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4 Making Second Quantization work.

4.1 Field Operators.

$$\langle x|a^\dagger_p|p\rangle = \sum_p \langle x|p\rangle\langle p|a^\dagger|p\rangle$$

$$= \sum_p$$

$$\langle x|a^\dagger_p = \sum_p \langle x|a^\dagger_p\rangle\langle p|$$

$$= \sum_p \langle x|p\rangle\langle p|a^\dagger$$

$$= \frac{1}{\sqrt{V}}\sum_p e^{ip\cdot x}\langle p|a^\dagger$$

$$\frac{1}{\sqrt{V}}\sum_p a^\dagger_p e^{-ip\cdot x} = \sum_p a^\dagger_p \langle p|x\rangle$$

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$$\hat{\psi}^\dagger(x) = \frac{1}{\sqrt{V}}\sum_p a_p^\dagger e^{-ip\cdot x}$$

creates part @ x.

$$\hat{\psi}(x) = \frac{1}{\sqrt{V}}\sum a_p e^{ip\cdot x}$$

annihilates part @ x.

Ex 4.1 $\quad |\Psi\rangle = \hat{\psi}^\dagger(x)|0\rangle$

$$|\Psi\rangle = \psi^\dagger(x)|0\rangle = \frac{1}{\sqrt{V}}\sum_p e^{-ip\cdot x} a_p^\dagger|0\rangle$$

$$\sum_q \hat{n}_q|\Psi\rangle = \frac{1}{\sqrt{V}}\sum_{qp} a_q^\dagger a_q a_p^\dagger|0\rangle e^{-ip\cdot x}$$

$$a_q a_p^\dagger|0\rangle = \delta_{qp}|0\rangle$$

$$\Rightarrow \sum_q \hat{n}_q|\Psi\rangle = \frac{1}{\sqrt{V}}\sum_{qp} a_q^\dagger \delta_{qp}|0\rangle e^{-ip\cdot x}$$

$$= \frac{1}{\sqrt{V}}\sum_p a_p^\dagger|0\rangle e^{-ip\cdot x}$$

$$= |\Psi\rangle$$

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$$\langle y|\Phi\rangle = \langle y|\psi^{\dagger}(x)|0\rangle$$

$$= \frac{1}{\sqrt{V}}\sum_{p} e^{-ip\cdot x}\langle y|p\rangle$$

$$= \frac{1}{V}\sum_{p} e^{-ip(x-y)}$$

$$= \delta^{(3)}(x-y).$$

Ex 4.2

$$[\psi(x), \psi^\dagger(y)] = \frac{1}{V}\sum_{pq} e^{i(p\cdot x - q\cdot y)}[a_p, a_q^\dagger]$$

$$= \frac{1}{V}\sum_{p} e^{i(p\cdot(x-y))}$$

$$= \delta^{(3)}(x-y)$$

$$[\psi^\dagger(x), \psi^\dagger(y)] = [\psi(x), \psi(y)] = 0$$

$$\{\psi(x), \psi^\dagger(y)\} = \delta^{(3)}(x-y)$$

$$\{\psi^\dagger(x), \psi^\dagger(y)\} = \{\psi(x), \psi(y)\} = 0.$$