Charlottesville High School

AP Physics C Lab 05

An Unparalleled Examination of a Series of Kirchoff's Rules

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I. ABSTRACT

Resistance is a property of a material that is defined, in sensible materials, as $\frac{V}{I}$. It is a hindrance to current and releases energy, via heat, to provide a drop in voltage. This series or parallel of experiments are aimed at confirming Kirchhoff's rules for circuits.

II. NOTATION

R_{eq}: Combined resistance of one or more resistors

III. INTRODUCTION

A. General Analysis

Kirchoff's rules provide a framework for understanding circuits and they are as follows: all voltage gains equal voltage drops in a circuit, loop rule, and the sum of currents entering a node exit equal to the sum of currents exiting a node, nodal rule:

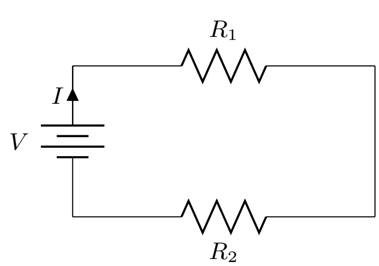
$$\sum_{k=1}^{n} V_k = 0 \tag{1}$$

$$\sum_{in=1}^{n} I_{in} = \sum_{out=1}^{n} I_{out} \tag{2}$$

Combined with Ohm's law, $V = IR$, these can be used to derive expressions for find R_{eq} for resistors in series and in parallel:

B. Resistors in Series

The following is a proof for finding R_{eq} for a circuit containing two resistors, R_1 and R_2, wired in series to a battery with voltage V. Since the resistors are in series they have the same current going through them.



Define R_{eq}

$$\frac{V}{I} = R_{eq}$$

Apply the loop rule

$$V = R_1 I + R_2 I$$

Solve for $\frac{V}{I}$

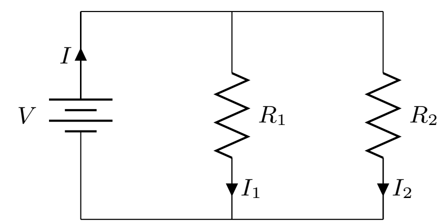
$$\frac{V}{I} = R_1 + R_2$$

Giving

$$R_{eq} = R_1 + R_2 \tag{3}$$

C. Resistors in Parallel

The following is a proof for finding R_{eq} for a circuit containing two resistors, R_1 and R_2, wired parallel to a bettery with voltage V. Since the resistors are in parallel they have the same potential difference V.



Define R_{eq}

$$\frac{V}{I} = R_{eq}$$

Apply the node rule

$$I_0 = I_1 + I_2$$

Apply Ohm's law

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

Solve for R_{eq} gives

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \tag{4}$$

IV. EXPERIMENTS

A. Error

For the measurements in all of our experiments, we used an ohmmeter with 1% error. For our percent error calculation we used the following formula:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial a_0}\sigma_{a_0}\right)^2 + \left(\frac{\partial f}{\partial a_1}\sigma_{a_1}\right)^2 + ... + \left(\frac{\partial f}{\partial a_n}\sigma_{a_n}\right)^2} \tag{5}$$

B. Resistors in Series

In this experiment, we attempt to verify our prediction about the R_{eq} of resistors wired in series. In order to do this, we retrieve two nominal 220Ω resistors, marking one to make them distinguishable, and measure them both with our digital ohmmeter. We then wire both resistors in series and measure them yielding $435 \pm 14.35\Omega$. Using Eqn. (3), we find that our experimental value fell within 0.17σ from our predicted value. We conclude that our data provides evidence for the formula derived for finding the R_{eq} of resistors wired in series.

Label:	Resistance (in Ω):
Resistor #1	215.00 ± 12.15
Resistor #2	217.00 ± 12.17
Measured R_{eq}	432.00 ± 17.20
Theoretical R_{eq}	435.00 ± 14.35

TABLE I. Results of Experiment B.

C. Resistors in Parallel

In this next experiment, we test the derived formula for finding the R_{eq} of resistors wired in parallel. We acquire two nominal 10kΩ resistors, marking one to make them distinguishable, and measure their resistance individually labeling them A and B with our digital ohmmeter. After measuring, we wire them in parallel and measure them yielding 4850.00 ± 148.5kΩ. Using Eqn. (4), we conclude that because our experimental value fell within 0.0σ away from my predicted value, our data supports confirms our derived formula for the R_{eq} of resistors in parallel.

Label:	Resistance (in Ω):
Resistor A	9700.00 ± 197.00
Resistor B	9720.00 ± 197.20
Measured R_{eq}	4850.00 ± 69.69
Theoretical R_{eq}	4850.00 ± 148.50

TABLE II. Results of Experiment C.

D. Multiple Resistors in Parallel

In this final experiment, we retest the derived formula for resistors wired in parallel. We retrieve eight additional nominal 220Ω resistors, reusing the resistors used in experiment B, and measure / marke them individually to obtain a total of ten nominal 220Ω resistors, labeling them 1-10 respectively. We then wire resistors 1 and 2 in parallel, measure them, and label the result trial 1. We then wire resistor 3 to trial 1s configuration in parallel and measure it labelling it trial 2. We continue adding one resistor at a time and increment trials until we have ten resistors wired in parallel and it is labelled trial 9. Displayed in FIG. 1, against theoretical values found using Eqn. (4), we find more evidence supporting our derived formula for R_{eq} of resistors measured in parallel.

Label:	Resistance (in Ω):
Resistance of 220 # 1	215.00 ± 12.15
Resistance of 220 # 2	217.00 ± 12.17
Resistance of 220 # 3	215.00 ± 12.15
Resistance of 220 # 4	214.00 ± 12.14
Resistance of 220 # 5	214.00 ± 12.14
Resistance of 220 # 6	213.00 ± 12.13
Resistance of 220 # 7	216.00 ± 12.16
Resistance of 220 # 8	216.00 ± 12.16
Resistance of 220 # 9	219.00 ± 12.19
Resistance of 220 # 10	217.00 ± 12.17

TABLE III. Results of Experiment C.

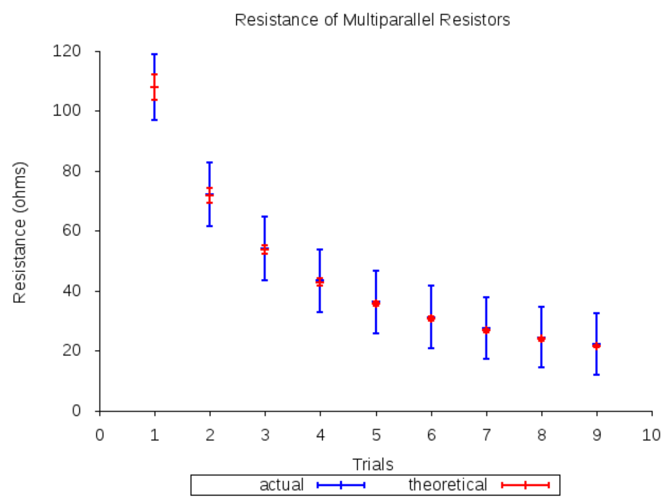


FIG. 1. Results of Experiment C.

V. CONCLUSION

In this report, we observed the R_{eq} of resistors wired in series and in parallel wired to an ohmmeter. The measured R_{eq} was found, in both cases, to confirm our predictions. We confirm that Kirchhoff's rules regarding circuits can be used to accurately predict R_{eq} of a circuit with resistors. Additionally, we find that in order to most accurately measure resistance in series resistors they should be measured once wired and to most accurately measure resistance in parallel they should be measured individually predicted using Kirchoff's rules.