

Yet Another 2-opt

Fast And Powerful Approximation For
The Traveling Salesperson Problem

Jacob Thomsen

`jakethom02@gmail.com`

John Tappen

`jtappen@gmail.com`

Jonah Simmons

`jonahksimmons@gmail.com`

Professor Tony Martinez
Brigham Young University
April 18, 2023

Abstract

The 2-opt algorithm is an optimization algorithm that can be applied to any approximate solution to the Traveling Salesperson Problem. We will explore and discuss our testing and variation on the algorithm as well as possible improvements to it.

1. Introduction

The Traveling Salesperson Problem (TSP) is arguably the most well known NP-Hard problem in the world. From its formalizing in the 19th century, it seems all possible solutions have been exhausted: from optimal exponential solutions to fast heuristic approximations. We will be focusing on an approximation that uses a greedy approach to find a fast solution. Then, we will improve upon it using an optimization known as *2-opt*.

2. Greedy

Greedy algorithms solve problems with the "best now" mentality. It looks at all current options, assigns some rank to each, and chooses the best. For example, a greedy chess solver would capture a pawn rather

than sacrificing a knight for a check-mate in three moves.

2.1. Advantages

Greedy approaches usually return good enough solutions for most problems. From the tests we ran, the greedy solutions were hard to beat by much.

In addition to returning a good value, it does it very fast. It does not require any knowledge of the future and does not care about the past. Because of these traits, it means that greedy approaches are usually computationally cheap, especially compared to other options. The main issue with guaranteed optimal approaches is that it must take into account all other possibilities to ensure the best solution; this takes an impractical amount of time.

2.2. Disadvantages

While greedy might seem ideal, it has several flaws. As it does not take into account future decisions, it usually is not guaranteed to return the optimal for most problems, TSP being one of those. The solution from a greedy approach can be vastly worse than the optimal and sometimes is almost random.

2.3. Time Complexity

Algorithm 1 Greedy algorithm

```
1: procedure GREEDY  $\rightarrow$  PATH
2:   let current = starting city
3:   while not visited all cities do
4:     let best = closest neighbor to current that has not been visited
5:     path.add(best)
6:     current = best
7:   end while
8: end procedure
```

Given the pseudo-code, we can see the time complexity is $O(V \times E)$, where V is the number of cities and E is the number of out edges. The outer loop goes through all nodes, $O(V)$, and each iteration looks at all neighbors to get the nearest unvisited neighbor, $O(E)$.

2.4. Purpose

The purpose of including the greedy algorithm is to offer a benchmark and comparison to the other algorithms. It produces fine results, but can be improved with a little bit of work. Due to its fast but imperfect nature, it will be used as the starting point for our 2-opt.

mization that works for TSP is 2-opt which makes minor changes to a given solution; in our case the initial solution is greedy. The basic idea of 2-opt is swapping two edges, if it would improve the overall cost, then re-ordering the rest of the graph to make it a valid path.

This approach was chosen because of its flexibility and speed. It's flexible because it can be applied to any other approximate solution, no matter how it was solved. This "back end" optimization is especially helpful for very difficult problems, such as the TSP, and as long as it's fast, has few drawbacks; whenever it is able to be used, it should be used.

3. 2-opt

Optimizations are very helpful in solving problems. One such opti-

3.1. Disadvantages

The issues with this approach are that it must have a valid solution to begin with and it only works for

bi-directional graphs. For many real world problems, this requirement can be a big problem. In addition to this requirement, it is also highly limited by its initial solution.

3.2. Advantages

Considering its shortcomings, 2-opt is still a very good approach. Since it is an optimization, it will always return a valid solution that is as good or better than your initial solution. In addition, 2-opt is rather fast.

3.3. Improvements

While the base algorithm is fine, there are some changes we can make to overcome some of its disadvantages. Since it is a local algorithm, or only looks at changes that are "close", it can easily miss a better solution that is further away.

To get out of these local minimums, we used a Monte Carlo approach. We would simulate a coin flip to determine whether or not there would be a swap (if the normal conditions aren't met). This randomness gives it the ability to explore a new

path that might lead to an overall better solution.

From testing of this coin flip approach, we determined that it did not improve the final results by any significant amount and it dramatically increased the runtime. Since this did not return a result in a reasonable amount of time, we decided to remove it.

The other key issue with 2-opt is how the initial solution strongly effects the final solution. If a random or purely convenient initial solution was chosen, it would not guarantee a good final result. Because of this, we had to be careful to pick an initial solution that was fast but still good. From trying a few different options, we settled on the best of a few greedy solutions each starting at different points. Since the greedy was quite fast but determined on starting point, running it multiple times will produce a fairly good solution without increasing the overall time complexity.

With these slight improvements, we were able to find very good solutions in a practical amount of time.

3.4. Time Complexity

Algorithm 2 2-opt algorithm

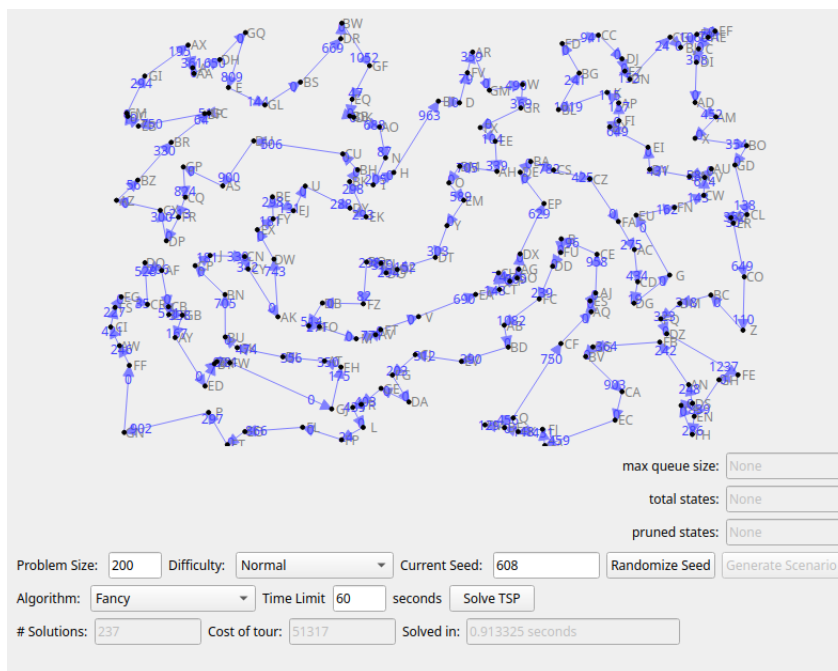
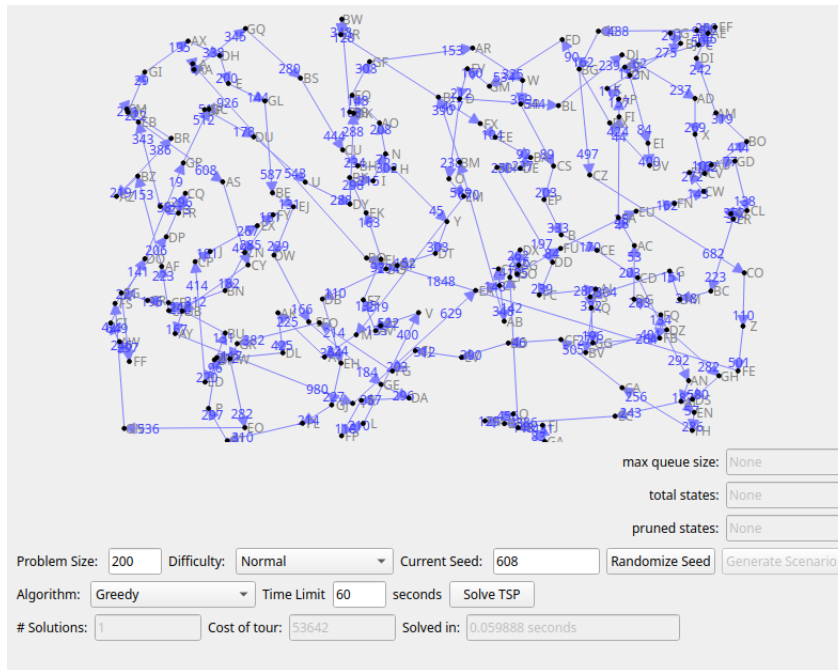
```
1: procedure 2OPT  $\rightarrow$  TOUR
2:   let tour = initial solution
3:   while tour can be improved do
4:     for edge1, edge2  $\in$  tour do
5:       if swapping the destinations improves cost then
6:         swap them
7:         reverse intermediate edges
8:       end if
9:     end for
10:  end while
11:  return tour
12: end procedure
```

As the pseudo-code shows, the number of iterations is unknown as the condition for the while loop is based on uncertain state. For each iteration, it must loop over all pairs of edges which is $O(E^2)$, where E is the number of edges. So the time complexity of 2-opt is $O(P \times E^2)$, where P is the unknown number of iterations until the tour cannot be im-

proved anymore.

Since 2-opt has to use an initial solution, the total time complexity must account for that as well. For our implantation, we use the best of 10 runs of our greedy algorithm, so the time complexity would be $O(V \times E + P \times E^2)$, assuming P is a constant, is equivalent to $O(V \times E + E^2)$.

4. Examples



From the pictures, you can clearly see the geometric effect that 2-opt has on the graph. As you see, it re-

moves any edges that cross over each other, which generates a shorter overall path.

5. Results

	Random		Greedy			Branch And Bound			2-Opt		
# Cities	Time (sec.)	Path Len.	Time	Path Len.	% of Rand.	Time	Path Len.	% of Greedy	Time	Path Len.	% of Greedy
15	0.000322	19779	0.002036	11981	0.61	26.28168	8596	0.72	0.017996	9053	0.76
30	0.000403	41811	0.006015	49021	1.17	timeout	n/a	n/a	0.037149	13216	0.27
60	0.000924	82175	0.019917	25143	0.31	timeout	n/a	n/a	0.083923	20366	0.81
100	0.001210	130287	0.027821	34668	0.27	timeout	n/a	n/a	0.207034	30022	0.87
200	0.001575	259415	0.074636	53202	0.21	timeout	n/a	n/a	0.810407	49153	0.92
300	0.003534	394327	0.01854	71524	0.18	timeout	n/a	n/a	2.561	69795	0.98
4000 (Max)	0.005198	5357576	62.49547	359732	.0671	timeout	n/a	n/a	timeout	n/a	n/a

5.1. Analysis Of Results

One thing we learn from the data is that there is real difference. Considering how fast 2-opt runs, it is impressive that there were noticeable improvements. From this we can

learn that deliberate effort can make a meaningful impact. From further testing, we discover that the upper limit of practicality is around 4,000 cities where it takes more than 10 minutes. Considering how hard a problem TSP is and how it consis-

tently improves over other approximations, these are very impressive results. We found that the sweet spot was around 3000 cities, where the 2-opt stopped improving on the greedy by any significant amount.

6. Future Work

For further attempts, there are some considerations. The main concern is escaping local minimums with 2-opt. From our testing, this would require a more sophisticated approach than

Monte Carlo. One possible approach would be running 2-opt on multiple initial solutions that might not be immediately as desirable, but might have a lower final solution.

Another direction would be to use an entirely different solver for initial solutions than greedy. There are many approximations to this problem, each with their respective pros and cons. Much additional testing could discover the best future course of action for either of these shortcomings.