 0))$

However, for any  $d > 0$  define  $f = \left\lceil \frac{1}{d} \right\rceil$ . Then,  $|a_f - b_f| = \frac{1}{f} \leq d$ .