MAT257 PSET 18—Question 3

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a) Consider $M = D_d^3 \setminus \operatorname{int} D_c^3$. This is a compact manifold with boundary, with boundary $\partial M = S_d^2 - S_c^2$, since S_c^2 has the opposite orientation. The integral

$$\int_M \mathrm{d}\omega = \int_{\partial M} \omega = \int_{S_d^2} \omega - \int_{S_c^2} \omega = (a + b/d) - (a + b/c) = b/d - b/c.$$

b) If ω is closed, $\mathrm{d}\omega = 0$. As ω is a 2-form on $\mathbb{R}^3 \setminus \{0\}$, we can use the result from part a, with $\displaystyle\int_M \mathrm{d}\omega = 0$ since ω is closed. Then we have $b/d = b/c$ for any $d > c > 0$ hence $b = 0$.