1.3: Variables and Sets

Exercise 1:

(a) $D(x) = 3$ is a divisor of x

$D(6) \wedge D(9) \wedge D(15)$

(b) $D(x, y) =$ x is divisible by y

$D(x,2) \wedge D(x,3) \wedge \neg D(x,4)$

(c) $N(x)$: x is a natural number ; $P(x)$: x is prime

$[N(x) \wedge N(y)] \wedge [P(x) + P(y)]$

Exercise 2:

(a) $M(x)$: x is a man ; $T(x,y)$: x is taller than y

$(M(x) \wedge M(y)) \wedge (T(x,y) \vee T(y,x))$

(b) $B(x)$: x has brown eyes; $R(x)$: x has red hair

$(B(x) \vee B(y)) \wedge (R(x) \vee R(y))$

(c) $(B(x) \wedge R(x)) \vee (B(y) \wedge R(y))$

Exercise 3:

(a) $\{x \mid x \text{ is a planet}\}$

(b) $\{x \mid x \text{ is an ivy league university}\}$

(c) $\{x \mid x \text{ is a state}\}$

(d) $\{x \mid x \text{ is a Canadian Province}\}$

Exercise 4:

(a) $\{x \mid \sqrt{x} \in \mathbb{N}\}$ or $\{x^2 \mid x > 0 \text{ and } x \in \mathbb{N}\}$

(b) $\{x \mid x = 2^n \text{ such that } n \in \mathbb{N}\}$ or $\{2^x \mid x \in \mathbb{N}\}$

(c) $\{x \mid x = n \text{ such that } 10 \leq n \leq 19 \text{ and } n \text{ is an integer}\}$ or $\{x \in \mathbb{N} \mid 10 \leq x \leq 19\}$

Exercise 5:

ⓐ No free variables. The statement is true.

ⓑ No free variables. The statement is false.

ⓒ X is a bound variable; c is a free variable.

Exercise 6:

ⓐ $(w \in \mathbb{R}) \wedge (13 - 2(w) > c)$. w and c are free.

ⓑ $(4 \in \mathbb{R}) \wedge (13 - 2(4) \in P)$. No free variables. True.

ⓒ $(4 \in P) \wedge (13 - 2(4) > 1)$. No free variables. False.

Exercise 7:

ⓐ $(x \in \mathbb{R}) \wedge (2x^2 + x - 1 = 0)$

$(x \in \mathbb{R}) \wedge ((x+2)(x-1) = 0)$

$\{-2, 1\}$.

ⓑ $(x \in \mathbb{R}^+) \wedge (2x^2 + x - 1 = 0)$

$(x \in \mathbb{R}^+) \wedge ((x+2)(x-1) = 0)$

$\{1\}$

ⓒ $(x \in \mathbb{Z}) \wedge (2x^2 + x - 1 = 0)$

$(x \in \mathbb{Z}) \wedge ((x+2)(x-1) = 0)$

$\{-2, 1\}$

ⓓ $(x \in \mathbb{N}) \wedge (2x^2 + x - 1 = 0)$

$(x \in \mathbb{N}) \wedge ((x+2)(x-1) = 0)$

$\{1\}$

Exercise 8: $\{x \mid$ Elizabeth Taylor was once married to $x\}$.

ⓐ Everyone who Elizabeth Taylor has been married to: {Conrad Hilton Jr., Michael Wilding, ...}

ⓑ $\{x \mid x$ is a logical connective studied in §1.1$\}$: $\{\neg, \wedge, \vee\}$

ⓒ $\{x \mid x$ is the author of this book$\}$: {Daniel J. Velleman}

Exercise 9:

ⓐ $\{x \in \mathbb{R} \mid x^2 - 4x + 3 = 0\}$: $\{1, 3\}$

ⓑ $\{x \in \mathbb{R} \mid x^2 - 2x + 3 = 0\}$: $\{-1, 3\}$

ⓒ $\{x \in \mathbb{R} \mid 5 \in \{y \in \mathbb{R} \mid x^2 + y^2 < 50\}\}$

$\{x \in \mathbb{R} \mid x^2 + 25 < 50\}$:

$\{x \in \mathbb{R} \mid x^2 < 25\}$: $\{-4, -3, -2, \dots\}$

[redacted]