

Let $f = \mathbf{1}_A$

- For any $\delta > 0$, consider the point $a = \left(u, \frac{u^2}{2}\right)$ where $u > 0$ is selected such that $\delta = 2\sqrt{u^2 + \frac{u^4}{4}}$ so that $|a| < \delta$.

$f(a) = 1$, thus, for $\epsilon = 1$, there does not exist any $\delta > 0$ where $|(x, y)| < \delta \implies |f(x, y) - f(0, 0)| < \epsilon$. Hence, f is not continuous at $(0, 0)$.

- If the domain was restricted to any line $D = \{(\alpha t, \beta t) : \alpha, \beta, t \in \mathbb{R}\}$. WLOG, assume $\alpha^2 + \beta^2 = 1$.

If $\alpha \neq 0$, choose $\delta = \frac{\beta}{\alpha^2}$. Then, $|(\alpha t, \beta t)| < \delta \implies |t| < \delta = \frac{\beta}{\alpha^2} \implies \beta t > (\alpha t)^2 \implies f(\alpha t, \beta t) = 0 = f(0, 0)$.

If $\alpha = 0$, $f(0, \beta t) = 0$, and it is known that a constant function is continuous.

Thus, the restriction of f to any line $x = \alpha t, y = \beta t$ [which crosses $(0, 0)$] is continuous at $(0, 0)$.