Mixing of vector and axial mesons at finite temperature: an indication towards chiral symmetry restoration

M. Dey
Maulana Azad College, Calcutta 13, India

V.L. Eletsky and B.L. Ioffe
ITEP, SU-117 259 Moscow, USSR

Received 14 September 1990; revised manuscript received 4 October 1990

Correlators of vector and axial currents are considered at low temperatures $T \ll T_c$, $T^2/6F_\pi^2 \ll 1$. Only the lowest states, massless pions, are taken into account in the sum over excited states in the Gibbs averages. Using the PCAC, current algebra and saturating the correlators by ρ and a_1 mesons it is shown that m_ρ increases and m_{a_1} decreases with the increase of temperature indicating towards the chiral symmetry restoration. At $T \neq 0$ a massless scalar meson contributes to the longitudinal part of the vector correlator with the residue proportional to T^2.

There is presently a considerable interest in finite temperature theories. It is connected with the expected phase transition from hadrons to quark–gluon plasma at a critical temperature T_c, which ranges from 100 to 300 MeV, according to different estimates (see, e.g. ref. [1]). It is believed that this transition is accompanied by the restoration of chiral symmetry which is spontaneously broken in the low temperature hadronic phase. This phase was investigated by Leutwyler and his co-workers by using effective chiral lagrangians [2,3]. They obtained low temperature expansions for thermodynamic properties such as pressure and energy density, and for the quark condensate. In particular, the quark condensate was shown [2] to decrease with rising temperature indicating towards the restoration of chiral symmetry at T_c. One may ask what happens to hadron masses as the temperature rises. However, there seems to be no unique way to define the mass of a particle at finite temperature (for a discussion see ref. [4]). For example, it is possible to interpret the mass either as a pole in the propagator or as the inverse correlation length at large space-like distances, which need not coincide at $T \neq 0$. These two definitions were used in refs. [5,6] in discussing the temperature dependence of the nucleon mass.

In this paper we shall address the problem of temperature dependence of vector and axial meson masses by considering correlators of two vector and two axial currents in the euclidean region. In the case of the vector channel this problem was first considered by Bochkarev and Shaposhnikov [7] who wrote a sum rule for a finite temperature correlator of two vector currents and obtained the temperature dependence of the ρ meson mass as well as that of the continuum threshold in the vector channel. In the paper by Dosch and Narrison [8] it was claimed that the results of ref. [7] depend strongly on the value of zero temperature condensates. Also, finite temperature sum rules for the axial channel were considered by Bochkarev [9] and by Dominguez and Loewe [10]. However, the authors of refs. [7–10] accounted for the temperature dependence of the quark loop, the leading term in the operator product expansion, using the quark basis. This yields temperature corrections only of order T^2/m_ρ^2 and $T^2/m_{a_1}^2$. In our opinion, this approach is inadequate. Indeed, particles in a heat bath have energies of the order of temperature. Thus, calculating Gibbs averages in the quark basis at small T implies dealing with soft on-shell quarks, which does not make much sense because such quarks are usually referred to as condensates. At tempera-

Elsevier Science Publishers B.V. (North-Holland)

tures well below the phase transition point the sum over excited states in Gibbs averages should be performed using the basis of hadronic states. Among them the main role is played by pionic states which comprise soft quark degrees of freedom. Below we shall see that temperature corrections to the correlators obtained in this way are of order T^2/F_π^2, where $F_\pi = 93$ MeV is the pion decay constant (see also refs. [2,3,5,6]).

Let us consider the following thermal correlators:

$$C^{V}_{\mu\nu}(q,T) = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{iqx} \times \sum_n \langle n|TV^a_\mu(0)V^a_\nu(x)\exp[(\Omega-H)/T]|n\rangle ,$$

$$C^{A}_{\mu\nu}(q,T) = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{iqx} \times \sum_n \langle n|TA^a_\mu(0)A^a_\nu(x)\exp[(\Omega-H)/T]|n\rangle , \tag{1}$$

where $\mathrm{e}^{-\Omega} = \sum_n \langle n|\exp(-H/T)|n\rangle$, and the sum is over the full set of eigenstates of the hamiltonian H. The currents V^a_μ and A^a_μ are isovector vector and axial currents, respectively. Note, that we do not need to specify the explicit form of the currents for the time being. We will consider these correlators at euclidean momentum, $q^2 = -Q^2 < 0$.

To evaluate $C^V_{\mu\nu}(q,T)$ and $C^A_{\mu\nu}(q,T)$ at low temperature, $T^2 \ll Q^2$, we shall take into account in the sums in eq. (1) the vacuum state and the lowest excited states of the hadron gas, the pions, which are assumed to be massless. The matrix elements $\langle\pi|TV^a_\mu(0)V^a_\nu(x)|\pi\rangle$ and $\langle\pi|TA^a_\mu(0)A^a_\nu(x)|\pi\rangle$ are easily evaluated applying reduction formulas to the pions and using PCAC to express the pion field through the divergence of the axial current. Equal time commutators of the axial current which arise are determined by the current algebra relations

$$[A^a_0, V^b_\mu] = f^{abc}A^c_\mu , \quad [A^a_0, A^b_\mu] = f^{abc}V^c_\mu , \tag{2}$$

where the f^{abc} are SU(2) structure constants, $f^{+-0} = \sqrt{2}$. Using eq. (2) and integrating over pionic phase space with account of the pionic distribution function,

$$\int \frac{\mathrm{d}^3k}{(2\pi)^3 2k} \frac{1}{\mathrm{e}^{k/T}-1} = \tfrac{1}{24}T^2 , \tag{3}$$

we obtain

$$C^V_{\mu\nu}(q,T) = (1-\epsilon)C^V_{\mu\nu}(q,0) + \epsilon C^A_{\mu\nu}(q,0) ,$$

$$C^A_{\mu\nu}(q,T) = (1-\epsilon)C^A_{\mu\nu}(q,0) + \epsilon C^V_{\mu\nu}(q,0) , \tag{4}$$

where $\epsilon = T^2/6F_\pi^2$ is the expansion parameter in powers of the pion density, and we must have $\epsilon \ll 1$ for our approximation to be valid.

Thus, at temperatures $T \leqslant F_\pi$ the thermal correlators $C^V_{\mu\nu}(q,T)$ and $C^A_{\mu\nu}(q,T)$ are expressed through their values at $T=0$ and have covariant form. This latter circumstance is due to the fact that we are considering temperatures which are much less than the virtualities in the correlator and the T dependence only comes from the phase space of thermal pions.

One observes from eq. (4) that direct interaction of thermal pions with the currents induces mixing of vector and axial correlators, the mixing parameter being ϵ. Admixture of opposite parity channel arises when the two pions interact with two currents at points 0 and x. The interaction of two pions with one current decreases the contribution of the initial channel. These interactions are represented diagrammatically in figs. 1a and 1b. Solid lines there correspond to zero temperature correlators $\langle 0|TV^a_\mu(0) \times V^a_\nu(x)|0\rangle$ and $\langle 0|TA^a_\mu(0)A^a_\nu(x)|0\rangle$. Dashed lines with a mark denote thermal pions. There are also other diagrams corresponding to the interaction of thermal pions with a state which saturates the correlator (see figs. 1c and 1d). However, according to the Adler theorem these diagrams lead to extra powers of soft thermal pion momentum in the numerator and are suppressed in the euclidean region as compared to figs. 1a and 1b by powers of T [5].

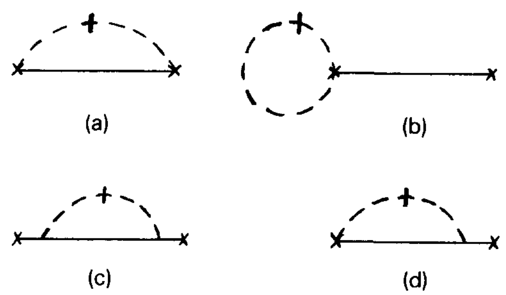

Fig. 1. One-loop corrections to correlators in the thermal pion gas. Solid lines denote zero temperature correlators. Dashed lines with a mark correspond to thermal pions.

From eq. (4) it follows that the sum of the vector and axial correlators does not change when the temperature is switched on:

$$C^{\mathrm{V}}_{\mu\nu}(q,T)+C^{\mathrm{A}}_{\mu\nu}(q,T)=C^{\mathrm{V}}_{\mu\nu}(q,0)+C^{\mathrm{A}}_{\mu\nu}(q,0)\,. \tag{5}$$

Let us present $C^{\mathrm{V}}_{\mu\nu}$ and $C^{\mathrm{A}}_{\mu\nu}$ as the sums of the isotropic and longitudinal parts:

$$C^{\mathrm{V}}_{\mu\nu}(q,T)=C^{\mathrm{V}}_{\mathrm{i}}(q^2,T)q^2g_{\mu\nu}+C^{\mathrm{V}}_{\ell}(q^2,T)q_\mu q_\nu\,,$$
$$C^{\mathrm{A}}_{\mu\nu}(q,T)=C^{\mathrm{A}}_{\mathrm{i}}(q^2,T)q^2g_{\mu\nu}+C^{\mathrm{A}}_{\ell}(q^2,T)q_\mu q_\nu\,. \tag{6}$$

In the chiral limit both $C^{\mathrm{V}}_{\mu\nu}$ and $C^{\mathrm{A}}_{\mu\nu}$ are transverse at any T, $q_\mu C^{\mathrm{V}}_{\mu\nu}=q_\mu C^{\mathrm{A}}_{\mu\nu}=0$. At $T=0$ both the isotropic and longitudinal parts of $C^{\mathrm{V}}_{\mu\nu}$ and $C^{\mathrm{A}}_{\mu\nu}$ obtain contributions from ρ and a_1 mesons, respectively. The longitudinal part of $C^{\mathrm{A}}_{\mu\nu}$ contains also the contribution from the pion pole. As shown in ref. [2], in the chiral limit the pion remains massless even at $T\neq 0$. Thus, C^{A}_{ℓ} is dominated by the pion pole both at $T=0$ and at $T\neq 0$. Therefore, from the second of eqs. (4) we get the temperature dependence of the pion decay constant:

$$F_\pi(T)=F_\pi\left(1-\frac{T^2}{12F_\pi^2}\right), \tag{7}$$

which is just the result of ref. [2]. Then, from eq. (5) it is clear that in the chiral limit C^{V}_{ℓ} at $T\neq 0$ obtains a contribution from a pole at $Q^2=0$ corresponding to a scalar Goldstone with the residue which is expressed through the temperature correction to F_π and proportional to T^2. In our approximation the residues of scalar and pseudoscalar Goldstones become equal at $T\approx 160$ MeV.

Let us now focus on the isotropic structure functions $C^{\mathrm{V}}_{\mathrm{i}}(q,T)$ and $C^{\mathrm{A}}_{\mathrm{i}}(q,T)$. At euclidean momentum, $Q^2=-q^2\gg T^2$, we can write the dispersion relations

$$C^{\mathrm{V(A)}}_{\mathrm{i}}=\frac{1}{\pi}\int_0^\infty\frac{\rho_i^{\mathrm{V(A)}}(s)\,\mathrm{d}s}{s+Q^2}+(\text{subtraction terms})\,, \tag{8}$$

where ρ^{V}_i and ρ^{A}_i are the corresponding spectral densities. Then, applying to C^{V}_i and $C^{\mathrm{A}}_{\mathrm{i}}$ the Borel transformation [11]

$$f_{\mathrm{B}}(M^2)=\frac{1}{\pi M^2}\int_0^\infty\exp\left(-\frac{q^2}{M^2}\right)\mathrm{Im}\,f(q^2)\,\mathrm{d}q^2\,, \tag{9}$$

which suppresses contributions from higher states and cancels subtraction terms, we obtain from eq. (5) the following relation:

$$\int_0^\infty\mathrm{d}s\,\exp\left(-\frac{s}{M^2}\right)[\delta_T\rho^{\mathrm{V}}(s)+\delta_T\rho^{\mathrm{A}}(s)]=0\,, \tag{10}$$

where $\delta_T\rho^{\mathrm{V}}$ and $\delta_T\rho^{\mathrm{A}}$ are the temperature dependent parts of spectral densities. From the above equation it is clear that with switching on the temperature, the masses of vector and axial mesons start moving in opposite directions if their couplings to the currents are kept fixed. Further, it is seen from eq. (4) that they converge, and not diverge, due to the specific signs of the terms proportional to ϵ. On the contrary, if the masses are kept fixed, then the couplings start converging. In reality, of course, both the masses and the couplings move with temperature.

Now, let us proceed more accurately. We shall separate in the dispersion relations (8) the contributions of the ρ and a_1 mesons. At $Q^2\geqslant 1$ GeV^2 due to asymptotic freedom we can approximate the continuum contributions to the spectral densities by quark loops starting from some thresholds. Assuming the usual quark representation for the currents,

$$V^a_\mu=\frac{1}{\sqrt{2}}\bar{q}\tau^a\gamma_\mu q\,,\quad A^a_\mu=\frac{1}{\sqrt{2}}\bar{q}\tau^a\gamma_\mu\gamma_5 q\,, \tag{11}$$

we have

$$C^{\mathrm{V}}_{\mathrm{i}}(T)=\frac{\lambda_\rho^2(T)}{Q^2+m_\rho^2(T)}+\frac{1}{4\pi^2}\int_{s_0^\rho}^\infty\frac{\mathrm{d}s}{s+Q^2}$$
$$+\text{subtraction terms}\,,$$
$$C^{\mathrm{A}}_{\mathrm{i}}(T)=\frac{\lambda_{\mathrm{a}_1}^2(T)}{Q^2+m_{\mathrm{a}_1}^2(T)}+\frac{1}{4\pi^2}\int_{s_0^{\mathrm{a}_1}}^\infty\frac{\mathrm{d}s}{s+Q^2}$$
$$+\text{ subtraction terms}\,, \tag{12}$$

where λ_ρ and λ_{a_1} are the residues of the currents into ρ and a_1 mesons, s^0_{V} and s^0_{A} are continuum thresholds in the corresponding channels. We are interested in the thermal variations $\delta m_\rho^2=m_\rho^2(T)-m_\rho^2(0)$, $\delta m_{\mathrm{a}_1}^2=m_{\mathrm{a}_1}^2(T)-m_{\mathrm{a}_1}^2(0)$, $\delta\lambda_\rho^2=\lambda_\rho^2(T)-\lambda_\rho^2(0)$, $\delta\lambda_{\mathrm{a}_1}^2=\lambda_{\mathrm{a}_1}^2(T)-\lambda_{\mathrm{a}_1}^2(0)$, $\delta s_0^\rho=s_0^\rho(T)-s_0^\rho(0)$, $\delta s_0^{\mathrm{a}_1}=s_0^{\mathrm{a}_1}(T)$

$-s_0^{a_1}(0)$. Substituting eq. (12) into eq. (4) and applying the Borel transformation (9) we obtain

$$\frac{\delta m_\rho^2}{\epsilon} = \frac{\lambda_{a_1}^2}{\lambda_\rho^2}(m_{a_1}^2 - m_\rho^2)\exp\left(-\frac{m_{a_1}^2 - m_\rho^2}{M^2}\right)$$
$$+\frac{\delta s_0^\rho}{\epsilon}\frac{1}{4\pi^2\lambda_\rho^2}(s_0^\rho - m_\rho^2)\exp\left(-\frac{s_0^\rho - m_\rho^2}{M^2}\right)$$
$$+\frac{M^2}{4\pi^2\lambda_\rho^2}\exp\left(\frac{m_\rho^2}{M^2}\right)$$
$$\times\left\{(m_\rho^2 - M^2)\left[\exp\left(-\frac{s_0^\rho}{M^2}\right) - \exp\left(-\frac{s_0^{a_1}}{M^2}\right)\right]\right\},$$
$$-M^2\left[s_0^\rho\exp\left(-\frac{s_0^\rho}{M^2}\right) - s_0^{a_1}\exp\left(-\frac{s_0^{a_1}}{M^2}\right)\right],$$
$$\frac{\delta\lambda_\rho^2}{\epsilon} = -\lambda_\rho^2 + \lambda_{a_1}^2\left(1 + \frac{m_{a_1}^2 - m_\rho^2}{M^2}\right)\exp\left(-\frac{m_{a_1}^2 - m_\rho^2}{M^2}\right)$$
$$+\frac{\delta s_0^\rho}{\epsilon}\frac{1}{4\pi^2}\left(1 + \frac{s_0^\rho - m_\rho^2}{M^2}\right)\exp\left(-\frac{s_0^\rho - m_\rho^2}{M^2}\right)$$
$$-\frac{1}{4\pi^2}\exp\left(\frac{m_\rho^2}{M^2}\right)\left\{(2M^2 - m_\rho^2)\right.$$
$$\times\left[\exp\left(-\frac{s_0^\rho}{M^2}\right) - \exp\left(-\frac{s_0^{a_1}}{M^2}\right)\right]$$
$$\left. + s_0^\rho\exp\left(-\frac{s_0^\rho}{M^2}\right) - s_0^{a_1}\exp\left(-\frac{s_0^{a_1}}{M^2}\right)\right\}, \tag{13}$$

where all masses, residues and thresholds are the zero temperature ones. Expressions for $\delta m_{a_1}^2$ and $\delta\lambda_{a_1}^2$ are obtained from the above formulas by substituting $\rho \leftrightarrow a_1$. The thermal variations of masses and residues given by eq. (13) are functions of the Borel parameter M^2 and of the continuum threshold variations δs_0^ρ and $\delta s_0^{a_1}$. Following the standard ideology of zero temperature QCD sum rules we shall vary δs_0^ρ and $\delta s_0^{a_1}$ and look at the T dependence of δm_ρ^2, $\delta\lambda_\rho^2$ and $\delta m_{a_1}^2$, $\delta\lambda_{a_1}^2$. Zero temperature QCD sum rules for both vector and axial channels are known [11,12] to be stable for $M^2 \geqslant 0.8$ GeV^2. At lower M^2 power corrections become uncontrolable. However, here we are not so pressed from the side of low M^2, because we do not use operator product expansion. The only restriction in our case is $M^2 \gg 4m_\pi^2$, since we are considering the chiral limit. At $M^2 \geqslant 1.5$ GeV^2 the continuum contribution approximated by the quark loop turns out to be considerable and increases the uncertainty. Thus, we shall look for the values of δs_0^ρ and $\delta s_0^{a_1}$ at which respectively δm_ρ^2, $\delta\lambda_\rho^2$ and $\delta m_{a_1}^2$, $\delta\lambda_{a_1}^2$ as functions of M^2 have plateaus at 0.8 GeV$^2 < M^2 < 1.5$ GeV^2. For the masses we take the experimental values $m_\rho = 0.77$ GeV and $m_{a_1} = 1.26$ GeV. Residues and thresholds are taken from zero temperature sum rules [11,12]: $s_0^\rho = 1.5$ GeV^2, $\lambda_\rho^2 = 2m_\rho^2/g_\rho^2 = 0.041$ GeV^2; $s_0^{a_1} = 2.3$ GeV^2, $\lambda_{a_1}^2 = 2m_{a_1}^2/f_{a_1}^2 = 0.045$ GeV^2. In the case of the ρ meson the residue and threshold are fixed rather tightly. For the a_1 meson the situation is not so unambiguous and the sum rules can be saturated for a certain range of the residue and threshold values, giving slightly different values of m_{a_1}. The values of $s_0^{a_1}$ and $\lambda_{a_1}^2$ given above are obtained from the sum rules of ref. [12] in which we have accounted for the M^2 dependence of α_s and demanded the sum rules to reproduce the experimental value of m_{a_1}. However, a shift of axial channel parameters allowed by the zero temperature sum rules yields uncertainties that do not exceed the ones given below. We have also tried slightly different values of quark condensates advocated in ref. [13]. The results turned out to be practically insensitive to this change. In fig. 2 we present the M^2 dependence of δm_ρ^2 and $\delta\lambda_\rho^2$ at different values of δs_0^ρ. The same for a_1 is shown in fig. 3. The final results are

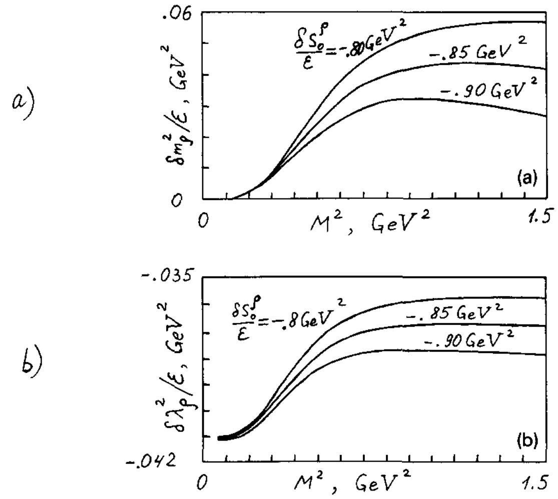

Fig. 2. M^2 dependence of δm_ρ^2 (a) and $\delta\lambda_\rho^2$ (b) at different values of δs_0^ρ.

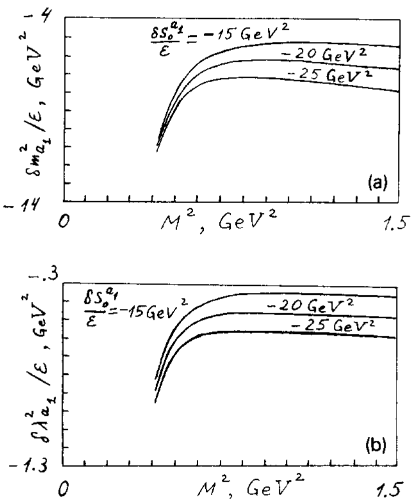

Fig. 3. Same as in fig. 2, but for a_1.

$$\delta m_\rho^2 = (0.045 \pm 0.015)\epsilon\ \mathrm{GeV}^2\,,$$
$$\delta\lambda_\rho^2 = -(0.037 \pm 0.002)\epsilon\ \mathrm{GeV}^2\,,$$
$$\delta s_0^\rho = -(0.85 \pm 0.10)\epsilon\ \mathrm{GeV}^2\,,$$
$$\delta m_{a_1}^2 = -(6.0 \pm 1.5)\epsilon\ \mathrm{GeV}^2\,,$$
$$\delta\lambda_{a_1}^2 = -(0.45 \pm 0.15)\epsilon\ \mathrm{GeV}^2\,,$$
$$\delta s_0^{a_1} = -(20 \pm 5)\epsilon\ \mathrm{GeV}^2\,. \qquad (14)$$

We see that with switching on the temperature the masses of ρ and a_1 mesons start converging, the a_1 mass moving considerably faster. This can be considered as an indication towards the chiral symmetry restoration at a higher temperature. Since the a_1 mass moves considerably faster than the ρ mass, the critical temperature for the chiral phase transition may be roughly estimated by putting $\delta m_{a_1}^2(T_c) = m_\rho^2 - m_{a_1}^2$. Then, from (14) we get $T_c \approx 100$ MeV.

In refs. [14,15] correlators of quark currents with hadronic quantum numbers were investigated numerically in lattice simulations in the vicinity of the phase transition. It was observed that the screening lengths, which describe the long distance behaviour of these correlators, tend to become degenerate in parity. Apparently, our results describe the on-set of this tendency at low temperatures.

We acknowledge useful discussions with H. Leutwyler, Ulf-G. Meißner, N. Paver and A.V. Smilga. M.D. wishes to thank the Indian and USSR Academies of Sciences for the hospitality at ITEP and also the DST, India, for the research grant No. SP/S2/K01/87.

References

[1] E.V. Shuryak, The QCD vacuum, hadrons and the superdense matter, World Scientific Lecture Notes in Physics, Vol. 8 (World Scientific, Singapore, 1988).
[2] J. Gasser and H. Leutwyler, Phys. Lett. B 184 (1987) 83; Nucl. Phys. B 307 (1988) 763.
[3] P. Gerber and H. Leutwyler, Nucl. Phys. B 231 (1989) 387.
[4] A.V. Smilga, Nucl. Phys. B 335 (1990) 569.
[5] V.L. Eletsky, Phys. Lett. B 245 (1990) 229.
[6] H. Leutwyler and A.V. Smilga, Nucl. Phys. B 342 (1990) 302.
[7] A.I. Bochkarev and M.E. Shaposhnikov, Nucl. Phys. B 160 (1986) 1113.
[8] H.G. Dosch and S. Narison, Phys. Lett. B 203 (1988) 155.
[9] A.I. Bochkarev, Sov. J. Nucl. Phys. 45 (1987) 517.
[10] C.A. Dominguez and M. Loewe, Phys. Lett. B 233 (1989) 201.
[11] M.A. Shifman, V.I. Zakharov and A.I. Vainshtein, Nucl. Phys. B 147 (1979) 385, 448, 519.
[12] L.J. Reinders, S. Yazaki and H.R. Rubinstein, Nucl. Phys. B 196 (1982) 125.
[13] B.L. Ioffe, in: Proc. XXII Intern. Conf. on High energy physics (Leipzig, 1984), Vol. II, p. 176.
[14] C. DeTar and J.B. Kogut, Phys. Rev. Lett. 59 (1987) 399; Phys. Rev. D 36 (1987) 2828.
[15] S. Gottlieb, W. Liu, D. Toussaint, R.L. Renken and R.L. Sugar, Phys. Rev. Lett. 59 (1987) 1881.