

# IN-MEDIUM SPECTRAL FUNCTIONS OF VECTOR- AND AXIAL-VECTOR MESONS

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[Jung, FR, Tripolt, von Smekal, Wambach, hep-ph/1610.08754]  
[FR, hep-ph/1504.03585]



**DFG** Deutsche  
Forschungsgemeinschaft

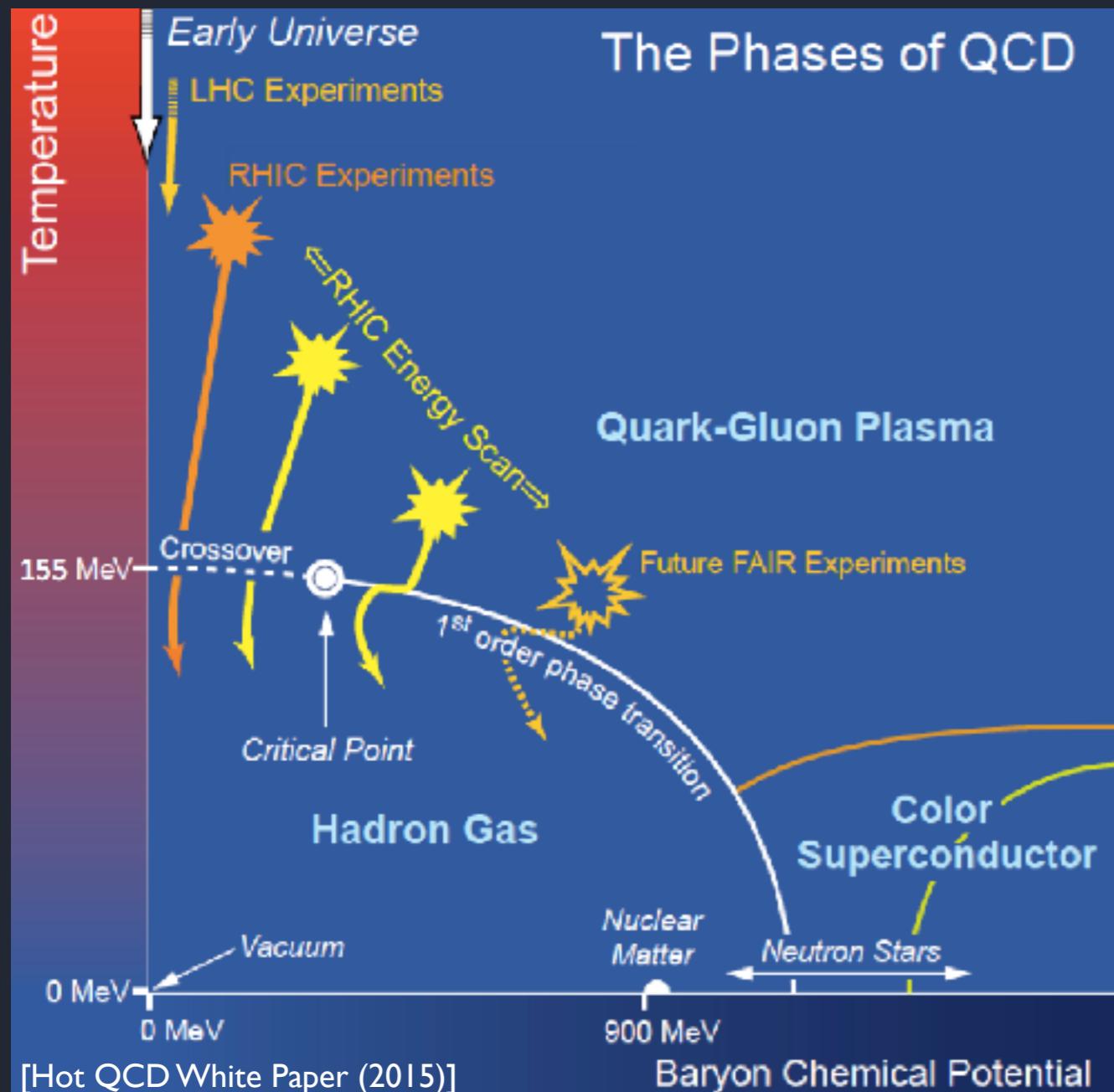
— RHIC/AGS ANNUAL USERS' MEETING —

BNL 12/06/2018

# OUTLINE

- Chiral symmetry restoration, dilepton spectra & vector mesons
- Spectral functions from the functional renormalization group
- (Axial) vector meson spectral functions in the medium
- Beyond low-energy modeling: dynamical hadronization

# QCD PHASE DIAGRAM



## Theory

compute order parameters

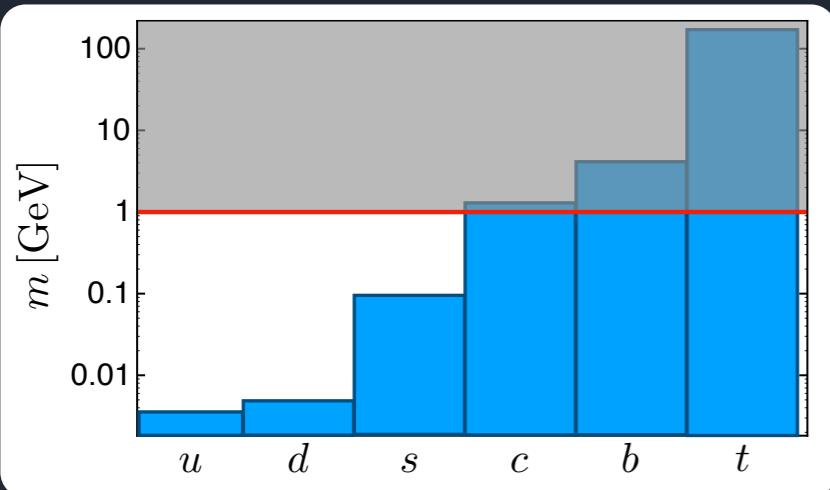
- confinement:  $L = \frac{1}{N_c} \left\langle \text{Tr}_f \mathcal{P} e^{ig \int_0^\beta d\tau A_0(\tau)} \right\rangle$
- $\chi$ SB:  $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle$

## Experiment

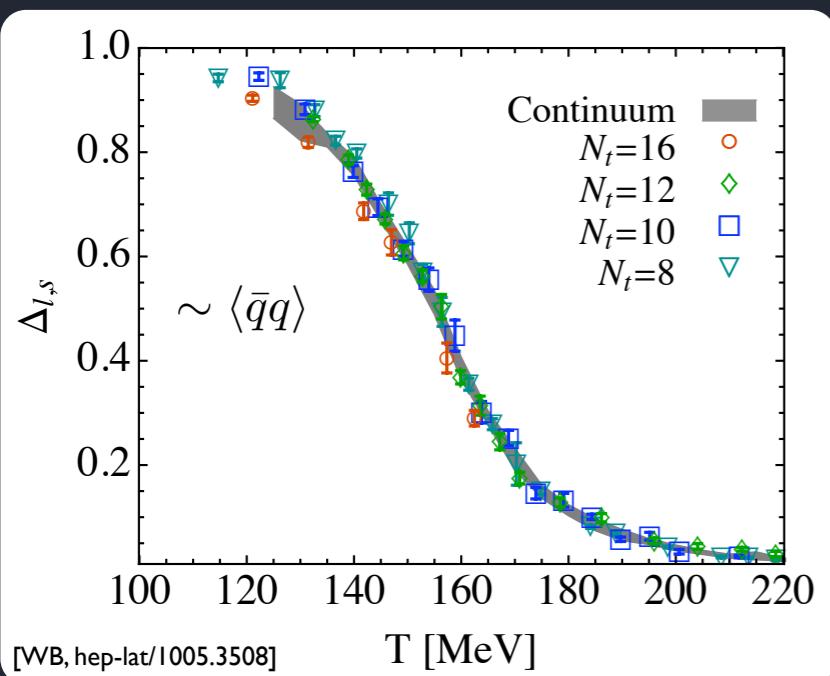
measure hadronic final states

→ phase transitions difficult to grasp!

# CHIRAL SYMMETRY BREAKING



- 3 (2) (super) light quark flavors  
→ approximate QCD flavor symmetry:  
 $SU(3)_L \times SU(3)_R$



- spontaneous symmetry breaking down to  $SU(3)_{L+R}$   
→ origin of hadron masses
  - 8 light pseudoscalar mesons  $\pi, K, \eta$  (pseudo Goldstones)
  - nucleon masses via coupling to chiral condensate
  - mass-splitting of chiral partners ...

**But: no experimental verification of chiral symmetry restoration in QGP yet!**

# DILEPTON SPECTRA

How to observe chiral symmetry restoration?

- dileptons escape fireball essentially without interaction
 

—————> ideal probes for high T/mu regions formed in early stages of collisions

- low- / intermediate-mass regime of dilepton spectra dominated by vector meson decays

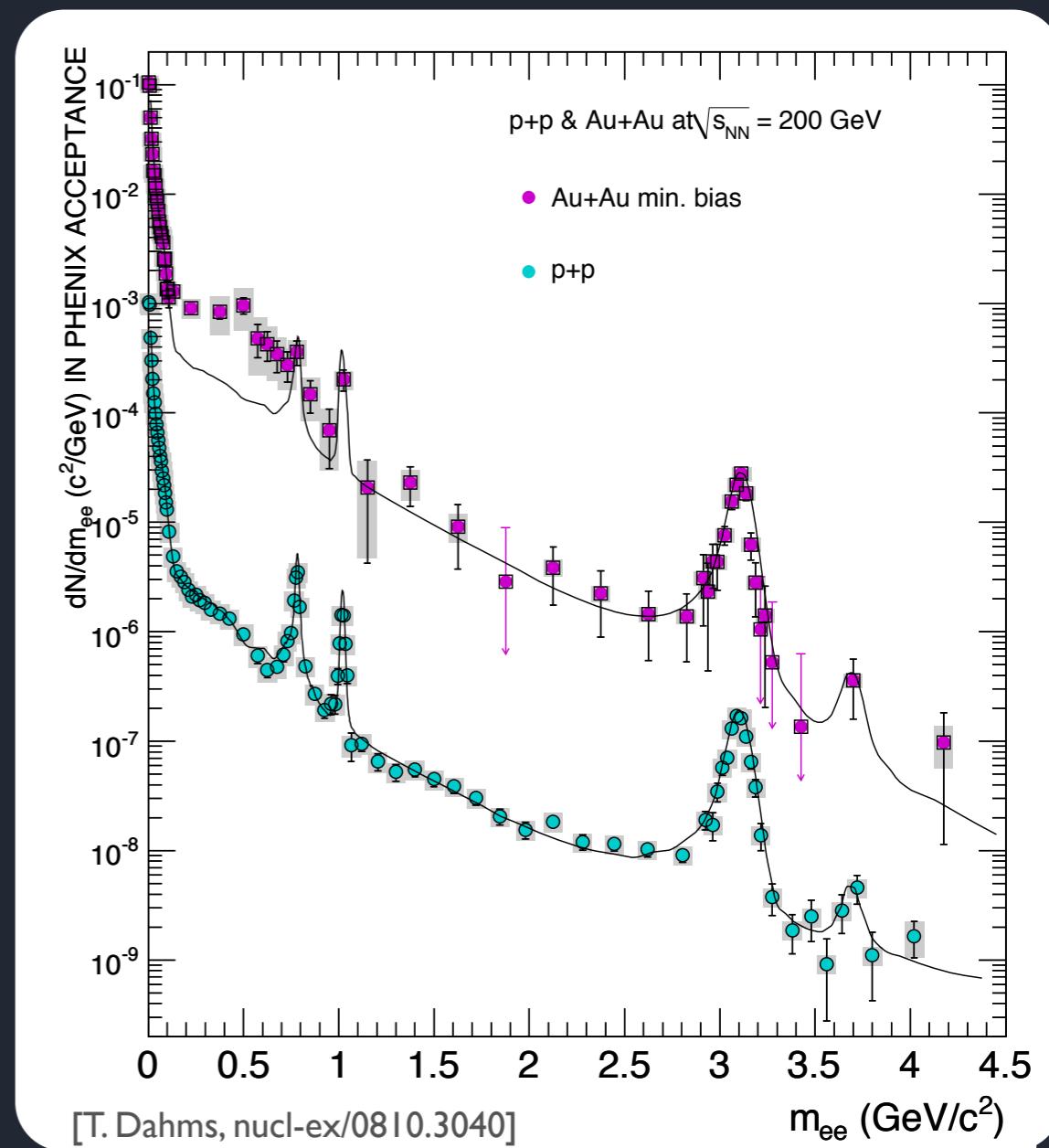
$$\frac{dN_{l+l-}}{d^4x d^4q} \sim \alpha_{\text{EM}}^2 \rho_{\text{EM}} \stackrel{\text{VMD}}{\sim} \alpha_{\text{EM}}^2 \left( \rho_\rho + \frac{1}{9} \rho_\omega + \frac{2}{9} \rho_\phi \right)$$

[Feinberg ('76), McLerran, Toimela ('85), Weldon ('90)]  
 [Sakurai ('69)]

- in-medium modification of rho-peak
 

—————> chiral transition?  
 (proposed by Pisarski in '81)

- suggestions:
  - dropping  $\rho$  mass at chiral restoration scale [Brown, Rho ('91)]
  - rising  $\rho$  mass [Pisarski ('95)]
  - melting  $\rho$  resonance [Rapp, Wambach ('99)]



# DILEPTON SPECTRA

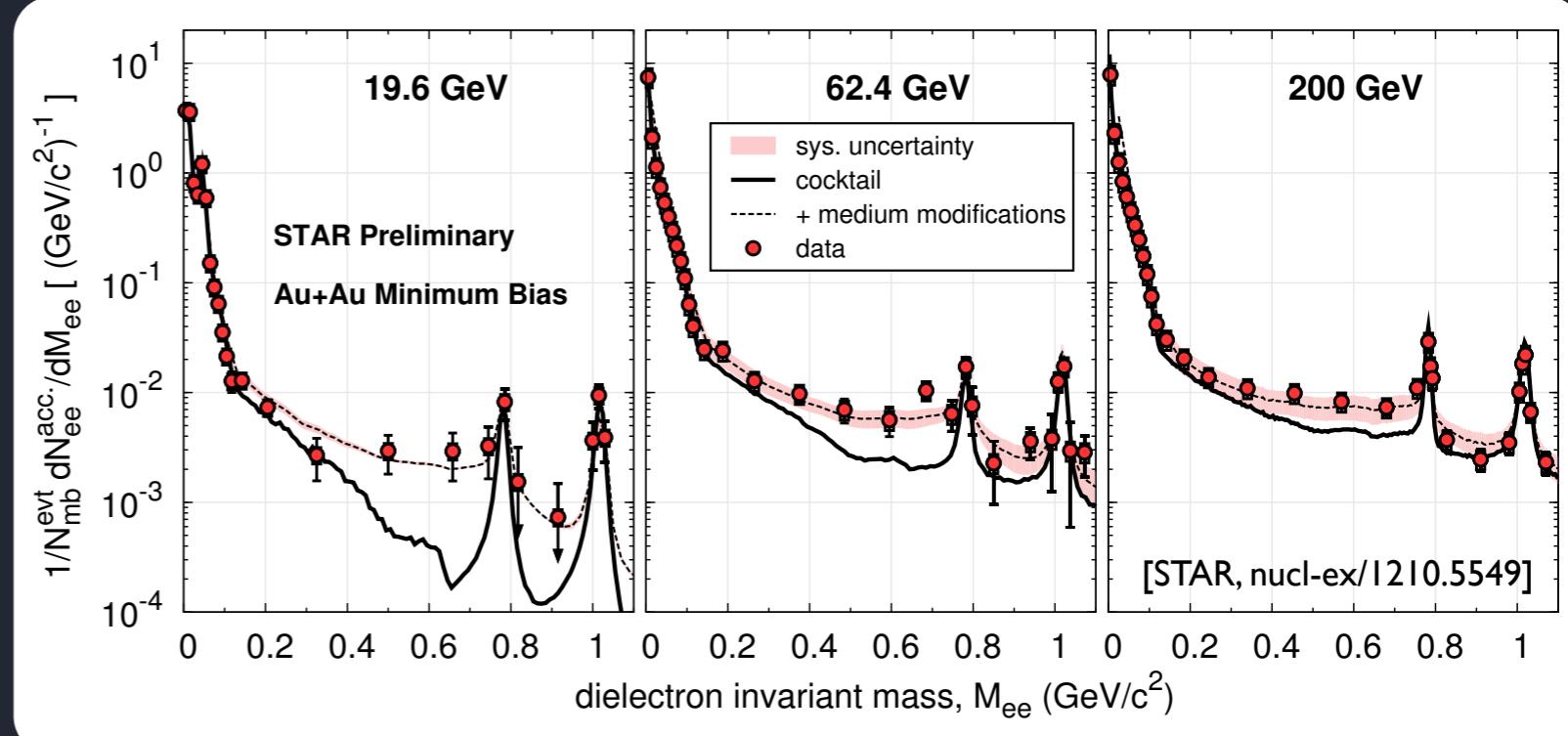
- melting scenario works well for describing dilepton data!

→ connection to  $\chi$ SR?

- Weinberg sum rule:

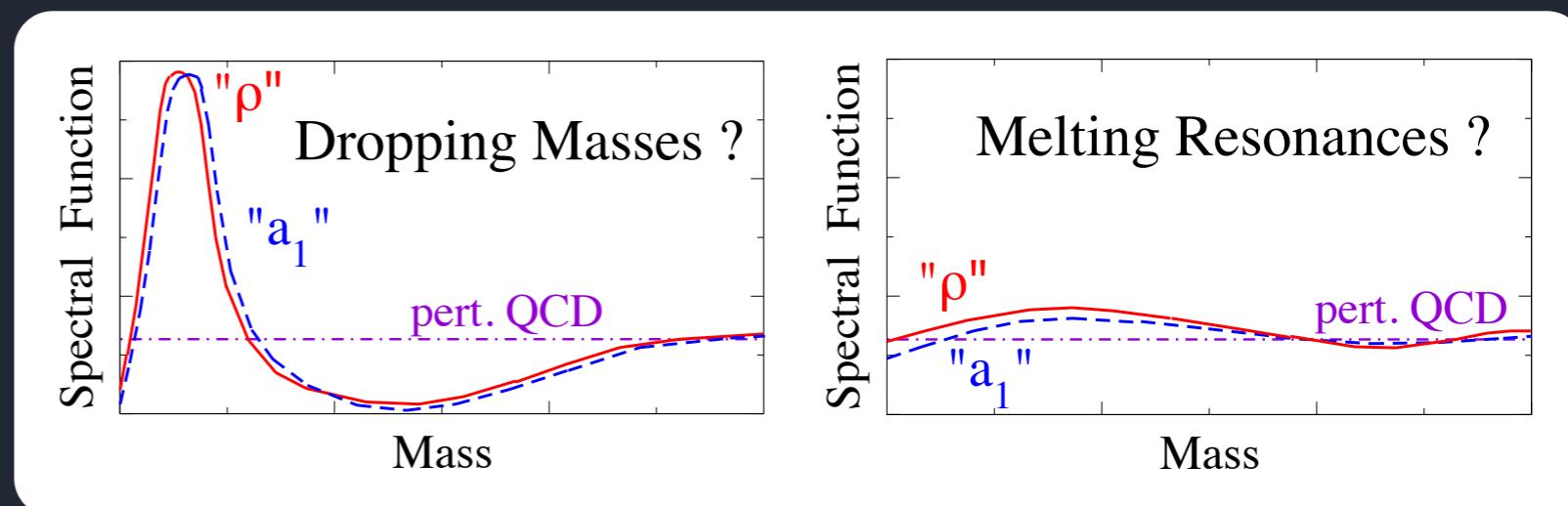
$$\int_0^\infty ds [\rho_V(s) - \rho_A(s)] = -2m_q \langle \bar{q}q \rangle$$

vector & axial-vector spectral functions      chiral condensate



need spectral function of  $\rho$  and its chiral partner  $a_1$

→ degeneration:  $\chi$ SR!



[Rapp, Wambach, van Hees, hep-ph/0901.3289]

# WHAT WE NEED

**We need a framework to compute thermodynamic properties / the phase structure and spectral properties on the same footing!**

Requirements:

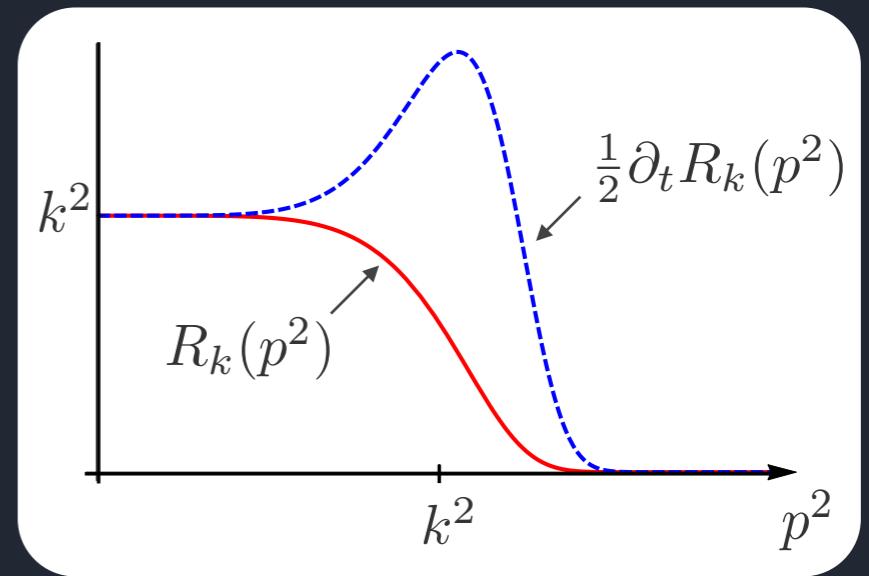
- non-perturbative method to describe QCD at low energies
- capture symmetries and their breaking patterns
- direct access to spectral properties (avoid analytic continuation of numerical data)
- quantum fluctuations (beyond mean-field approximations)
- access to finite  $\mu$  (no sign problem)

# FUNCTIONAL RG

- introduce regulator to partition function to suppress momentum modes below energy scale  $k$  (Euclidean space):

$$Z_k[J] = \int \mathcal{D}\varphi e^{-S[\varphi] - \Delta S_k[\varphi] + \int_x J\varphi}$$

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$



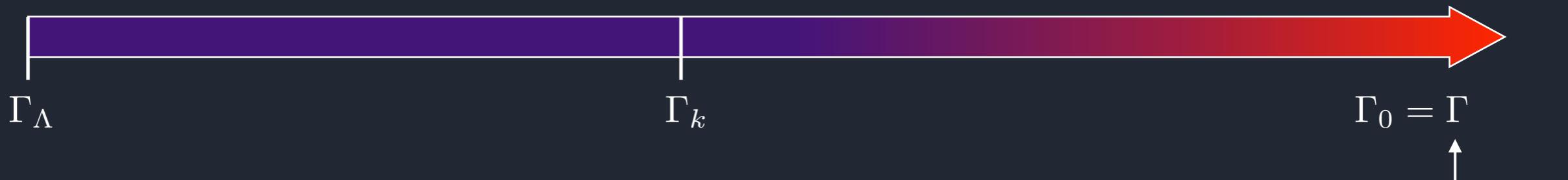
- scale dependent effective action:

$$\Gamma_k[\phi] = \sup_J \left\{ \int_x J(x)\phi(x) - \ln Z_k[J] \right\} - \Delta S_k[J] \quad \phi = \langle \varphi \rangle_J$$

- evolution equation for  $\Gamma_k$ :  
[Wetterich 1993]

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right] \quad \partial_t = k \frac{d}{dk}$$

successively integrate out fluctuations from UV to IR (Wilson RG)



→  $\Gamma_k$  is eff. action that incorporates all fluctuations down to scale  $k$

→ lowering  $k$ : zooming out / coarse graining

full quantum effective action  
(generates IPI correlators)

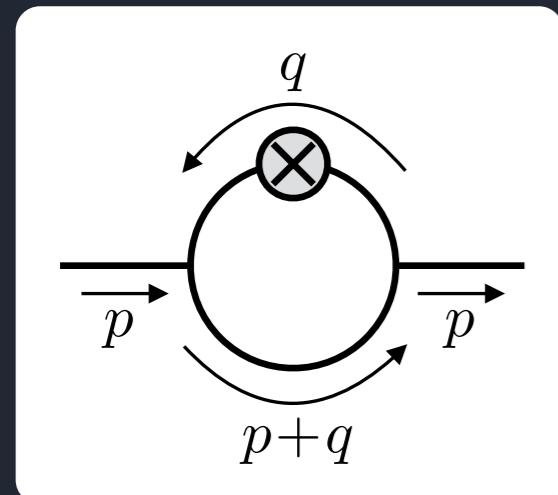
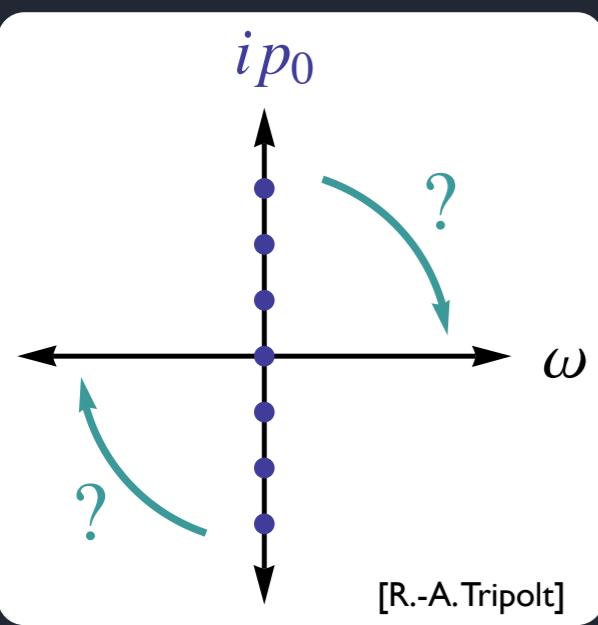
# ANALYTIC CONTINUATION

- one-loop like structure of the FRG allows for straightforward analytic continuation!

[Floerchinger, hep-ph/1112.4374]

[Tripolt, Strodthoff, von Smekal, J.Wambach, hep-ph/1311.0630]

[Pawlowski, Strodthoff, hep-ph/1508.01160]



1. start from Euclidean spacetime (Matsubara formalism for finite T)

→ Euclidean 2-pt function  $\partial_t \Gamma_k^{(2),E}(p_0, \vec{p})$   
 $\sim n \in \mathbb{Z}$

2. exploit the periodicity of the occupation numbers

$$n_{B/F}(E + ip_0) = n_{B/F}(E)$$

3. rotate frequency to the real axis → retarded 2-pt function

$$\partial_t \Gamma_k^{(2),R}(\omega, \vec{p}) = - \lim_{\epsilon \rightarrow 0} \partial_t \Gamma_k^{(2),E}(p_0 = -i(\omega + i\epsilon), \vec{p})$$

4. spectral function

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \text{Im } G^R(\omega, \vec{p}) = \frac{1}{\pi} \frac{\text{Im } \Gamma^{(2),R}(\omega, \vec{p})}{|\Gamma^{(2),R}(\omega, \vec{p})|^2}$$

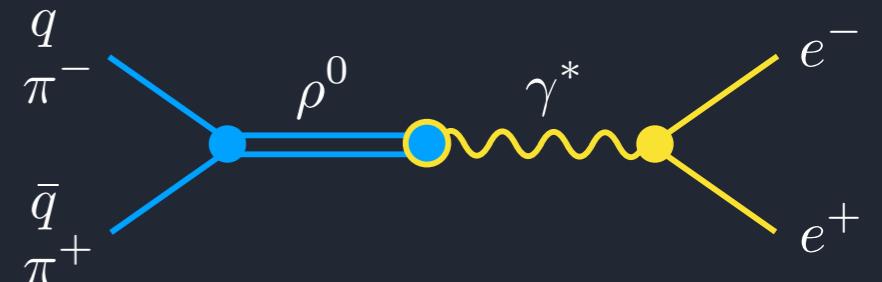
→ direct non-perturbative computation of spectral functions (without sign-problem)

# A LOW ENERGY MODEL

- need hadronic electromagnetic current for dilepton spectrum:  $\frac{dN_{l^+l^-}}{d^4x d^4p} \sim \rho_{\text{EM}} \sim \langle j_{\text{EM}} j_{\text{EM}} \rangle$
- vector meson dominance:  $j_{\text{EM}}^\mu \Big|_{\text{hadrons}} = \frac{e}{g} (m_\rho^2 \rho^\mu + m_\phi^2 \phi^\mu + m_\omega^2 \omega^\mu)$
- VMD can be realized by promoting the flavor symmetry to a gauge symmetry; vector mesons emerge as gauge bosons  
[Sakurai ('60), Kroll, Lee, Zumino ('68), Lee, Nieh ('68)]
- 2 flavor (no  $\Phi$ ) quark-meson model based on VMD (only iso-triplet vector mesons: no  $\omega$ ):

$$\Gamma_k = \int d^4x \left\{ \bar{q} [\gamma_\mu \partial^\mu - \mu \gamma_0 + h_S (\sigma + i \gamma_5 \vec{\tau} \vec{\pi}) + i h_V (\gamma_\mu \vec{\tau} \vec{\rho}^\mu + \gamma_\mu \gamma_5 \vec{\tau} \vec{a}_1^\mu)] q \right. \\ \left. + U_k(\phi^2) - c\sigma + \frac{1}{2} (D_\mu \phi)^2 + \frac{1}{8} \text{tr} V_{\mu\nu} V^{\mu\nu} + \frac{1}{4} m_V^2 \text{tr} V_\mu V^\mu \right\} + \Delta \Gamma_{\pi a_1}$$

[Jung, FR, Tripolt, von Smekal, Wambach, hep-ph/1610.08754]



$$\phi = (\vec{\pi}, \sigma)$$

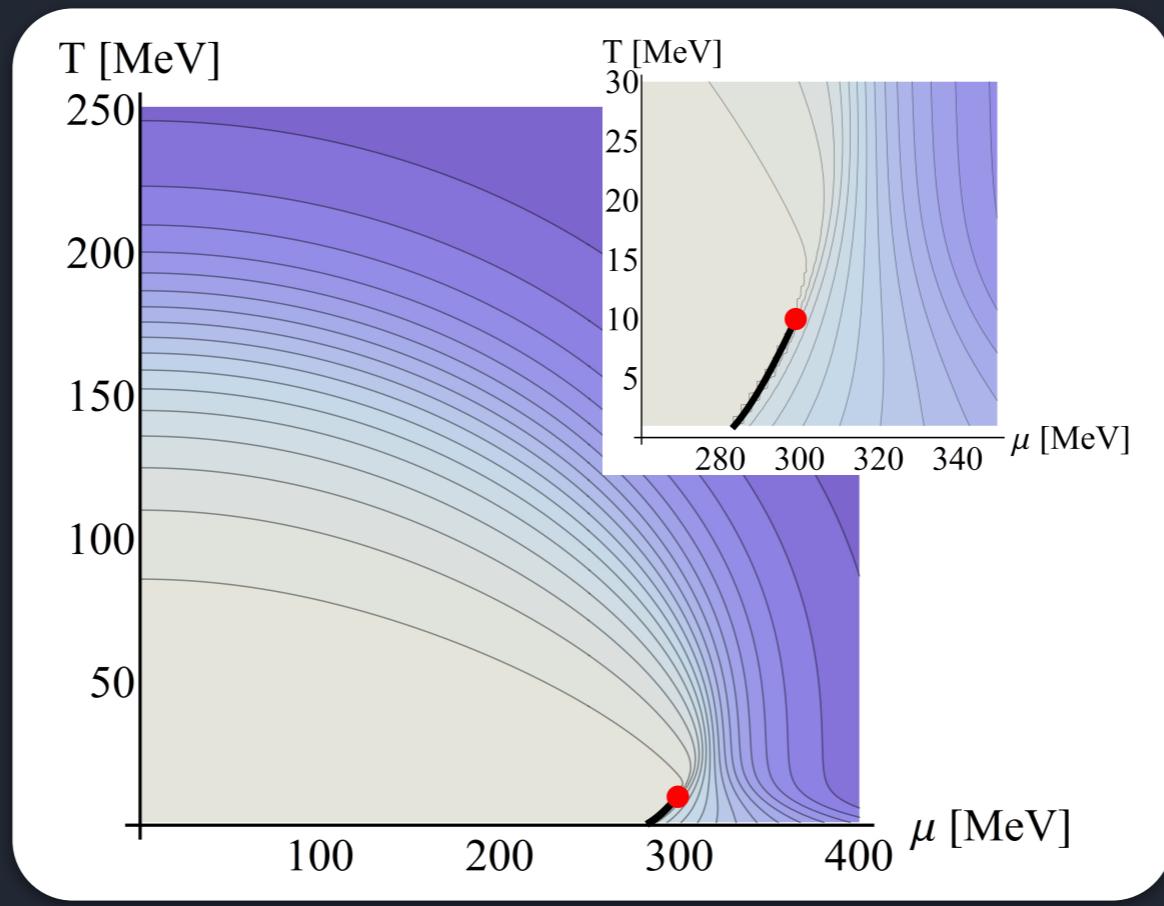
$$V^\mu = \vec{\rho}^\mu \vec{T} + \vec{a}_1^\mu \vec{T}^5$$

- covariant derivative:  $\frac{1}{2} (D_\mu \phi)^2 = \frac{1}{2} (\partial_\mu \phi)^2 - i g_V \partial_\mu \phi \cdot V_\mu \phi - \frac{g_V^2}{2} (V_\mu \phi) \cdot (V_\mu \phi)$
- field strength:  $\frac{1}{8} \text{tr} V_{\mu\nu} V_{\mu\nu} = \frac{1}{8} \text{tr} (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \frac{i g_V}{2} \text{tr} \partial_\mu V_\nu [V_\mu, V_\nu] - \frac{g_V^2}{8} \text{tr} [V_\mu, V_\nu]^2$

# A LOW ENERGY MODEL

[Jung, FR, Tripolt, von Smekal, Wambach, hep-ph/1610.08754]

phase diagram:



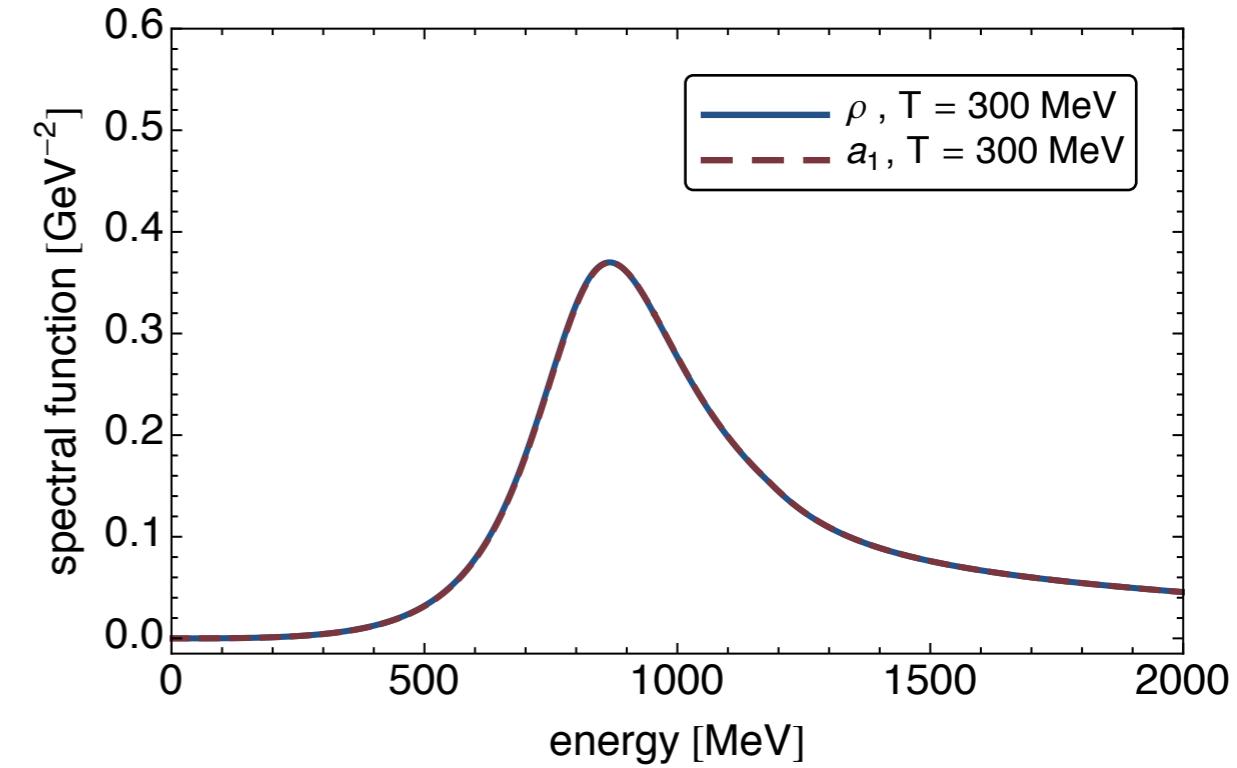
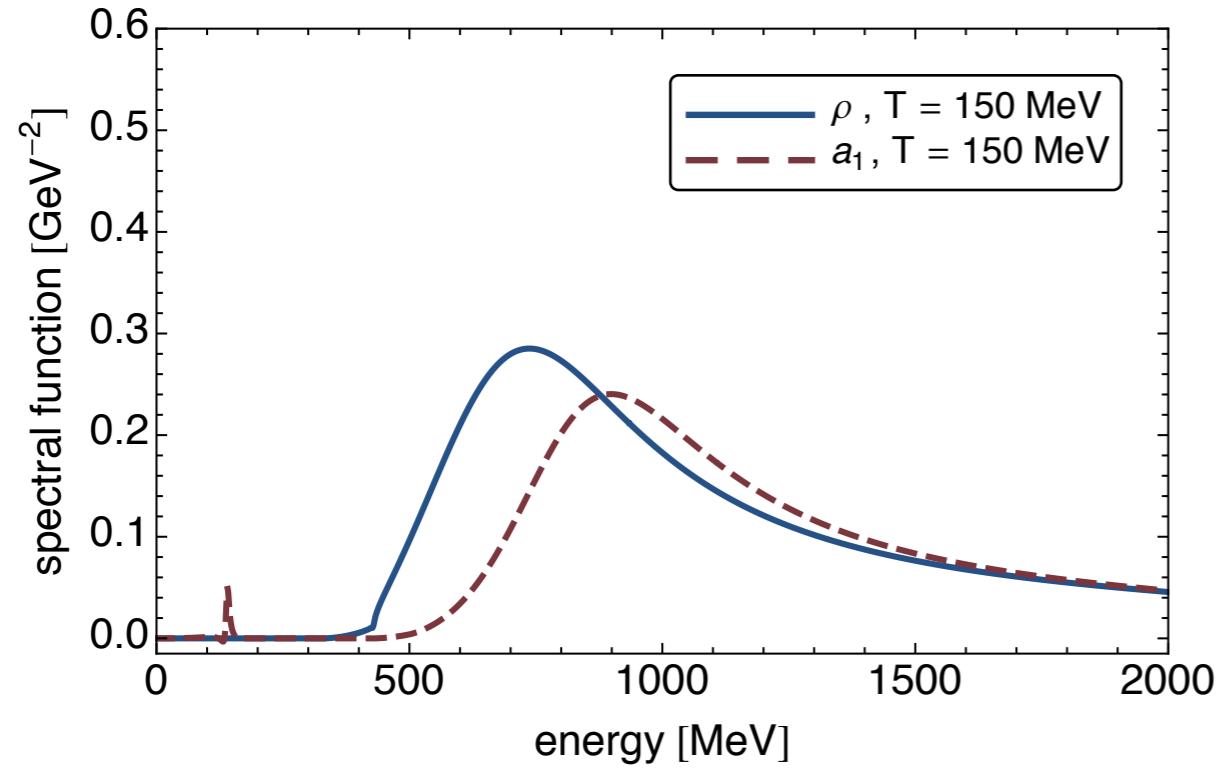
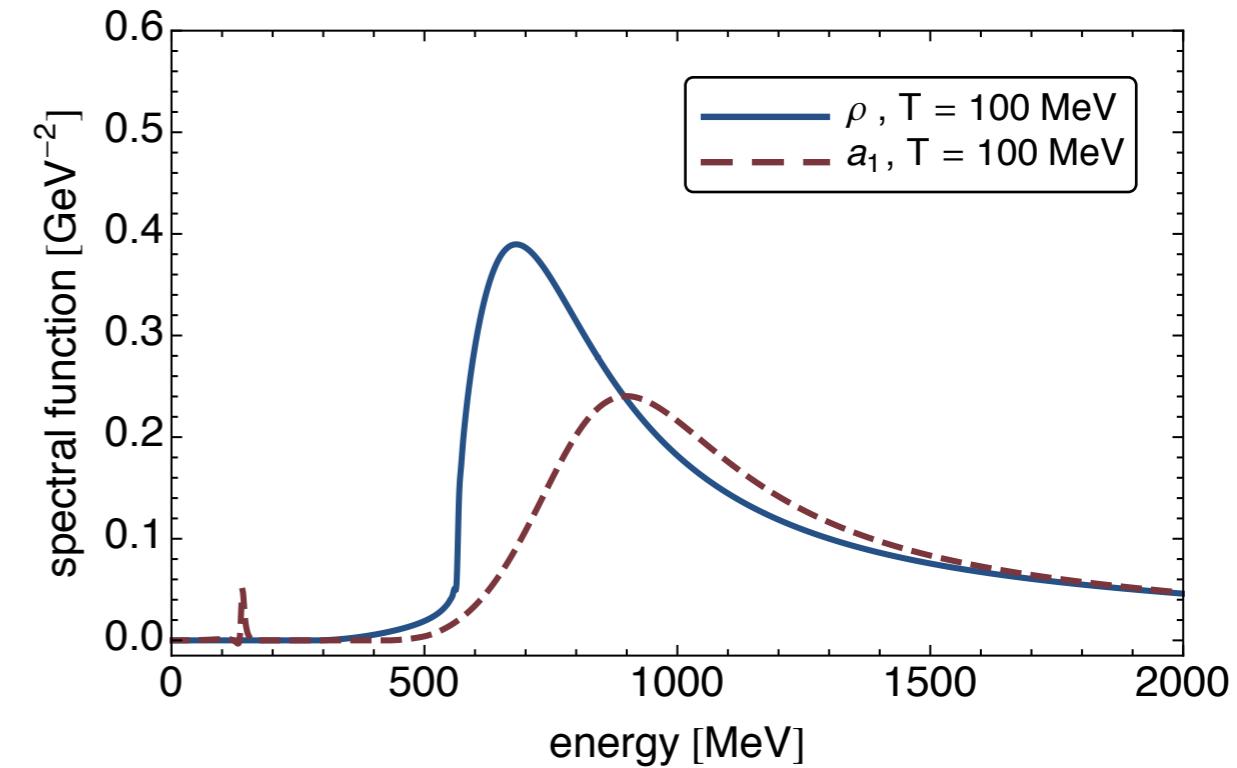
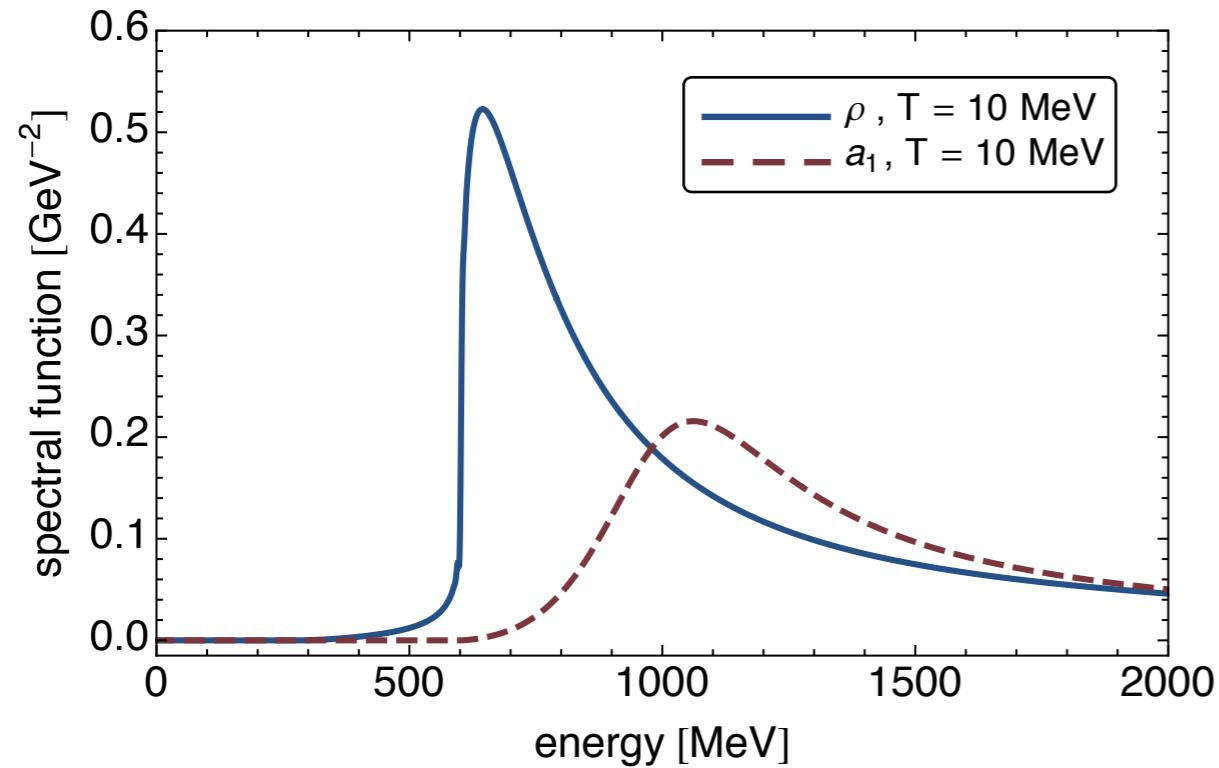
RG flow of the two-point functions:

$$\partial_k \Gamma_{\rho,k}^{(2)} = -\frac{1}{2} \left[ \text{Diagram with } \rho \text{ loop, } \pi \text{ external lines} \right] - 2 \left[ \text{Diagram with } \rho \text{ loop, } \psi \text{ external lines} \right]$$
$$\partial_k \Gamma_{a_1,k}^{(2)} = \left[ \text{Diagram with } a_1 \text{ loop, } \sigma \text{ external lines} \right] + \left[ \text{Diagram with } a_1 \text{ loop, } \pi \text{ external lines} \right] - \frac{1}{2} \left[ \text{Diagram with } a_1 \text{ loop, } \pi \text{ external lines} \right] - \frac{1}{2} \left[ \text{Diagram with } a_1 \text{ loop, } \sigma \text{ external lines} \right] - 2 \left[ \text{Diagram with } a_1 \text{ loop, } \psi \text{ external lines} \right]$$

approx.: no internal vector mesons

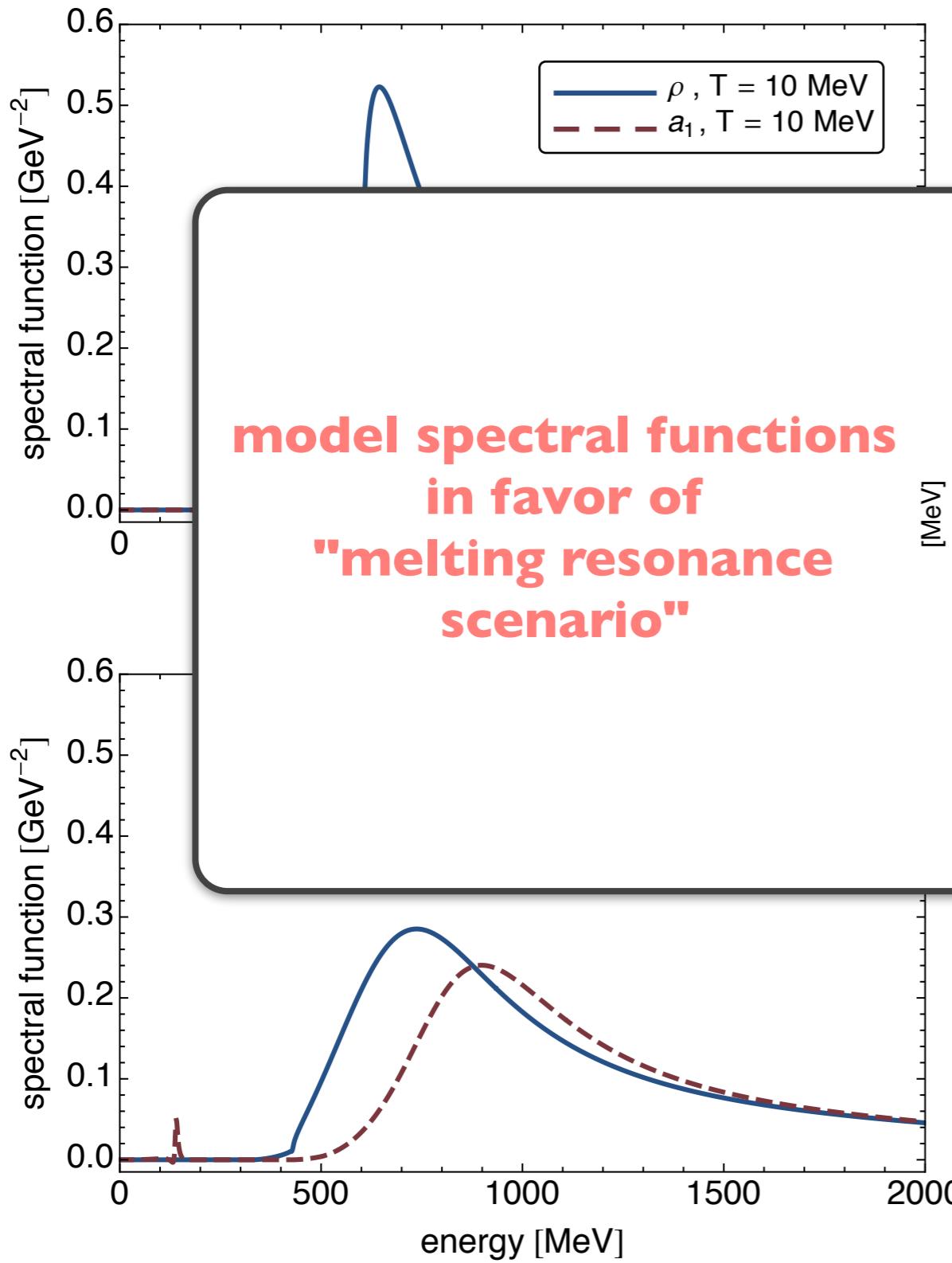
# SPECTRAL FUNCTIONS

Jung, FR, Tripolt, von Smekal, Wambach, hep-ph/1610.08754]

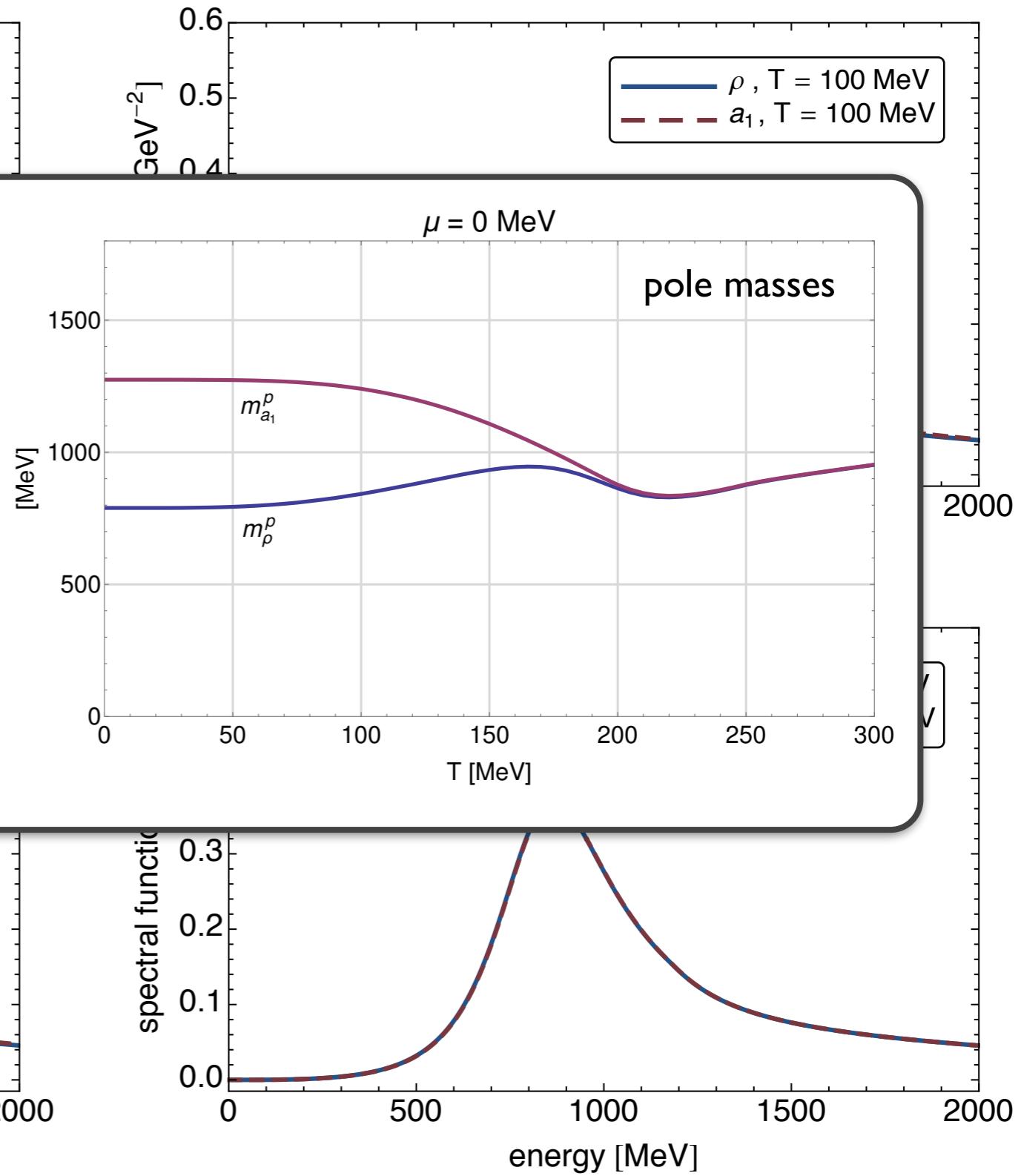


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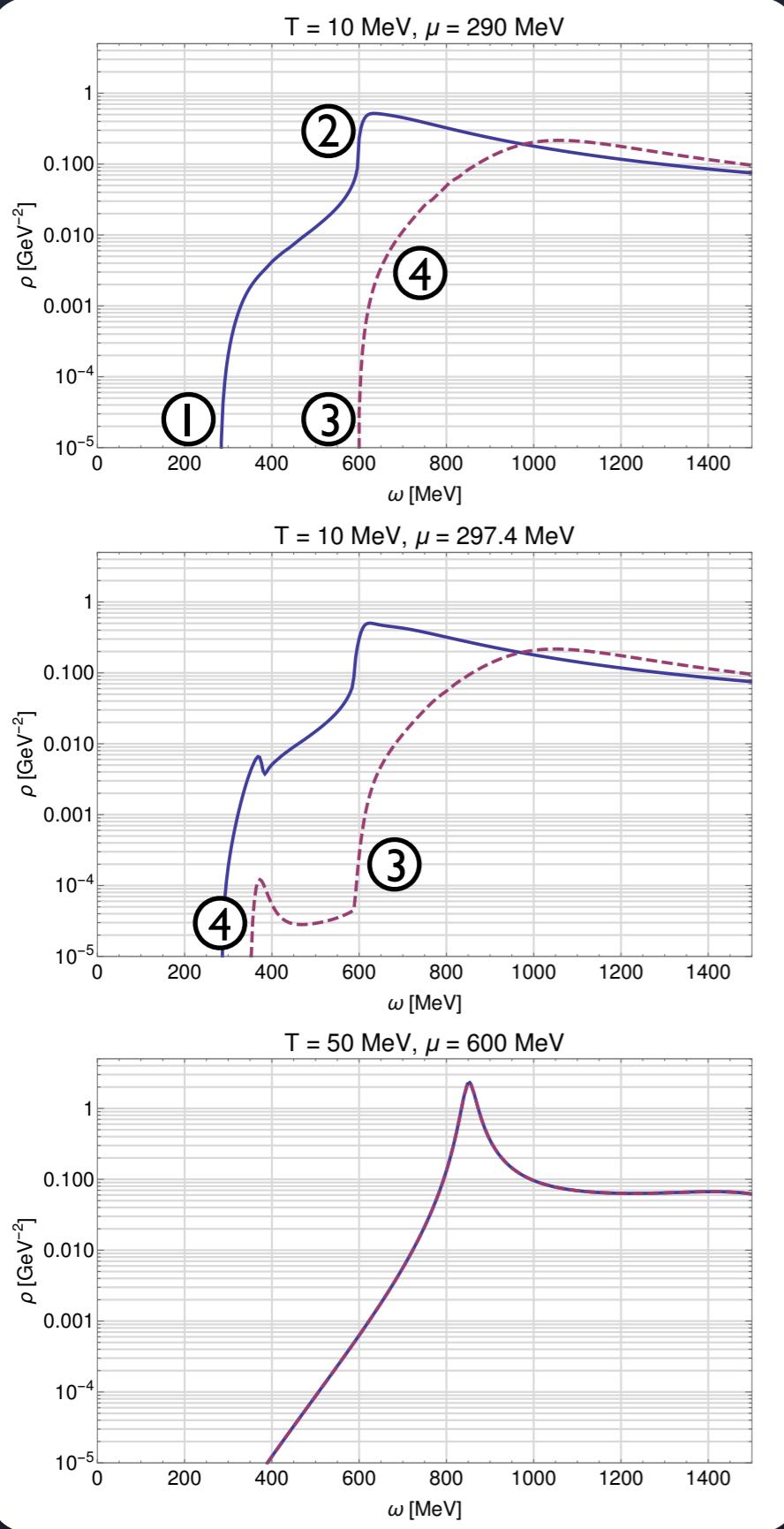


**model spectral functions  
in favor of  
"melting resonance  
scenario"**

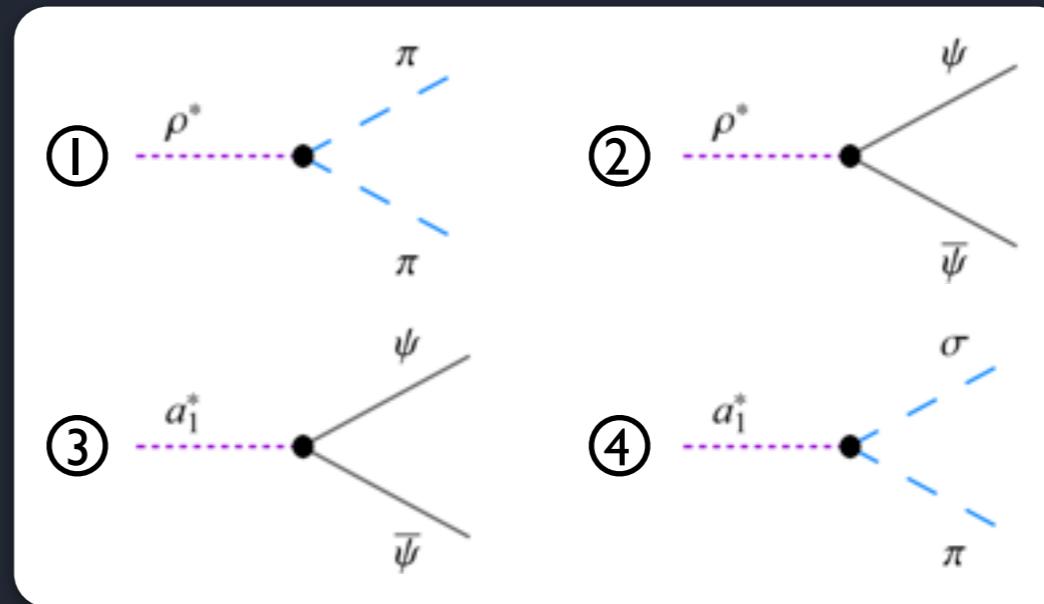


# SPECTRAL FUNCTIONS

[Jung, FR, Tripolt, von Smekal, Wambach, hep-ph/1610.08754]



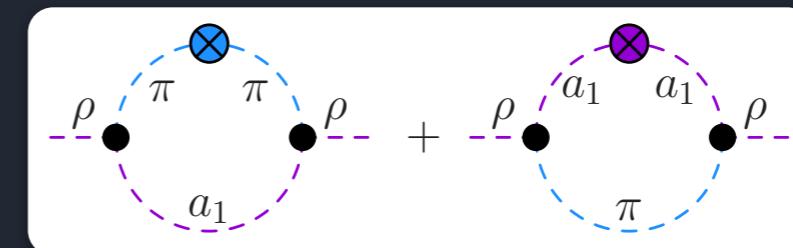
- some decay channels:



- manifestation of CEP in dilepton spectra:

- $a_1$  couples to critical mode  $\sigma$
- include vector meson fluctuations

→ feedback on  $\rho$  spectral function



- similar contribution from  $\pi$ 's
- fine-tuning of  $T$  and  $\mu$  to CEP necessary

# DYNAMICAL HADRONIZATION

$\Lambda \gg 1 \text{ GeV}$

Up to now only low-energy model; what happens if gluon fluctuations are included?

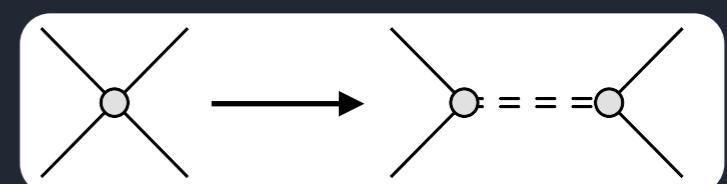
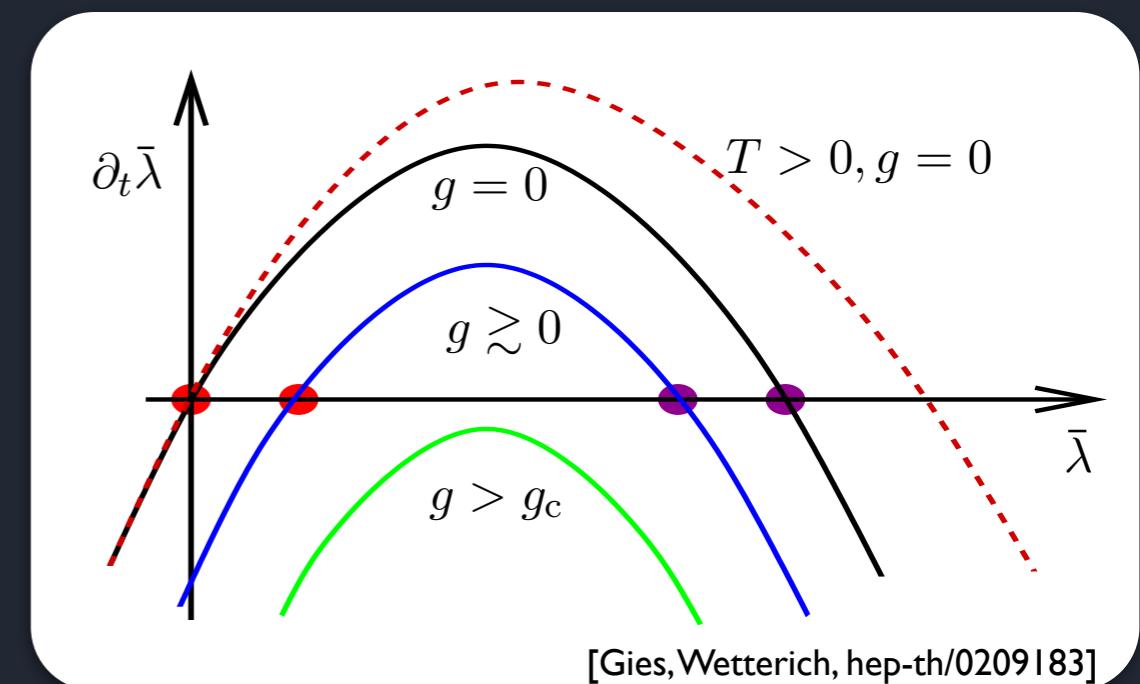
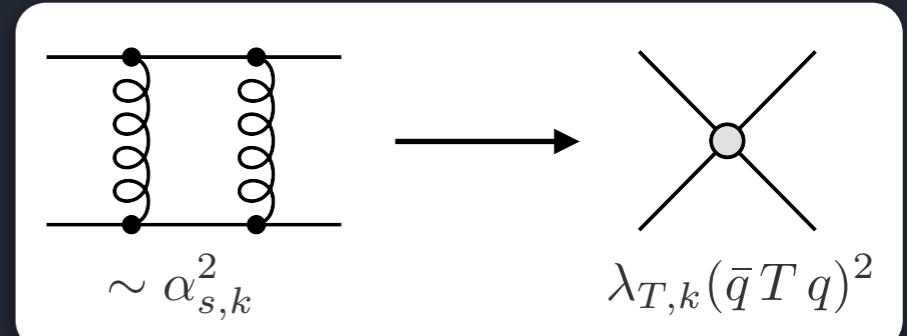


- initial action: gauge fixed QCD
- effective four-quark interaction channels are generated:
- $\lambda_{T,k}$  grows with decreasing scale
- RG-flow controlled by IR-attractive fixed point for small  $g$
- strong coupling exceeds critical value: **chiral symmetry breaking**  
→ **four-quark interaction diverges**
- introduce mesons via bosonization:  
(similar: baryonization from 6-quark interactions)

Yukawa coupling

$$\lambda_{T,k} = \frac{h_{T,k}^2}{m_{\phi,k}^2}$$

meson mass parameter



# DYNAMICAL HADRONIZATION

[Gies, Wetterich, Phys.Rev. D '02, Pawłowski, Ann.Phys. '07, Floerchinger, Wetterich, Phys.Lett. B '09]

[Braun, Fister, Pawłowski, FR, hep-ph/1412.1045]

- bosonize 4-quark interaction in each RG step: **dynamical hadronization**

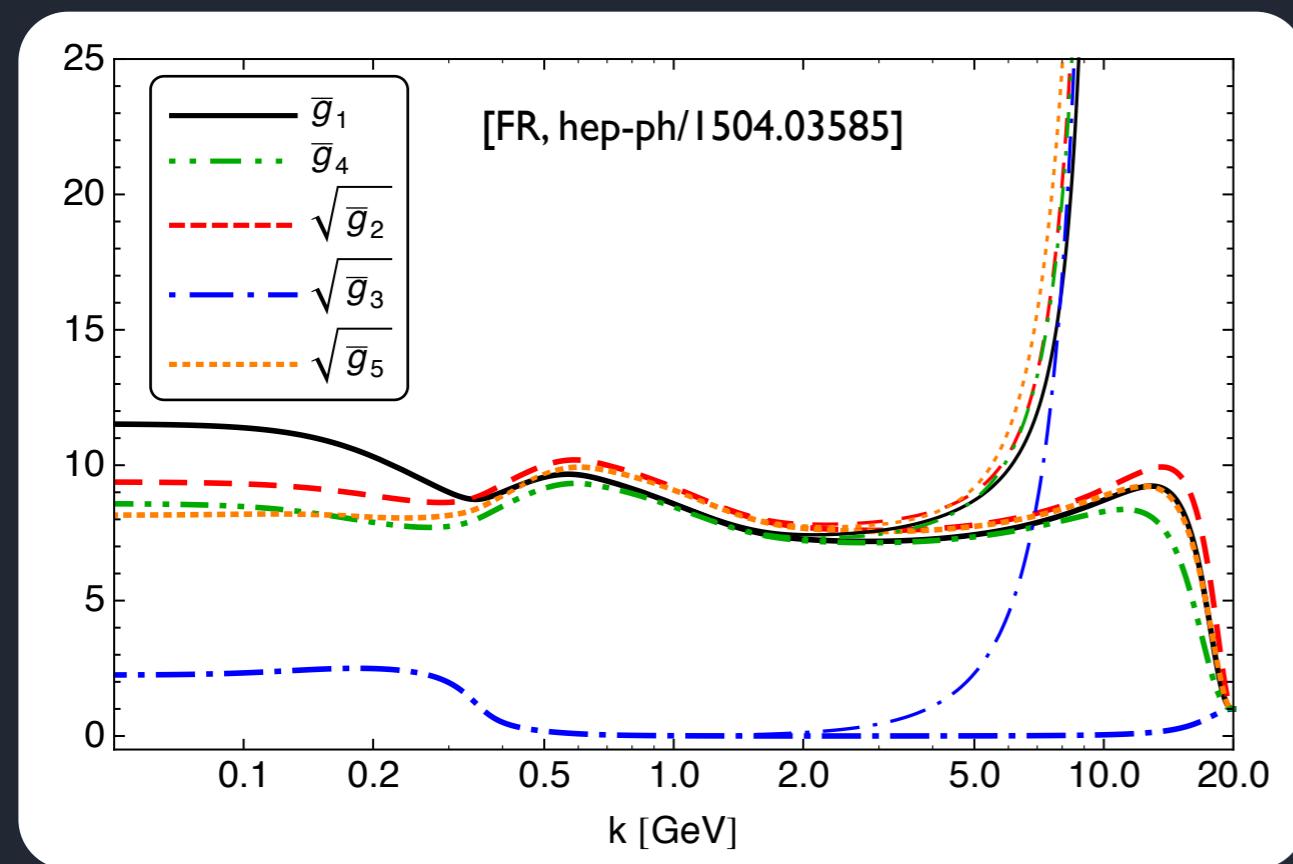
→ ‘mesons’ become scale dependent fields

$$\partial_t \phi_k \sim \frac{\partial_t \lambda_{T,k}}{h_{T,k}} (\bar{q} T q)$$

e.g. for  $\rho$

$$T = \gamma_\mu \vec{\tau}$$

→ low-energy parameters are predicted; only  $\alpha_s$  and quark masses as input  
(intermediate fixed point in 4-quark interaction!)



- no VMD assumed here:  
vector meson couplings different a priori

- VMD would imply

$$g_V = g_1 = \sqrt{g_2} = g_4 = \sqrt{g_5}$$
$$g_3 = 0$$

# SUMMARY & OUTLOOK

- in-medium vector- and axial-vector meson spectral functions
  - low-energy model based on VMD
  - direct, non-perturbative computation
  - degeneration of vector and axial-vector spectral functions above the chiral transition
  - consistent with melting resonance scenario
- including gauge fields: dynamical hadronization
  - no model parameter dependence
  - ‘test’ of VMD

Outlook / work in progress:

- ‘marry’ dynamical hadronization and analytic continuation
- vector meson fluctuations
- baryons & more decay channels
- confinement
- dilepton rates

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Outlook / work in progress

- ‘marry’ dynamical hadronization and analytic continuation
- vector meson form factors
- baryons & more decay channels
- confinement
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thank you