First, we show T is linear. For any $x, y, z \in \mathbb{R}^n$ and $a, b \in \mathbb{R}$,

$$[T(ax + by)]z = \langle ax + by, z\rangle = a\langle x, z\rangle + b\langle y, z\rangle = [aT(x) + bT(y)]z$$

Next, by way of contradiction assume $\exists x, y \in \mathbb{R}^n, x \neq y : Tx = Ty$.

$$\implies \forall z \in \mathbb{R}^n, \langle x, z\rangle = \langle y, z\rangle.$$

Choose n vectors to be z. The standard basis vectors $e_1, \ldots, e_n$.

$$\begin{gathered}\langle x, e_i\rangle = x_i, \langle y, e_i\rangle = y_i \\ \implies \forall i = 1, \ldots, n, x_i = y_i \\ \therefore x = y\end{gathered}$$

This is a contradiction thus T is one-to-one. For linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$, injective, surjective, and bijective are equivalent.