Contents

1 What is an Electric Circuit

Definition: An electric circuit is an interconnection of circuit elements (incl. Conductors, semi-conductors, non-conductors).

2 Electrical Variables

To help us analyze electric circuits, we define several electrical variables:

2.1 Current

Definition: Electric currents: The "movement" or rate of change of electrical charge.

$$i \equiv \frac{dq}{dt} \tag{1}$$

1. S.I. unit: Ampere = Columb per second.
2. The current has a direction, and its direction is defined as the direction of positive charge. (Can be shown with $\leftarrow$ or $\rightarrow$)

- When performing circuit analysis, the direction of positive current is either given or we are required to guess a direction of positive current and verify it later.
- A positive current means the direction you guessed is correct. A negative current means the actual direction is the direction opposiing the current.

2.2 Voltage

Definition: Voltage: The energy required for 1C of charge between point A and B in a circuit is called the voltage between points A and B.

$$V \equiv \frac{dw}{dq} \tag{2}$$

1. S.I. unit: Volt = Joule per columb.
2. The voltage also have polarity (+ or −). Positive polarity means energy is consumed when the charge moves from A to B.

- If the polarity is not given, guess a polarity
- a positive voltage indicates the guess is correct and a negative voltage indicates the guess is incorrect.

2.3 Power

Definition: Power: The rate of delivering or absorbing energy.

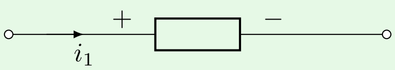
$$p \equiv \frac{dw}{dt} \tag{3}$$

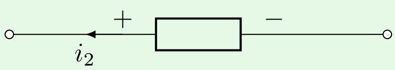
1. S.I. unit: Watt = Joule per second.
2. Using chain rule,

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = iv \tag{4}$$

- **Note:** When a problem asks for a current or voltage, the direction/polarity **must** be indicated otherwise full credit will not be given.

Definition: For a pair of v and i, PSC holds if the current direction **enters** the positive side of voltage polarity. If PSC holds, $p = +vi$; else $p = -vi$





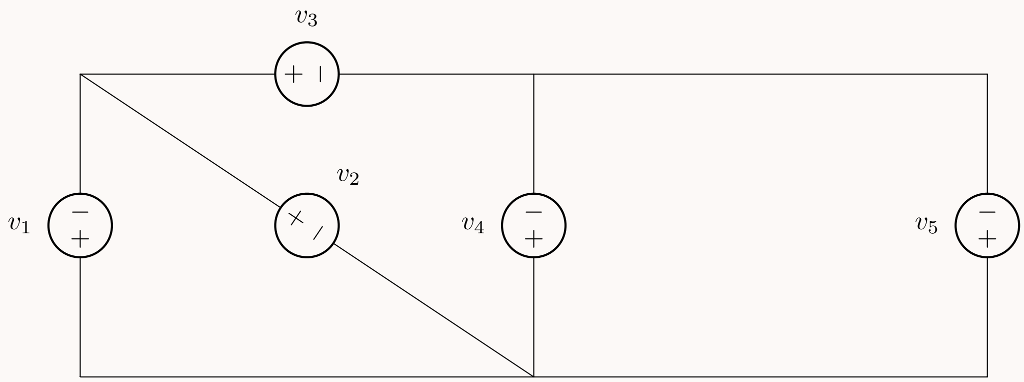
PSC holds on the left and does not hold on the right.

- The consequences of this convention are
  1. $p > 0 \implies$ power is absorbed
  2. $p < 0 \implies$ power is delivered (generated by the circuit element)

2.4 Examples

Example 1 (1)

Identify the devices that generate power in the circuit below and find the unknowns in the table.

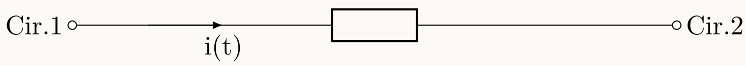


Theorem: **Conservation of Power** states that the algebraic sum of the power of all elements in a circuit is zero. (algebraic means the signs are preserved)

$$\sum p = 0 \tag{5}$$

Proof: Will not be presented at this point in the course.

Example 2 (2)



Given the above circuit and

$$v(t) = 50(1 - e^{-5000t})\text{V} \tag{6}$$
$$i(t) = 10e^{-5000t}\text{A} \tag{7}$$

Find the total energy transfered to this device after $t = 0$.

Solution 1 (2): PSC holds. Thus,

$$p(t) = v(t)i(t) \tag{8}$$
$$p(t) = 50(1 - e^{-5000t})10(e^{-5000t})\text{W} \tag{9}$$

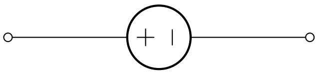
From the definition of power,

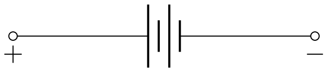
$$p(t) = \frac{dw}{dt} \therefore dw = p(t)dt \tag{10}$$
$$w = \int_0^\infty p(t)\text{d}t = \int_0^\infty 50(1 - e^{-5000t})10(e^{-5000t})\text{d}t = \frac{1}{20}\text{J} = 50\text{mJ} \tag{11}$$

3 Circuit Elements

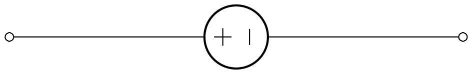
3.1 Independent sources

1. *Independent voltage sources:* A circuit element that has a specific voltage independent of the current that flows through it.
   - For example, the voltage can be a fixed voltage like $v_s = 2\text{V}$ or a variable voltage source like $v_s = 2\sin(50t + 2)$.
   - On diagrams, generic voltage source is shown on the left and fixed (DC) voltage sources is shown on the right. For DC sources, the uppercase letter V is commonly used.

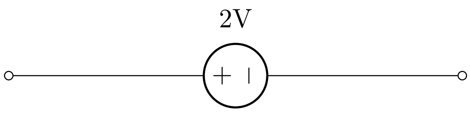




- For DC source, the longer lines denotes positive voltage polarity and shorter lines denote negative voltage polarity.
- An sinsudal voltage is drawn as



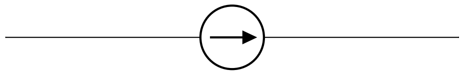
- Questions
  (a) What is the direction of i for the following voltage source?



  Answer: You cannot determine.
  (b) Does a voltage source always generate power?
  Answer: No. Take the voltage source above. Take a current $i_1 = 2\text{A} \rightarrow$. Then, PSC holds and $p = vi = (2\text{V})(2\text{A}) = 4\text{W}$. Since $p > 0$, the voltage source absorbs power.

2. *Independent current source:* It gives a specific current independent of the voltage across it. Can be constant like $i_s = 5\text{A}$ or time dependent like $i_s = 10\cos(15t)$

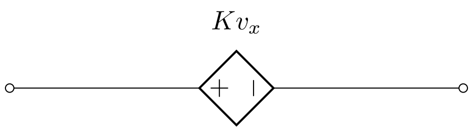
- The independent current source is drawn as



- Questions
  (a) What is the polarity of voltage across a current source?
  (b) Does a current source always generate power?
- Do **not** let the word *source* decieve you. A source is not associated with generating power.

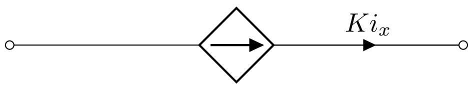
3.2 Dependent Sources

1. *Voltage-dependent voltage source:*



The voltage of this source depends on voltages somewhere else in the circuit. i.e. $v_s = Kv_s$

2. *Current-dependent voltage source:*



The voltage of this source depends on the current somewhere else in a circuit i.e. $v_s = ki_x$. Note k has the dimensions Current/Voltage.

3. *Voltage-dependent current source:*
4. *Current-dependent current source:*

3.3 Resistor

A resistor is a circuit element that keeps the ratio between the voltage and current constant.

$$R \equiv \frac{v}{i} \tag{12}$$

- S.I. unit: Volt per Amp or Ohm (Ω).
- The resistor is drawn like this. If PSC holds, $v = Ri$ else $v = -Ri$. This is also known as Ohm's Law.

- The inverse of the resistence R is called the conductance G

$$G \equiv \frac{1}{R} = \pm\frac{i}{v} \tag{13}$$

- The S.I. unit for conductance is 1/Ohm, also refered to as "mho" or "siemens" (Si).
- We could not find a generic relation for the power of a source, since $P = vi$ (assuming PSC holds). Since for a independent voltage source, we don't know the current and vise versa. For a resistor however, assume PSC holds,

$$P = vi = (Ri)i = Ri^2 \tag{14}$$

$$= v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{15}$$

If PSC doesn't hold,

$$P = -vi = -(-Ri)i = Ri^2 \tag{16}$$

$$= -v\left(-\frac{v}{R}\right) = \frac{v^2}{R} \tag{17}$$

Thus, the power of a resistor is independent of whether or not PSC holds. For physical resistors, $R > 0$ hence $P > 0$ which means power is always absorbed.

3.4 Short Circuit

A short circuit is the limiting behavior for a resistor as $R \to 0$. Since $v = Ri$, the voltage is zero independent of the current. This is denoted with a solid line:

- Other names include zero ohm path or ideal conductor.
- **In analysis, all parts of a circuit that are connected using ideal conductor can be considered the same point in the circuit.**

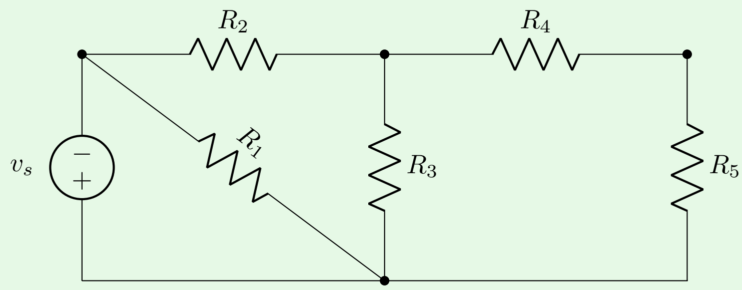
3.5 Open Circuit

The limiting behavior for a resistor as $R \to \infty$. Since $v = Ri$, the current is zero independent of the voltage.

For example, the air between the ground and transmission line can be treated like a open circuit, since air is not a conductor.

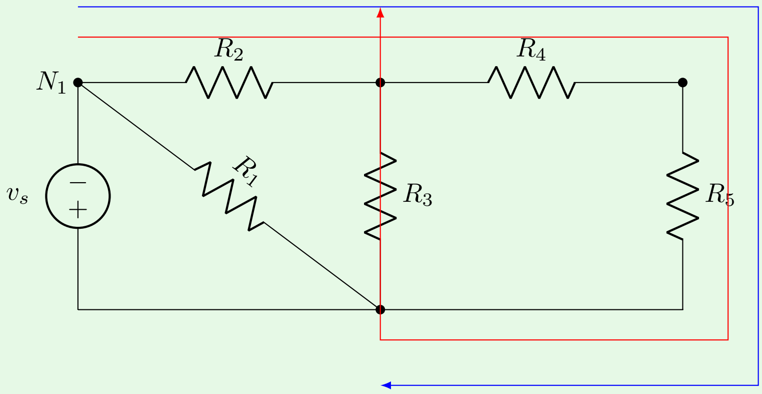
4 Circuit Analysis Definitions

Definition: A **Node** is a junction of two or more circuit elements.



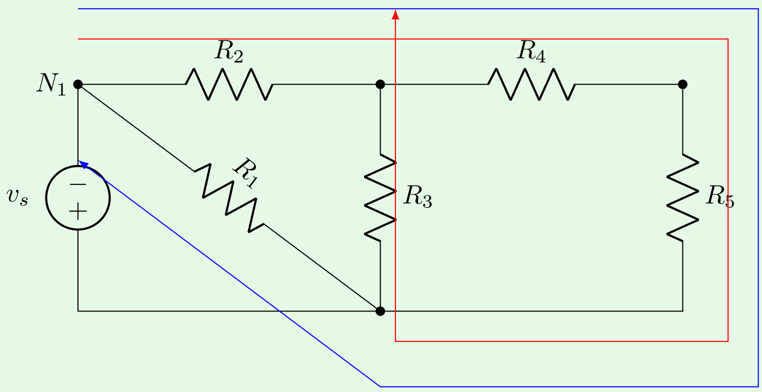
The labelled points are nodes: top-left connects v_s, R_1 and R_2; top-center connects R_2, R_3, and R_4; top-right connects R_4 and R_5; as well as the region at the bottom (that are connected by short circuit as the junction of v_s, R_1, R_3, R_5).

Definition: Start moving from one node towards the other nodes. As long as no node is passed (**unless** it is a loop when start and end nodes are the same) more than once, the set of nodes passed is called a **Path**.



The blue one is a path, the red one is not.

Definition: If the beginning and end of a path is one node, that path is a **loop**.



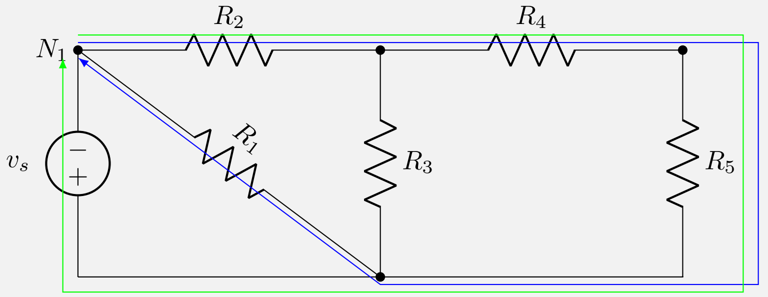
The blue one is a loop, the red one is not a loop.

4.1 Examples

Example 3 (1)

Identify the loops in the circuit above circuit

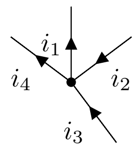
Solution 2 (1): Arrows are too hard to draw



5 Circuit Analysis Laws

5.1 Kirchoff's Current Law (KCL)

- KCL states that the algebraic sum of the currents entering a node is zero. This is based on the conservation of charge which will not be covered in this course.



- Define a sign convention for the algebraic sum. Either positive sign for current for entering the node or leaving the node is sufficient.
- For instance, define the sign convention as positive current entering the node. Then, KCL requires

$$i_1 - i_2 + i_3 - i_4 = 0 \tag{18}$$

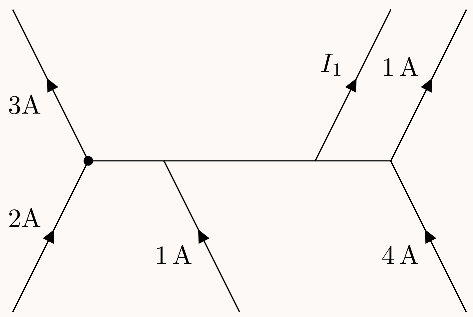
- An equivalent statement of KCL is: "The sum of the currents entering a node is equal to the sum of the currents leaving the node." This statement would require

$$i_2 + i_3 = i_1 + i_4 \tag{19}$$

  which is equivalent to (18).

Example 4 (2)

Find I_1 in the following circuit

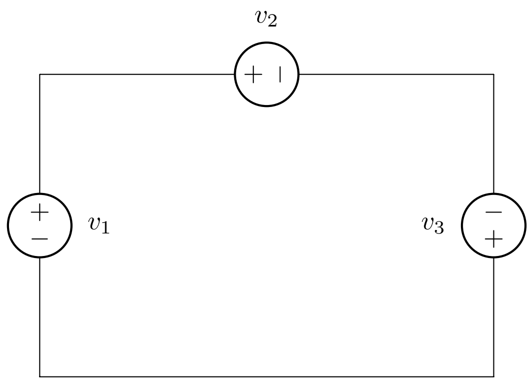


Solution 3 (2): All the currents are entering/leaving one node. KCL requires the currents entering and leaving the node to be equal. Therefore,

$$2 + 1 + 4 = 3 + I_1 + 1 \implies I_1 = 3\,\text{A} \tag{20}$$

5.2 Kirchoff's Voltage Law (KVL)

- KVL states that the algebraic sum of the voltages around any loop is zero.
  1. If the loop direction enters a voltage source from positive voltage polarity, voltage used for KVL is positive.
  2. Else, the voltage used for KVL is negative.



- For the same loop, the starting point and direction of performing the analysis is irrelevent. i.e. starting from the top left in the clockwise direction, KVL requires

$$+ v_2 - v_3 - v_1 = 0 \tag{21}$$

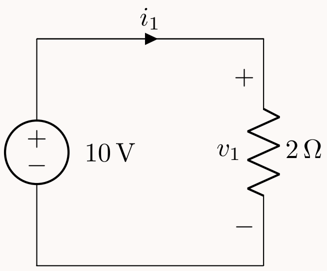
  Starting from the top right in the anti-clockwise direction,

$$- v_2 + v_1 + v_3 = 0 \tag{22}$$

- When writing KVL for circuits with resistors, it can be combined with Ohm's law in the following fashion. Note that this is independent of voltage polarity defined for the resistor.
  1. When the current direction entering a resistor is the same as the loop direction, the voltage of the resistor for KVL is $+iR$
  2. When the current direction entering a resistor is opposing the loop direction, the voltage of the resistor for KVL is $-iR$

Example 5 (1)

Use KVL and Ohm's law to find the current of the resistor.



Solution 4 (1): Applying KVL in the clockwise direction,

$$v_1 - 10 = 0 \implies v_1 = 10\,\mathrm{V} \tag{23}$$

Using Ohm's law. Since current enters the + polarity, PSC holds. Thus,

$$v_1 = 2i_1 \implies 10 = 2i_1 \implies i_1 = 5\,\mathrm{A} \tag{24}$$

Using the "shortcut", again applying KVL in a clockwise direction. Since the current direction is the same as the loop direction,

$$+ 2i_1 - 10 = 0 \implies i_1 = 5\,\mathrm{A} \tag{25}$$

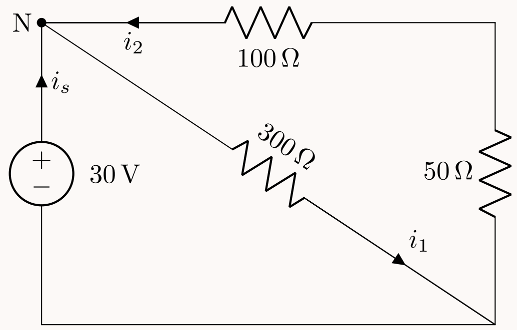
6 Circuit Analysis Tips

- If a circuit **does not** include a voltage-dependent current source or a current-dependent voltage source, you may use any of the following units systems to perform your analysis.
  1. V, A, Ω, W
  2. V, mA, kΩ, mW
  3. kV, mA, Ω, W
  4. Other consistent units can be used but are not as common.
- It is simpler to use a consistent system of units to analyze the circuits. Thus, units consistent with the voltage-dependent current source or a current-dependent voltage source should be used.

6.1 Examples

Example 6 (2)

Find the power of the voltage source.



Solution 5 (2): Since PSC does not hold for the voltage source,

$$P = -30i_s \tag{26}$$

KVL for the bottom left loop (clockwise)

$$-30 + 300i_1 = 0 \implies i_1 = 0.1\,\mathrm{A} \tag{27}$$

KVL for the outer loop (clockwise)

$$-30 - 100i_2 - 50i_2 = 0 \implies i_2 = -0.2\,\mathrm{A} \tag{28}$$

KCL at node N

$$i_s + i_2 = i_1 \implies i_s = i_1 - i_2 = 0.1 - (-0.2) = 0.3\,\mathrm{A} \tag{29}$$

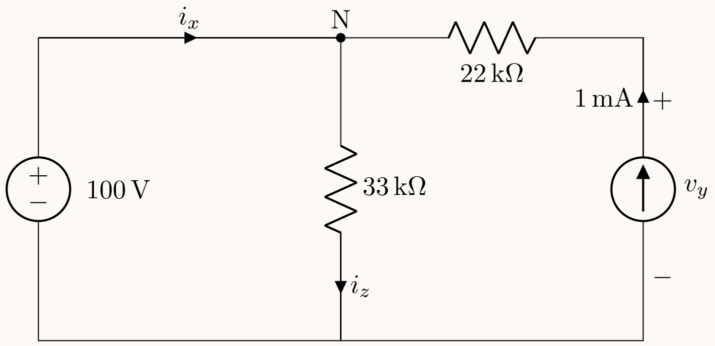
Find the power

$$P = -30i_s = -30(0.3) = -9\,\mathrm{W} \tag{30}$$

Since $P < 0$, power is generated.

Example 7 (3)

Find the power of each source.



Solution 6 (3): KVL for the left loop (clockwise)

$$-100 + 33i_z = 0 \implies i_z = 3\,\text{mA} \tag{31}$$

KCL for N, i_x and $1\,\text{mA}$ are entering N and $i_z = 3\,\text{mA}$ is leaving N.

$$i_x + 1 = i_z \implies i_x = 2\,\text{mA} \tag{32}$$

For the voltage source, PSC does not hold. Thus,

$$P = -vi = -100 \times 2 = -200\,\text{mW} \tag{33}$$

KVL for outer loop (clockwise)

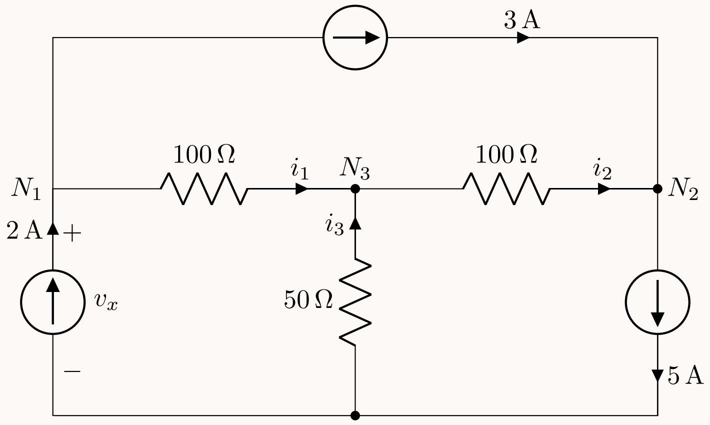
$$-100 - (1 \times 22) + v_y = 0 \implies v_y = 122\,\text{V} \tag{34}$$

For the current source, PSC does not hold. Thus,

$$P = -vi = -122 \times 1 = -122\,\text{mW} \tag{35}$$

Example 8 (4)

Find the power of each source.



Solution 7 (4): KCL at N_1

$$2 = 3 + i_1 \implies i_1 = -1\,\text{A} \tag{36}$$

KCL at N_2

$$3 + i_2 = 5 \implies i_2 = 2\,\text{A} \tag{37}$$

KCL at N_2

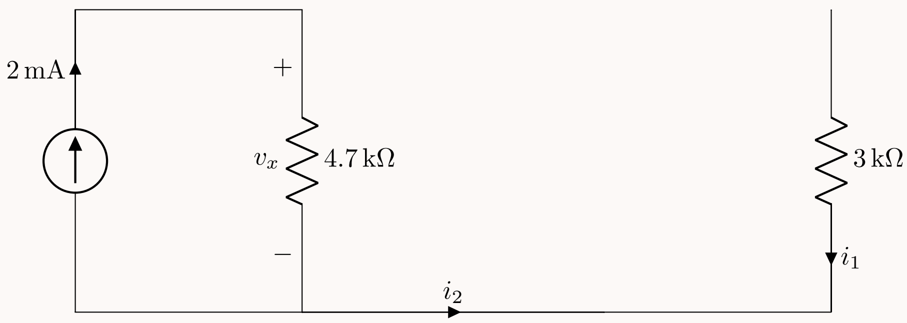
$$i_1 + i_3 = i_2 \implies i_3 = 3\,\text{A} \tag{38}$$

KVL for the bottom left loop (clockwise)

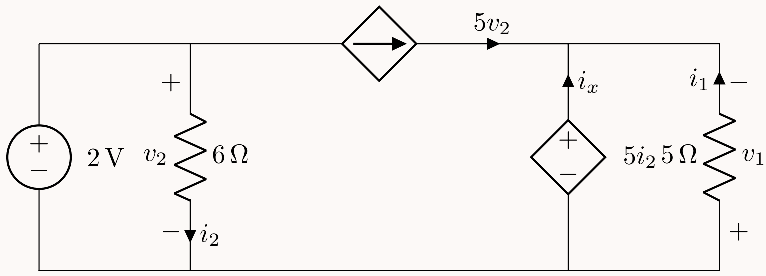
$$-v_x + 100i_1 - 50i_3 = 0 \implies v_x = 100 \times -1 - 50 \times 3 = -250\,\text{V} \tag{39}$$

The rest is left as an exercise.

Example 9 (5)



Example 10 (6)



Solution 8 (6): Use KVL on the left most loop (clockwise).

$$-2 + 6i_2 = 0 \implies i_2 = 0.33\,\mathrm{A} \tag{40}$$

Using Ohm's law on the resistor, PSC holds thus

$$v_2 = 6i_2 = 2\,\mathrm{V} \tag{41}$$

The current provided by the dependent current source would be 10 A and the voltage provided by the dependent voltage source would be 1.67 V.
Using KVL on the right most loop (anti-clockwise),

$$v_1 + 5i_2 = 0 \implies v_1 = -5i_2 = -1.67\,\mathrm{V} \tag{42}$$

Using Ohm's law on this resistor to find i_1. As PSC holds,

$$i_1 = v_1/5 = 0.33\,\mathrm{A} \tag{43}$$

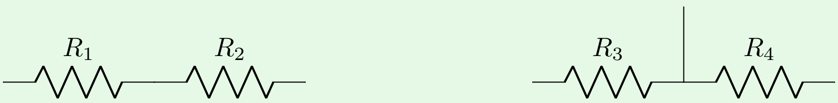
Finally using KCL at the junction,

$$i_x + i_1 + 5v_2 = 0 \implies i_x = -10.33\,\mathrm{A} \tag{44}$$

7 Equivalent circuits for parallel and series resistors

7.1 Series Connected Resistors

Definition: Series Connection: Two circuit elements are connect in series if and only if they are connected back to back, and at their point of connection, there is no other current path.



R_1 and R_2 are connected in series. R_3 and R_4 are also connected in series since the path between the two resistence is a open circuit, thus the current is zero so it is not a current path.

Theorem: For resistors in series, the equivalent resistence is

$$R_{eq} = \sum_k R_k \tag{45}$$

Proof: Take a circuit with 2 resistors in series.
Using KVL (clockwise),

$$-v_{tot} + v_1 + v_2 = 0 \tag{46}$$

Using Ohm's law,

$$v_1 = R_1 i_{tot} \tag{47}$$

$$v_2 = R_2 i_{tot} \tag{48}$$

Combining the results,

$$v_{tot} = v_1 + v_2 = (R_1 + R_2) i_{tot} \tag{49}$$

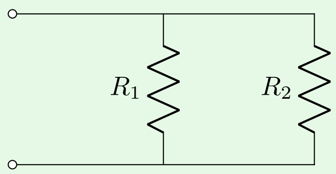
For an equvilent resistor from Ohm's law,

$$v_{tot} = R_{eq} i_{tot} \implies R_{eq} = R_1 + R_2 \tag{50}$$

From induction, the equivalent resistence of any series connected resistors is equal to the sum of their resistences.

7.2 Parellel Connected Resistors

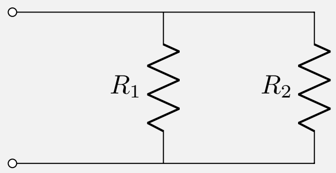
Definition: Parallel connection: Two circuit elements are connected in parallel if they share two common nodes.



Theorem: For parellel resistors, the equivalent resistence is

$$R_{eq} = \left[\sum_k \frac{1}{R_k}\right]^{-1} \tag{51}$$

Proof:



Write KCL at N

$$i_{tot} = i_1 + i_2 \tag{52}$$

Using KVL

$$-v_{tot} + v_1 = 0 \implies v_{tot} = v_1 = v_2 \tag{53}$$

Using Ohm's law, PSC holds

$$i_{tot} = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_{tot} \tag{54}$$

Using ohm's law for the equivalent resistor,

$$i_{tot} = \frac{v_{tot}}{R_{eq}} \implies \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \implies R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \tag{55}$$

Consider two resistors in parellel with $R_2 = 0$. Thus,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{0}{R_1} = 0 \tag{56}$$

You can consider that the current is "sane". When given an option to flow with no resistence, it will take that path. Thus, if $R_2 = 0$, no current will flow through R_1 and the equivalent resistence is 0

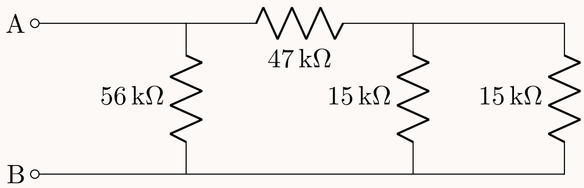
Corollary: Recall the definition of conductance is $G = 1/R$. Thus,

$$G_{eq} = \sum_k G_k \tag{57}$$

7.3 Examples

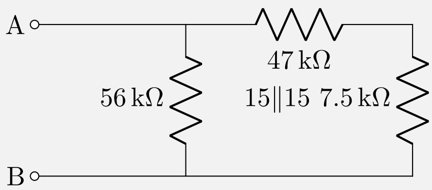
Example 11 (1)

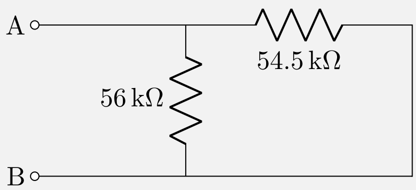
Find the equivalent resistence between A and B



Solution 9 (1): Simplify the circuit in several steps (starting from the right)

1. The two right most resistors are correct in parellel.



2. 

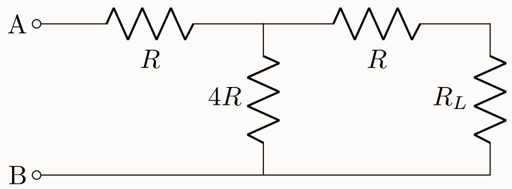
3.

$$R_{eq} = 56||54.5 = \frac{56 \times 54.5}{56 + 54.5} = 27.62\,\mathrm{k\Omega} \tag{58}$$

Note that in the original circuit, the 15 kΩ resistors are **not** connected in series to the 47 kΩ resistor as there exist (one) current path between them.

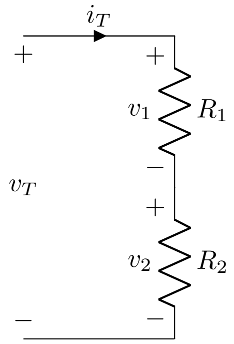
Example 12 (2)

Select R such that $R_{AB} = R_L$



8 Voltage Division

Suppose there are two resistor connected in series. With voltage through both as v_T and i_T.



Then the voltage division principle states

$$v_1 = \frac{R_1}{R_1 + R_2} v_T \tag{59}$$

In general for N series connected resistors,

$$v_i = \frac{R_i}{\sum_k R_k} v_T \tag{60}$$

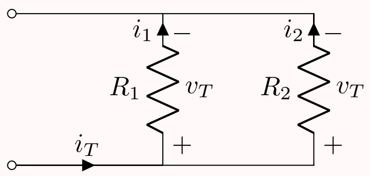
Note the polarity of the voltage v_i **must** be the same as v_T, otherwise the $-$ sign must be introduced.

As a consequence, the resistor with the higher resistence will have the higher voltage.

Proof: The equivalent resistence of the two resistors is $R_T = R_1 + R_2$. Using Ohm's law, $v_T = i_T(R_1 + R_2)$.

9 Current Division

Derivation: Suppose there are two resistors connected in parellel.



Use Ohm's Law

$$v_T = R_1 i_1 \tag{61}$$

$$v_T = \frac{R_1 R_2}{R_1 + R_2} \tag{62}$$

Since the LHS is the same, the RHS must be the same as well. Thus,

$$i_1 = \frac{R_2}{R_1 + R_2} i_T \tag{63}$$

Equation (63) is known as the current division principle. This can also be expressed in terms of conductances

$$i_1 = \frac{G_1}{G_1 + G_2} i_T \tag{64}$$

For the general case of N parellel resistors, it is possible to use the equation with the conductances similar to the voltage division principle.

$$i_i = \frac{G_i}{\sum_k G_k} i_T \tag{65}$$

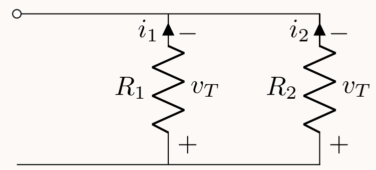
It is possible to write the other resistors as an equivalent resistor with resistence R'_{eq}, then the current at the resistor of interest R is

$$i = \frac{R'_{eq}}{R + R'_{eq}} i_T \tag{66}$$

The directions of i_T and i_1 **must** be similar. Otherwise, the $-$ sign must be introduced.

Example 13 ()

Find v_0 and i_0



10 Nodal Analysis

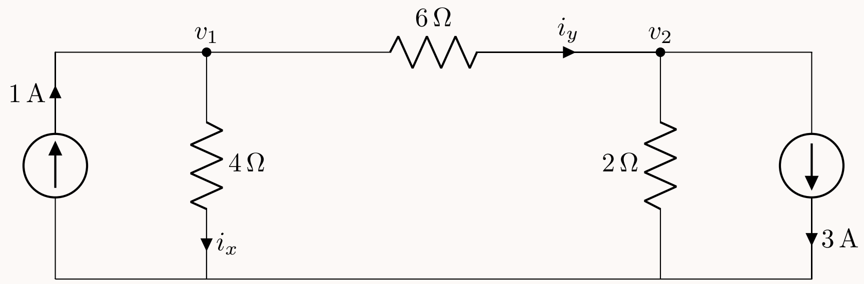
- An algorithmic approcah to solve circuits.
- Objective: Find node voltages.
- Methodology: KCL for all the nod in terms of the **node voltage**: The voltage with respect to a reference node or ground node.
- The common convention is to assume a negative sign for current entering node and positive sign for current leaving node. It is also customary to write the current leaving the node at a resistor.

Definition:

1. V_{AB} is the voltage with the $+$ polarity at point A and $-$ polarity at point B.
2. V_A is the voltage with the $+$ polarity at point A and $-$ at the reference node or ground.
3. The voltage of the ground node is equal to zero.

Example 14 (1)

Perform nodal analysis on the following circuit.



Solution 10 (1): Using ohm's law, $i_x = v_1/4$, $i_y = (v_1 - v_2)/6$. (labelled for analysis of node 1)
Execute KCL for node 1

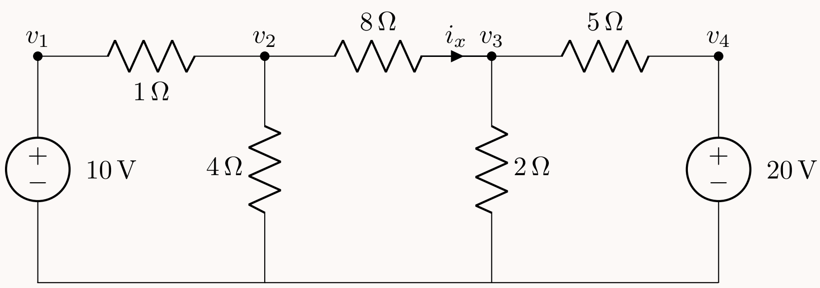
$$-1 + \frac{v_1}{4} + \frac{v_1 - v_2}{6} = 0 \tag{67}$$

Execute KCL for node 2. It is recommended to start fresh for each node.

$$+3 + \frac{v_2}{2} + \frac{v_2 - v_1}{6} = 0 \tag{68}$$

Now there are two equations for two unknowns. This can be solved using computer to obtain $v_1 = 2/3\,\mathrm{V}, v_2 = -13/2\,\mathrm{V}$. Note that v_1 represents the voltage measured by using a voltmeter with the positive terminal of node 1 and negative terminal at the ground.
The power of nodal analysis is that now it is possible to easily find the voltage between any two points in the circuit. For example, the voltage of the 3 A is just $v_2 - 0 = -13/2\,\mathrm{A}$

Example 15 (2)



Solution 11 (): In this circuit, there are voltage sources! Hence, we do not know the current of the at certain nodes thus writing KCL writing there would not be productive. However, the voltage at nodes 1 and 4 are actually known!

$$v_1 = +10\,\mathrm{V} \tag{69}$$
$$v_2 = +20\,\mathrm{V} \tag{70}$$

Note to pay attention to the direction the voltage sources are oriented.
KCL for node 2

$$\frac{v_2 - v_1}{1} + \frac{v_2 - 0}{4} + \frac{v_2 - v_3}{8} = 0 \tag{71}$$

KCL at node 3

$$\frac{v_3 - v_4}{5} + \frac{v_3 + 0}{2} + \frac{v_3 - v_2}{8} = 0 \tag{72}$$

It may seem like there are too many unknowns, but v_1 and v_2 are already known. Hence, only $v_2 = 7.82\,\mathrm{V}, v_3 = 6.03\,\mathrm{V}$ needs to be found.
To find i_x, use the ohm's law

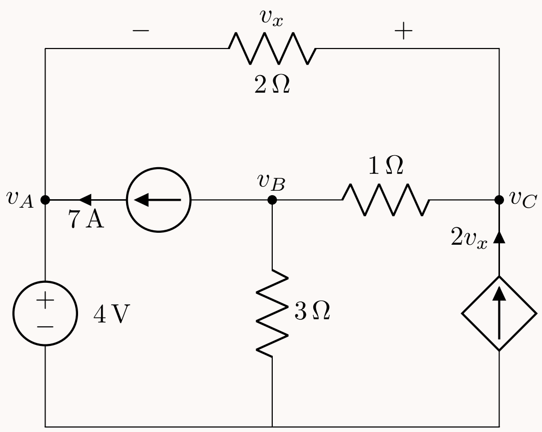
$$i_x = \frac{v_2 - v_3}{8} = 0.223\,\mathrm{A} \tag{73}$$

10.1 Circuits with Dependent Sources

- Before writing the KCL for dependent source, first write the parameters it depends on in terms of the *node voltages*.

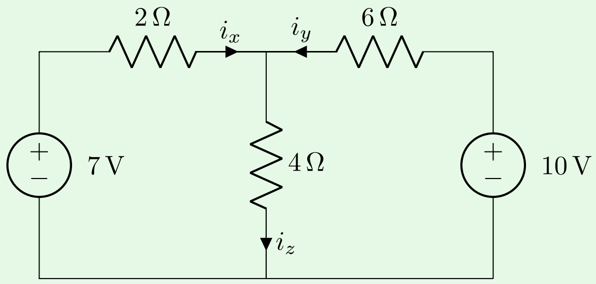
Example 16 ()

Perform nodal analysis on the circuit.



11 Mesh Analysis

Definition: A **Mesh** is a loop that does not include any other loops inside it.



1. A **mesh current** is a hypothetical circulating current about eash mesh. Note this is **not** a real current, just a variable for circuit analysis.
2. A **branch current** is a physical current that flows through a branch.

- Branch currents can be expressed in terms of mesh currents. Note that if the mesh current is in the same direction as the branch current, it can be added directly otherwise a negative sign must be included.
- The $2\,\Omega$ resistor only belongs to the mesh with mesh current i_1. Thus, $i_x = i_1$
- The $6\,\Omega$ resistor only belongs to the mesh with mesh current i_2. However, the i_y is in the opposite direction as the mesh current; thus, $i_y = -i_2$
- The $4\,\Omega$ resistor belongs to both meshes. i_z is in the same direction as the mesh current i_1 but opposite direction of the mesh current i_2; thus, $i_z = i_1 - i_2$.

- Mesh analysis is a different circuit analysis technique that is generally more efficient for circuits with lots of series connections.
- The objective for mesh analysis is to find all the mesh currents.
- Methodology for mesh analysis:
  1. Identify all the meshes in the circuit.
  2. Associate a mesh current with each mesh. It is conventional to choose a clockwise direction for all the mesh current.
  3. If there are dependent sources, identify the parameters they depend on based on the mesh currents.
  4. Identify current sources and the meshes they effect.
     (a) If at the periphery (part of only one mesh), the current of that mesh is determined.

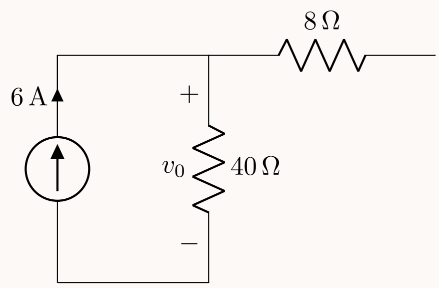
(b) If not at the periphery, the current sources should provide 1 equation. The other equation will come from KVL of a larger loop (*supermesh*) including to meshes the current source effects.

5. Write KVL for all the meshes in terms of the mesh current. It is conventional to write KVL in the clockwise direction.

- For mesh 1:
$$-7 + 2i_1 + 4(i_1 - i_2) = 0 \tag{74}$$
For mesh 2:
$$4(i_2 - i_1) + 6i_2 + 10 = 0 \tag{75}$$
Solving the equations, $i_1 = 5/22\,\mathrm{A}, i_2 = 29/22\,\mathrm{A}$. The branch currents can be found.

- For meshes with current sources, KVL can no longer be written for those meshes. However, knowing that the current at the current source is constant, they provide a constraint for the mesh currents; hence, the circuit is still determined.

Example 17 ()

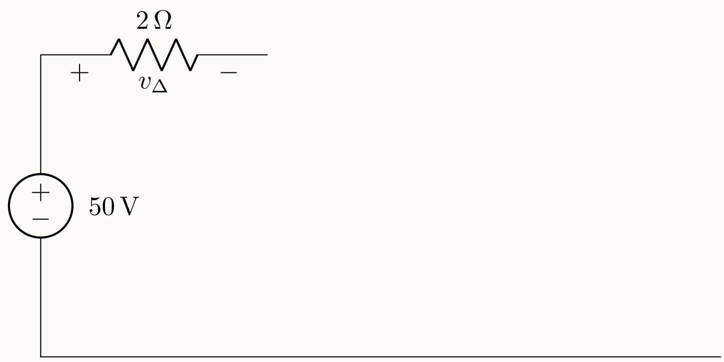


11.1 Circuits with Dependent Sources

- For analysis with dependent sources, the parameters the dependent sources depend on must be written first in terms of mesh current.

Example 18 ()

Use mesh analysis to find which sources are generating power.



Solution 12 (): First, write parameter the dependent sources depend on in terms of mesh currents.
$$v_\Delta = +2i_1 \tag{76}$$
$$i_\Delta = +i_2 \tag{77}$$
A dependent current source is on the periphery of the circuit. Thus, it is unhelpful to write KVL for that mesh as the current of that mesh is already known, as
$$i_3 = -1.7(2i_1) = 3.4i_1 \tag{78}$$

Now, KVL can be written for mesh 1 and mesh 2 respectively

$$0 = -50 + 2i_1 + 4(i_1 - i_2) + 9(i_2) \tag{79}$$
$$0 = -9(i_2) + 4(i_2 - i_1) + 5i_2 + 20(i_2 - i_3) \tag{80}$$

Solving the equation to obtain $i[\,] = \{-5, 16, 17\}$A. Then, $v_\Delta = -10\,\mathrm{V}, i_\Delta = 16\,\mathrm{A}$ For the 50 V independent source,

$$P = -50i_1 = -50(-5) = 250\,\mathrm{W} \tag{81}$$

For the $9i_\Delta$ dependent source,

$$P = -(i_2 - i_1)(9i_\Delta) = -3024\,\mathrm{W} \tag{82}$$

For the $1.7v_\Delta$ current source, firstly find the KVL for mesh 3 to find the voltage across the current source.

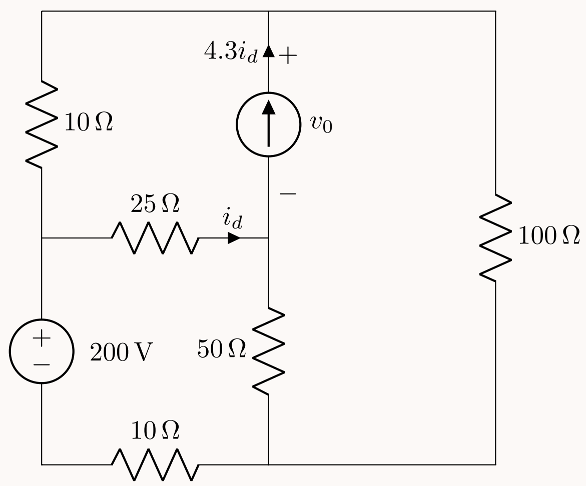
$$+20(i_3 - i_2) + v_y = 0 \implies v_y = -20\,\mathrm{V} \tag{83}$$

$$P = -v_y(1.7v_\Delta) = -340\,\mathrm{W} \tag{84}$$

Thus, only the dependent voltage source and dependent current sources are generating power.

Example 19 ()

Use mesh analysis to find v_0.



Solution 13 (): Firstly find $i_d = i_1 - i_2$ as it is a parameter for a dependent source. Then write KVL for mesh 1.

$$0 = 10i_1 - 200 + 25i_1 + 50i_1 \tag{85}$$

This current source is not at the periphery of the circuit as it is part of more than one mesh. For the current source,

$$4.3i_d = 4.3(i_1 - i_2) = i_3 - i_2 \tag{86}$$

Write KVL for the "supermesh".

$$0 = +50(i_3 - i_1) + 25(i_2 - i_1) + 10i_2 + 100i_3 \tag{87}$$

$i[\,] = \{4.6, 5.7, 0.97\}$A. Mesh analysis is complete. Finding v_0 is trivial by writing KVL for mesh 2.

$$0 = v_0 + 25(i_2 - i_1) + 10i_2 = 0 \implies v_0 = -84.5\,\mathrm{V} \tag{88}$$

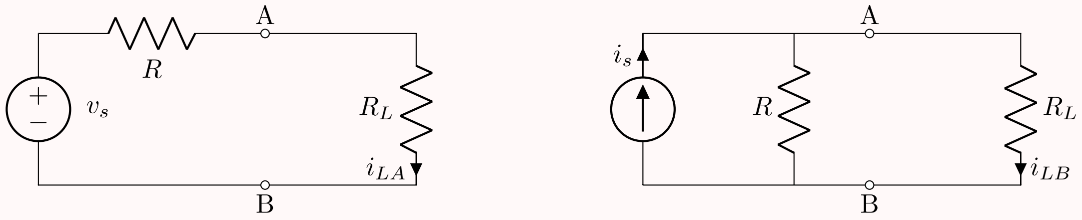
12 Source Transformation

- A voltage source in series with a resistor is equivalent to a current source in parellel with the same resistor. One can be transformed to the other using source transformation with magnitude

$$v_s = Ri_s \tag{89}$$

- The direction of i_s and v_s do **not** conform to PSC.

Derivation: Consider the following two circuits.



Using Ohm's law on for the circuit on the left and current division for the circuit on the right,

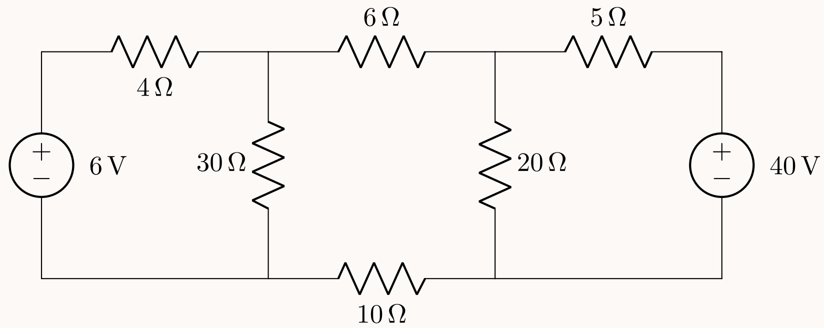
$$i_{LA} = \frac{v_s}{R + R_L} \tag{90}$$

$$i_{LB} = i_s \frac{R}{R + R_L} \tag{91}$$

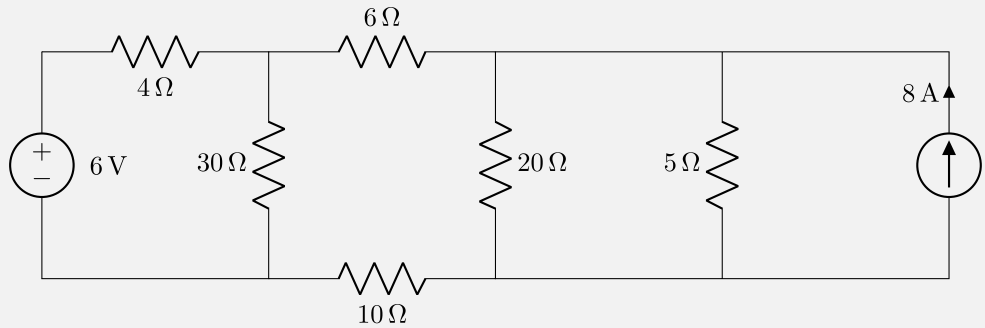
For the two sources to be equivalent, $i_{LA} = i_{LB}$. Thus, $v_s = Ri_s$. Note that v_s and i_s does not conform to PSC, as noted above. As R_L cancels out in the expression, the two circuits are equivalent regardless of what is connected to terminals A and B.

Example 20 (1)

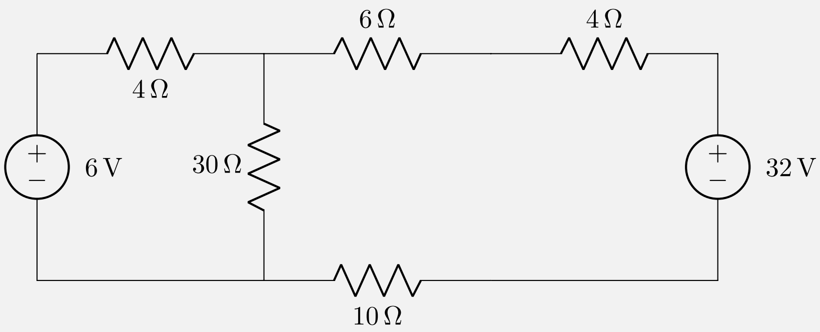
Use the Source Transformation to find the power of the 6 V voltage source.



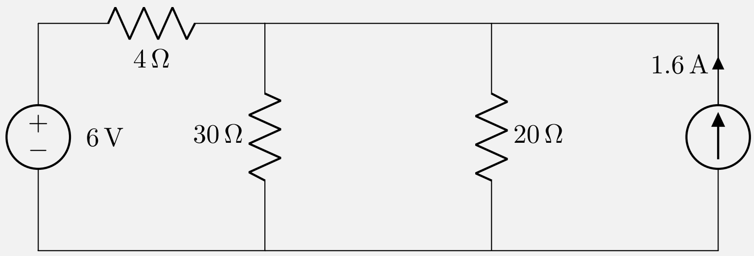
Solution 14 (1): First, simplify the circuit that is connected to the 6 V voltage source. Firstly, simplify the series connection of 40 V voltage source and 5 Ω resistor with a parellel connection of a current source and resistor.



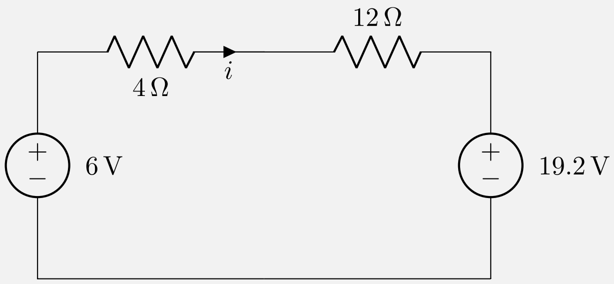
Now, the 5 Ω resistor is connected in parellel to 20 Ω resistor. The equivalent resistence is 4 Ω. Then, the current source connected parellel to the 4 Ω resistor can be transformed by a voltage source connected series to the 4 Ω resistor.



Now the $6\,\Omega$ resistor, $4\,\Omega$ resistor, $32\,\mathrm{V}$ voltage source, $10\,\Omega$ resistor are connected in series. Because the order of the circuit elements connected in series are not relevent to the terminals, the three resistors can be combined to a equivalent resistor of $20\,\Omega$. Now the source transformation can be applied



The $30\,\Omega$ and $20\,\Omega$ resistors are connected in parellel. Using a source transformation the circuit can be simplified to a simple circuit.



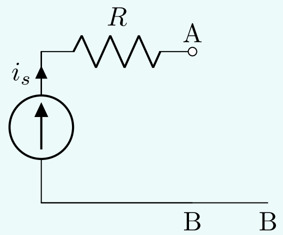
Now writing KVL clockwise for the circuit,

$$-6 + 4i + 12i + 19.2 = 0 \implies i = -0.825\,\mathrm{A} \tag{92}$$

Since the current direction does not conform to PSC for the $6\,\mathrm{V}$ source,

$$P = -vi = 4.95\,\mathrm{W} \tag{93}$$

Corollary: For resistors connected in parellel with a voltage source or in series with a current source can be eliminated from the circuit. This is done by opening the resistor for parellel connection and shorting the resistor for series connection.



13 Thevenin and Norton Equivilant Circuit

Just as an interconnection of resistors can be replaced by a equivalent resistence, an interconnection of resistors and sources can be replaced by an equivalent circuit

13.1 Thevenin equivalent circuit

Theorem: For any linear electrical network containing only voltage sources, current sources and resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}.

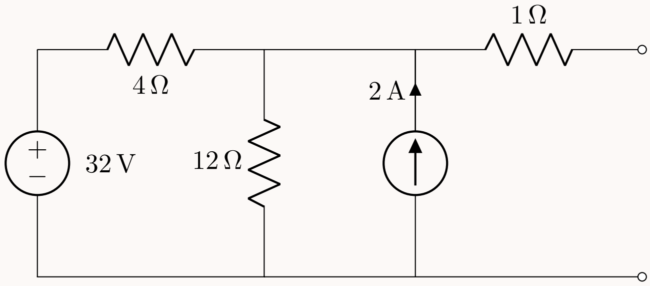
- The Thevenin voltage V_{th} is the open circuit voltage between A and B in the original circuit.
- There are three different methods of finding the Thevenin resistence R_{th}.

13.1.1 Method 1: For circuits without dependent sources

1. Deactivate all the independent sources in the circuit.
   - Deactivate a voltage source by shorting it.
   - Deactivate a current source by opening it.
2. Find the equivalent resistance, which would be the Thevenin resistance.

Example 21 ()

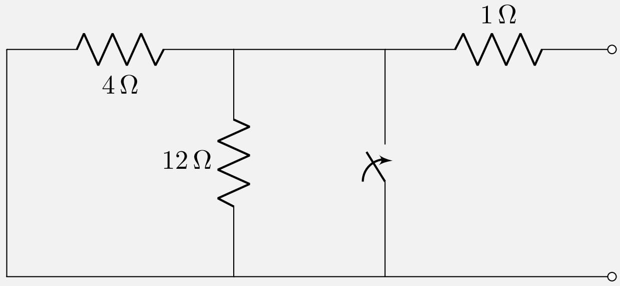
Find the thevenin equivalent of the circuit connected to R_L



Solution 15 (): Firstly find the open circuit voltage using nodal analysis.

$$\frac{v_1 - 32}{4} \tag{94}$$

To find R_{th}, deactivate all the sources



Now the $4\,\Omega$ resistor and $12\,\Omega$ resistor are connected in parellel. Thus, the equivalent resistence in the circuit is $4\,\Omega$, which is the Thevenin resistance.

13.2 Norton Equivalent Circuit

- Norton equivalent circuit is the dual of the Thevenin equivalent circuit with a resistor of resistance R_N and current source with current I_N connected in parellel.

- To find the equivalent resistance, $R_N = R_{th}$
- There are two methods to find the Norton current I_N
  1. I_N is equal to the short circuit current between the two terminals. The direction of the Norton current must be consistent with the direction of i_{sc}.
  2. Perform source transformation on Thevenin equivalent circuit; thus $I_N = V_{th}/R_{th}$ with the direction of the current source not conforming to PSC of the voltage source (in Thevenin equivilant circuit).
- Note that you **cannot** use the same circuit analysis for short circuit current and open circuit voltage.

Example 22 ()

Find the Norton equivilant circuit for previous example

Solution 16 (): KVL @1

$$\frac{v_1 - 32}{4} + \frac{v_1}{12} - 2 + \frac{v_1}{1} = 0 \implies v_1 = 7.5\,\mathrm{V} \tag{95}$$

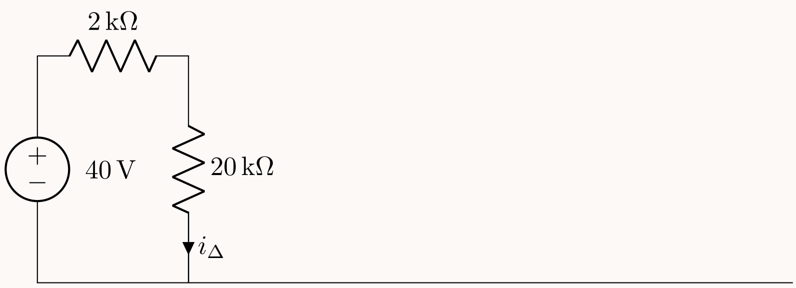
The short circuit current is the same as the current in the $1\,\Omega$ resistor. Thus, $i_{sc} = v_1/1 = 7.5\,\mathrm{A}$.

13.2.1 Method 2: For circuits that includes at least one independent source

$$R_{th} = \frac{v_{oc}}{i_{sc}} \tag{96}$$

Where v_{oc} and i_{sc} **must** conform to PSC.

Example 23 ()



13.3 Method 3: Most general method

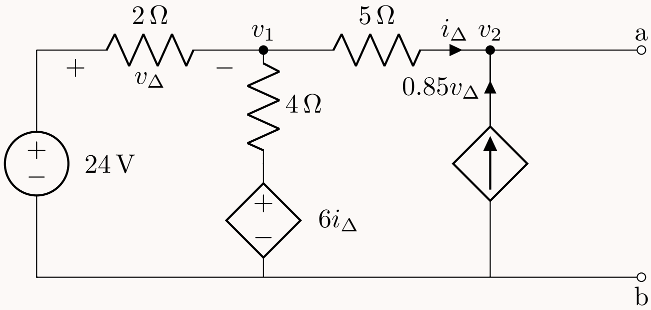
- If there are no independent source in the circuit, the circuit is not energized; thus, $v_{oc} = 0, i_{sc} = 0$. This calls for a new method.
  1. Deactivate all independent sources.
  2. Connect a test current source between the terminals you want to find the Thevenin resistance, terminals A and B. For simplicity, common practice is using the test current source of 1 A.
  3. Find the voltage across the test current source using any circuit analysis technique.

  $$R_{th} = \frac{V_T}{I_T} \tag{97}$$

  Note that PSC must **not** hold for V_T and I_T.
- Note that the test source is only added to the circuit for evaluating R_{th} and must be disconnected after the calculation.
- Note that some texts add a test voltage source instead of a test current source. This method also works but you may have to do the long division.

Example 24 ()

Find the Thevenin equivilent of the circuit connected to R_L



Solution 17 (): Using nodal analysis, first write the dependent sources as a function of the node voltages.

$$v_\Delta = 24 - v_1 \tag{98}$$

$$i_\Delta = \frac{v_1 - v_2}{5} \tag{99}$$

$$\tag{100}$$

KCL at node 1:

$$\frac{v_1 - 24}{2} + \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{5} = 0 \tag{101}$$

KCL at node 2:

$$\frac{v_2 - v_1}{5} - 0.85(24 - v_1) + 0 = 0 \tag{102}$$

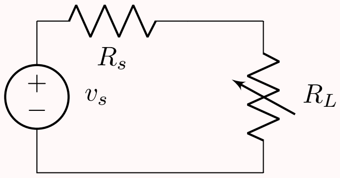
Node 3:

$$v_3 = 6i_\Delta = 6\left(\frac{v_1 - v_2}{5}\right) \tag{103}$$

$v_1 = 5.538\,\text{V}, v_2 = 84\,\text{V}, v_3 = -94.154\,\text{V} \therefore v_{th} = v_2 = 84\,\text{V}$. To find the thevenin resistence, deactivate the independent source

13.4 Maximum Power Transfer

Derivation:



Consider this circuit. Choose R_L such that maximum power is transfered. Note the power relation for the resistor.

$$P_L = R_L i_L^2 \tag{104}$$

$$v_s = (R_s + R_L) i_L \tag{105}$$

$$P_L = R_L \frac{v_s^2}{(R_s + R_L)^2} \tag{106}$$

To maximize, find the partial derivative with respect to R_L

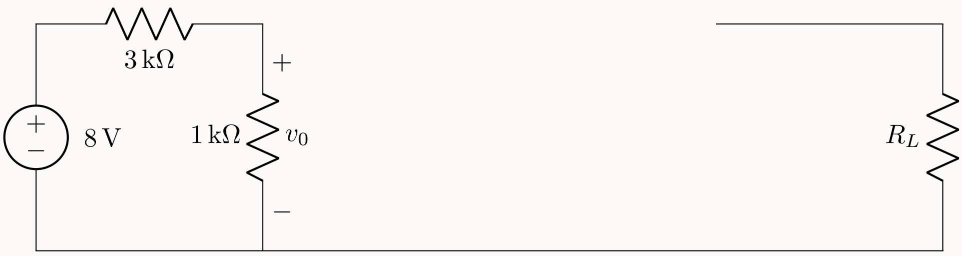
$$\frac{\partial P_L}{\partial R_L} = v_s^2 \frac{R_S - R_L}{(R_S + R_L)^3} = 0 \tag{107}$$

To maximize power transfer, $R_S = R_L$. Thus, the maximum power transfer is

$$P_L = \frac{v_s^2}{4R_L} \tag{108}$$

Example 25 ()

Find R_L that results in maximum power transfer.



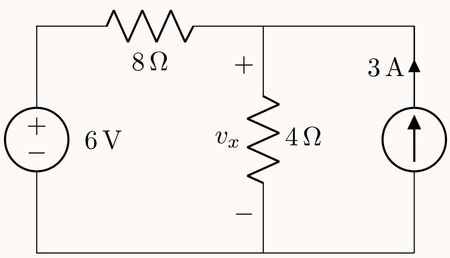
14 Superposition Principle

Definition: A linear circuit that consists of independent sources, linear dependent sources, and linear elements.

Theorem: Superposition Principle: The response of a linear circuit to multiple independent sources is equal to the algebraic sum of the responses caused by each independent source acting alone.

Example 26 ()

Consider the following circuit and find v_x



Solution 18 (): First deactivate the current source. Then, using voltage division principle

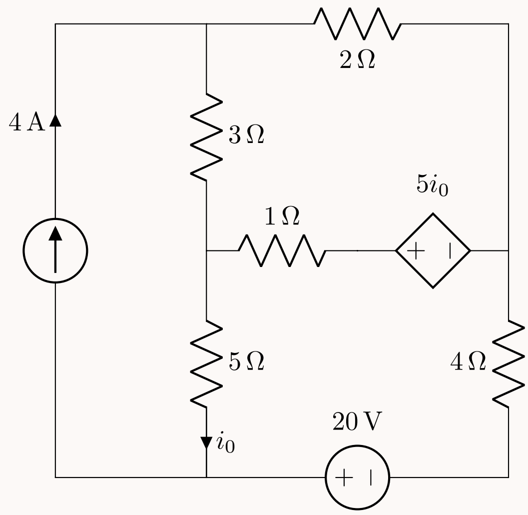
$$v_{x1} = 6\frac{4}{4+8} = 2\,\mathrm{V} \tag{109}$$

Then, deactivate the voltage source. Then, using current division principle

$$i_{x1} = 3\frac{8}{4+8} = 2\,\mathrm{A} \tag{110}$$

From ohm's law, $v_{x2} = iR = 4 \times 2 = 8\,\mathrm{V}$. Then, the total voltage would be $8 + 2 = 10\,\mathrm{V}$. This result can be confirmed using nodal analysis.

Example 27 ()



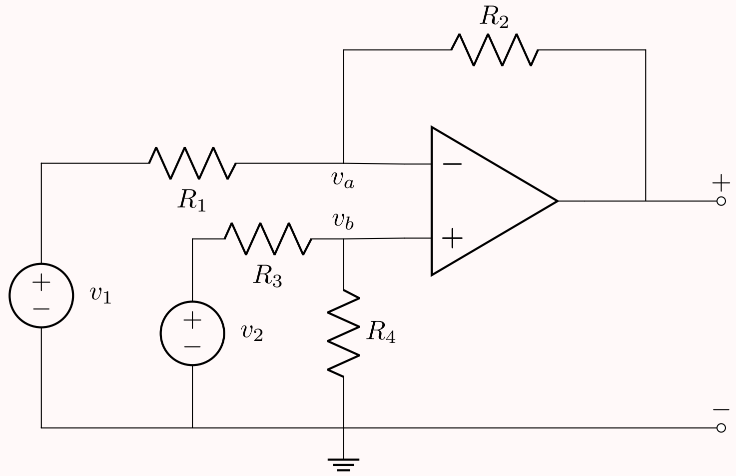
- **Note: Never** deactivate the dependent sources when using superposition principle.

Solution 19 (): Phase I: Deactivate the voltage source and perform mesh analysis.

15 Operational Amplifier: Op-Amp

15.1 Difference Amplifier

Derivation:



Use KCL at node A.

$$\frac{v_a - v_1}{R_1} + \frac{v_a - v_0}{R_2} = 0 \tag{111}$$

Use KCL at node B.

$$\frac{v_b}{R_4} + \frac{v_b - v_2}{R_3} = 0 \tag{112}$$

Solving for v_0 using computer algebra software yields

$$v_0 = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}\, v_2 - \frac{R_2}{R_1}\, v_1 \tag{113}$$

16 Capacitors

Definition: A capacitor is a circuit element that consists of two conducting plates that are seperated by an insulator. It stores energy in the elecric field.

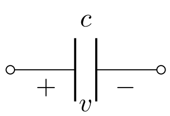
- When connected to a voltage source, positive and negative charges ($\pm q$) are deposited on the conductive plates.

$$q = cv \tag{114}$$

- Note q and v can be time dependent, but C must be constant. This is known as the capacitance.
- The S.I. unit for capacitance is columb per volt, called a "Farad" with the symbol F. Practically, 1 F is a very large capacitance and not often seen in normal circuits. In practice, the capacitances are usually expressed in µF,mF,nF, or pF.
- Differentiating both sides of (114),

$$i = c\dot{v} \tag{115}$$

- In a circuit, a capacitor is drawn with this symbol



- **Note** that (114) and (115) holds when PSC is held. Otherwise, an minus sign must be introduced to these equations.
- If the capacitance and the current is known, the voltage betweeen two different times can be found with

$$v(t_2) - v(t_1) = \frac{1}{C}\int_{t_1}^{t_2} i(t)\mathrm{d}t \tag{116}$$

- This equation is often written with $t_2 = t, t_1 = 0$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)\mathrm{d}\tau \tag{117}$$

- From (115), we note the following:
  1. If the voltage is constant (DC), then it doesn't change with respect to time hence $\dot{v} = 0$. Then, the current becomes zero and the capacitor behaves like an open circuit.
  2. The voltage must be a differentiable function of time. Hence at undifferentiable points, the current is undefined.

16.1 Energy of a capacitor

Derivation: Since the current is known in terms of the voltage,

$$P = vi = Cv\dot{v} \tag{118}$$

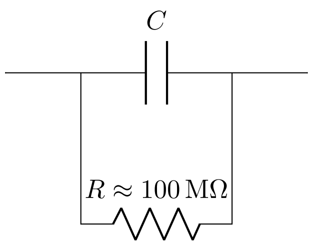
It is also known that $P = \dot{w}$. Integrating both sides from t_1 to t_2,

$$w(t_2) - w(t_1) = C\int_{t_1}^{t_2} v\mathrm{d}v = \frac{1}{2}C(v(t_2)^2 - v(t_1)^2) \tag{119}$$

Again, using $t_2 = t, t_1 = 0, w(t_1) = 0$, then

$$w(t) = \frac{1}{2}cv(t)^2 \tag{120}$$

- An ideal capacitor does not dissipate energy. It only stores energy then delivers the energy.
- A real capacitor has some leakage resistance. This is modelled by connecting the leakage resistance in parellel with an ideal capacitor.



Example 28 ()

Determine the voltage across a $2\,\mathrm{\mu F}$ capacitor if the current through it is $i(t) = 6\exp(-3000t)\mathrm{mA}$, and $v(0) = 0$

Solution 20 ():

$$v(t) = v(0) + \frac{1}{2} \tag{121}$$

16.2 Equivalent Capacitances

Derivation: For parellel connected capacitors,
For series connected capacitors, use KVL to find:

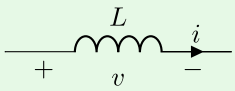
$$v = v_1 + v_2 + \cdots + v_n \tag{122}$$

Then,

$$v(t) = \sum v_i(0) + (\frac{1}{C_i}) \int_0^t i(\tau)\mathrm{d}t \tag{123}$$

17 Inductors

Definition: An inductor consists of a coil of conducting wire. The core of this coil could be air, iron.... Inductors store energy in its magnetic field with the given properties: (If v and i conform to PSC)



$$v = L\dot{i} \tag{124}$$

- L is called the inductance. The unit for inductance is Henry (H).
- From (124),
  1. For DC circuits, the inductor voltage is zero, and an inductor behaves like a short-circuit.
  2. The current of an inductor must be differentiable with respect to time.
- If the inductance and the voltage (as a function of time) is known, then the difference in current at two different times is

$$i(t_2) - i(t_1) = \frac{1}{L} \int_{t_1}^{t_2} v(t)\mathrm{d}t \tag{125}$$

17.1 Energy of an inductor

Derivation:

$$P = vi = Li\dot{i} \tag{126}$$
$$P = \dot{w} \tag{127}$$
$$\tag{128}$$

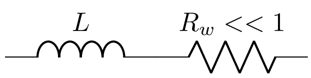
Integrating both sides yield

$$w(t_2) - w(t_1) = \frac{1}{2}L(i(t_2)^2 + i(t_1)^2) \tag{129}$$

If the energy is initially uncharged, $w(0) = 0$, the energy stored in an inductor at time t is

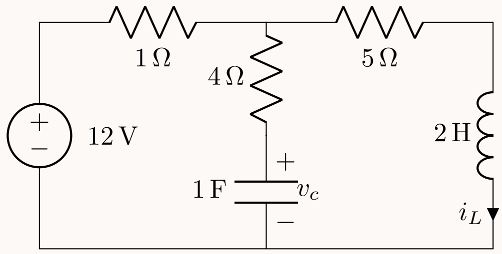
$$w(t) = \frac{1}{2}Li(t)^2 \tag{130}$$

- An ideeal inductor does not dissipate energy like an ideal capacitor. It only stores and delivers energy.
- A real inductor includes a series resistor:



Example 29 ()

Find the energy stored in the capacitor and inductor at steady state DC.



Solution 21 (): At steady state, open the capacitor and short the inductor. Write KVL on the outer loop.

$$6i_L = 12 \implies i_L = 2\,\text{A} \tag{131}$$

The energy stored in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2} \times 2 \times 2^2 = 4\,\text{J} \tag{132}$$

Write KVL for the right loop,

$$v_c = 5i_L = 10\,\text{V} \tag{133}$$

The energy stored in the capacitor is

$$w_C = \frac{1}{2}Cv^2 = \frac{1}{2} \times 1 \times 10^2 = 50\,\text{J} \tag{134}$$

17.2 Equivalent Inductances

Derivation: In series,

$$L_{eq} = L_1 + L_2 + \cdots + L_n \tag{135}$$

In parellel,

$$L_{eq} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \tag{136}$$

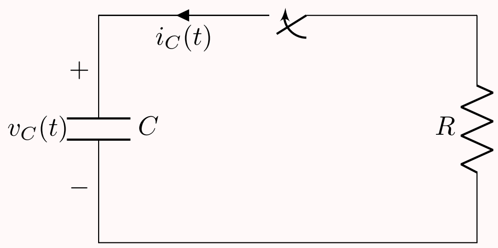
18 First order circuits

Definition: A first order circuit is a circuit where the voltage and current relations is can be described by a system of first order differential equations.

- The passive elements of these circuits are either R & C or R & L.

18.1 Source-free RC Circuits

Derivation: The capacitor has been connected to an arbiturary external circuit before time $t = 0$ such that it gains a voltage $v_C(0^-) = V_0$. At time $t = 0$, the capacitor is disconnected to the external circuit and the switch is closed.



We can write KVL for this loop (counter clockwise)

$$v_C(t) + i_C(t)R = 0 \tag{137}$$

For a capacitor, $i_C = C\dot{v}_C$. Substituting into the previous relation,

$$v_C + RC\dot{v}_C = 0 \tag{138}$$

R and C are time-independent, thus, this is a simple seperable differential equation and the general solution is

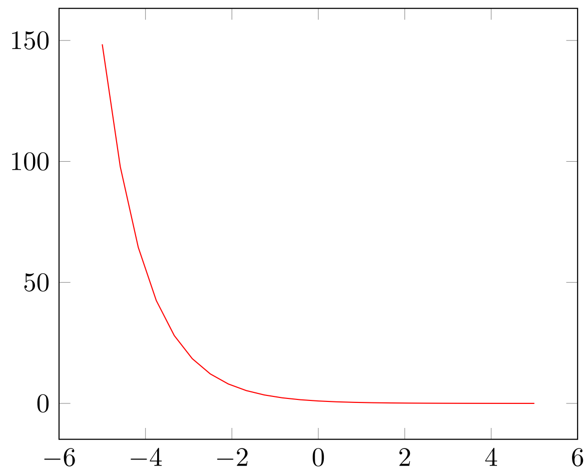
$$v_C(t) = A\exp\left(-\frac{t}{RC}\right) \tag{139}$$

for arbituary constant A. Noting that $v_C(0^-) = V_0$ and the voltage across a capacitor cannot change abruptly, the capacitor voltage is

$$v_C(t) = V_0\exp\left(-\frac{t}{RC}\right) \tag{140}$$

From (137), note that the initial current in the capacitor is $i_C(0) = -\frac{V_0}{R} \equiv I_0$. Thus, the current flowing through the capacitor is

$$i_C(t) = I_0\exp\left(-\frac{t}{RC}\right) \tag{141}$$



Definition: We define the *time constant*

$$\tau \equiv RC \tag{142}$$

as it has the dimensions of time.

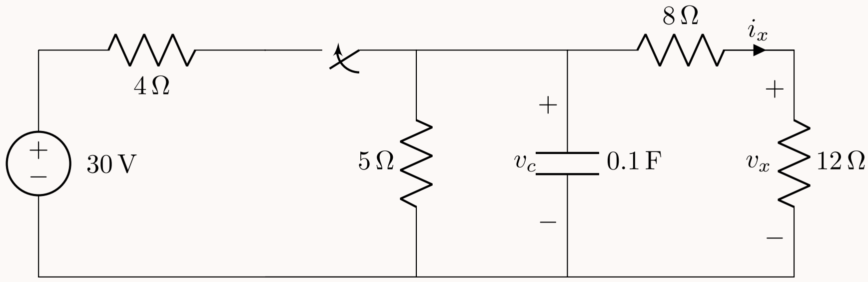
- This indicate how long it takes for the tangent line (of the voltage) to cross the time axis.
- This is also the amount of time for the voltage or current to drop to $1/e$ of its original value.
- A larger time constant means the voltage and current decays slower.
- There is some initial energy in the circuit that gradually dissipate through the resistor

$$w(0) = \frac{1}{2}CV_0^2 \tag{143}$$

- If a capacitor that is connected to an interconnection of resistors (in series), the capacitor voltage can be found by substituting the Thevenin resistance for R in (140).

Example 30 ()

The switch was closed for a long time before time $t = 0$. At $t = 0$, the switch is opened. Find the transient behavior of v_x and i_x.



Solution 22 (): At steady state, the capacitor behaves like a open circuit. Opening the capacitor, the resistors on the right side of the circuit is connected in series with an equivalent resistance of $20\,\Omega$. This is connected to a parellel with $5\,\Omega$ resistor with the equivalent resistance of $4\,\Omega$. Using the voltage division principle, $v_c = 15\,\text{V}, t < 0$.

For $t > 0$ when the switch is opened, the left hand side of the circuit is disconnected from the circuit, thus, the circuit becomes a source free RC circuit. The equivalent resistance is now $R_{eq} = 5||(8 + 12) = 4\,\Omega$. Since the capacitor voltage cannot change abruptly, use (114) to find

$$v_c(t) = 15e^{-2.5t}\,\text{V} \tag{144}$$

Use the voltage division principle to find

$$v_x(t) = v_c(t)\frac{12}{12+8} = 9e^{-2.5t}\,\text{V} \tag{145}$$

Using ohm's law

$$i_x(t) = \frac{v_x(t)}{12} = 0.75e^{2.5t}\,\text{A} \tag{146}$$

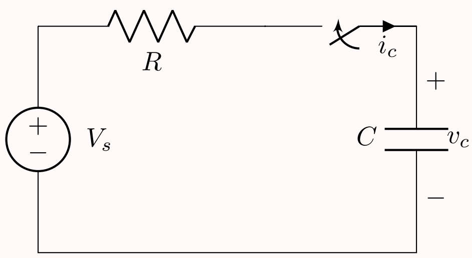
18.2 Step Response of an RC Circuit

Definition: A step response, given by the Heaviside theta function (or step function) is a common signal for circuit analysis.

$$\Theta(x) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \tag{147}$$

This is the reasoning behind the name of this circuit, as the voltage as a function of time behaves like the step function.

Derivation:



Use KVL in the clockwise direction,

$$v_c - V_s + Ri_c = 0 \tag{148}$$

Replace i_c with $C\dot{v}_c$,

$$v_c - V_s + RC\dot{v}_c = 0 \tag{149}$$

This is again a seperable first order differential equation with the general solution

$$v_c(t) = V_s + A\exp\left(-\frac{t}{RC}\right) \tag{150}$$

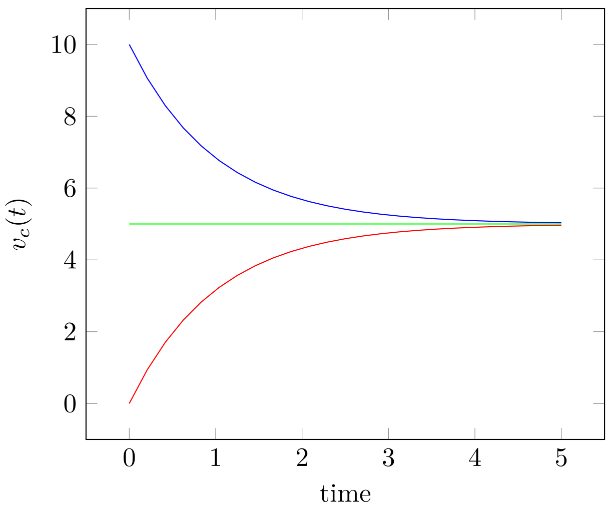
With the initial condition $v_c(0) = V_0$,

$$v_c(t) = V_s + (V_0 - V_s)e^{-\frac{t}{RC}} \tag{151}$$

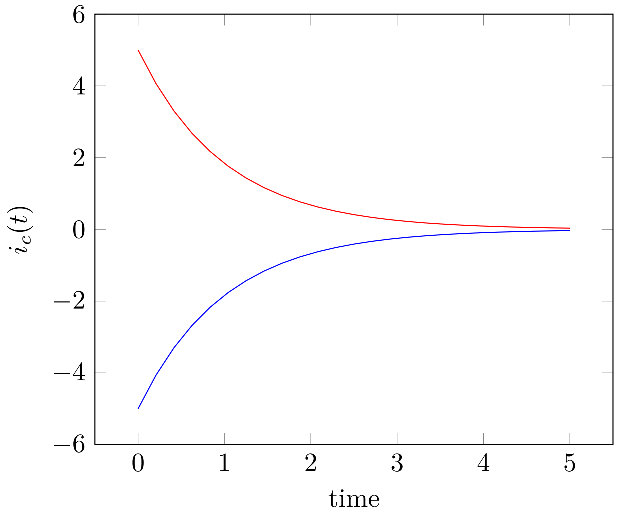
For the current,

$$i_c(t) = C\dot{v}_c(t) = \frac{V_s - V_0}{R}e^{-\frac{t}{RC}} \tag{152}$$

- For $V_s > V_0$, here $V_s = 5, V_0 = 10$ (red); and $V_0 < V_s$: $V_s = 5, V_0 = 0$ (blue). The source voltage is denoted in green.



- The current for the conditions would be



- For an arbiturary circuit connected to a capacitor, find the Thevenin equivalent and substitute V_{th} and R_{th} in place of V_s and R.

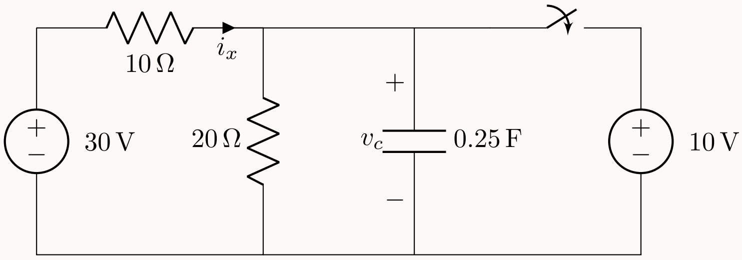
$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))\exp\left(\frac{-t}{R_{th}C}\right) \tag{153}$$

- This can be split into the forced response (first) and the natural response term (second).

$$v_c(t) = V_s(1 - e^{-\frac{t}{RC}}) + V_0e^{-\frac{t}{RC}} \tag{154}$$

Example 31 ()

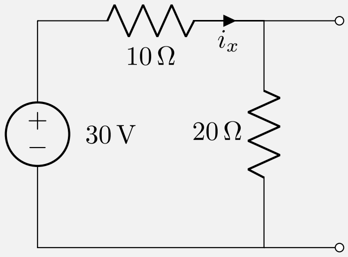
The switch has been closed for a long time and opened at $t = 0$. Find the transient behavior or i_x.



Solution 23 (): For time $t < 0$, $v_c = 10\,\mathrm{V}$. Write KVL for the outer loop,

$$-30 + 10i_x + 10 = 0 \implies i_x = 2\,\mathrm{A} \tag{155}$$

For time $t > 0$, the switch is opened and the 10 V voltage source is disconnected from the circuit. We know the initial voltage is $v_c(0) = 10\,\mathrm{V}$. We need to just find the Thevenin equivalent to the circuit connected to the capacitor.



The resistors are connected in series. The open circuit voltage is the same as the voltage across the 20 Ω resistor. Using the voltage division principle,

$$v_{th} = 30\,\frac{20}{20+10} = 20\,\mathrm{V} \tag{156}$$

Use the first method to find the Thevenin resistance. Deactivating the voltage source, the two resistors are connected in parellel. Thus, the Thevenin resistance is

$$R_{th} = \frac{20 \times 10}{20+10} = \frac{20}{3}\,\Omega \tag{157}$$

Now using (153),

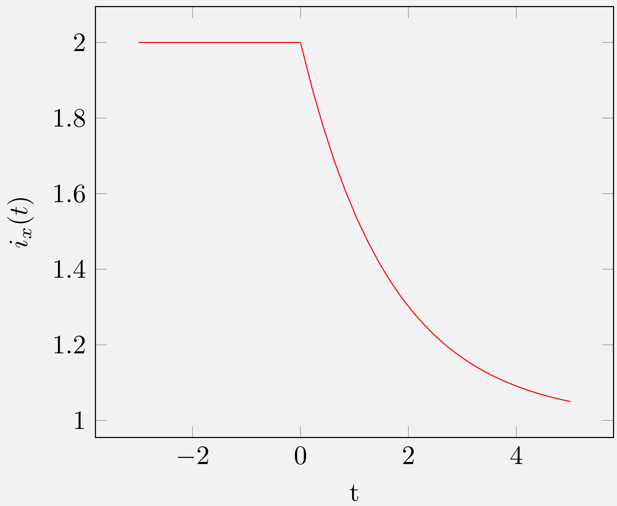
$$v_c(t) = 20 + (10-20)\exp\left(-\frac{t}{(\frac{20}{3})(20)}\right) = 20 - 10e^{-0.6t}\,\mathrm{V} \tag{158}$$

To find i_x, write KVL for the outer loop,

$$-30 + 10i_x + v_c(t) = 0 \tag{159}$$

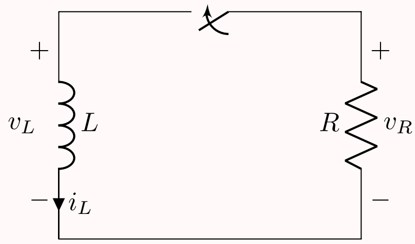
$$i_x = \frac{30 - v_c(t)}{10} = 1 + e^{-0.6t}\mathrm{A} \tag{160}$$

Plotting the current,



18.3 Source Free RL Circuits

Derivation: Before $t = 0$, the switch is open and the circuit is attached to an external circuit such that the inductor current is $i_L(0) = I_0$. At time $t = 0$, the external circuit is disconnected and the switch is closed.



We know for an inductor $v_L = L\dot{i}_L$ and for a resistor $v_R = -Ri_L$. Writing KVL in the counterclockwise direction yields

$$0 = v_L - v_R = L\dot{i}_L + Ri_L \tag{161}$$

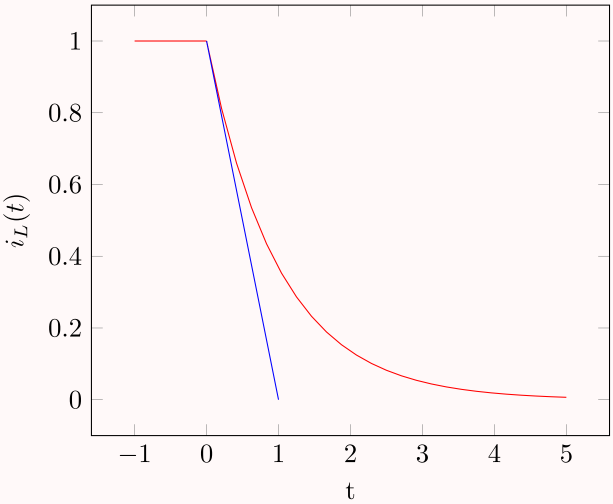
This is again a simple seperable first order differential equation with the general solution

$$i_L(t) = Ae^{-tR/L} \tag{162}$$

Using the initial condition $i_L(0) = I_0$,

$$i_L(t) = I_0 e^{-tR/L} \tag{163}$$

Plotting this,



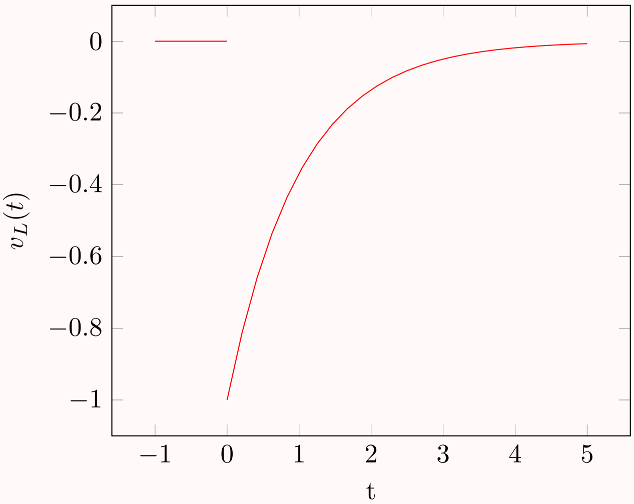
The time constant is

$$\tau = \frac{L}{R} \tag{164}$$

To find the voltage, use

$$v_L = L\dot{i} = -RI_0 e^{-tR/L} \tag{165}$$

Plotting the voltage,

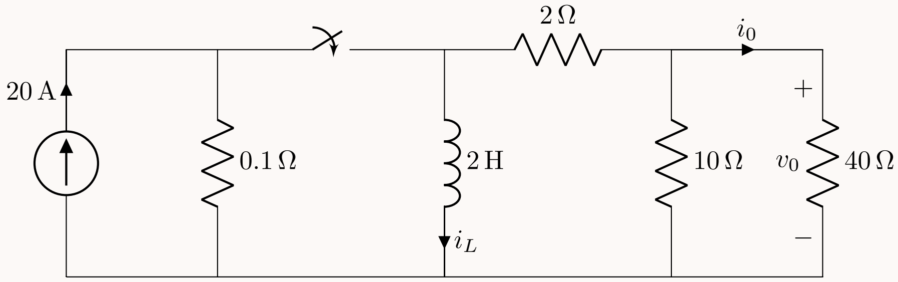


Note that the inductor voltage changes abruptly at $t = 0$, which is okay. The inductor current cannot change abruptly.

Example 32 ()

The switch has been closed for a long time. It is opened at $t = 0$.

1. Find $i_0(t)$
2. What percentage of the energy stored in the inductor is dissipated in the $10\,\Omega$ resistor



Solution 24 (): First find the current for time $t < 0$. Since the switch is closed for a long time, the inductor behaves like a short circuit path. Thus, all the current fro the current source would path through the inductor, so $i_L = 20\,\text{A}$, $i_0 = 0$

For $t > 0$, the left hand side of the switch is disconnected from the circuit as the switch is opened. We know that the current of an inductor cannot change abruptly, thus $i_L(0) = 20\,\text{A}$. The equivalent resistance of the three resistors is $10\,\Omega$. Using (163)

$$i_L(t) = I_0 e^{-tR/L} = 20e^{-10t/2} = 20e^{-5t} \tag{166}$$

Using the current division principle,

$$i_0(t) = -i_L(t)\frac{10}{10+40} = -4e^{-5t}\,\text{A} \tag{167}$$

Then, find the energy stored in the inductor at $t = 0$.

$$w_L = \frac{1}{2}Li_L^2(0) = \frac{1}{2} \times 2 \times 20^2 = 400\,\text{J} \tag{168}$$

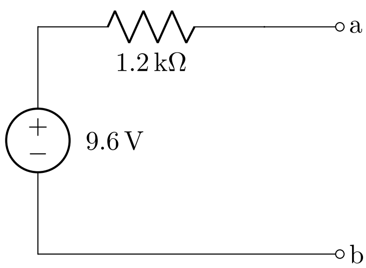
I don't know how he got here

$$v_0 = 40i_0 = -160e^{-5t} \tag{169}$$

$$P_{10\,\Omega} = v_0 i = \frac{v_0^2}{R} = 2560e^{-10t}\text{W} \tag{170}$$

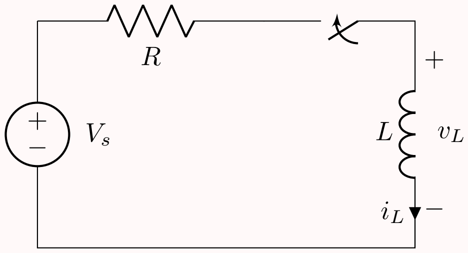
$$w_{10\,\Omega} = \int_0^\infty P_{10\,\Omega}\text{d}t = 256\,\text{J} \tag{171}$$

The ratio would be $256/400 = 64\%$.



18.4 Step Response of an RL Circuit

Derivation: Consider the following circuit at $t < 0$ is connected to an external circuit such that $i_L(0^-) = I_0$. At time $t = 0$ the external circuit is disconnected and the switch is closed.



Write KVL for this loop in the clockwise direction.

$$-V_s + Ri_L + L\dot{i}_L = 0 \tag{172}$$

This is a seperable first order differential equation with the general solution

$$i_L(t) = \frac{V_s}{R} + Ae^{-tR/L} \tag{173}$$

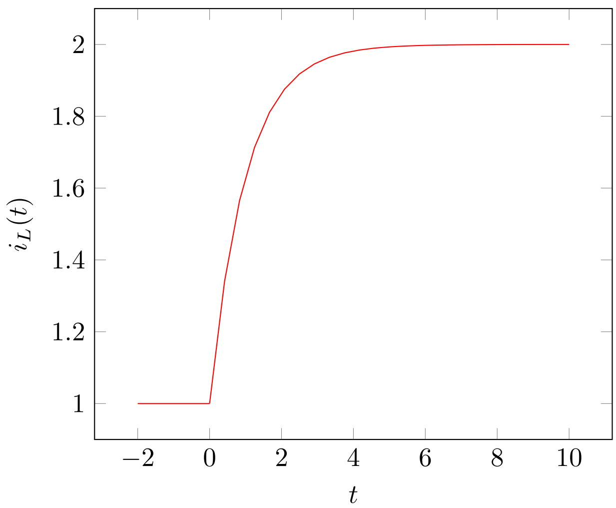
Using the initial condition $i_L(0) = I_0$,

$$i_L(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-tR/L} \tag{174}$$

The time constant for this circuit is

$$\tau = \frac{L}{R} \tag{175}$$

- Assuming $V_s/R > I_0$, here $V_s/R = 2$ and $I_0 = 1$.



- We can seperate the inductor current into natural response and forced response terms respectively.

$$i_L(t) = I_0 e^{-t/\tau} + \frac{V_s}{R}(1 - e^{-t/\tau}) \tag{176}$$

- If an inductor is connected to an interconnection of inductors and sources, we can reduce the circuit to a Thevenin equivalent circuit and it behaves like what is derived, with $R = R_{th}$ and $V = V_{th}$.
- Once an RL circuit with a source approaches steady state DC condition, the inductor can be replaced with a short circuit path. Thus,

$$i_L(\infty) = \frac{V_s}{R} \tag{177}$$

- Substituting this result gives another expression for the inductor current

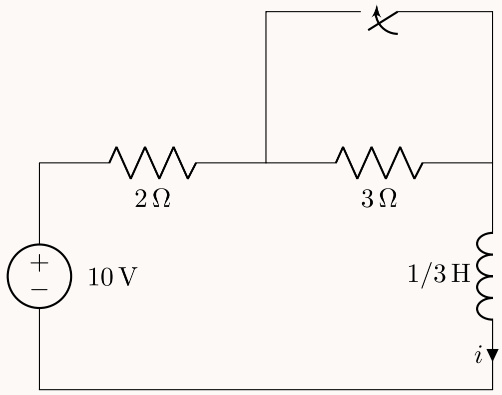
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} \tag{178}$$

- Suppose the switch is closed not at $t = 0$, but $t = t_0$, the inductor current would be

$$i_L(t) = i_L(\infty) + [i_L(t_0) - i_L(\infty)] \exp\left(\frac{t_0 - t}{\tau}\right) \tag{179}$$

Example 33 ()

The switch has been closed for a long time. It is opened at time $t = 0$. Find the relation for i.



Solution 25 (): For time $t < 0$, the switch is closed for a long time. Thus, no current flows through the $3\,\Omega$ resistor and the inductor behaves like a short circuit. Thus, $i = 10/2 = 5\,\text{A}$.
For time $t > 0$, the switch is opened. The Thevenin resistance is just the equivalent resistence $5\,\Omega$. Thus the time constant is $\tau = L/R = 1/15\,\Omega$. At time $t \to \infty$, the circuit is in steady state DC condition thus $i = V/R = 2\,\text{A}$. We know the inductor current is

$$i(t) = i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 2 + 3e^{-15t} \tag{180}$$

To verify the answer, it is possible to write KVL (clockwise) for the loop

$$-10 + 5i_L + \frac{1}{3}\dot{i}_L = 0 \tag{181}$$

If the solution solves the differential equation and $i_L(0)$ is correct, then the solution is correct.

19 AC Circuits

19.1 Complex numbers

- Electric engineers like to claim that they are different and they use j as the imaginary number where $j^2 = -1$.
- A complex number can be represented in two ways:
  1. Rectangular form: $z = x + jy$. (x, y) can be seen as the coordinates of a point on the complex plane.
  2. Polar form: $z = re^{i\theta} \equiv r\angle\theta$. r is the distance from the origin, θ is the angle from the positive real axis.
- To convert a complex number from rectangular to polar form

$$r = \sqrt{x^2 + y^2} \tag{182}$$

$$\theta = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \pi + \arctan\left(\frac{y}{x}\right) & x < 0 \end{cases} \tag{183}$$

- To convert a complex number from polar to rectangular form,

$$x = r\cos\theta \tag{184}$$

$$y = r\sin\theta \tag{185}$$

- To perform arithmitic on complex numbers $z_1 = x_1 + jy_1 = r_1 e^{i\theta_1}, z_2 = x_2 + jy_2 = r_2 e^{i\theta_2}$,

$$\begin{aligned} z_1 \pm z_2 &= (x_1 \pm x_2) + j(y_1 \pm y_2) && (186)\\ z_1 z_2 &= r_1 r_2 e^{j(\theta_1+\theta_2)} && (187)\\ &= x_1x_2 - y_1y_2 + j(x_1y_2 + x_2y_1) && (188)\\ \frac{z_1}{z_2} &= \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)} && (189)\\ &= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2} && (190) \end{aligned}$$

- Note the convention for writing complex numbers should not have i in the denominator. To divide two complex numbers, multiply by the numerator and denominator by the complex conjugate of the denominator.

$$z_1^* \equiv \overline{z_1} = x_1 - jy_1 = r_1 e^{-i\theta_1} \tag{191}$$

- The norm-square of a complex number is $zz^* = x^2 + y^2 = r^2$

19.2 Trignometric Identities

$$\begin{aligned} \sin(\alpha \pm 180^\circ) &= -\sin\alpha && (192)\\ \cos(\alpha \pm 180^\circ) &= -\cos\alpha && (193)\\ \sin(\alpha \pm 90^\circ) &= \pm\cos\alpha && (194)\\ \sin(\alpha \pm 90^\circ) &= \mp\sin\alpha && (195) \end{aligned}$$

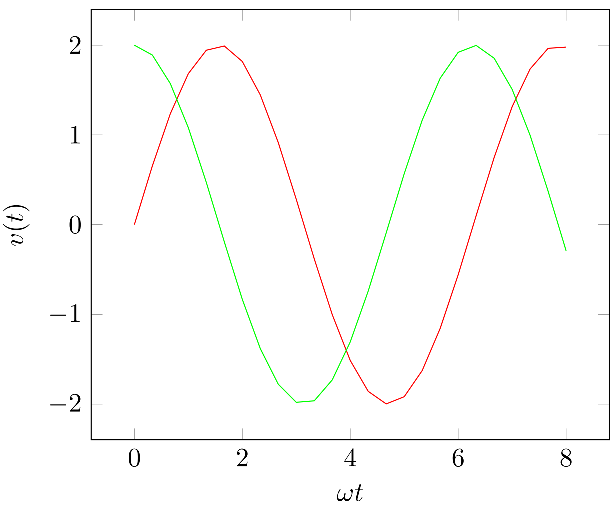
19.3 Sinusoids and Phasors

Definition: AC Circuits are circuits with the voltage and current signals changes with respect to time. AC circuits we will analyze will be those that vary sinusoidally, since all signals can be decomposed into sinusoidal voltage and current sources (using methods like fourier or laplace transforms)

- The **general** form of a sinusoidal signal (voltage):

$$v(t) = V_m \sin(\omega t + \alpha) \tag{196}$$

  - V_m is the magnitude or the amplitude of the signal. The unit here is volt. The sin function is dimensionless, and the argument is also dimensionless.
  - ω is the angular frequency, with units radians per second.
  - t is time, with common unit of second.
  - α is the initial phase angle or phase, commonly in radians.
- **Note** if α is expressed in degrees, it must be converted to radians by multiplying by $\pi/180$ before it is added to ωt.



- Two plots of sinusoidal voltages are shown with amplitude $V_m = 2$. The red plot has $\alpha = 0$ and the green plot has $\alpha = \frac{\pi}{2}$
- A common way to show the difference between two sinusoidal signal is called the *Phase Lead* or *Phase Lag*.
  - The green signal leads the red signal by $\frac{\pi}{2}$ (or 90°).
  - The red signal lags the green the signal by $\frac{\pi}{2}$ (or 90°).
- Usually, only angles between 0 to π is used to express phase lead or phase lag of sinusoidal signals.

Definition:

- The **fundamental period** of a sinusoidal signal is the smallest positive period

$$T_0 = \frac{2\pi}{\omega} \tag{197}$$

- The **frequency** is defined as

$$f = \frac{1}{T_0} = \frac{\omega}{2\pi} \tag{198}$$

The unit of frequency is in s^{-1} or Hz. In north america, the standard frequency is 60 Hz and in the rest of the world, the frequency is 50 Hz.

Example 34 ()

What is the phase difference between the following sinusoidal signals.

1.

$$\begin{aligned} v_1(t) &= A_1 \sin(\omega_1 t + 10°) \\ v_2(t) &= A_2 \sin(\omega_2 t + 70°) \end{aligned}$$

2.

$$\begin{aligned} v_1(t) &= -10\cos(\omega t + 50°) \\ v_2(t) &= 12\sin(\omega t - 10°) \end{aligned}$$

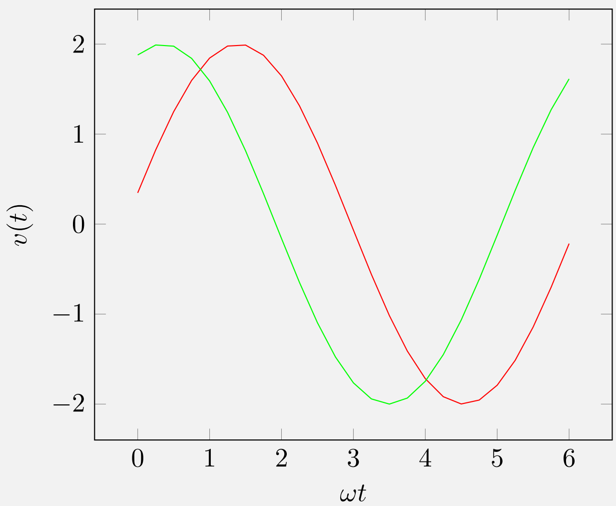
Solution 26 ():

1. We first notice that the frequency given for v_1 is ω_1 and v_2 is ω_2. Note that **if $\omega_1 \neq \omega_2$, the phase difference is not defined for v_1 and v_2.**

   If $\omega_1 = \omega_2$, the phase difference

$$\Delta\alpha = 70° - 10° = 60° \tag{199}$$

   Since we subtracted the phase angle of v_2 from that of v_1, thus this shows v_2 **leads** v_1 by 60°.



2. Note when comparing the phase difference between two sinusoidal signals, the trignometric function used must be the same as well as the magnitude of the ampliutude. Thus, $v_1(t)$ must be converted using the trignometric identities to

$$v_1(t) = 10\sin(\omega t - 40^\circ) \tag{200}$$

Then, comparing that with the phase of v_2,

$$\Delta\alpha = (-10^\circ) - (-40^\circ) = 30^\circ \tag{201}$$

Therefore, v_2 lags v_1 by 30°, or equivalently, v_1 lags v_2 by 30°.

Definition: A **Phasor** is an object used for AC circuit analysis used to describe sinusoidal signals. Take the voltage

$$v(t) = V_m\cos(\omega t + \alpha) = \mathrm{Re}(V_m e^{j(\omega t+\alpha)}) = \mathrm{Re}(V_m e^{j\alpha}e^{j\omega t}) \tag{202}$$

If the angular frequency is known, $v(t)$ can be uniquely determined using a complex number whose magnitude and angle are V_m and α respectively. This complex number

$$\mathbf{V} = V_m e^{j\alpha} \tag{203}$$

$\mathbf{V}$ is known as the phasor for $v(t)$. The phasor is usually denoted with an uppercase letter (also boldface in print, but just uppercase for written).

- Phasors are used because it is very difficult to do arithmitic on sinusoidal, but for complex number arithmitic is much simpler.
- To solve an AC circuit, it is common to first transform the problem from the time domain to the phasor domain, solve it there, then transform the solution back into the time domain.
- Since a phasor is determined by a magnitude and angle, they can be treated as vectors.

Example 35 ()

Show the phasor diagram for the following AC voltage and current. Find their phase difference.

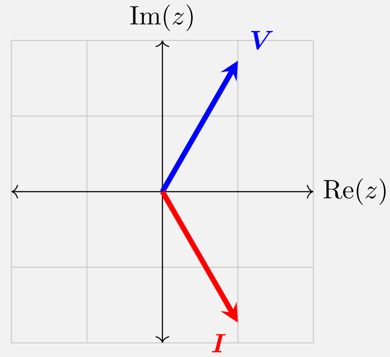
$$v(t) = V_m\cos(377t + 60^\circ)$$
$$i(t) = I_m\sin(377t + 30^\circ)$$

Solution 27 (): For the voltage phasor,

$$\mathbf{V} = V_m e^{j60^\circ} = V_m\angle 60^\circ \tag{204}$$

For the current phasor, first change the sin function to cos function using the trignometric identity. Then,

$$\mathbf{I} = I_m e^{-j60^\circ} = I_m\angle{-60^\circ} \tag{205}$$



Example 36 ()

Using phasors, find $i_1 + i_2$.

$$i_1(t) = 4\cos(\omega t + 30^\circ)$$
$$i_2(t) = 5\sin(\omega t - 20^\circ)$$

Solution 28 (): First convert these into phasors. Note to use the trignometric identity for i_2 to obtain

$$\mathbf{I}_1 = 4\angle 30^\circ$$
$$\mathbf{I}_2 = 5\angle{-110^\circ}$$

Add in the rectangular form,

$$\begin{aligned}\mathbf{I}_1 + \mathbf{I}_2 &= 4\angle 30^\circ + 5\angle{-110^\circ} \\ &= 4\cos(30^\circ) + j4\sin(30^\circ) + 5\cos(-110^\circ) + j5\cos(-110^\circ) \\ &= 1.754 + j2.698 \\ &= 3.218\angle{-56.98^\circ}\end{aligned} \tag{206}$$

The sum of the two signals is then

$$i_1 + i_2 = 3.218\cos(\omega t - 56.98^\circ) \tag{207}$$

19.4 Calculus with Phasors

Proposition: For a phasor $\mathbf{Z}$,

$$\dot{\mathbf{Z}} = j\omega\mathbf{Z} \tag{208}$$
$$\int \mathbf{Z}\,\mathrm{d}t = \frac{\mathbf{Z}}{j\omega} \tag{209}$$

Proof: Take the phasor $\mathbf{V} = V_m\angle\phi = V_m e^{j\phi}$. Take the derivative of what the phasor represents

$$v(t) = V_m\cos(\omega t + \phi) \tag{210}$$
$$\dot{v}(t) = -V_m\omega\sin(\omega t + \phi) = +V_m\omega\cos(\omega t + \phi + 90^\circ) \tag{211}$$

The phasor for $\dot{v}(t)$ is

$$\dot{\mathbf{V}} = V_m\omega e^{\omega t}e^{j\phi}e^{j90^\circ} = j\omega(V_m e^{j\phi}) = j\omega\mathbf{V} \tag{212}$$

Taking the integral of what the phasor represent,

$$\int v(t)\mathrm{d}t = \frac{V_m}{\omega}\sin(\omega t + \phi) = \frac{V_m}{\omega}\cos(\omega t + \phi - 90^\circ) \tag{213}$$

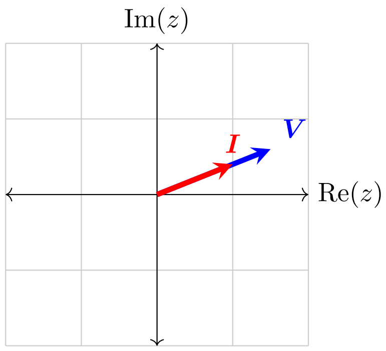
For the phasor,

$$\int \mathbf{V}\mathrm{d}t = \frac{V_m}{\omega}e^{j(\phi - 90^\circ)} = \frac{1}{j\omega}(V_m e^{j\phi}) = \frac{\mathbf{V}}{j\omega} \tag{214}$$

19.5 Phasor relation for the voltage and current of R, L, and C

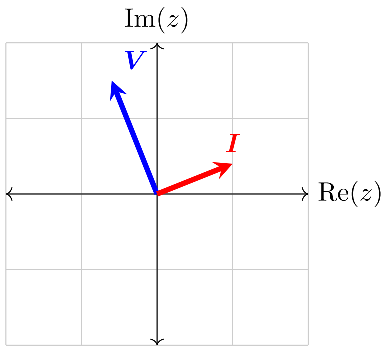
- For a resistor, consider the current through a resistor $\mathbf{I} = I_m\angle\alpha$. Applying Ohm's law,

$$\mathbf{V} = RI_m\angle\alpha = R\mathbf{I} \tag{215}$$



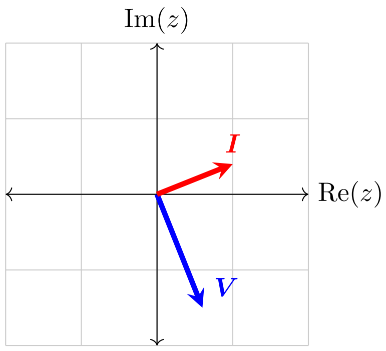
- For an inductor, consider the current phasor in the inductor as $\mathbf{I} = I_m \angle \alpha$. It is known that $v(t) = L\dot{i}$, thus

$$\mathbf{V} = j\omega L \mathbf{I} \tag{216}$$



- For a capacitor, consider the voltage phasor in the capacitor as $\mathbf{V} = V_m \angle \alpha$. It is known that $i(t) = C\dot{v}$,

$$\mathbf{V} = \frac{\mathbf{I}}{j\omega C} \tag{217}$$



19.6 KVL and KCL the phasor domain

Proposition: Suppose there is a loop in the circuit of N circuit elements with voltage phasors $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V_N}$ with the same (angluar) frequency ω. Then, KVL for phasors requires

$$\sum \mathbf{V}_n = \mathbf{V}_1 + \mathbf{V}_2 + \ldots \mathbf{V}_N = 0 \tag{218}$$

Proposition: Suppose a node with N current paths and incoming current phasors $\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I_N}$ with the same (angluar) frequency ω. Then, KCL for phasors requires

$$\sum \mathbf{I}_n = \mathbf{I}_1 + \mathbf{I}_2 + \ldots \mathbf{I}_N = 0 \tag{219}$$

20 Impedence

Definition: Take an arbiturary circuit element with a sinusoidal voltage and current with the same frequency. The voltage and current phasors are $\mathbf{V}$ and $\mathbf{I}$ respectively. Then,

$$Z \equiv \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle (\theta_v - \theta_i) \tag{220}$$

Rewriting the impedence in rectangular form,

$$Z \equiv R + jX \tag{221}$$

- The real part of the impedence R is called *resistance*.
- The imaginary part of impedence X is called *reactance*.
- The units for Z, R, X is Ohm.
- **Note** if the passive sign convention does not hold for $\mathbf{V}$ and $\mathbf{I}$, then $Z = -\mathbf{V}/\mathbf{I}$.

$$\det(\lambda \mathbb{1}_{n\times n}) = \lambda^n \tag{222}$$

- For DC circuits there is resistance and conductance. For AC circuits,

Definition: The admittance of an circuit element is with current phasor $\mathbf{I}$ and voltage phasor $\mathbf{V}$ is

$$Y \equiv \frac{\mathbf{I}}{\mathbf{V}} = \frac{I_m}{V_m} \angle (\theta_i - \theta_v) \tag{223}$$

Rewriting this in the rectangular form,

$$Y = G + jB \tag{224}$$

- The real part of the admittance G is called *conductance*.
- The imaginary part of the admittance B is called

20.1 Series and parellel connected impedances

Proposition:

- The equivalent impedance of multiple circuit elements connected in series and parellel behaves the same as the resistance in DC circuits.
- The equivalent admittance for multiple circuit elements connected in parellel behaves the same as conductance in DC circuits.

20.2 The impedance for the resistor, inductor, and capacitor

- For a resistor,

$$Z_R = \frac{\mathbf{V}}{\mathbf{I}} = \frac{R\mathbf{I}}{\mathbf{I}} = R \tag{225}$$

- The impedance of a resistor is the resistance itself lmao. In the complex plane, this is a positive real number.
- For an inductor,

$$Z_L = \frac{\mathbf{V}}{\mathbf{I}} = \frac{j\omega L\mathbf{I}}{\mathbf{I}} = j\omega L \tag{226}$$

- In the complex plane, the impedance of an inductor is on the positive imaginary axis.

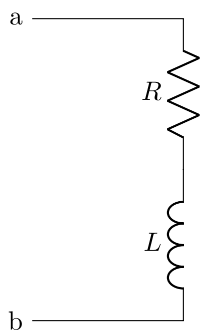
- Note that for the same inductor, the impedance is also dependent on the frequency of the circuit.
- In the limit where $\omega \to 0$, the impedance approaches 0, which is the reason that under steady state DC conditions, an inductor behaves like a short circuit.
- For a capacitor,

$$Z_C = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\frac{-j}{\omega C}\mathbf{I}}{\mathbf{I}} = \frac{-j}{\omega C} = \frac{1}{\omega C} \tag{227}$$

- In the complex plane, the impedance of a capacitor is on the negative imaginary axis.

20.3 The impedance of RL, RC, LC, and RLC circuits

20.3.1 RL Circuit



$$Z_{RL} = Z_R + Z_L = R + j\omega L \tag{228}$$

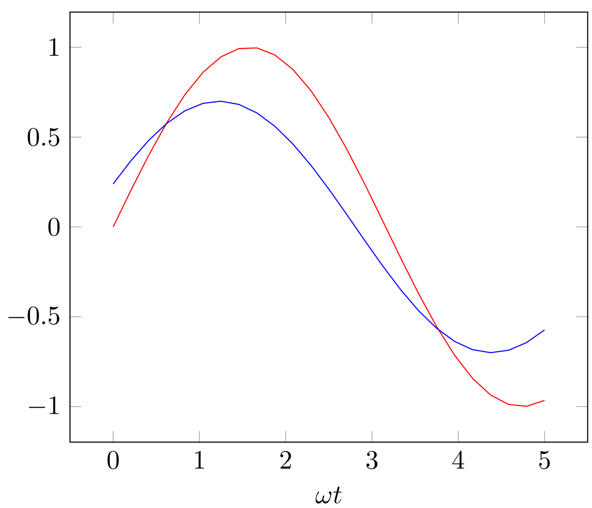
- Z_{RL} is located in the first quadrant of the complex plane. The angle of the impedance is

$$\angle Z_{RL} = \arctan\left(\frac{\omega L}{R}\right) \tag{229}$$

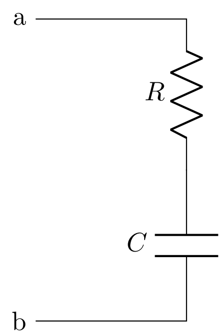
- If $Z_R >> Z_L \iff R >> \omega L \implies \angle Z_{RL} \to 0°$
- If $Z_R << Z_L \iff R << \omega L \implies \angle Z_{RL} \to 90°$
- Because of this, the voltage of an RL circuit leads the current by an angle between 0° to 90°

$$Z_{RL} = \frac{V_m}{I_m}\angle(\theta_v - \theta_i) \implies 0° < \theta_v - \theta_i < 90° \tag{230}$$

- In the time domain, the voltage will lead the current by a quarter of a period. (blue is voltage, red is current)



20.3.2 RC Circuit



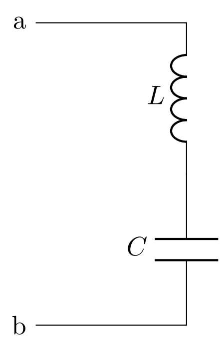
$$Z_{RC} = Z_R + Z_C = R - \frac{j}{\omega C} \tag{231}$$

- Z_{RC} is located in the fourth quadrant of the complex plane. The angle of the impedance is

$$\angle Z_{RC} = \arctan\left(-\frac{1}{\omega RC}\right) \tag{232}$$

- If $R >> \frac{1}{\omega C} \implies \angle Z_{RC} \to 0°$
- If $R << \frac{1}{\omega C} \implies \angle Z_{RC} \to -90°$
- $-90° < \theta_v - \theta_i < 0°$,
- Because of this, the voltage of an RC circuit lags the current by an angle between 0° to 90°.

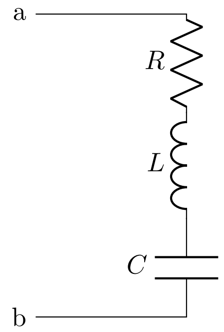
20.3.3 LC Circuit



$$Z_{LC} = Z_L + Z_C = j\omega L - \frac{j}{\omega c} = j\left(\omega L - \frac{1}{\omega C}\right) \tag{233}$$

- The impedance is purely imaginary.
- If $\omega L > \frac{1}{\omega C}$, the impedance will be on the positive imaginary axis. These circuit are called **inductive** LC circuits.
- If $\omega L < \frac{1}{\omega C}$, the impedance will be on the negative imaginary axis. These circuit are called **capacitive** LC circuits.
- The voltage can either lead or lag the current, depending on the if it is an inductive or capacitive circuit.

20.3.4 RLC Circuit



$$Z_{RLC} = Z_R + Z_L + Z_C = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{234}$$

- The real part of the impedance is positive, but the imaginary part can be either positive or negative. Thus, the complex number can be located in first or fourth quadrant of the complex plane.
- If Z_{RLC} is in the first quadrant ($\omega L > \frac{1}{\omega C}$), the RLC circuit is called an **inductive** (RL) circuit.
- If Z_{RLC} is in the fourth quadrant ($\omega_L < \frac{1}{\omega C}$), the RLC circuit is called an **capacitive** (RC) circuit.

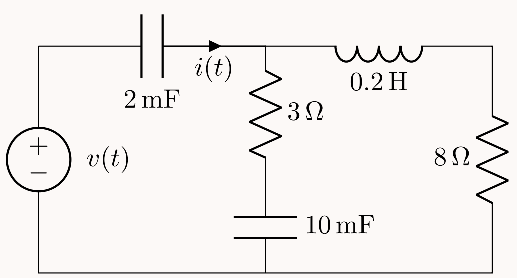
Example 37 ()

How do RL, RC, LC, and RLC circuits behave if they are connected in parellel?

21 Analysis of AC Circuits

- Application of KVL and KCL in the phasor domain
- The function of $Z = \mathbf{V}/\mathbf{I}$ relation in AC circuit analysis is similar to the function of $R = v/i$ in DC circuit analysis.
- All the circuit analysis techniques for DC circuits (nodal analysis, mesh analysis, voltage division, superposition, etc) are applicable to AC circuits as well. The only difference is that complex numbers must be used (phasors and impedances)

Example 38 ()



$v(t) = 20\cos(50t)$. Find $i(t)$.

Solution 29 (): First we must convert all voltage and current to phasors, and all the circuit elements to impedances. The voltage source has a voltage phasor $\mathbf{V} = 20\angle 0$.
To find the impedances, we call the impedance of the 2 mF capacitor Z_1, the RC component Z_2 and the RL

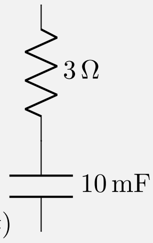
component Z_3. The frequency of the circuit is dependent on the source, which here is $\omega = 50$

$$Z_1 = \frac{-j}{\omega C_1} = \frac{-j}{50 \times 0.002} = -j10\,\Omega \tag{235}$$

$$Z_2 = R_2 - \frac{j}{\omega C_2} = 3 - \frac{j}{50 \times 0.01} = 3 - j2\Omega \tag{236}$$

$$Z_3 = R_3 + j\omega L_2 = 8 + j50 \times 0.2 = 8 + j10\Omega \tag{237}$$

Representing the circuit in the phasor domain,



We can find the equivalent impedance, note Z_2 and Z_3 is connected in parellel, which is connected in series with Z_1. Thus,

$$Z_{eq} = Z_1 + Z_2 || Z_3 = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = 3.22 + j11.07\Omega = 11.53\angle{-73.5}\,\Omega \tag{238}$$

Now using ohm's law for phasors,

$$\mathbf{I} = \frac{\mathbf{V}}{Z_{eq}} = \frac{20\angle 0}{11.53\angle - 73.5°} = 1.73\angle 73.5° \tag{239}$$

Converting this back to the time domain,

$$i(t) = 1.73\cos(50t + 73.5°) \tag{240}$$

- Note that this circuit can be replaced with an equivalent RL or RC.
- If the reactance is positive, it can be replaced with RL, otherwise it can be replaced with RC. (Note this is possible because the frequency remains constant)
- In the example above, $R_{eq} = 3.22\,\Omega, C_{eq} = 1.81\,\text{mF}$.