MAT257 PSET 4—Question 6

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Consider $h_x(t) = f(tx)$. Since f is differentiable, taking the derivative of h_x is the same as taking the partial derivative of f with respect to t, as x in h_x is constant. Using chain rule for f, $h_x'(t) = \partial_t f(tx) = f'(tx) \cdot x$.

Next, note that by one variable fundamental theorem of calculus $\displaystyle\int_0^1 h_x'(t)\mathrm{d}t = h(1) - h(0) = f(x) - f(0) = f(x)$ as $f(0) = 0$.

Also, $\displaystyle f(x) = \int_0^1 h_x'(t)\mathrm{d}t = \int_0^1 f'(tx) \cdot x\mathrm{d}t = \sum_{i=1}^n x_i \int_0^1 Df_i(tx)\mathrm{d}t$. So, define $\displaystyle g_i(x) := \int_0^1 Df_i(tx)\mathrm{d}t$, then $\displaystyle f(x) = \sum_{i=1}^n x_i g_i(x)$.