Note that

1. $0 \in A$ and $1 \in A$ because they are rational.
2. $[0,1] \cap \mathbb{Q} \subset A$ as A contains every rational number in $[0,1]$.

Suppose $\exists x \in [0,1] \notin A$. Then, $x \in A^C$. As A is closed, A^C is open.

Hence, $\exists$ open rectangle $R = (a,b) \subset A^C$. WLOG, assume $0 \leq a < b \leq 1$. By the density of $\mathbb{Q}$, $\exists y \in R \cap \mathbb{Q} \subset [0,1] \cap \mathbb{Q} \subset A$.

But also, $y \in R \subset A^C$, which contradicts $y \in A$; hence, there cannot exist $x \in [0,1] \notin A$ so $[0,1] \subset A$.