(d) False. It would have no solutions.

Exercise 32:

(a) False. $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$

(b) False $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(c) False $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) False $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

1.3: Matrices and Matrix Operations

Exercise 1:

(a) $(4\times5)(4\times5)$ undefined

(b) $(4\times5)(5\times2) + (4\times2)$

$(4\times2)+(4\times2)$ defined (4×2)

(c) $(4\times5)(5\times2) + (4\times5)$

$(4\times2)+(4\times5)$ undefined.

(d) $(4\times5)(4\times5) + (4\times5)$ undefined.

(e) $(5\times4)((4\times5)+(4\times5))$

$(5\times4)(4\times5)$ defined (5×5)

(f) $(5\times4)((4\times5)(5\times2))$

$(5\times4)(4\times2)$ defined (5×2)

(g) $(5\times4)^T(4\times5)$

$(4\times5)(4\times5)$ undefined.

(h) $((4\times5)^T+(5\times4))(4\times2)$

$((5\times4)+(5\times4))(4\times2)$

$(5\times4)(4\times2)$ defined (5×2)

Exercise 2:

$a-b=8$

$b+c=1$

$3d+c=7$

$2a-4d=6$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 8 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 & 7 \\ 2 & 0 & 0 & -4 & 6 \end{bmatrix} \xrightarrow[\text{RREF}]{\text{To}} \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$a=5,\ b=-3,\ c=4,\ d=1$

Exercise 3:

ⓐ $\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}$

ⓑ $\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 4 & -1 \\ -2 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

ⓒ $5 \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}$

ⓓ $-7 \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{bmatrix}$

ⓔ $2 \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

$\begin{bmatrix} 8 & -2 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \Rightarrow$ undefined.

ⓕ $4 \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 24 & 4 & 12 \\ -4 & 4 & 8 \\ 16 & 4 & 12 \end{bmatrix} - \begin{bmatrix} 2 & 10 & 4 \\ -2 & 0 & 2 \\ 6 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$

ⓖ $-3 \times \left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + 2 \times \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) = -3 \times \left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 12 & 2 & 6 \\ -2 & 2 & 4 \\ 8 & 2 & 6 \end{bmatrix} \right) = -3 \times \begin{bmatrix} 13 & 7 & 8 \\ -3 & 2 & 5 \\ 11 & 4 & 10 \end{bmatrix} = \begin{bmatrix} -39 & -21 & -2 \\ 9 & -6 & -1 \\ -33 & -12 & - \end{bmatrix}$

ⓗ $A - A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

ⓘ $tr\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \right) = 1 + 0 + 4 = 5$

ⓙ $tr\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - 3 \times \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) = tr\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix} \right) = tr\left(\begin{bmatrix} -17 & 2 & -7 \\ 2 & -3 & -5 \\ -9 & -1 & -5 \end{bmatrix} \right) = -17 - 3 - 5$
$= -25$

ⓚ $4 \times tr\left(7 \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}\right) = 4 \times tr\left(\begin{bmatrix} 28 & -7 \\ 0 & 14 \end{bmatrix} \right) = 4 \times (28 + 14) = 4 \times 42 = 168$

ⓛ $tr(A) \Rightarrow$ undefined.

Exercise 4:

ⓐ $2 \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}^T * \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = 2 \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

(c) $\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right)^T = \begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}^T + 5 \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} + 5 \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 15 \\ 20 & 5 \\ 10 & 25 \end{bmatrix} \Rightarrow$ undefined.

(e) $\frac{1}{2} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}^T - \frac{1}{4} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ 2 & 1/2 \\ 1 & 5/2 \end{bmatrix} - \begin{bmatrix} 3/4 & 0 \\ -1/4 & 1/2 \\ 1/4 & 1/4 \end{bmatrix} = \begin{bmatrix} -1/4 & 3/2 \\ 9/4 & 0 \\ 3/4 & 9/4 \end{bmatrix}$

(f) $\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(g) $2 \times \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}^T - 3 \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T = 2 \times \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - 3 \times \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 9 \\ 15 & 0 & 6 \\ 6 & 3 & 12 \end{bmatrix} = \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$

(h) $\begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & 6 \end{bmatrix}^T = \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$

Exercise 5:

(a) $\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow$ ondefined.

(c) $\left(3 \times \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$

(d) $\left(\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \right) \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \times \left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \right) = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

(g) $\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 0 & 12 \\ -2 & 1 \\ 11 & 8 \end{bmatrix}^T = \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$

(h) $\left(\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}^T \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}\right) \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}^T = \left(\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}\right) \times \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{bmatrix} \times \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix}$

(i) $tr\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T\right) = tr\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 30 & 1 & 21 \\ 1 & 2 & 1 \\ 21 & 1 & 29 \end{bmatrix}\right) = 61$

(j) $tr\left(4 \times \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}^T - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}\right) = tr\left(4 \times \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 24 & -4 & 16 \\ 4 & 4 & 4 \\ 12 & 8 & 12 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}\right)$

$= tr\left(\begin{bmatrix} 23 & -9 & 14 \\ 5 & 4 & 3 \\ 9 & 6 & 8 \end{bmatrix}\right) = 35$

(k) $tr\left(\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}^T \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}^T + 2\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}^T\right) = tr\left(\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}\right)$

$= tr\left(\begin{bmatrix} 3 & 5 & 4 \\ 12 & -2 & 5 \\ 6 & 8 & 7 \end{bmatrix} + \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 15 & 3 & 12 \\ 14 & 0 & 7 \\ 12 & 12 & 13 \end{bmatrix}\right) = 28$

Exercise 6:

(a) $\left(2 \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right) \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \left(2 \times \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right) \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} =$

$\left(\begin{bmatrix} 2 & -2 & 6 \\ 10 & 0 & 4 \\ 4 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right) \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix}$

(b) $\left(4 \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}\right) \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} + 2\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 16 & -4 \\ 0 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 8 & -2 \\ 0 & 4 \end{bmatrix} = \underset{\text{matrix}}{2 \times 3} + \underset{\text{matrix}}{2 \times 2} \Rightarrow$ undefined

(c) $\left(-\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}\right)^T + 5\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T = \left(\begin{bmatrix} -3 & 0 \\ 1 & -2 \\ -1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}\right)^T + 5\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} -3 & -12 & -6 \\ -5 & 2 & -8 \\ -4 & -5 & -7 \end{bmatrix}^T + \begin{bmatrix} 5 & -5 & 15 \\ 25 & 0 & 10 \\ 10 & 5 & 20 \end{bmatrix}$

$= \begin{bmatrix} -3 & -5 & -4 \\ -12 & 2 & -5 \\ -6 & -8 & -7 \end{bmatrix} + \begin{bmatrix} 5 & -5 & 15 \\ 25 & 0 & 10 \\ 10 & 5 & 20 \end{bmatrix} = \begin{bmatrix} 2 & -10 & 11 \\ 13 & 2 & 5 \\ 4 & -3 & 13 \end{bmatrix}$

(d) $\left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}^T - 2\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}\right)^T = \left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 8 & 4 \\ 6 & 2 & 10 \end{bmatrix}\right)^T = \left(\begin{bmatrix} 12 & -6 & 3 \\ 0 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 8 & 4 \\ 6 & 2 & 10 \end{bmatrix}\right)^T$

$= \begin{bmatrix} 10 & -14 & -1 \\ -6 & 2 & -8 \end{bmatrix}^T = \begin{bmatrix} 10 & -6 \\ -14 & 2 \\ -1 & -8 \end{bmatrix}$

ⓔ $\begin{bmatrix}4 & -1\\0 & 2\end{bmatrix}^T \times \left(\begin{bmatrix}1 & 4 & 2\\3 & 1 & 5\end{bmatrix} \times \begin{bmatrix}1 & 4 & 2\\3 & 1 & 5\end{bmatrix}^T - \begin{bmatrix}3 & 0\\-1 & 2\\1 & 1\end{bmatrix}^T \times \begin{bmatrix}3 & 0\\-1 & 2\\1 & 1\end{bmatrix}\right) = \begin{bmatrix}4 & 0\\-1 & 2\end{bmatrix} \times \left(\begin{bmatrix}1 & 4 & 2\\3 & 1 & 5\end{bmatrix} \times \begin{bmatrix}1 & 3\\4 & 1\\2 & 5\end{bmatrix} - \begin{bmatrix}3 & -1 & 1\\0 & 2 & 1\end{bmatrix} \times \begin{bmatrix}3 & 0\\-1 & 2\\1 & 1\end{bmatrix}\right) =$

$\begin{bmatrix}4 & 0\\-1 & 2\end{bmatrix} \times \left(\begin{bmatrix}21 & 17\\17 & 35\end{bmatrix} - \begin{bmatrix}11 & -1\\-1 & 5\end{bmatrix}\right) = \begin{bmatrix}4 & 0\\-1 & 2\end{bmatrix} \times \begin{bmatrix}10 & 18\\18 & 30\end{bmatrix} = \begin{bmatrix}40 & 72\\26 & 42\end{bmatrix}$

ⓕ $\begin{bmatrix}1 & 5 & 2\\-1 & 0 & 1\\3 & 2 & 4\end{bmatrix}^T \times \begin{bmatrix}6 & 1 & 3\\-1 & 1 & 2\\4 & 1 & 3\end{bmatrix}^T - \left(\begin{bmatrix}6 & 1 & 3\\-1 & 1 & 2\\4 & 1 & 3\end{bmatrix} \times \begin{bmatrix}1 & 5 & 2\\-1 & 0 & 1\\3 & 2 & 4\end{bmatrix}\right)^T = \begin{bmatrix}1 & -1 & 3\\5 & 0 & 2\\2 & 1 & 4\end{bmatrix} \times \begin{bmatrix}6 & -1 & 4\\1 & 1 & 1\\3 & 2 & 3\end{bmatrix} - \left(\begin{bmatrix}14 & 36 & 25\\4 & -1 & 7\\12 & 26 & 21\end{bmatrix}\right)^T$

$\begin{bmatrix}14 & 4 & 12\\36 & -1 & 26\\25 & 7 & 21\end{bmatrix} - \begin{bmatrix}14 & 4 & 12\\36 & -1 & 26\\25 & 7 & 21\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$

Exercise 7:

ⓐ $\begin{bmatrix}3 & -2 & 7\end{bmatrix} \times \begin{bmatrix}6 & -2 & 4\\0 & 1 & 3\\7 & 7 & 5\end{bmatrix} = \begin{bmatrix}67 & 41 & 41\end{bmatrix}$

ⓑ $\begin{bmatrix}0 & 4 & 9\end{bmatrix} \times \begin{bmatrix}6 & -2 & 4\\0 & 1 & 3\\7 & 7 & 5\end{bmatrix} = \begin{bmatrix}63 & 67 & 57\end{bmatrix}$

ⓒ $\begin{bmatrix}3 & -2 & 7\\6 & 5 & 4\\0 & 4 & 9\end{bmatrix} \times \begin{bmatrix}-2\\1\\7\end{bmatrix} = \begin{bmatrix}41\\21\\67\end{bmatrix}$

ⓓ $\begin{bmatrix}6 & -2 & 4\\0 & 1 & 3\\7 & 7 & 5\end{bmatrix} \times \begin{bmatrix}3\\6\\0\end{bmatrix} = \begin{bmatrix}6\\6\\63\end{bmatrix}$

ⓔ $\begin{bmatrix}0 & 4 & 9\end{bmatrix} \times \begin{bmatrix}3 & -2 & 7\\6 & 5 & 4\\0 & 4 & 9\end{bmatrix} = \begin{bmatrix}24 & 56 & 97\end{bmatrix}$

ⓕ $\begin{bmatrix}3 & -2 & 7\\6 & 5 & 4\\0 & 4 & 9\end{bmatrix} \times \begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}76\\98\\97\end{bmatrix}$

Exercise 8:

ⓐ $6\begin{bmatrix}3\\6\\0\end{bmatrix} + 0\begin{bmatrix}-2\\5\\4\end{bmatrix} + 7\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}67\\64\\63\end{bmatrix}$

$-2\begin{bmatrix}3\\6\\0\end{bmatrix} + 1\begin{bmatrix}-2\\5\\4\end{bmatrix} + 7\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}41\\21\\67\end{bmatrix}$

$4\begin{bmatrix}3\\6\\0\end{bmatrix} + 3\begin{bmatrix}-2\\5\\4\end{bmatrix} + 5\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}41\\59\\57\end{bmatrix}$

ⓑ $3\begin{bmatrix}6\\0\\7\end{bmatrix} + 6\begin{bmatrix}-2\\1\\7\end{bmatrix} + 0\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}6\\6\\63\end{bmatrix}$

$-2\begin{bmatrix}6\\0\\7\end{bmatrix} + 5\begin{bmatrix}-2\\1\\7\end{bmatrix} + 4\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}-6\\17\\41\end{bmatrix}$

$7\begin{bmatrix}6\\0\\7\end{bmatrix} + 4\begin{bmatrix}-2\\1\\7\end{bmatrix} + 9\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}70\\31\\122\end{bmatrix}$

Exercise 9:

$yA = \begin{bmatrix}y_1a_{11} & y_1a_{12} & \cdots & y_1a_{1n}\\y_2a_{21} & y_2a_{22} & \cdots & y_2a_{2n}\\\vdots & \vdots & & \vdots\\y_ma_{m1} & y_ma_{m2} & \cdots & y_ma_{mn}\end{bmatrix} = y_1\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\end{bmatrix} + y_2\begin{bmatrix}a_{21} & a_{22} & \cdots & a_{2n}\end{bmatrix} + \cdots + y_m\begin{bmatrix}a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix}$

Exercise 10:

(a)
$$3\begin{bmatrix}6 & -2 & 4\end{bmatrix} + -2\begin{bmatrix}0 & 1 & 3\end{bmatrix} + 7\begin{bmatrix}7 & 7 & 5\end{bmatrix} = \begin{bmatrix}67 & 41 & 41\end{bmatrix}$$
$$6\begin{bmatrix}6 & -2 & 4\end{bmatrix} + 5\begin{bmatrix}0 & 1 & 3\end{bmatrix} + 4\begin{bmatrix}7 & 7 & 5\end{bmatrix} = \begin{bmatrix}64 & 21 & 59\end{bmatrix}$$
$$0\begin{bmatrix}6 & -2 & 4\end{bmatrix} + 4\begin{bmatrix}0 & 1 & 3\end{bmatrix} + 9\begin{bmatrix}7 & 7 & 5\end{bmatrix} = \begin{bmatrix}63 & 67 & 57\end{bmatrix}$$

(b)
$$6\begin{bmatrix}3 & -2 & 7\end{bmatrix} + -2\begin{bmatrix}6 & 5 & 4\end{bmatrix} + 4\begin{bmatrix}0 & 4 & 9\end{bmatrix} = \begin{bmatrix}6 & -6 & 70\end{bmatrix}$$
$$0\begin{bmatrix}3 & -2 & 7\end{bmatrix} + 1\begin{bmatrix}6 & 5 & 4\end{bmatrix} + 3\begin{bmatrix}0 & 4 & 9\end{bmatrix} = \begin{bmatrix}6 & 17 & 31\end{bmatrix}$$
$$7\begin{bmatrix}3 & -2 & 7\end{bmatrix} + 7\begin{bmatrix}6 & 5 & 4\end{bmatrix} + 5\begin{bmatrix}0 & 4 & 9\end{bmatrix} = \begin{bmatrix}63 & 41 & 122\end{bmatrix}$$

Exercise 11:

$$\begin{bmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{bmatrix}\left(\begin{bmatrix}1 & 5 & 2\\ -1 & 0 & 1\\ 3 & 2 & 4\end{bmatrix} \times \begin{bmatrix}6 & 1 & 3\\ -1 & 1 & 2\\ 4 & 1 & 3\end{bmatrix}\right) = \begin{bmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{bmatrix}\begin{bmatrix}- & - & 19\\ - & - & 0\\ - & - & 25\end{bmatrix} = \begin{bmatrix}- & -\\ - & 182\end{bmatrix} \Longrightarrow 182$$

Exercise 12:

(a) Let A be any matrix of size $m \times n$ and B be any matrix of size $p \times q$. Suppose that AB and BA are both defined. Thus, AB is of size $m \times q$ and $n = p$. Also, BA is of size $p \times n$ and $q = m$. Since $n = p$, BA is a square matrix, and since $q = m$, AB is a square matrix.

(b) Let A be an $m \times n$ matrix and B be a $p \times q$ matrix. Suppose that $A(BA)$ is a defined matrix. BA is defined and of size $p \times n$, and $q = m$. Since $A(BA)$ is defined, both $n = p$ and $A(BA)$ is of size $m \times n$. Since $q = m$ and $n = p$, B is a matrix of size $n \times m$.

Exercise 13:

(a) $A = \begin{bmatrix}2 & -3 & 5\\ 9 & -1 & 1\\ 1 & 5 & 4\end{bmatrix}$ $x = \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$ $b = \begin{bmatrix}7\\ -1\\ 0\end{bmatrix}$

(b) $A = \begin{bmatrix}4 & 0 & -3 & 1\\ 5 & 1 & 0 & -8\\ 2 & -5 & 9 & -1\\ 0 & 3 & -1 & 7\end{bmatrix}$ $x = \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$ $b = \begin{bmatrix}1\\ 3\\ 0\\ 2\end{bmatrix}$

Exercise 14:

ⓐ
$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 2 \\ 4x_1 + 3x_2 + 7x_3 &= -1 \\ -2x_1 + x_2 + 5x_3 &= 4 \end{aligned}$$

ⓑ
$$\begin{aligned} 3w - 2x \qquad + z &= 0 \\ 5w \qquad + 2y - 2z &= 0 \\ 3w + x + 4y + 7z &= 0 \\ -2w + 5x + y + 6z &= 0 \end{aligned}$$

Exercise 15:

ⓐ
$$AB = \left[\begin{array}{c|c} \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 7 & -1 \\ 0 & 3 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 1 \end{bmatrix}\begin{bmatrix} 7 & -1 \\ 0 & 3 \end{bmatrix} & \begin{bmatrix} 1 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix} \end{array}\right] =$$

$$\left[\begin{array}{c|c} \begin{bmatrix} -8 & 9 \\ -9 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 14 \\ 28 & 2 \end{bmatrix} & \begin{bmatrix} 0 \\ -6 \end{bmatrix} + \begin{bmatrix} -10 \\ 14 \end{bmatrix} \\ \hline \begin{bmatrix} -13 & 26 \end{bmatrix} + \begin{bmatrix} 42 & -3 \end{bmatrix} & \begin{bmatrix} 14 \end{bmatrix} + \begin{bmatrix} 27 \end{bmatrix} \end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix} -1 & 23 \\ 37 & -13 \end{bmatrix} & \begin{bmatrix} -10 \\ 8 \end{bmatrix} \\ \hline \begin{bmatrix} 29 & 23 \end{bmatrix} & \begin{bmatrix} 41 \end{bmatrix} \end{array}\right] = \begin{bmatrix} -1 & 23 & -10 \\ 37 & -13 & 8 \\ 29 & 23 & 41 \end{bmatrix}$$

ⓑ
$$AB = \left[\begin{array}{c|c} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 5 \\ 7 & -1 \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 0 & 3 \end{bmatrix} & \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} -3 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & -3 & 4 \\ 1 & 5 & 6 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 5 \\ 7 & -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 3 \end{bmatrix} & \begin{bmatrix} 0 & -3 & 4 \\ 1 & 5 & 6 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} -3 \end{bmatrix} \end{array}\right] =$$

$$\left[\begin{array}{c|c} \begin{bmatrix} -1 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 15 \end{bmatrix} & \begin{bmatrix} 5 \end{bmatrix} + \begin{bmatrix} -15 \end{bmatrix} \\ \hline \begin{bmatrix} 37 & -19 \\ 29 & 20 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 0 & 3 \end{bmatrix} & \begin{bmatrix} 14 \\ 44 \end{bmatrix} + \begin{bmatrix} -6 \\ -3 \end{bmatrix} \end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix} -1 & 23 \end{bmatrix} & \begin{bmatrix} -10 \end{bmatrix} \\ \hline \begin{bmatrix} 37 & -13 \\ 29 & 23 \end{bmatrix} & \begin{bmatrix} 8 \\ 41 \end{bmatrix} \end{array}\right] = \begin{bmatrix} -1 & 23 & -10 \\ 37 & -13 & 8 \\ 29 & 23 & 41 \end{bmatrix}$$

Exercise 16:

ⓐ
$$\left[\begin{array}{c|c} \begin{bmatrix} 3 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \end{bmatrix}\begin{bmatrix} 2 \end{bmatrix} & \begin{bmatrix} 3 & -1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} -3 \end{bmatrix}\begin{bmatrix} 1 & 4 \end{bmatrix} \\ \hline \begin{bmatrix} 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 2 \end{bmatrix} & \begin{bmatrix} 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 1 & 4 \end{bmatrix} \end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix} 3 \end{bmatrix} + \begin{bmatrix} -6 \end{bmatrix} & \begin{bmatrix} -12 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -12 \end{bmatrix} \\ \hline \begin{bmatrix} 11 \end{bmatrix} + \begin{bmatrix} 10 \end{bmatrix} & \begin{bmatrix} -20 & 24 \end{bmatrix} + \begin{bmatrix} 5 & 20 \end{bmatrix} \end{array}\right]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix} -3 \end{bmatrix} & \begin{bmatrix} -15 & -11 \end{bmatrix} \\ \hline \begin{bmatrix} 21 \end{bmatrix} & \begin{bmatrix} -15 & 44 \end{bmatrix} \end{array}\right] = \begin{bmatrix} -3 & -15 & -11 \\ 21 & -15 & 44 \end{bmatrix}$$

(b) $$\left[\begin{array}{c|c} \begin{bmatrix}2\\1\\0\end{bmatrix}\begin{bmatrix}2\end{bmatrix}+\begin{bmatrix}-5\\3\\5\end{bmatrix}\begin{bmatrix}0\end{bmatrix} & \begin{bmatrix}2\\1\\0\end{bmatrix}\begin{bmatrix}-1&3&-4\end{bmatrix}+\begin{bmatrix}-5\\3\\5\end{bmatrix}\begin{bmatrix}1&5&7\end{bmatrix} \\ \hline \begin{bmatrix}1\end{bmatrix}\begin{bmatrix}2\end{bmatrix}+\begin{bmatrix}4\end{bmatrix}\begin{bmatrix}0\end{bmatrix} & \begin{bmatrix}1\end{bmatrix}\begin{bmatrix}-1&3&-4\end{bmatrix}+\begin{bmatrix}4\end{bmatrix}\begin{bmatrix}1&5&7\end{bmatrix}\end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix}4\\2\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}-2&6&-8\\-1&3&-4\\0&0&0\end{bmatrix}+\begin{bmatrix}-5&-25&-35\\3&15&21\\5&25&35\end{bmatrix} \\ \hline \begin{bmatrix}2\end{bmatrix}+\begin{bmatrix}0\end{bmatrix} & \begin{bmatrix}-1&3&-4\end{bmatrix}+\begin{bmatrix}4&20&28\end{bmatrix}\end{array}\right]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix}4\\2\\0\end{bmatrix} & \begin{bmatrix}-7&-19&-43\\2&18&17\\5&25&35\end{bmatrix} \\ \hline \begin{bmatrix}2\end{bmatrix} & \begin{bmatrix}3&23&24\end{bmatrix}\end{array}\right] = \begin{bmatrix}4&-7&-19&-43\\2&2&18&17\\0&5&25&35\\2&3&23&24\end{bmatrix}$$

(c) $$\left[\begin{array}{c|c} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}3\\-1\\1\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} & \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}\begin{bmatrix}-2\\6\end{bmatrix} \\ \hline \begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}3\\-1\\1\end{bmatrix}+\begin{bmatrix}2&0\\-1&2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} & \begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+\begin{bmatrix}2&0\\-1&2\end{bmatrix}\begin{bmatrix}-2\\6\end{bmatrix}\end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix}3\\-1\\1\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}3\\4\\5\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix} \\ \hline \begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}4\\0\end{bmatrix} & \begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}-4\\14\end{bmatrix}\end{array}\right]$$

$$= \left[\begin{array}{c|c} \begin{bmatrix}3\\-1\\1\end{bmatrix} & \begin{bmatrix}3\\4\\5\end{bmatrix} \\ \hline \begin{bmatrix}4\\0\end{bmatrix} & \begin{bmatrix}-4\\14\end{bmatrix}\end{array}\right] = \begin{bmatrix}3&3\\-1&4\\1&5\\4&-4\\0&14\end{bmatrix}$$

Exercise 17:

(a) No. The first block requires matrix multiplication of a size 2×3 by a size 2×2. This is undefined.

(b) $$\left[\begin{array}{c|c|c} \begin{bmatrix}-1&2&1&5\\0&-3&4&2\end{bmatrix}\begin{bmatrix}2\\-3\\7\\0\end{bmatrix} & \begin{bmatrix}-1&2&1&5\\0&-3&4&2\end{bmatrix}\begin{bmatrix}1\\5\\-1\\3\end{bmatrix} & \begin{bmatrix}-1&2&1&5\\0&-3&4&2\end{bmatrix}\begin{bmatrix}4\\2\\5\\-3\end{bmatrix} \\ \hline \begin{bmatrix}1&5&6&1\end{bmatrix}\begin{bmatrix}2\\-3\\7\\0\end{bmatrix} & \begin{bmatrix}1&5&6&1\end{bmatrix}\begin{bmatrix}1\\5\\-1\\3\end{bmatrix} & \begin{bmatrix}1&5&6&1\end{bmatrix}\begin{bmatrix}4\\2\\5\\-3\end{bmatrix}\end{array}\right] = \left[\begin{array}{c|c|c} \begin{bmatrix}-1\\37\end{bmatrix} & \begin{bmatrix}23\\-13\end{bmatrix} & \begin{bmatrix}-10\\8\end{bmatrix} \\ \hline \begin{bmatrix}29\end{bmatrix} & \begin{bmatrix}23\end{bmatrix} & \begin{bmatrix}31\end{bmatrix}\end{array}\right] = \begin{bmatrix}-1&23&-10\\37&-13&8\\29&23&41\end{bmatrix}$$

Exercise 18:

(a) Let A be an $m\times n$ matrix and B be a $p\times q$ matrix. Suppose that A has a row of zeros and AB is defined. Thus, AB has size $m\times q$. If row A_m is the row of zeros, entry $(AB)_{m1} = A_{m1}B_{11} + A_{m2}B_{21} + \dots + A_{mp}B_{p1} = 0$. By similar calculations for the other entries of $(AB)_m$, each entry will be zero. Thus, AB has a row of zeros.

(b) If B has a column of zeros and AB is defined, then AB also has a column of zeros.

Exercise 19:

Suppose $K \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$

Now suppose $K \neq 0$ and $A \neq 0$. Thus, A has at least one nonzero entry. When this nonzero entry is multiplied by K, it results in a nonzero entry. Therefore $KA \neq 0$. But $KA = 0$. Thus, $K = 0$ or $A = 0$.

Exercise 20:

I is a matrix such that $I_{11} = 1$; $I_{22} = 1, \ldots, I_{nn} = 1$, and all other entries are zero.
Let A be any $n \times n$ matrix.

$(AI)_{11} = A_{11}I_{11} + A_{12}I_{21} + \cdots + A_{1n}I_{n1} = A_{11}(1) + 0 + \cdots + 0 = A_{11}$

$(AI)_{12} = A_{11}I_{12} + A_{12}I_{22} + \cdots + A_{1n}I_{n2} = 0 + A_{12}(1) + \cdots + 0 = A_{12}$

$(AI)_{nn} = A_{n1}I_{1n} + A_{n2}I_{2n} + \cdots + A_{nn}I_{nn} = 0 + 0 + \cdots + A_{nn}(1) = A_{nn}$

Thus, $AI = A$.

$(IA)_{11} = I_{11}A_{11} + I_{12}A_{21} + \cdots + I_{1n}A_{n1} = (1)A_{11} + 0 + \cdots + 0 = A_{11}$

$(IA)_{12} = I_{11}A_{12} + I_{12}A_{22} + \cdots + I_{1n}A_{n2} = (1)A_{12} + 0 + \cdots + 0 = A_{12}$

$(IA)_{nn} = I_{n1}A_{1n} + I_{n2}A_{2n} + \cdots + I_{nn}A_{nn} = 0 + 0 + \cdots + (1)A_{nn} = A_{nn}$.

Thus, $IA = A$.

Therefore $AI = IA = A$.

Exercise 21:

(a) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(b) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(c) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 & 0 & 0 \\ a_{13} & a_{23} & a_{33} & 0 & 0 & 0 \\ a_{14} & a_{24} & a_{34} & a_{44} & 0 & 0 \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} & 0 \\ a_{16} & a_{26} & a_{36} & a_{46} & a_{56} & a_{66} \end{bmatrix}$

(d) $\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$

Exercise 22:

(a) $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$

Exercise 23:

Proof: Suppose A and B are $n \times n$ matrices.

Note that $A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nn}+b_{nn} \end{bmatrix}$ Thus, $tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \cdots + (a_{nn}+b_{nn})$

$tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$ and $tr(B) = b_{11} + b_{22} + \cdots + b_{nn}$

Thus $tr(A) + tr(B) = a_{11} + b_{11} + a_{22} + b_{22} + \cdots + a_{nn} + b_{nn} = tr(A+B)$

$\square$

Exercise 24:

Consider the matrices: $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Method 1 – Product Definition: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 14 & 14 \\ 14 & 14 & 14 \\ 14 & 14 & 14 \end{bmatrix}$

Method 2 – Linear Combinations:

$1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 14 \\ 14 \end{bmatrix} \Rightarrow \text{repeat } 2\times \Longrightarrow \begin{bmatrix} 14 & 14 & 14 \\ 14 & 14 & 14 \\ 14 & 14 & 14 \end{bmatrix}$

Method 3 – Block Multiplication: $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \hline 1 & 2 & 3 \end{array}\right]\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \\ \hline 3 & 3 & 3 \end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix}\begin{bmatrix} 3 & 3 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix}\begin{bmatrix} 3 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix}\begin{bmatrix} 3 & 3 \end{bmatrix} & \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix}\begin{bmatrix} 3 \end{bmatrix} \end{array}\right]$

$= \left[\begin{array}{c|c} \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} + \begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix} & \begin{bmatrix} 5 \\ 5 \end{bmatrix} + \begin{bmatrix} 9 \\ 9 \end{bmatrix} \\ \hline \begin{bmatrix} 5 & 5 \end{bmatrix} + \begin{bmatrix} 9 & 9 \end{bmatrix} & \begin{bmatrix} 5 \end{bmatrix} + 9 \end{array}\right] = \left[\begin{array}{c|c} \begin{bmatrix} 14 & 14 \\ 14 & 14 \end{bmatrix} & \begin{bmatrix} 14 \\ 14 \end{bmatrix} \\ \hline \begin{bmatrix} 14 & 14 \end{bmatrix} & \begin{bmatrix} 14 \end{bmatrix} \end{array}\right] = \begin{bmatrix} 14 & 14 & 14 \\ 14 & 14 & 14 \\ 14 & 14 & 14 \end{bmatrix}$

Exercise 25:

Just one: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Exercise 26:

There are none.

Exercise 27:

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(b) Note that if $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$:

$a^2+bc=5 \quad ab+bd=0$

$ac+cd=0 \quad bc+d^2=9$

Since, $A = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$, c and b must be 0.

Thus, $a^2=5$ and $d^2=9$, and so $a=\pm\sqrt{5}$ $d=\pm 3$

There are 4 square roots of A: $\begin{bmatrix} \pm\sqrt{5} & 0 \\ 0 & \pm 3 \end{bmatrix}$

(c) No. For example: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$a^2+bc=-1 \quad ab+bd=0$

$ac+cd=0 \quad bc+d^2=1$

Thus, c and b must be zero, so $a^2=-1$ and $d^2=1$.

undefined.

Exercise 28:

(a) $AA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$a^2+bc=0 \quad ab+bd=0$

$ac+cd=0 \quad bc+d^2=0$

$A \neq 0 \Rightarrow A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

(b) $AA=A$ and $A \neq 0$

$a^2+bc=a \quad ab+bd=b$

$ac+cd=c \quad bc+d^2=d$

Thus, $a+d=1$

The following matrices are acceptable: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Exercise 29:

(a) True. let a be any $n \times n$ matrix. A^T is a $n \times m$ matrix. Thus (AA^T) is $m \times m$ and (A^TA) is $n \times n$.

(b) True

(c) False

(d) True.

Exercise 30:

(a) True.

(b) False (e.g. $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$).

(c) True

(d) True