Introduction

Exercise 1: 2^{15}

(a) $2^{15}-1 = 32{,}767$

$a = 3$, $b = 5$

$xy = (2^5-1)\cdot(1+2^5+2^{10})$

$xy = 31\cdot(1+32+1024)$

$xy = 31\cdot 1057$

(b) $2^{32{,}767}-1$ $\quad a = 1057$, $b = 31$

$x = 2^{31}-1 = 2{,}147{,}483{,}647$

Exercise 2:

n	Is n prime?	3^n-1	Is 3^n-1 prime?
2	yes	8	no: $8 = 4\cdot 2$
3	yes	26	no: $26 = 2\cdot 13$
4	no: $4 = 2\cdot 2$	80	no: $80 = 2\cdot 40$
5	yes	242	no: $242 = 2\cdot 121$
6	no: $6 = 2\cdot 3$	728	no: $762 = 2\cdot 364$
7	yes	2,186	no: $2{,}186 = 2\cdot 1093$
8	no: $8 = 4\cdot 2$	6,560	no: $6{,}560 = 2\cdot 3{,}280$
9	no: $9 = 3\cdot 3$	19,682	no: $19{,}682 = 2\cdot 9{,}841$
10	no: $10 = 2\cdot 5$	59,049	no: $59{,}048 = 2\cdot 29{,}524$

Conjecture: Suppose n is an integer larger than 1 and n is prime. Then 3^n-1 is not prime.

Exercise 3

ⓐ $(2 \times 3 \times 5 \times 7) + 1 = 211$

ⓑ $2 + 1 = 3$ or $(2 \times 3) + 1 = 7$ or $(2 \times 3 \times 5 \times 7 \times 11) + 1 = 2311$

Exercise 4:

$n = 5$

$x = (5 + 1)! + 2$

$x = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) + 2$

$x = 722$

Thus: 722, 723, 724, 725, 726

Exercise 5: If $2^n - 1$ is prime, then $2^{n-1}(2^n - 1)$ is perfect.

- $2^{(5-1)}(2^5 - 1) = 2^4(31) = \boxed{496}$
- $2^{(7-1)}(2^7 - 1) = 2^6(127) = \boxed{8128}$

Exercise 6:

Proof: Let n be any prime number greater than 3. Consider the sequence: $n, n+2, n+4$. Note that every third consecutive number is divisible by 3. Two cases (since n is prime):

- Case 1: $n/3$ has a remainder of 1. Thus, $n+2$ is divisible by 3.
- Case 2: $n/3$ has a remainder of 2. Thus, $n+4$ is divisible by 3.

Thus, either $n+2$ or $n+4$ is divisible by 3, and therefore it cannot be the case that $n, n+2, n+4$ are all prime.

□

Exercise 7:

$(\Rightarrow)$ $1+2+4+5+10+11+20+22+44+55+110=284$

$(\Leftarrow)$ $1+2+4+71+142=220$