MAT257 PSET 15—Question 4

Jonah Chen

$$\omega = xy\mathrm{d}x + 3\mathrm{d}y - yz\mathrm{d}z \tag{1}$$

$$\mathrm{d}\omega = y\mathrm{d}x \wedge \mathrm{d}x + x\mathrm{d}y \wedge \mathrm{d}x - z\mathrm{d}y \wedge \mathrm{d}z - \mathrm{d}z \wedge \mathrm{d}z = -x\mathrm{d}x \wedge \mathrm{d}y - z\mathrm{d}y \wedge \mathrm{d}z \tag{2}$$

$$\mathrm{d}(\mathrm{d}\omega) = -\mathrm{d}x \wedge \mathrm{d}x \wedge \mathrm{d}y - \mathrm{d}z \wedge \mathrm{d}y \wedge \mathrm{d}z = 0 \tag{3}$$

$$\eta = x\mathrm{d}x - yz^2\mathrm{d}y + 2x\mathrm{d}z \tag{4}$$

$$\omega \wedge \eta = (6x - y^2z^3)\mathrm{d}y \wedge \mathrm{d}z + (-xyz - 2x^2y)\mathrm{d}z \wedge \mathrm{d}x + (-3 - xy^2z^2)\mathrm{d}x \wedge \mathrm{d}y \tag{5}$$

$$\mathrm{d}\eta = 2yz\mathrm{d}y \wedge \mathrm{d}z - 2\mathrm{d}z \wedge \mathrm{d}x \tag{6}$$

$$\mathrm{d}(\omega \wedge \eta) = (6 - xz - 2x^2 - 2xy^2z)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z \tag{7}$$

$$(\mathrm{d}\omega) \wedge \eta = (-2x^2 - xz)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z \tag{8}$$

$$\omega \wedge (\mathrm{d}\eta) = (2xy^2z - 6)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z \tag{9}$$

$$(\mathrm{d}\omega) \wedge \eta - \omega \wedge (\mathrm{d}\eta) = (-2x^2 - xz - 2xy^2z + 6)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z = \mathrm{d}(\omega \wedge \eta) \tag{10}$$

The direct computations (3) and (10) verified the two theorems respectively.