Chapter 2: Quantificational Logic

2.1: Quantifiers

Exercise 1:

(a) $\forall x\,(\exists y(F(x,y)) \rightarrow S(x))$ where $F(x,y)$: x has forgiven y and $S(x)$: x is a saint.

(b) $\neg\exists x\,(C(x) \wedge \forall y\,(D(y) \rightarrow S(x,y)))$ where $C(x)$: x is in Calc., $D(y)$: y is in Discrete, $S(x,y)$: x is smarter than y

(c) $\forall x\,((x \neq m) \rightarrow L(x,m))$ where m: Mary, $L(x,y)$: x likes y.

(d) $\exists x\,(P(x) \wedge S(j,x)) \wedge \exists y\,(P(y) \wedge S(r,y))$ where $P(x)$: x is a police officer, $S(x,y)$: x saw y; j: Jane, r: Roger

(e) $\exists x\,(P(x) \wedge S(j,x) \wedge S(r,x))$ same key as previous.

Exercise 2:

(a) $\forall x\,(C(x) \rightarrow \exists y\,(R(y) \wedge U(y,x)))$ where $C(x)$: x bought a RR w/ cathy, $R(x)$: x is rich, $U(x,y)$: x is y's uncle

(b) $\exists x\,(D(x) \wedge M(x)) \rightarrow \forall y\,(\exists z\,(D(z) \wedge F(y,z)) \rightarrow Q(y))$ where $D(x)$: x lives in the dorms, $M(x)$: x has measles, $F(x,y)$: x is friends w/ y, $Q(x)$: x has to quarantine.

(c) $\neg\exists x\,F(x) \rightarrow \forall y\,(A(y) \rightarrow \exists z\,(D(z) \wedge T(y,z))$ where $F(x)$: x failed the test, $A(x)$: x got an A, $D(x)$: x got a D, $T(x,y)$: x will tutor y.

(d) $\exists x\,C(x) \rightarrow C(j)$ where $C(x)$: x can do it, j: Jones

(e) $C(j) \rightarrow \forall x\,C(x)$ same key as above.

Exercise 3:

(a) $\forall z\,((z > x) \rightarrow (z > y))$ x and y are free variables.

(b) $\forall a \exists x\,((ax^2 + 4x - 2 = 0) \leftrightarrow (a \geq -2))$ No free variables

(c) $\forall x\,((x^3 - 3x < 3) \rightarrow (x < 10))$ No free variables.

(d) $(\exists x\,(x^2 + 5x = w) \wedge \exists y\,(4 - y^2 = w)) \rightarrow (-10 < w < 10)$ w is a free variable.

Exercise 4:

(a) All married men are unhappy.

(b) y is the sister of one of x's parents.

Exercise 5:

(a) All prime numbers that are not equal to 2, are odd.

(b) There is a perfect number greater than or equal to all perfect numbers.

Exercise 6:

(a) $x^2+2x+3=0$ and $x^2+2x-3=0$ are inconsistent. This is true.

(b) There is no x such that $x^2+2x+3=0$ or there is no x such that $x^2+2x-3=0$. This is false.

(c) There is no x such that $x^2+2x+3=0$ and there is no x such that $x^2+2x-3=0$. This is false.

Exercise 7:

(a) Someone is a parent of everyone. (false)

(b) Everyone is a parent to someone (false)

(c) No one is a parent. (false)

(d) Someone is a parent of no one. (true)

(e) Someone is not a parent (true)

Exercise 8:

(a) True

(b) False

(c) False

(d) True

(e) True

(f) False

Exercise 9:

(a) True

(b) False

(c) True

(d) False

(e) ~~True~~

(f) True

Exercise 10:

(a) True

(b) False

(c) False

(d) True

(e) True

(f) True