

$$\text{int}A_1 = \{x \in \mathbb{R}^n : |x| < 1\}$$

$$\text{ext}A_1 = \{x \in \mathbb{R}^n : |x| > 1\}$$

$$\text{bd}A_1 = \{x \in \mathbb{R}^n : |x| = 1\}$$

Consider a point $x \in \text{int}A_1$. \exists open ball $B = \{x \in \mathbb{R}^n : |x| < 1\}$ of radius 1 centered at the origin where $x \in B \subset A_1$.

Consider a point $x \in \text{ext}A_1$. Choose an open ball B of radius $\frac{1}{2}(|x| - 1)$ centered at x . For any $y \in B$, by triangle inequality, $|x| \leq |x - y| + |y| \implies |y| \geq |x| - |x - y| = \frac{1}{2}|x| + \frac{1}{2}$. As $|x| > 1$, $|y| > 1$. Thus, $B \subset \text{ext}A_1 \subset A^C$.

Consider a point $x \in \text{bd}A_1$ and any open ball B of radius $2r$ centered about the point. WLOG, take $2r < 1$. Then, the points $a = (1 - r)x$ and $b = (1 + r)x$ are contained in B as $|a - x| = |(1 - r)x - x| = r < 2r$, $|b - x| = |(1 + r)x - x| = r < 2r$. However, $a \in A_1$ as $|a| = 1 - r$ but $b \in A_1^C$ as $|b| = 1 + r$. Thus, x is on the boundary of A_1 .

$$\text{int}A_2 = \emptyset$$

$$\text{ext}A_2 = \{x \in \mathbb{R}^n : |x| \neq 1\}$$

$$\text{bd}A_2 = \{x \in \mathbb{R}^n : |x| = 1\}$$

Take any open ball $B \subset \mathbb{R}^n$ of radius $2r$ centered at y . Then, let $d = \frac{r}{|y|}$. The points $a = (1 - d)y$ and $b = (1 + d)y$ are contained in B as $|a - y| = |(1 - d)y - y| = |d||y| = r$ and $|b - y| = |(1 + d)y - y| = |d||y| = r$, but $|a| = |y| - r$, $|b| = |y| + r$. Thus, for any open there are points with different norms, so, there is no open ball that is contained in A_2 . Hence, $\text{int}A_2$ is empty.

Finding the boundary of A_2 involves the same as the proof for the boundary of A_1 .

As $\text{int}A_2 \cup \text{bd}A_2 \cup \text{ext}A_2 = \mathbb{R}^n$, $\text{ext}A_2 = \mathbb{R}^n \setminus (\text{int}A_2 \cup \text{bd}A_2)$.

$$\text{int}A_3 = \emptyset$$

$$\text{ext}A_3 = \emptyset$$

$$\text{bd}A_3 = \mathbb{R}^n$$

Since \mathbb{Q}^n is dense in \mathbb{R}^n , any open rectangle in \mathbb{R}^n will contain points in A_3 and A_3^C . Thus, there are no points in the interior or exterior of A_3 , and the boundary of A_3 is the full set \mathbb{R}^n .