MAT257 PSET 7—Question 3

Jonah Chen

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Let $f, g : A \to \mathbb{R}$ be integrable and $f \leq g$. Then, for any partition P of A,

$$L(f,P) = \sum_{S\in P} v(S)m_S(f) \leq \sum_{S\in P} v(S)m_S(g) = L(g,P)$$
$$U(f,P) = \sum_{S\in P} v(S)M_S(f) \leq \sum_{S\in P} v(S)M_S(g) = U(g,P)$$

so $\sup\limits_P L(f,P) \leq \sup\limits_P L(g,P)$ and $\inf\limits_P U(f,P) \leq \inf\limits_P U(g,P)$.

As f and g are both integrable, $\displaystyle\int_A f = \sup_P L(f,P) = \inf_P U(f,P) \leq \inf_P U(g,P) = \sup_P L(f,P) = \int_A g$.