MAT257 PSET 6—Question 3

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WLOG, assume $\dfrac{\partial(f,g)}{\partial(y,z)}$ has rank 2 at p_0, hence it is invertible (as $\dfrac{\partial(f,g)}{\partial(x,y,z)}$ is rank 2 at p_0, so the statement must true for at least one of $(x,y),(x,z),(y,z)$ and the rest of the proof is the same no matter which choice was made).

Let $F = (f,g)$. f and g are C^1 so F is C^1, the implicit function theorem guarentees the existance of $h(x)$ such that $F(x, h_1(x), h_2(x)) = 0$ in an open neighborhood about p_0.

Thus locally, the curve described by $f(x,y,z) = 0 \wedge g(x,y,z) = 0$ can be parameterized with $x = x, y = h_1(x), z = h_2(x)$.