Contents

1 Lecture 1

1.1 An Electric Circuit

An electric circult is an interconnection of circuit elements (incl. Conductors, semi-conductors, non-conductors).

1.2 Electrical Variables

To help us analyze electric circuits, we define several electrical variables:

Definition: Electric currents: The "movement" or rate of change of electrical charge.

$$i \equiv \frac{dq}{dt} \tag{1}$$

1. S.I. unit: Ampere = Columb per second.
2. The current has a direction, and its direction is defined as the direction of positive charge. (Can be shown with $\leftarrow$ or $\rightarrow$)

When performing circuit analysis, the direction of positive current is either given or we are required to guess a direction of positive current and verify it later. A positive current means the direction you guessed is correct. A negative current means the actual direction is the direction opposiing the current.

Definition: Voltage: The energy required for 1C of charge between point A and B in a circuit is called the voltage between points A and B.

$$V \equiv \frac{dw}{dq} \tag{2}$$

1. S.I. unit: Volt = Joule per columb.
2. The voltage also have polarity ($+$ or $-$). Positive polarity means energy is consumed when the charge moves from A to B.

If the polarity is not given, guess an polarity and a positive voltage indicates the guess is correct and a negative voltage indicates the guess is incorrect.

Definition: Power: The rate of delivering or absorbing energy.

$$p \equiv \frac{dw}{dt} \tag{3}$$

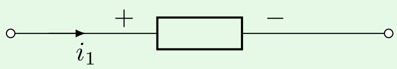
1. S.I. unit: Watt = Joule per second.
2. Using chain rule,

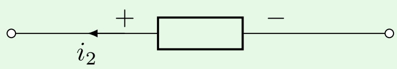
$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = iv \tag{4}$$

Note: When a problem asks for a current or voltage, the direction/polarity **must** be indicated otherwise full credit will not be given.

1.3 Passive Sign Convention

Definition: For a pair of v and i, PSC holds if the current direction **enters** the positive side of voltage polarity. If PSC holds, $p = +vi$; else $p = -vi$





PSC holds on the left and does not hold on the right.

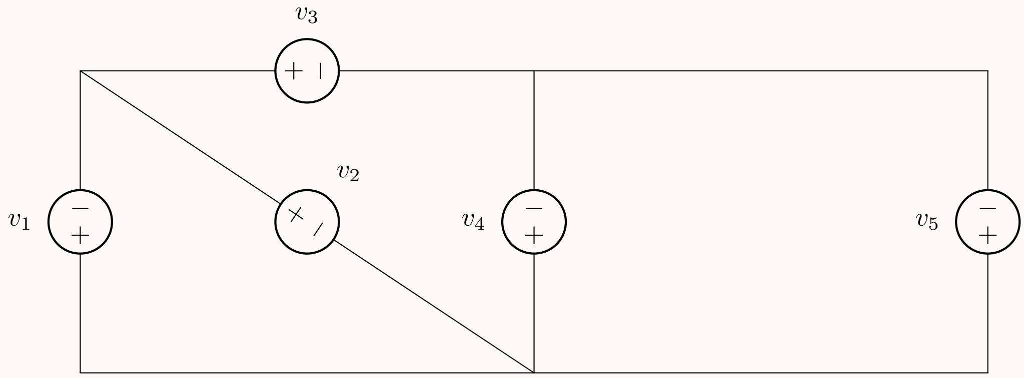
The consequences of this convention are

1. $p > 0 \implies$ power is absorbed
2. $p < 0 \implies$ power is delivered (generated by the circuit element)

2 Lecture 2

Example 1 (1)

Identify the devices that generate power in the circuit below and find the unknowns in the table.

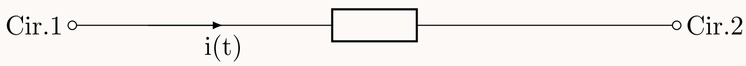


Theorem: Conservation of Power states that the algebraic sum of the power of all elements in a circuit is zero. (algebraic means the signs are preserved)

$$\sum p = 0 \tag{5}$$

Proof. Will not be presented at this point in the course. □

Example 2 (2)



Given the above circuit and

$$v(t) = 50(1 - e^{-5000t})\text{V} \tag{6}$$
$$i(t) = 10e^{-5000t}\text{A} \tag{7}$$

Find the total energy transfered to this device after $t = 0$.

Solution 1 (2): PSC holds. Thus,

$$p(t) = v(t)i(t) \tag{8}$$
$$p(t) = 50(1 - e^{-5000t})10(e^{-5000t})\text{W} \tag{9}$$

From the definition of power,

$$p(t) = \frac{dw}{dt} \therefore dw = p(t)dt \tag{10}$$
$$w = \int_0^\infty p(t)\text{d}t = \int_0^\infty 50(1 - e^{-5000t})10(e^{-5000t})\text{d}t = \frac{1}{20}\text{J} = 50\text{mJ} \tag{11}$$

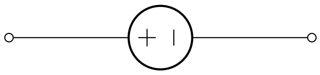
3 Lecture 3

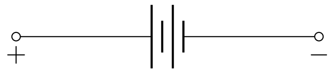
3.1 Circuit Elements

1. Independent sources:

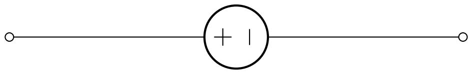
   (a) *Independent voltage sources:* A circuit element that has a specific voltage independent of the current that

flows through it.
For example, the voltage can be a fixed voltage like $v_s = 2\text{V}$ or a variable voltage source like $v_s = 2\sin(50t + 2)$.
On diagrams, generic voltage source is shown on the left and fixed (DC) voltage sources is shown on the right.
For DC sources, the uppercase letter V is commonly used.



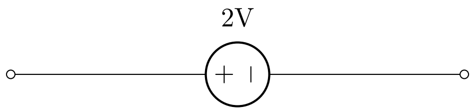


For DC source, the longer lines denotes positive voltage polarity and shorter lines denote negative voltage polarity.
An sinsudal voltage is drawn as



Questions

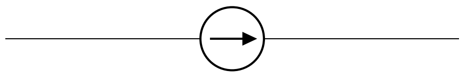
i. What is the direction of i for the following voltage source?



Answer: You cannot determine.

ii. Does a voltage source always generate power?
Answer: No. Take the voltage source above. Take a current $i_1 = 2\text{A} \rightarrow$. Then, PSC holds and $p = vi = (2\text{V})(2\text{A}) = 4\text{W}$. Since $p > 0$, the voltage source absorbs power.

(b) *Independent current source:* It gives a specific current independent of the voltage across it. Can be constant like $i_s = 5\text{A}$ or time dependent like $i_s = 10\cos(15t)$
The independent current source is drawn as



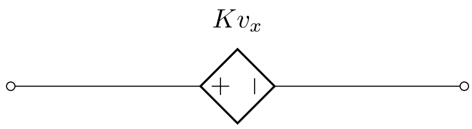
Questions

i. What is the polarity of voltage across a current source?
ii. Does a current source always generate power?

Do not let the word *source* decieve you. A source is not associated with generating power.

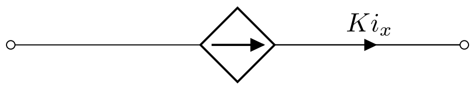
(c) Dependent Sources:

i. *Voltage-dependent voltage source:*



The voltage of this source depends on voltages somewhere else in the circuit. i.e. $v_s = Kv_s$

ii. *Current-dependent voltage source:*



The voltage of this source depends on the current somewhere else in a circuit i.e. $v_s = ki_x$. Note k has the dimensions Current/Voltage.

iii. *Voltage-dependent current source:*
iv. *Current-dependent current source:*

4 Lecture 4

4.1 Circuit Elements (cont.)

1. Resistor: A circuit element that has keeps a constant ratio between a voltage and current, such that

$$R \equiv \frac{v}{i} \tag{12}$$

S.I. unit: Volt per Amp or Ohm (Ω).

The resistor is drawn like this. If PSC holds, $v = Ri$ else $v = -Ri$. This is also known as Ohm's Law.

The inverse of the resistence R is called the conductance G

$$G \equiv \frac{1}{R} = \pm\frac{i}{v} \tag{13}$$

The S.I. unit for conductance is 1/Ohm, also refered to as "mho" or "siemens" (Si).

We could not find a generic relation for the power of a source, since $P = vi$ (assuming PSC holds). Since for a independent voltage source, we don't know the current and vise versa. For a resistor however, assume PSC holds,

$$P = vi = (Ri)i = Ri^2 \tag{14}$$

$$= v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{15}$$

If PSC doesn't hold,

$$P = -vi = -(-Ri)i = Ri^2 \tag{16}$$

$$= -v\left(-\frac{v}{R}\right) = \frac{v^2}{R} \tag{17}$$

Thus, the power of a resistor is independent of whether or not PSC holds. For physical resistors, $R > 0$ hence $P > 0$ which means power is always absorbed.

2. Short Circuit: The limiting behavior for a resistor as $R \to 0$. Since $v = Ri$, the voltage is zero independent of the current. This is denoted with a solid line:

   Other names include zero ohm path or ideal conductor.

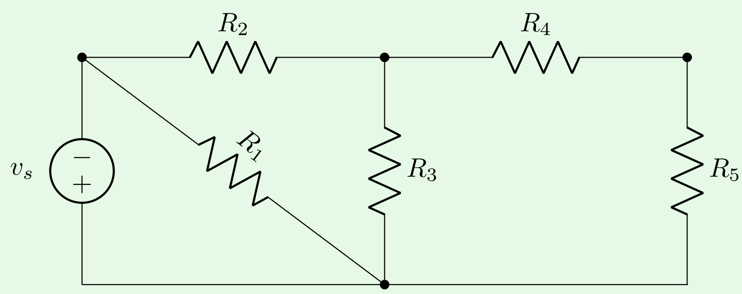
   In analysis, all parts of a circuit that are connected using ideal conductor can be considered the same point in the circuit.

3. Open Circuit: The limiting behavior for a resistor as $R \to \infty$. Since $v = Ri$, the current is zero independent of the voltage.

   For example, the air between the ground and transmission line can be treated like a open circuit, since air is not a conductor.

4.2 Circuit Analysis Definitions

Definition: A **Node** is a junction of two or more circuit elements.

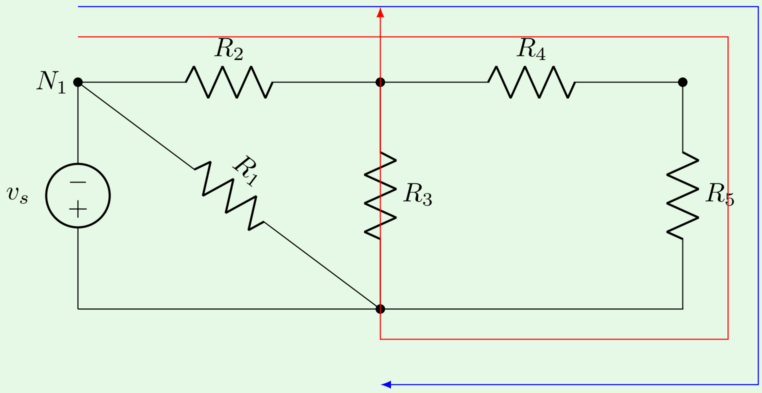


The labelled points are nodes: top-left connects v_s, R_1 and R_2; top-center connects R_2, R_3, and R_4; top-right connects R_4 and R_5; as well as the region at the bottom (that are connected by short circuit as the junction of v_s, R_1, R_3, R_5).

5 Lecture 5

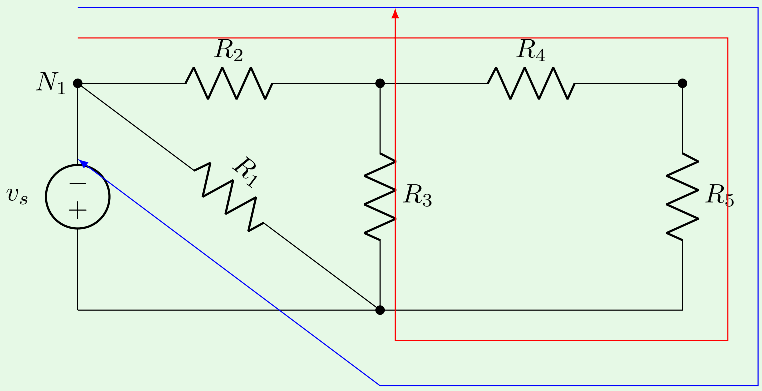
5.1 Circuit Analysis Definitions cont.

Definition: Start moving from one node towards the other nodes. As long as no node is passed (**unless** it is a loop when start and end nodes are the same) more than once, the set of nodes passed is called a **Path**.



The blue one is a path, the red one is not.

Definition: If the beginning and end of a path is one node, that path is a **loop**.

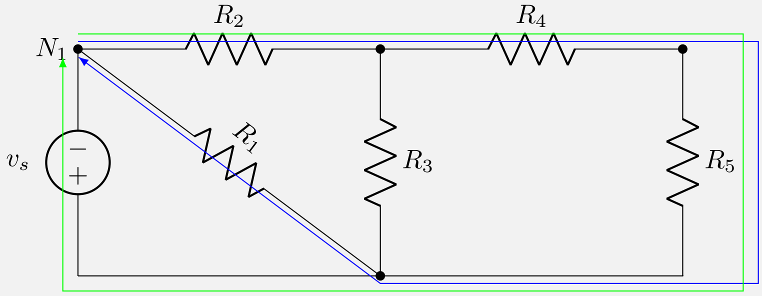


The blue one is a loop, the red one is not a loop.

Example 3 (1)

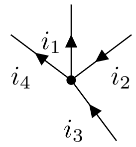
Identify the loops in the circuit above circuit

Solution 2 (1): Arrows are too hard to draw



5.2 Circuit Analysis Laws

1. *Kirchoff's Current Law (KCL)*: The algebraic sum of the currents entering a node is zero.



Define a sign convention for the algebraic sum. Either positive sign for current for entering the node or leaving the node is sufficient. For instance, define the sign convention as positive current entering the node. Then, KCL requires

$$i_1 - i_2 + i_3 - i_4 = 0 \tag{18}$$

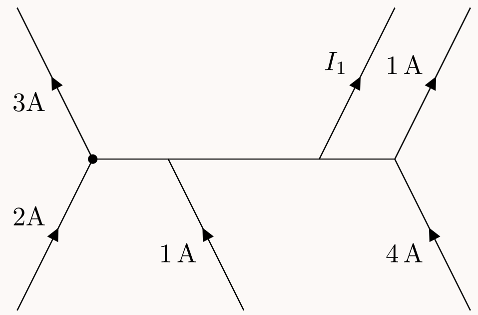
An equivilent statement of KCL is: "The sum of the currents entering a node is equal to the sum of the currents leaving the node." This statement would require

$$i_2 + i_3 = i_1 + i_4 \tag{19}$$

which is equivilent to (18).

Example 4 (2)

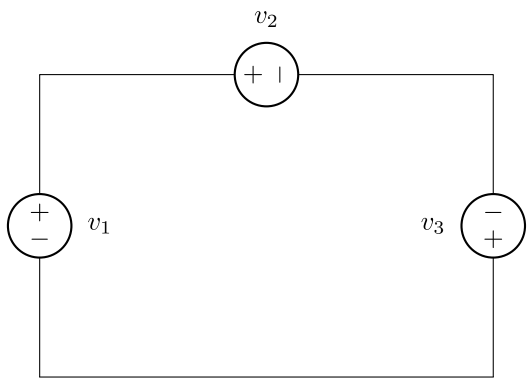
Find I_1 in the following circuit



6 Lecture 6

6.1 Circuit Analysis Laws (cont.)

1. *Kirchoff's Voltage Law (KVL)*: The algebraic sum of the voltages around any loop is zero.
   (a) If the loop direction enters a voltage source from positive voltage polarity, voltage used for KVL is positive.
   (b) Else, the voltage used for KVL is negative.



For the same loop, the starting point and direction of performing the analysis is irrelevent. i.e. starting from the top left in the clockwise direction, KVL requires

$$+v_2 - v_3 - v_1 = 0 \tag{20}$$

Starting from the top right in the anti-clockwise direction,

$$-v_2 + v_1 + v_3 = 0 \tag{21}$$

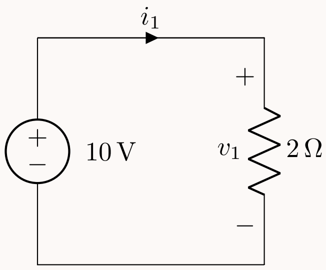
When writing KVL for circuits with resistors, it can be combined with Ohm's law in the following fashion. Note that this is independent of voltage polarity defined for the resistor.

(a) When the current direction entering a resistor is the same as the loop direction, the voltage of the resistor for KVL is $+iR$

(b) When the current direction entering a resistor is opposing the loop direction, the voltage of the resistor for KVL is $-iR$

Example 5 (1)

Use KVL and Ohm's law to find the current of the resistor.



Solution 3 (1): Applying KVL in the clockwise direction,

$$v_1 - 10 = 0 \implies v_1 = 10\,\mathrm{V} \tag{22}$$

Using Ohm's law. Since current enters the + polarity, PSC holds. Thus,

$$v_1 = 2i_1 \implies 10 = 2i_1 \implies i_1 = 5\,\mathrm{A} \tag{23}$$

Using the "shortcut", again applying KVL in a clockwise direction. Since the current direction is the same as the loop direction,

$$+2i_1 - 10 = 0 \implies i_1 = 5\,\mathrm{A} \tag{24}$$

6.2 Circuit Analysis Tips

If a circuit **does not** include a voltage-dependent current source or a current-dependent voltage source, you may use any of the following units systems to perform your analysis.

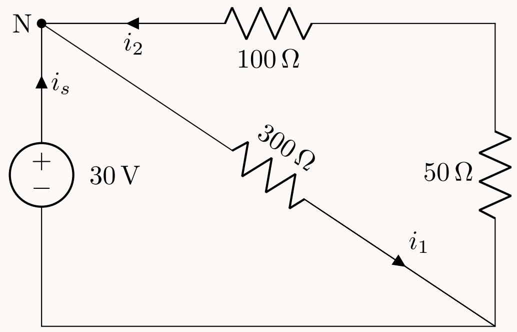
1. V, A, Ω, W
2. V, mA, kΩ, mW
3. kV, mA, Ω, W
4. Other consistent units can be used but are not as common.

It is simpler to use a consistent system of units to analyze the circuits. Thus, units consistent with the voltage-dependent current source or a current-dependent voltage source should be used.

6.3 Some Examples

Example 6 (2)

Find the power of the voltage source.



Solution 4 (2): Since PSC does not hold for the voltage source,

$$P = -30i_s \tag{25}$$

KVL for the bottom left loop (clockwise)

$$-30 + 300i_1 = 0 \implies i_1 = 0.1\,\mathrm{A} \tag{26}$$

KVL for the outer loop (clockwise)

$$-30 - 100i_2 - 50i_2 = 0 \implies i_2 = -0.2\,\mathrm{A} \tag{27}$$

KCL at node N

$$i_s + i_2 = i_1 \implies i_s = i_1 - i_2 = 0.1 - (-0.2) = 0.3\,\mathrm{A} \tag{28}$$

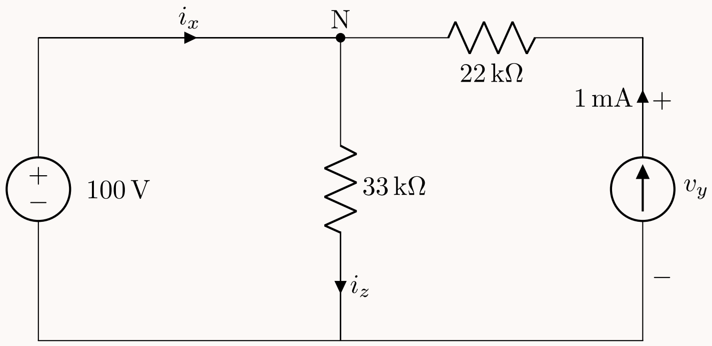
Find the power

$$P = -30i_s = -30(0.3) = -9\,\mathrm{W} \tag{29}$$

Since $P < 0$, power is generated.

Example 7 (3)

Find the power of each source.



Solution 5 (3): KVL for the left loop (clockwise)

$$-100 + 33i_z = 0 \implies i_z = 3\,\mathrm{mA} \tag{30}$$

KCL for N, i_x and $1\,\mathrm{mA}$ are entering N and $i_z = 3\,\mathrm{mA}$ is leaving N.

$$i_x + 1 = i_z \implies i_x = 2\,\mathrm{mA} \tag{31}$$

For the voltage source, PSC does not hold. Thus,

$$P = -vi = -100 \times 2 = -200\,\mathrm{mW} \tag{32}$$

KVL for outer loop (clockwise)

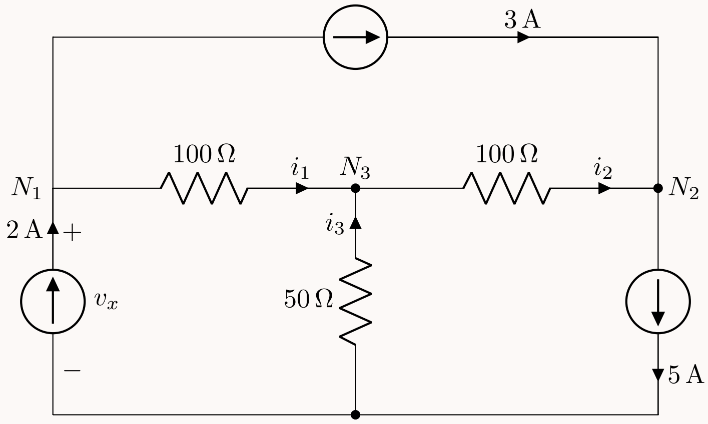
$$-100 - (1 \times 22) + v_y = 0 \implies v_y = 122\,\mathrm{V} \tag{33}$$

For the current source, PSC does not hold. Thus,

$$P = -vi = -122 \times 1 = -122\,\mathrm{mW} \tag{34}$$

Example 8 (4)

Find the power of each source.



Solution 6 (4): KCL at N_1

$$2 = 3 + i_1 \implies i_1 = -1\,\mathrm{A} \tag{35}$$

KCL at N_2

$$3 + i_2 = 5 \implies i_2 = 2\,\mathrm{A} \tag{36}$$

KCL at N_2

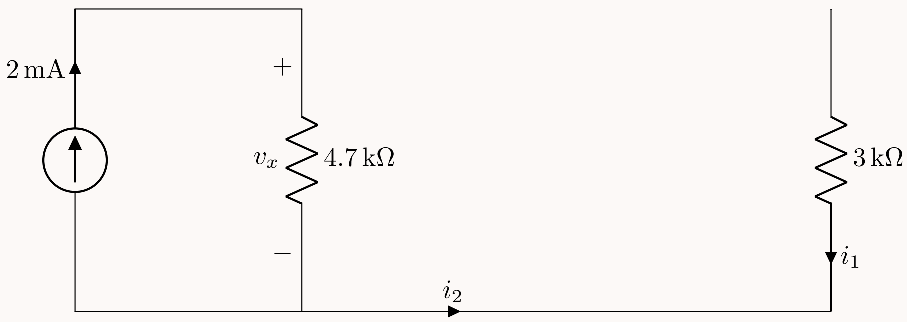
$$i_1 + i_3 = i_2 \implies i_3 = 3\,\mathrm{A} \tag{37}$$

KVL for the bottom left loop (clockwise)

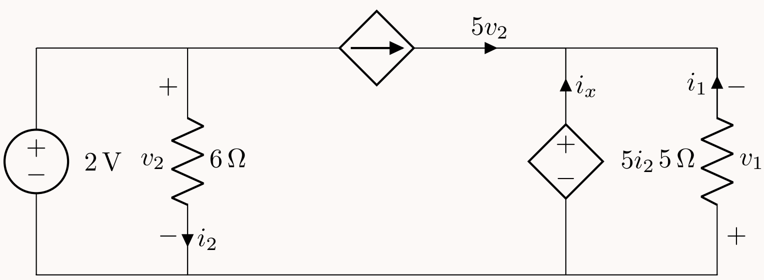
$$-v_x + 100i_1 - 50i_3 = 0 \implies v_x = 100 \times -1 - 50 \times 3 = -250\,\mathrm{V} \tag{38}$$

The rest is left as an exercise.

Example 9 (5)



Example 10 (6)



Solution 7 (6): Use KVL on the left most loop (clockwise).

$$-2 + 6i_2 = 0 \implies i_2 = 0.33\,\mathrm{A} \tag{39}$$

Using Ohm's law on the resistor, PSC holds thus

$$v_2 = 6i_2 = 2\,\mathrm{V} \tag{40}$$

The current provided by the dependent current source would be 10 A and the voltage provided by the dependent voltage source would be 1.67 V.
Using KVL on the right most loop (anti-clockwise),

$$v_1 + 5i_2 = 0 \implies v_1 = -5i_2 = -1.67\,\mathrm{V} \tag{41}$$

Using Ohm's law on this resistor to find i_1. As PSC holds,

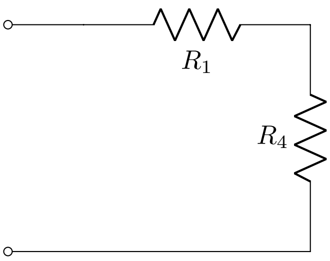
$$i_1 = v_1/5 = 0.33\,\mathrm{A} \tag{42}$$

Finally using KCL at the junction,

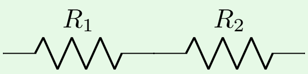
$$i_x + i_1 + 5v_2 = 0 \implies i_x = -10.33\,\mathrm{A} \tag{43}$$

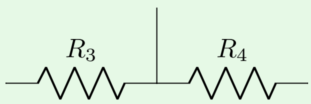
7 Lecture 7

7.1 Equivilent circuits for parallel and series resistors



Definition: Series Connection: Two circuit elements are connect in series if and only if they are connected back to back, and at their point of connection, there is no other current path.





R_1 and R_2 are connected in series. R_3 and R_4 are also connected in series since the path between the two resistence is a open circuit, thus the current is zero so it is not a current path.

Theorem: For resistors in series, the equivilent resistence is

$$R_{eq} = \sum_k R_k \tag{44}$$

Proof. Take a circuit with 2 resistors in series.

Using KVL (clockwise),

$$-v_{tot} + v_1 + v_2 = 0 \tag{45}$$

Using Ohm's law,

$$v_1 = R_1 i_{tot} \tag{46}$$
$$v_2 = R_2 i_{tot} \tag{47}$$

Combining the results,

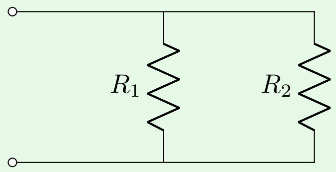
$$v_{tot} = v_1 + v_2 = (R_1 + R_2)i_{tot} \tag{48}$$

For an equvilent resistor from Ohm's law,

$$v_{tot} = R_{eq}i_{tot} \implies R_{eq} = R_1 + R_2 \tag{49}$$

From induction, the equivilent resistence of any series connected resistors is equal to the sum of their resistences. □

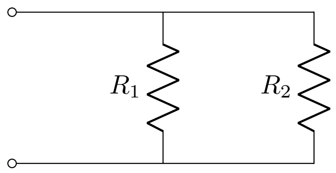
Definition: Parallel connection: Two circuit elements are connected in parallel if they share two common nodes.



Theorem: For parellel resistors, the equivilent resistence is

$$R_{eq} = \left[\sum_k \frac{1}{R_k}\right]^{-1} \tag{50}$$

Proof.



Write KCL at N

$$i_{tot} = i_1 + i_2 \tag{51}$$

Using KVL

$$-v_{tot} + v_1 = 0 \implies v_{tot} = v_1 = v_2 \tag{52}$$

Using Ohm's law, PSC holds

$$i_{tot} = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_{tot} \tag{53}$$

Using ohm's law for the equivilent resistor,

$$i_{tot} = \frac{v_{tot}}{R_{eq}} \implies \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \implies R_{eq} = \frac{R_1R_2}{R_1 + R_2} \tag{54}$$

□

Consider two resistors in parellel with $R_2 = 0$. Thus,

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{0}{R_1} = 0 \tag{55}$$

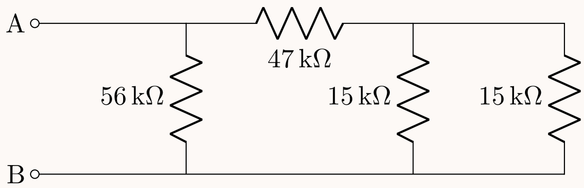
You can consider that the current is "sane". When given an option to flow with no resistence, it will take that path. Thus, if $R_2 = 0$, no current will flow through R_1 and the equivilent resistence is 0

Correlary: Recall the definition of conductance is $G = 1/R$. Thus,

$$G_{eq} = \sum_k G_k \tag{56}$$

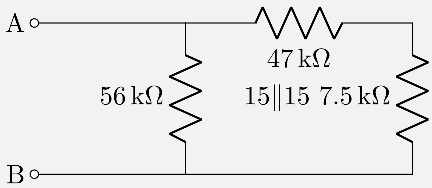
Example 11 (1)

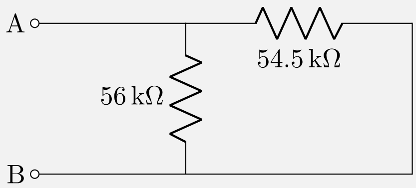
Find the equivilent resistence between A and B



Solution 8 (1): Simplify the circuit in several steps (starting from the right)

1. The two right most resistors are correct in parellel.



2. 

3. $$R_{eq} = 56 \| 54.5 = \frac{56 \times 54.5}{56 + 54.5} = 27.62\,\mathrm{k\Omega} \tag{57}$$

Note that in the original circuit, the $15\,\mathrm{k\Omega}$ resistors are **not** connected in series to the $47\,\mathrm{k\Omega}$ resistor as there exist (one) current path between them.

Example 12 (2)

Select R such that $R_{AB} = R_L$