MAT257 PSET 10—Question 4

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Consider the linear transformation $G = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$. Then, $G([0,1]^3) = T$. As G is a linear transformation, G is C^1 and $G' = G$ so $\det(G') = \det(G) = -2$, which is never zero. As T's boundary is content zero, we can integrate over $(0,1)^3$ to get the same answer.

Note that $f(x,y,z) = x+2y-z$ is the linear transformation from $F : \mathbb{R}^3 \to \mathbb{R}$ where $F = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix}$. Then, $F \circ G = \begin{pmatrix} 2 & 0 & 0 \end{pmatrix}$. Therefore, by change of variables theorem,

$$\int_T x + 2y - z = \int_{(0,1)^3} (F \circ G)|\det(G')| = 2\int_{(0,1)^3} \begin{pmatrix} 2 & 0 & 0 \end{pmatrix}$$

By fubini's theorem

$$2\int_{[0,1]^3} \begin{pmatrix} 2 & 0 & 0 \end{pmatrix} = 2\int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}y \int_0^1 2x\mathrm{d}x = 2$$