1.2: Truth Tables

Exercise 1:

(a)

P	Q	¬P	∨	Q
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

(b)

S	G	(S ∨ G)	∧	(¬S ∨ ¬G)
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Exercise 2:

(a)

P	Q	¬	[P	∧	(Q	∨	¬P)]
T	T	F	T	T	T	T	F
T	F	T	T	F	F	F	F
F	T	T	F	F	T	T	F
F	F	T	F	F	F	T	F

(b)

P	Q	R	(P	∨	Q)	∧	(¬P	∨	R)
T	T	T	T	T	T	T	F	T	T
T	T	F	T	T	T	F	F	T	F
T	F	T	T	T	F	T	F	T	T
T	F	F	T	T	F	F	F	T	F
F	T	T	F	T	T	T	T	T	T
F	T	F	F	T	T	F	T	F	F
F	F	T	F	F	F	T	T	T	T
F	F	F	F	F	F	F	T	F	F

Exercise 3:

(a)

P	Q	P + Q
T	T	F
T	F	T
F	T	T
F	F	F

(b)

P	Q	(P ∨ Q)	∧	¬	(P	∧	Q)
T	T	T	F	F	T	T	T
T	F	T	T	T	T	F	F
F	T	T	T	T	F	F	T
F	F	F	F	T	F	F	F

Exercise 4:

P	Q	¬	(¬P	∧	¬Q)	P	∨	Q
T	T	T	F	F	F	T	T	T
T	F	T	F	T	F	T	T	F
F	T	T	T	F	F	F	T	T
F	F	F	T	T	T	F	F	F

Exercise 5:

(a)

P	Q	P	↓	Q
T	T	T	F	T
T	F	T	F	F
F	T	F	F	T
F	F	F	T	F

(b)

P	Q	$\neg P$	$\wedge$	$\neg Q$
T	T	F	F	T
T	F	F	F	F
F	T	T	F	T
F	F	T	T	F

(c)

P	$\neg P$	$P \downarrow P$	$\neg P$
T	F	F	T
F	T	T	F

P	Q	$(P \downarrow Q) \downarrow (P \downarrow Q)$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

P	Q	$(P \downarrow P) \downarrow (Q \downarrow Q)$	$P \wedge Q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Exercise 6:

(a)

P	Q	$P \mid Q$
T	T	F
T	F	T
F	T	T
F	F	T

(b)

P	Q	$\neg P \vee \neg Q$
T	T	F T F F T
T	F	F T T T F
F	T	T F T F T
F	F	T F T T F

(c)

P	$\neg P$	$P \mid P$
T	F T	T F T
F	T F	F T F

P	Q	$P \vee Q$	$(P \mid P) \mid (Q \mid Q)$
T	T	T T T	T T F T T F T
T	F	T T F	T F F T F F F
F	T	T T F	F T F T T F T
F	F	F F F	F T F F F T F

P	Q	$P \wedge Q$	$(P \mid Q) \mid (P \mid Q)$
T	T	T T T	T F F T T F F T
T	F	T F F	T T F F T T F
F	T	F F F	F T T F F T T
F	F	F F F	F T F F F T F

Exercise 7:

a is valid.

	J	P	C	¬(J ∧ P)	P ∨ C	J	∴ C
	T	T	T	F	T	T	T
	T	T	F	F	T	T	F
	T	F	T	T	T	T	T
	T	F	F	T	F	T	F
	F	T	T	T	T	F	T
	F	T	F	T	T	F	F
	F	F	T	T	T	F	T
	F	F	F	T	F	F	F

b is invalid

	B	F	P	C	B ∨ F	P ∨ C	¬(F ∧ C)	∴ ¬(B ∧ P)
	T	T	T	T	T	T	F	F
→	T	T	T	F	T	T	T	T
	T	T	F	T	T	T	F	F
	T	T	F	F	T	F	T	F
→	T	F	T	T	T	T	T	T
→	T	F	T	F	T	T	T	T
	F	F	T	T	F	T	F	F
	T	F	F	T	T	T	F	F
	F	F	F	T	F	F	T	F
	T	T	T	F	T	T	T	F
	F	T	T	F	T	T	T	F
	T	T	F	F	T	F	T	F
	F	T	F	F	T	F	T	F
	T	F	T	T	T	T	T	T
	F	F	T	T	F	T	F	F
	T	F	F	T	T	T	F	F
	F	F	F	T	F	F	T	F
	T	T	T	F	T	T	T	F
	F	T	T	F	T	T	T	F
	T	T	F	F	T	F	T	F
	F	T	F	F	T	F	T	F
	T	F	T	T	T	T	T	T
	F	F	T	T	F	T	F	F
	T	F	F	T	T	T	F	F
	F	F	F	T	F	F	T	F

© J B S $J \vee B$ $\neg S \vee \neg B$ $\therefore J \vee \neg S$

J	B	S	$J \vee B$	$\neg S \vee \neg B$	$J \vee \neg S$
T	T	T	T	F F F	T F
T	T	F	T	T T F	T T
T	F	T	T	F T T	T F
T	F	F	T	T T T	T T
F	T	T	T	F F F	F F
F	T	F	T	T T F	T T
F	F	T	F	F T T	F F
F	F	F	F	T T T	T T

c is valid.

(d) S E H $(S \wedge H) \vee (E \wedge \neg H) \therefore \neg(S \wedge E)$

S	E	H	$(S \wedge H) \vee (E \wedge \neg H)$				$\neg(S \wedge E)$	
T	T	T	T	T	F	F	F	T
T	T	F	F	T	T	T	F	T
T	F	T	T	T	F	F	T	F
T	F	F	F	F	F	T	T	F
F	T	T	F	F	F	F	T	F
F	T	F	F	T	T	T	T	F
F	F	T	F	F	F	F	T	F
F	F	F	F	F	F	T	T	F

d is invalid

Exercise 8:

P	Q	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$ (a)					$\neg P \vee Q$ (b)		$(P \vee \neg Q) \wedge (Q \vee \neg P)$ (c)				$\neg(P \vee Q)$ (d)		$(Q \wedge P) \vee \neg P$ (e)		
T	T	T	T	F	F	F	F	T	T	F	T	T	F	T	T	T	F
T	F	F	F	F	F	T	F	F	T	T	F	F	F	T	F	F	F
F	T	F	F	T	F	F	T	T	F	F	F	T	F	T	F	T	T
F	F	F	T	T	T	T	T	T	T	T	T	T	T	F	F	T	T

a and c are equivalent. b and e are equivalent.

Exercise 9:

P	Q	$(P \vee Q) \wedge (\neg P \vee \neg Q)$ ⓐ : neither	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$ ⓑ : contradiction	$(P \vee Q) \vee (\neg P \vee \neg Q)$ ⓒ : tautology
T	T	F F F F	F F F F	T T F F
T	F	T T F T	F F F T	T T F T
F	T	T T T F	F F T F	T T T F
F	F	F F T T	F F T T	F T T T

ⓓ : tautology

P	Q	R	$[P \wedge (Q \vee \neg R)] \vee (\neg P \vee R)$
T	T	T	T T T T F T
T	T	F	T T T T F F
T	F	T	F F F T F T
T	F	F	T T T T F F
F	T	T	F T F T T T
F	T	F	F T T T T T
F	F	T	F F F T T T
F	F	F	F T T T T T

Exercise 10:

ⓐ

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F T	F F F
T	F	F T	F F T
F	T	F T	T F F
F	F	T F	T T T

ⓑ

P	Q	R	P ∧ (Q ∨ R)	(P ∧ Q) ∨ (P ∧ R)	P ∨ (Q ∧ R)	(P ∨ Q) ∧ (P ∨ R)
T	T	T	T T T	T T T	T T T	T T T
T	T	F	T T T	T T F	T F T	T T T
T	F	T	T T T	F F T	T F T	T T T
T	F	F	F F F	F F F	T F F	T T F
F	T	T	F T T	F F F	T T T	T T T
F	T	F	F T F	F F F	F F F	T F F
F	F	T	F T T	F F F	F F T	F F T
F	F	F	F F F	F F F	F F F	F F F

Exercise 11:

ⓐ $\neg(\neg P \wedge \neg Q) :: \neg\neg(P \vee Q) : P \vee Q$

ⓑ $(P \wedge Q) \vee (P \wedge \neg Q) :: P \wedge (Q \vee \neg Q) :: P$

ⓒ $\neg(P \wedge \neg Q) \vee (\neg P \wedge Q) :: (\neg P \vee Q) \vee (\neg P \wedge Q) :: (\neg P \vee Q \vee \neg P) \wedge (\neg P \vee Q \vee Q)$
$:: (\neg P \vee Q) \wedge (\neg P \vee Q) :: \neg P \vee Q$

Exercise 12:

ⓐ $\neg(\neg P \vee Q) \vee (P \wedge \neg R) :: (P \wedge \neg Q) \vee (P \wedge \neg R) :: P \wedge (\neg Q \vee \neg R) :: P \wedge \neg(Q \wedge R)$

ⓑ $\neg(\neg P \wedge Q) \vee (P \wedge \neg R) :: (P \vee \neg Q) \vee (P \wedge \neg R) :: (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) ::$
$(P \vee \neg Q) \wedge (P \vee \neg Q \vee \neg R) :: (P \vee \neg Q)$

ⓒ $(P \wedge R) \vee [\neg R \wedge (P \vee Q)] :: (P \wedge R) \vee [(\neg R \wedge P) \vee (\neg R \wedge Q)] :: [(P \wedge R) \vee (\neg R \wedge P)] \vee (\neg R \wedge Q)$
$:: [((P \wedge R) \vee \neg R) \wedge ((P \wedge R) \vee P)] \vee (\neg R \wedge Q) :: [((P \vee \neg R) \wedge (R \vee \neg R)) \wedge ((P \wedge R) \vee P)] \vee (\neg R \wedge Q)$
$:: [(P \vee \neg R) \wedge ((P \wedge R) \vee P)] \vee (\neg R \wedge Q) :: [(P \vee \neg R) \wedge P] \vee (\neg R \wedge Q)$
$:: [(P \wedge P) \vee (\neg R \wedge P)] \vee (\neg R \wedge Q) :: [P \vee (\neg R \wedge P)] \vee (\neg R \wedge Q) :: P \vee (\neg R \wedge Q)$

Exercise 13:

$\neg(P \vee Q) :: \neg(\neg P \vee \neg Q) :: \neg\neg(\neg P \wedge \neg Q) :: \neg P \wedge \neg Q$

Exercise 14:

$[P \wedge (Q \wedge R)] \wedge S :: [(P \wedge Q) \wedge R] \wedge S :: (P \wedge Q) \wedge (R \wedge S)$

Exercise 15: 2^n lines.

Exercise 16:

P	Q	$P \vee \neg Q$
F	F	T
F	T	F
T	F	T
T	T	T

Exercise 17:

P	Q	$(P \wedge \neg Q) \vee (Q \wedge \neg P)$
F	F	F
F	T	T
T	F	T
T	T	F

Exercise 18:

Conc. is tautology: either all premises are true, or not all premises are false.
Conc. is contradiction: if all premises are true then argument is invalid; if any premise is false, then valid.
Premise is tautology: validity depends.
Premise is contradiction: argument is always valid (one premise is always false) (at least)