(a) We will first show $\langle Tx, Ty\rangle = \langle x, y\rangle \implies |Tx| = |x|$

$$
\begin{aligned}
|Tx|^2 &= \langle Tx, Tx\rangle = \langle x, x\rangle = |x|^2 \\
|Tx| &= |x|
\end{aligned}
$$

Then, we will show $|Tx| = |x| \implies \langle Tx, Ty\rangle = \langle x, y\rangle$.

$$
\begin{aligned}
|T(x+y)| &= |x+y| \\
|T(x+y)|^2 &= |x+y|^2 \\
\langle Tx + Ty, Tx + Ty\rangle &= \langle x+y, x+y\rangle \\
\langle Tx, Tx\rangle + \langle Ty, Ty\rangle + 2\langle Tx, Ty\rangle &= \langle x, x\rangle + \langle y, y\rangle + 2\langle x, y\rangle \\
|Tx|^2 + |Ty|^2 + 2\langle Tx, Ty\rangle &= |x|^2 + |y|^2 + 2\langle x, y\rangle \\
\langle Tx, Ty\rangle &= \langle x, y\rangle
\end{aligned}
$$

Hence, $\langle Tx, Ty\rangle = \langle x, y\rangle \iff |Tx| = |x|$

(b) If $|Tx| = |x|$, $|Tx| = 0 \iff |x| = 0$. Since the norm is positive definite, $Tx = 0 \iff x = 0$. As T is a linear transformation from $\mathbb{R}^n \to \mathbb{R}^n$ and $\ker T = \{0\}$, T must be invertible hence it must be 1-1 and onto.

As T is norm preserving and $TT^{-1} = 1$, $|T(T^{-1}x)| = |T^{-1}x| = |x|$.