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Mo Tu We Th Fr Sa Su

5.3: Classical Fields:

Classical field takes location in spacetime and returns a scalar, vector, tensor, or something more complicated that represents an amplitude in some way.

5.4 Lagrangian and Hamiltonian Density

Ex 5.5

$$H=\sum_j \frac{p_j^2}{2m}+\frac{1}{2}K(q_{j+1}-q_j)^2$$

$$L=\sum_j \frac{p_j^2}{2m}-\frac{1}{2}K(q_{j+1}-q_j)^2$$

$$\sum_j \to \frac{1}{\ell}\int dx$$

$$\sum_j \frac{1}{2}m\left(\frac{\partial q_j}{\partial t}\right)^2 \to \frac{1}{\ell}\int dx\, \frac{1}{2}m\left(\frac{\partial \phi(x,t)}{\partial t}\right)^2$$

$$=\int dx\, \frac{1}{2}\rho\left(\frac{\partial \phi}{\partial t}\right)^2$$

=

L

Then

$\mathcal{L}, \mathcal{H}$

Conjugate m

$\Pi(x)$

$$\frac{q_{j+1} - q_j}{l} \to \frac{\partial \phi(x,t)}{\partial x}$$

$$\Rightarrow \sum_j \frac{1}{2} K (q_{j+1} - q_j)^2 \to \int dx \, \frac{1}{2} \tau \left(\frac{\partial \phi}{\partial x}\right)^2,$$

$$\tau = Kl$$

$$\Rightarrow H = \int d^3x \left[\frac{1}{2} \rho \left(\frac{\partial \phi}{\partial t}\right)^2 + \frac{1}{2} \tau (\nabla \phi)^2 \right]$$

$$L = \int d^3x \left[\frac{1}{2} \rho \left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2} \tau (\nabla \phi)^2 \right]$$

Then

$$H = \int d^3x \, \mathcal{H}$$

$$L = \int d^3x \, \mathcal{L}$$

$\mathcal{L}$, $\mathcal{H}$ functions of ϕ, $\dot{\phi}$, and ϕ'

Conjugate momentum:

$$\Pi(x) \equiv \frac{\delta L}{\delta \dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

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Then $\mathcal{H} = \pi\dot{\phi} - \mathcal{L}$

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\delta S = 0 \Rightarrow \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0.$$

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$$\phi(x,t) = \sum_{k_n} a_{k_n} e^{-i(\omega t - k_n \cdot x)}$$

$\Rightarrow$ superpos of harm oscill $\Rightarrow$ quantized.

For EM field, quanta are photons