1.5. The Conditional and Biconditional Connective

Exercise 1:

(a) $(U \vee \neg E) \rightarrow \neg H$

(b) $(F \wedge H) \rightarrow D$

(c) $(F \rightarrow D) \wedge (H \rightarrow D)$

(d) $(x \neq 2) \rightarrow (P(x) \rightarrow O(x))$

Exercise 2:

(a) $S \rightarrow (P \wedge A)$

(b) $M \rightarrow (C \wedge D)$

(c) $\neg D \rightarrow S$

(d) $(D(x,4) \vee D(x,6)) \rightarrow \neg P(x)$

Exercise 3:

(a) $R \rightarrow (W \wedge \neg S)$

(b) $(W \wedge \neg S) \rightarrow R$ (equivalent to converse of a).

(c) $R \rightarrow (W \wedge \neg S)$ (equivalent to a).

(d) $(W \wedge \neg S) \rightarrow R$ (equivalent to converse of a).

(e) $(S \vee \neg W) \rightarrow \neg R$

$R \rightarrow \neg(S \vee \neg W)$

$R \rightarrow \neg(\neg W \vee S)$

$R \rightarrow (W \wedge \neg S)$ (equivalent to a).

(f) $R \rightarrow (W \wedge \neg S)$ (equivalent to a).

(g) $(W \rightarrow R) \vee (\neg S \rightarrow R)$

$(\neg W \vee R) \vee (S \vee R)$

$(R \vee R) \vee (\neg W \vee S)$

$R \vee \neg(W \wedge \neg S)$

$\neg R \rightarrow \neg(W \wedge \neg S)$

$(W \wedge \neg S) \rightarrow R$ (equivalent to converse of a).

Exercise 4:

(a)

	S	E	H	S ∨ E	S → H	E → ¬H	¬(S ∧ E)
	T	T	T	T	T	F F	F T
	T	T	F	T	F	T T	F T
→	T	F	T	T	T	T F	T F
	T	F	F	T	F	T T	T F
	F	T	T	T	T	F F	T F
→	F	T	F	T	T	T T	T F
	F	F	T	F	T	T F	T F
	F	F	F	F	T	T T	T F

Argument is valid.

(b)

	T	U	G	R	(T ∧ U) → R	G → ¬R	G ∧ T	¬U
	T	T	T	T	T T	F F	T	F
	T	T	T	F	T F	T T	T	F
	T	T	F	T	T T	T F	F	F
	T	T	F	F	T F	T T	F	F
	T	F	T	T	F T	F F	T	T
→	T	F	T	F	F T	T T	T	T
	T	F	F	T	F T	T F	F	T
	T	F	F	F	F T	T T	F	T
	F	T	T	T	F T	F F	F	F
	F	T	T	F	F T	T T	F	F
	F	T	F	T	F T	T F	F	F
	F	T	F	F	F T	T T	F	F
	F	F	T	T	F T	F F	F	T
	F	F	T	F	F T	T T	F	T
	F	F	F	T	F T	T F	F	T
	F	F	F	F	F T	T T	F	T

Argument is valid.

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	W	P	R	W ↔ (P ∧ R)		¬R	W ↔ P
	T	T	T	T	T	F	T
	T	T	F	F	F	T	T
	T	F	T	F	F	F	F
	T	F	F	F	F	T	F
	F	T	T	F	T	F	F
→	F	T	F	T	F	T	F
	F	F	T	T	F	F	T
→	F	F	F	T	F	T	T

The argument is invalid.

Exercise 5:

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	C	P	T	C → P	C → T	¬P ∨ T	
→	T	T	T	T	T	F	T
	T	T	F	T	F	F	F
	T	F	T	F	T	T	T
	T	F	F	F	F	T	T
→	F	T	T	T	T	F	T
→	F	T	F	T	T	F	F
→	F	F	T	T	T	T	T
→	F	F	F	T	T	T	T

The argument is invalid.

⑥

	D	R	B	(D ∨ R) ∧ ¬(D ∧ R)	B → D	R → ¬B
	T	T	T	T F F T	T	F F
	T	T	F	T F F T	T	T T
→	T	F	T	T T T F	T	T F
→	T	F	F	T T T F	T	T T
	F	T	T	T T T F	F	F F
→	F	T	F	T T T F	T	T T
	F	F	T	F F T F	F	T F
	F	F	F	F F T F	T	T T

The argument is valid.

Exercise 6:

ⓐ

P	Q	P ↔ Q	(P ∧ Q) ∨ (¬P ∧ ¬Q)
T	T	T	T T F F F
T	F	F	F F F F T
F	T	F	F F T F F
F	F	T	F T T T T

ⓑ

P	Q	R	(P → Q) ∨ (P → R)	P → (Q ∨ R)
T	T	T	T T T	T T
T	T	F	T T F	T T
T	F	T	F T T	T T
T	F	F	F F F	F F
F	T	T	T T T	T T
F	T	F	T T T	T T
F	F	T	T T T	T T
F	F	F	T T T	T F

Exercise 7:

(a)

P	Q	R	$(P \to R) \wedge (Q \to R)$	$(P \vee Q) \to R$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

(b)

P	Q	R	$(P \to R) \vee (Q \to R)$	$(P \wedge Q) \to R$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Exercise 8:

(a)

P	Q	R	$(P \to Q) \wedge (Q \to R)$	$(P \to R) \wedge [(P \leftrightarrow Q) \vee (R \leftrightarrow Q)]$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

(b)

P	Q	R	(P → Q)	v	(Q → R)
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	T

Exercise 9:

P	Q	P ∧ Q	¬(P → ¬Q)		
T	T	T	T	F	F
T	F	F	F	T	T
F	T	F	F	T	F
F	F	F	F	T	T

Exercise 10:

P	Q	P ↔ Q	¬((P → Q) → ¬(Q → P))				
T	T	T	T	T	F	F	T
T	F	F	F	F	T	F	T
F	T	F	F	T	T	T	F
F	F	T	T	T	F	F	T

Exercise 11:

(a)

P	Q	(P v Q)	↔	Q	P → Q
T	T	T	T		T
T	F	T	F		F
F	T	T	T		T
F	F	F	T		T

(b)

P	Q	$(P \wedge Q) \leftrightarrow Q$	$Q \rightarrow P$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

Exercise 12:

P	Q	R	(a) $P \rightarrow (Q \rightarrow R)$	(b) $Q \rightarrow (P \rightarrow R)$	(c) $(P \rightarrow Q) \wedge (P \rightarrow R)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

P	Q	R	(d) $(P \wedge Q) \rightarrow R$	(e) $P \rightarrow (Q \wedge R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

a, b, and d are equivalent.
c and e are equivalent