Chapter 1: Systems of Linear Equations and Matrices

1.1: Introduction to Systems of Linear Equations

Exercise 1: a, c, and f are linear equations.

Exercise 2: a, b, and c are linear equations.

Exercise 3:

(a) $x = t \qquad -5y = 3 - 7t$

$$y = -\frac{3}{5} + \frac{7}{5}t = \frac{7}{5}t - \frac{3}{5}$$

(b) $x_1 = s \qquad x_2 = t \qquad 4x_3 = 7 - 3s + 5t$

$$x_3 = \frac{7}{4} - \frac{3}{4}s + \frac{5}{4}t$$

(c) $x_1 = r \qquad x_2 = s \qquad x_3 = t \qquad 6x_4 = 1 + 8r - 2s + 5t$

$$x_4 = \frac{1}{6} + \frac{4}{3}r - \frac{1}{3}s + \frac{5}{6}t$$

(d) $v = q \qquad w = r \qquad x = s \qquad y = t \qquad 4z = -3q + 8r - 2s + t$

$$z = -\frac{3}{4}q + 2r - \frac{1}{2}s + \frac{1}{4}t$$

Exercise 4:

(a) $\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Exercise 5:

(a)
$$\begin{aligned} 2x_1 \quad\quad &= 0 \\ 3x_1 - 4x_2 &= 0 \\ -x_2 &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 3x_1 \quad\quad - 2x_3 &= 5 \\ 7x_1 + x_2 + 4x_3 &= -3 \\ -2x_2 + x_3 &= 7 \end{aligned}$$

(c)
$$\begin{aligned} 7x_1 + 2x_2 + x_3 - 3x_4 &= 5 \\ x_1 + 2x_2 + 4x_3 \quad\quad &= 1 \end{aligned}$$

(d)
$$\begin{aligned} x_1 \quad\quad &= 7 \\ x_2 \quad &= -2 \\ x_3 &= 3 \\ x_4 &= 4 \end{aligned}$$

Exercise 6:

ⓐ $x - 2y = 5$

ⓑ $x = t$ $\quad -2y = 5 - t$

$y = \frac{1}{2}t - \frac{5}{2}$

Exercise 7:

$$\left.\begin{aligned} ax_1^2 + bx_1 + c &= y_1 \\ ax_2^2 + bx_2 + c &= y_2 \\ ax_3^2 + bx_3 + c &= y_3 \end{aligned}\right\}$$

Since each of the points must satisfy the curve's equation.

Exercise 8:

$c = a + b$

$2x + y + 3z = (x + y + 2z) + (x + z)$

$2x + y + 3z = 2x + y + 3z$

Exercise 9:

$x_1 = c - kt$ $\quad x_2 = t$, where t is any real number.

so, $c - kt + \ell t = d$

$\ell t - kt = d - c$

$t(\ell - k) = d - c$, for all real numbers t.

Specifically, where $t = 0$, $d = c$; and where $t = 1$, $\ell = k$.

Exercise 10:

$x - y = 3 \Rightarrow 2x - 2y = 6$

$2x - 2y = k$

Thus, $k = 6 \Rightarrow$ infinitely many solutions

when $k \neq 6 \Rightarrow$ no solutions.

Never just one solution.

Exercise 11:

(a) The lines never intersect.

(b) The lines intersect at one point.

(c) The lines coincide.

Exercise 12:

If there is one solution, all three lines intersect at the same point. Eliminating one of the three lines will not affect this solution.
If there are infinitely many solutions, all three lines coincide. Getting rid of one line would still result in two coinciding lines.

Exercise 13:

$$ax + by = 0$$
$$cx + dy = 0$$
$$ex + fy = 0$$

Either the lines coincide or they intersect at one point: $(0, 0)$.

1.2: Gaussian Elimination

Exercise 1: a, b, c, d, h, i, j

Exercise 2: a, b, d, e.

Exercise 3:

(a) Both

(b) Neither

(c) Both

(d) Row-echelon form

(e) Neither

(f) Both