1.4: Operations On Sets

Exercise 1:

ⓐ $\{3, 12\}$

ⓑ $A \cup B = \{1, 3, 7, 12, 20, 35\}$

$(A \cup B) \setminus C = \{1, 12, 20, 35\}$

ⓒ $(B \setminus C) = \{12, 20\}$

$A \cup (B \setminus C) = \{1, 3, 12, 20, 35\}$

- None of the above sets are disjoint.
- Set a $\subseteq$ Set c and Set b $\subseteq$ Set c

Exercise 2:

ⓐ $A \cup B$ = {U.S., Germany, China, Australia, France, India, Brazil}.

ⓑ $(A \cap B)$ = {Germany}

$(A \cap B) \setminus C = \emptyset$

ⓒ $(B \cap C)$ = {Germany, France}

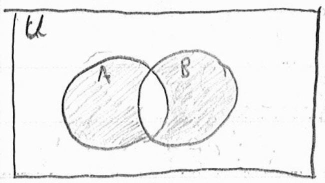
$(B \cap C) \setminus A$ = {France}

*Every set is disjoint with Set b.
- Set b is a subset of Sets a and c. Set c is a subset of Set a.

Exercise 3:

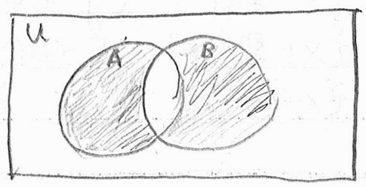
$x \in (A \cup B) \setminus (A \cap B)$

$(x\in A \vee x \in B) \wedge (x \notin A \vee x \notin B)$



$x \in (A\setminus B)\cup(B\setminus A)$

$(x\in A \wedge x\notin B) \vee (x \in B \wedge x \notin A)$



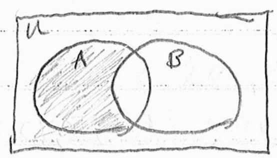
Exercise 4:

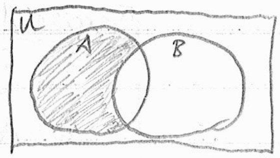
ⓐ $(x\in A)\wedge(x\notin(A\cap B))$

$x\in A \wedge (x\notin A \vee x\notin B)$

$(x\in A\wedge x\notin A)\vee(x\in A\wedge x\notin B)$

$(x\in A \wedge x\notin B)$

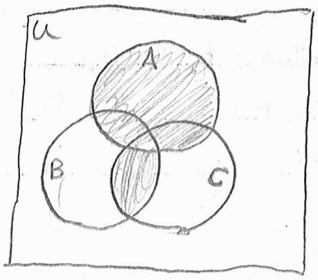




$A\setminus B \Rightarrow x\in A\wedge x\notin B$

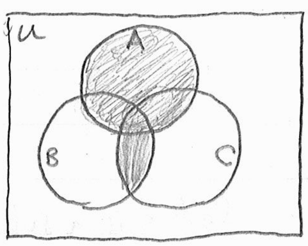
ⓑ $A\cup(B\cap C)$

$x\in A \vee (x\in B\wedge x\in C)$



$(A\cup B)\cap(A\cup C)$

$(x\in A\vee x\in B)\wedge(x\in A\vee x\in C) \Rightarrow x\in A\vee(x\in B\wedge x\in C)$



Exercise 5:

ⓐ See previous exercise.

ⓑ See previous exercise.

Exercise 6:

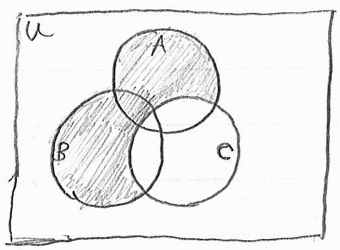
ⓐ $(A \cup B) \setminus C$

$(x \in A \vee x \in B) \wedge x \notin C$

$(A \setminus C) \cup (B \setminus C)$

$(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$

$(x \in A \vee x \in B) \wedge x \notin C$



ⓑ $A \cup (B \setminus C)$

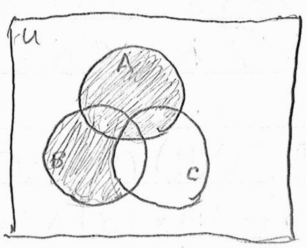
$x \in A \vee (x \in B \wedge x \notin C)$

$(A \cup B) \setminus (C \setminus A)$

$(x \in A \vee x \in B) \wedge (x \notin (C \setminus A))$

$(x \in A \vee x \in B) \wedge (x \notin C \vee x \in A)$

$x \in A \vee (x \in B \wedge x \notin C)$



Exercise 7:

ⓐ See previous exercise.

ⓑ See previous exercise.

Exercise 8:

ⓐ $(A \setminus B) \cap C$

$(x \in A \wedge x \notin B) \wedge x \in C$

$(x \in A \wedge x \in C) \wedge x \notin B$

$(A \cap C) \setminus B$

ⓑ $(A \cap B) \setminus B$

$(x \in A \wedge x \in B) \wedge x \notin B$

$x \in A \wedge (x \in B \wedge x \notin B)$ (contradiction)

$\varnothing$

ⓒ $A \setminus (A \setminus B)$

$x \in A \wedge (x \notin (A \setminus B))$

$x \in A \wedge (x \notin A \vee x \in B)$

$(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$

$(x \in A \wedge x \in B)$

$A \cap B$

Exercise 9:

ⓐ $(A \setminus B) \setminus C$

$(x \in A \wedge x \notin B) \wedge x \notin C$

$x \in A \wedge x \notin B \wedge x \notin C$

ⓑ $A \setminus (B \setminus C)$

$x \in A \wedge \neg(x \in B \wedge x \notin C)$

$x \in A \wedge (x \notin B \vee x \in C)$

ⓒ $(A \setminus B) \cup (A \cap C)$

$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \in C)$

$x \in A \wedge (x \notin B \vee x \in C)$

ⓓ $(A \setminus B) \cap (A \setminus C)$

$(x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)$

$x \in A \wedge x \notin B \wedge x \notin C$

ⓔ $A \setminus (B \cup C)$

$x \in A \wedge \neg(x \in B \vee x \in C)$

$x \in A \wedge (x \notin B \wedge x \notin C)$

$x \in A \wedge x \notin B \wedge x \notin C$

a, d, and e are equivalent.

b and c are equivalent.

Exercise 10:

ⓐ $(A \cup B) \setminus B$

$(x \in A \vee x \in B) \wedge x \notin B$

Example 1:

$A = \{x, y\}$

$B = \{y\}$

$(A \cup B) \setminus B = \{x\}$

$\{x\} \neq \{x, y\}$

Example 2:

$A = \{x\}$

$B = \{x\}$

$(A \cup B) \setminus B = \varnothing$

$\varnothing \neq \{x\}$

ⓑ Proof: Let A and B be any sets. Consider any x such that $x \in (A \cup B) \setminus B$.

Note that $(x \in A \vee x \in B) \wedge x \notin B$

$(x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin B)$

$x \in A \wedge x \notin B$

$x \in A \setminus B$

Thus, $(A \cup B) \setminus B = A \setminus B$

$\square$

Exercise 11:

$x \in (A \setminus B) \cup B$

$(x \in A \wedge x \notin B) \vee x \in B$

$(x \in A \vee x \in B) \wedge (x \notin B \vee x \in B)$

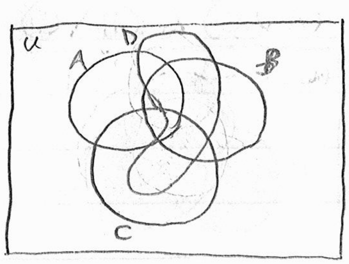
$x \in A \vee x \in B$

Thus, $A \subseteq (A \setminus B) \cup B$, and $(A \setminus B) \cup B = A \cup B$

Exercise 12:

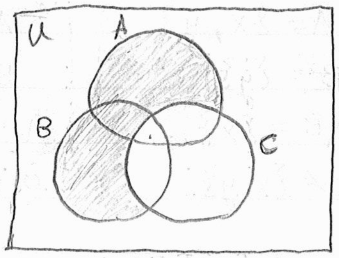
ⓐ The Venn diagram is missing sets. (i.e., $(A \cap D) \setminus (B \cup C)$).

ⓑ

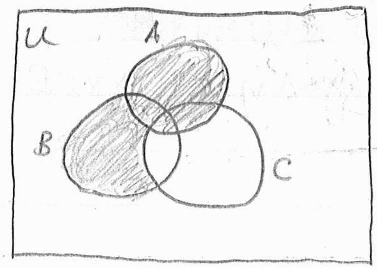


Exercise 13:

ⓐ



$(A \cup B) \setminus C$



$A \cup (B \setminus C)$

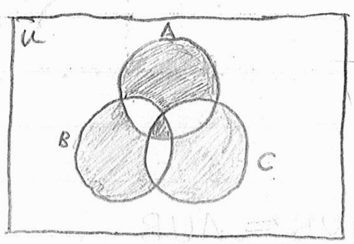
$$(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$$

ⓑ $A = \{x\}$ $\qquad (A \cup B) \setminus C = \varnothing$

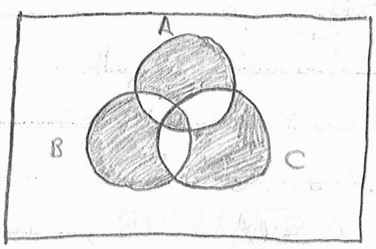
$B = \{x\}$ $\qquad A \cup (B \setminus C) = \{x\}$

$C = \{x\}$ $\qquad (A \cup B) \setminus C \neq A \cup (B \setminus C)$

Exercise 14:



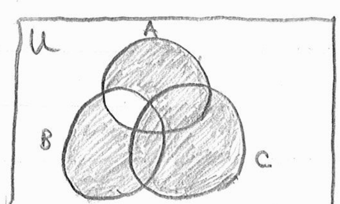
$A \triangle (B \triangle C)$

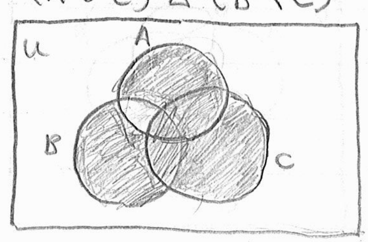


$(A \triangle B) \triangle C$

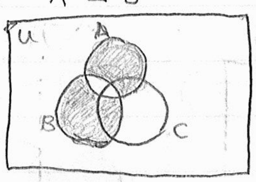
Exercise 15:

ⓐ $(A \triangle B) \cup C$ $\qquad (A \cup C) \triangle (B \setminus C)$

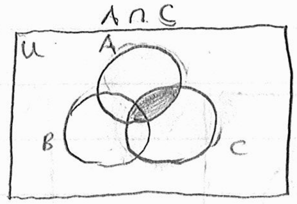




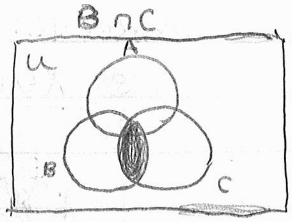
(b) $A \triangle B$



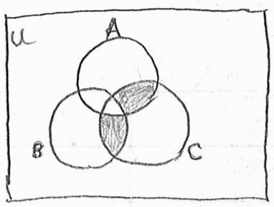
$A \cap C$



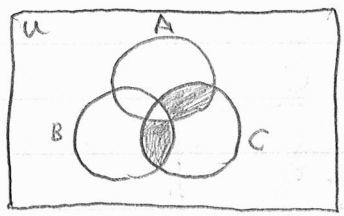
$B \cap C$



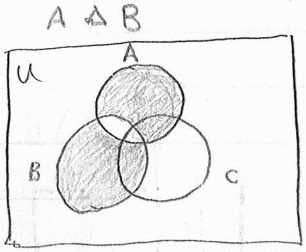
$(A \triangle B) \cap C$



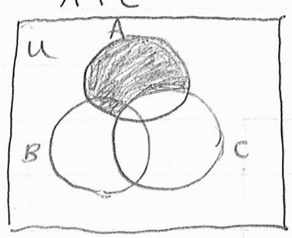
$(A \cap C) \triangle (B \cap C)$



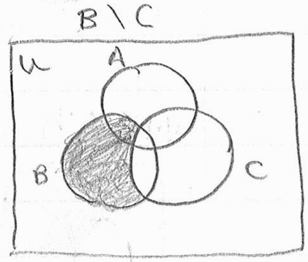
(c) $A \triangle B$



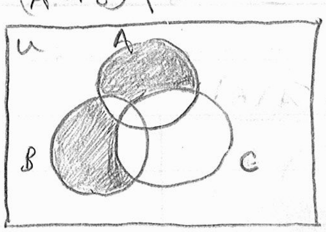
$A \setminus C$



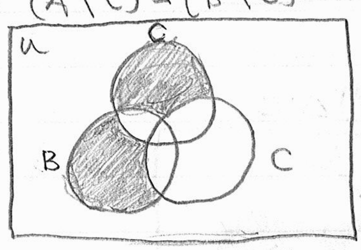
$B \setminus C$



$(A \triangle B) \setminus C$

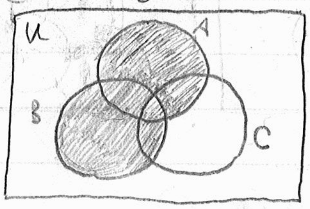


$(A \setminus C) \triangle (B \setminus C)$

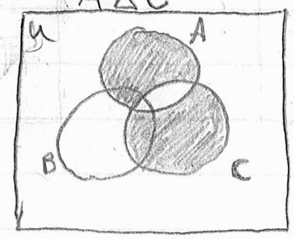


Exercise 16:

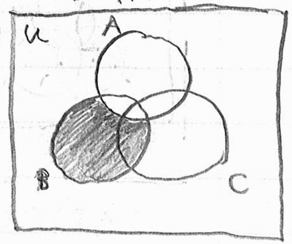
ⓐ $A \cup B$



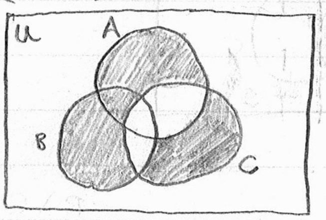
$A \Delta C$



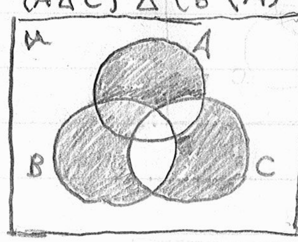
$B \setminus A$



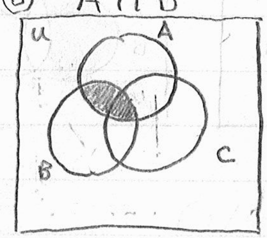
$(A \cup B) \Delta C$



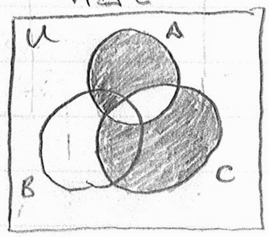
$(A \Delta C) \Delta (B \setminus A)$



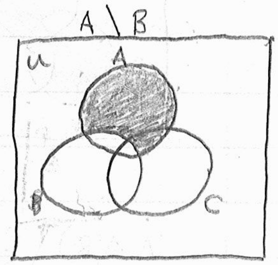
ⓑ $A \cap B$



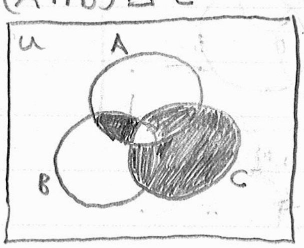
$A \Delta C$



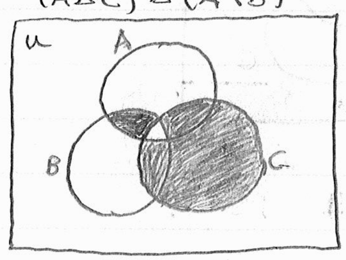
$A \setminus B$



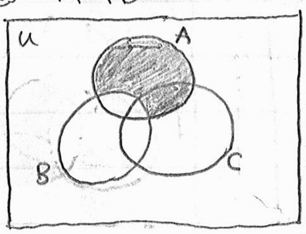
$(A \cap B) \Delta C$



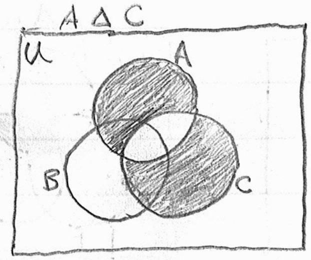
$(A \Delta C) \Delta (A \setminus B)$



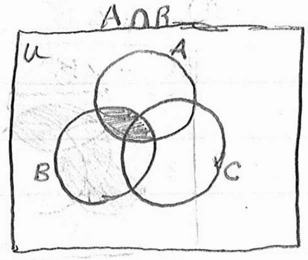
(c) $A \setminus B$



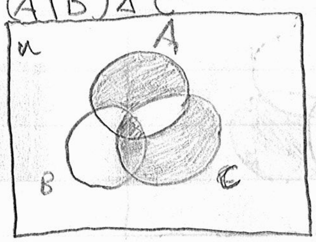
$A \triangle C$



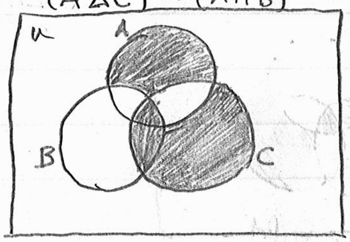
$A \cap B$



$(A \setminus B) \triangle C$

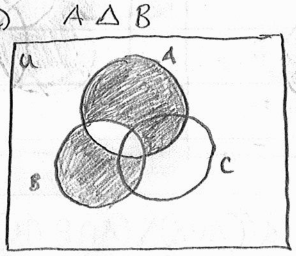


$(A \triangle C) \triangle (A \cap B)$

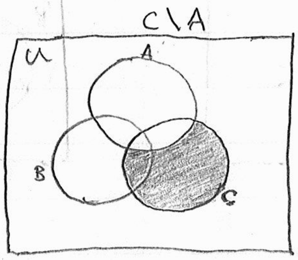


Exercise 17:

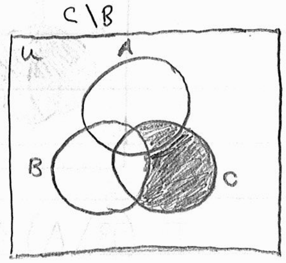
(a) $A \triangle B$



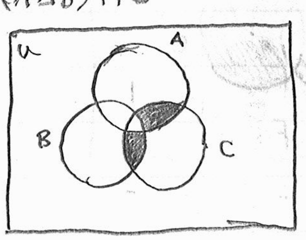
$C \setminus A$



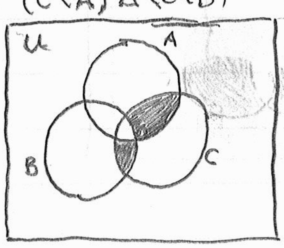
$C \setminus B$



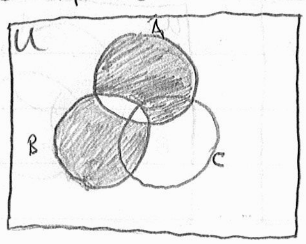
$(A \triangle B) \cap C$



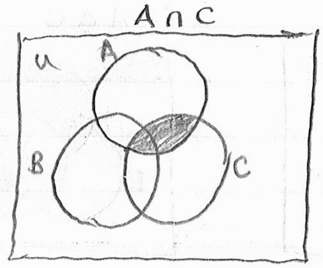
$(C \setminus A) \triangle (C \setminus B)$



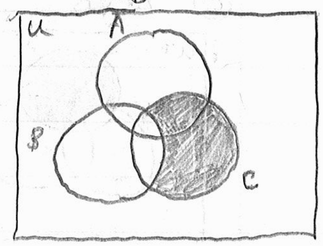
(b) $A \triangle B$



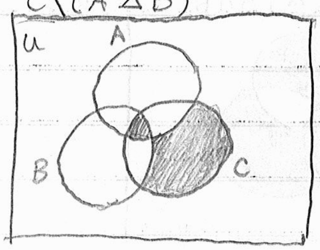
$A \cap C$



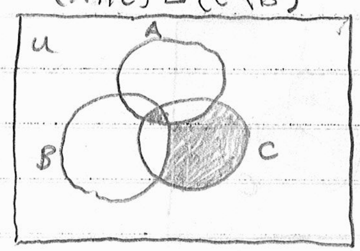
$C \setminus B$



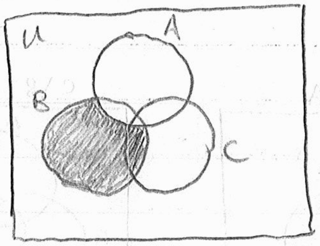
$C \setminus (A \triangle B)$



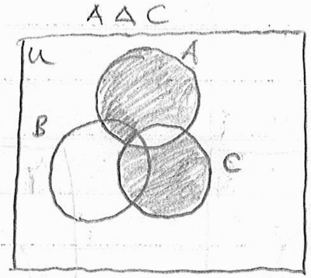
$(A \cap C) \triangle (C \setminus B)$



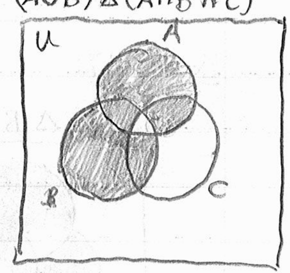
(c) $B \setminus A$



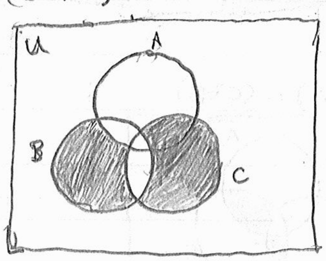
$A \triangle C$



$(A \cup B) \triangle (A \cap B \cap C)$



$(B \setminus A) \triangle C$



$(A \triangle C) \triangle ((A \cup B) \setminus (A \cap B \cap C))$