MAT257 PSET 8—Question 4

Jonah Chen

As g_1, g_2 are both C^1 functions,

$$D_1 f(x, y) = g_1(x, 0) + D_1 \int_0^y g_2(x, t)\mathrm{d}t \qquad \text{by FTC on } g_1$$

$$= g_1(x, 0) + \int_0^y D_1 g_2(x, t)\mathrm{d}t \qquad \text{by Leibnitz' rule}$$

$$= g_1(x, 0) + \int_0^y D_2 g_1(x, t)\mathrm{d}t \qquad \text{because } D_1 g_2 = D_2 g_1$$

$$= g_1(x, 0) + g_1(x, y) - g_1(x, 0) \qquad \text{by FTC on } g_2$$

$$= g_1(x, y)$$

As $\int_0^x g_1(t, 0)\mathrm{d}t$ is independent of the second variable y, by fundamental theorem of calculus,

$$D_2 f(x, y) = D_2 \int_0^y g_2(x, t)\mathrm{d}t = g_2(x, y)$$