MAT257 PSET 14—Question 3

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As $\gamma : [0,1] \to \mathbb{R}^2$ is differentiable, define coordinate functions $\gamma_1, \gamma_2 : [0,1] \to \mathbb{R}$ that are differentiable so that

$$\gamma(t) = \gamma_1(t)e_1 + \gamma_2(t)e_2.$$

As $|\gamma(t)| = 1$,$f(t) =: \langle \gamma(t), \gamma(t) \rangle = (\gamma_1(t))^2 + (\gamma_2(t))^2 = 1$. Hence,

$$f'(t) = 2\gamma_1(t)\gamma_1'(t) + 2\gamma_2(t)\gamma_2'(t) = 2\langle \gamma'(t), \gamma(t) \rangle = 0 \implies \langle \gamma'(t), \gamma(t) \rangle = 0$$

The tangent vector to the curve $\gamma(t)$ is $\gamma'(t)_{\gamma(t)}$,

$$\langle \gamma'(t)_{\gamma(t)}, \gamma(t)_{\gamma(t)} \rangle = \langle \gamma'(t), \gamma(t) \rangle = 0$$

so for any t, the tangent vector to the curve $\gamma'(t)_{\gamma(t)}$ is perpendicular to the $\gamma(t)_{\gamma(t)}$.