2.2: Equivalences Involving Quantitrens

Exercise 1:

O Vx Jy (Mx→ (Fxy ∧ Hy)) ¬VxJy (Mx → (Fxy ∧ Hy)) ∃x ¬Jy (Mx→ (Fxy ∧ Hy)) ∃x Vy ¬(¬Mx v (Fxy ∧ Hy)) ∃x Vy (¬¬Mx ∧¬(Fxy ∧ Hy)) Ix by (Mx 1 (7 Fxy v7 Hy))
There is a moth major whose Friends don't all need
help with their homework.

There is someone such that all of his roomder do not distitle someone.

There is an x s.t. XEAUB and XECID.

For all x, there is a greater y touch that for all z, z2+5z zy,

Exercise 2:

67(Vx Jy (Lxy) A 7 Jx Vy (Lxy))
7 Vx Jy (Lxy) V Jx Vy (Lxy)

Jx Vy(7 Lxy) V Jx Vy (Lxy)

Someone doesn't like anyone or someone likerononyone.

$$\bigcirc \neg (\forall x ((x \in A) \rightarrow \exists \forall ((y \in B) \land (x \in C \leftrightarrow y \in C))))$$

$$\exists x \neg (\neg (x \in A) \lor \exists y ((y \in B) \land (x \in C \leftrightarrow y \in C))))$$

$$\exists x ((x \in A) \land \forall y \neg ((y \in B) \land (x \notin C \leftrightarrow y \in C)))$$

$$\exists x ((x \in A) \land \forall y ((y \notin B) \lor (x \notin C \leftrightarrow y \in C)))$$

$$\exists x ((x \in A) \land \forall y ((y \in B) \rightarrow (x \notin C \leftrightarrow y \in C)))$$

(a)
$$= (4y > 0) = (ax^2 + bx + c = y)$$

$$= (4y (y > 0) = 3x (ax^2 + bx + c = y))$$

$$= 3y = (7(y > 0) \times 3x (ax^2 + bx + c = y))$$

$$= 3y ((y > 0) \times 4x = (ax^2 + bx + c = y))$$

$$= 3y ((y > 0) \times 4x = (ax^2 + bx + c = y))$$

Exercise 3:

@ This

1 False

O True

@ True

Exercise 4:

7 Hx P(x)

THX TIP(X)

(x)qrxErr

Jx 7P(x)

Exercise 5:

(x) JAXEAP(x)

TX (XEA ~ P(X))

Yx7 (xEANP(x))

Vx (X X X VTP(x))

YX (XEA -> 7P(X))

Thus, YXEATP(x).

Exercise 6:

(x) Q v (x) q) xE

((x) Q v (x) 9) x E r r

7 7x7(P(x)vQ(x))

74x (1P(x) ~ 7Q(x))

- (Ax - b(x) v Ax - Q(x))

74x - P(X) x 74x - Q(X)

EXP(X) V EXB(X)

Exercise 7:

 $\exists \times (P(x) \rightarrow Q(x))$

((X) V (X) Y F) XE

(X) DXEV (X) 9T XE

TYXP(X) V 3xQ(X)

YXP(X) -> 3xQ(X).

Exercise 8:

 $\forall x (x \in A \rightarrow P(x)) \land \forall x (x \in B \rightarrow P(x))$

> Yx(AUB)P(x)

 $\forall x((x \in A \rightarrow P(x)) \land (x \in B \rightarrow P(x)))$

YX ((XEA VP(X)) ^ (XEB V P(X)))

Yx ((x&A 1x&B) v P(x))

Yx ("(XKA NXEB) -> PCX))

Wx ((xeAvxeB) → PW))

Exercise 9:

Let P(x)= x is a person and Q(x)=x is a quitt. ∀x (P(x), Q(x)) => for all x, x is a person or x is a quit ∀xP(x)v ∀xQ(x) → all x are people or all x are quito. The propositions are not equivalent.

Exercise 10:

- (X) 98 × EN (X) 9A3 × E @ (x) A A P(x) V E V (x E A P(x)) XE 3x ((x-eA ^ P(x)) v (x & B ^ P(x))) Jx ((xeAvxeB) AP(x)) Jx (x & AUB MP(X)) ∃x € (AUB) P(x)
- BNO. 3x EAP(x) A 3x EBP(x): There is a person in A and a person in B. Ix e (AB)P(x): There is a person who is in both A and B.

Exercise !!

ASB Vx (xeA -> xeB) YX (X&A Y XEB)

YX7 (XEAAX&B)

73x (XEA NX & B)

A\B=Ø

Exercise 12:

CSAUB Vx (x∈C → x∈ AUB) Wx (xxc v xeA v x eB) ∀x ((X£C v X€A) v X€B)

~ = X (x EA ~ X EB) YX7 (XEANXEB) VX (XEAVX EB)

> DYX (-(XECAXEA)VXEB) VX ((X C ^ X XA) -> X EB) CLASB

Exercise 13:

@ ASB

Yx (x ∈ A → x ∈ B)

 $\forall x ((x \in A \lor x \in B) \neq x \in B)$ (by bercise 5. (1 (b))

Yx (XEAUB (XEB)

AUB = B

1 ASB

YX(XEA -XEB)

∀x((x∈An x∈B) ⇔ x∈A) (by Exercise 5.11(a))

 $\forall x (x \in A \cap B \leftrightarrow x \in A)$

AMB=A

Exercise 14:

AMB=Ø

77x(XEA AXEB)

Vx7(XEA > XEB)

Vx (X&A VX&B)

Yx (xeA - x & B)

 $\forall x ((x \in A^n \times B) \leftrightarrow x \in A)$ (by Exercise 5.11 (a))

Yx(X&A/B (x&A)

A/B = A

Exercise 15:

- a x is the teacher of only one student:
- @ Someone is the teacher of only one person.
- @ Only one person is the teacher of someone.
- a Only one person is the teacher of someone.
- @ Only one person is the teacher of only one person.

 (a) Only one teacher has only one student) equivalent.