

$$\frac{\delta F}{\delta f(x)} = \lim_{\epsilon \rightarrow 0} \frac{F[f(x) + \epsilon \delta(x-x)] - F[f(x)]}{\epsilon}$$

$$S = \int dt dx \mathcal{L}(q, \frac{\partial q}{\partial t}, \frac{\partial q}{\partial x})$$

$$\begin{aligned} \frac{\delta S}{\delta q} &= \lim_{\epsilon \rightarrow 0} \frac{S[q(x,t) + \epsilon \delta((x-t), (t-t))] - S[q(x,t)]}{\epsilon} \\ &= \lim_{\epsilon \rightarrow 0} \int dt dx \mathcal{L}(q + \epsilon \delta((x-t), (t-t)), \frac{\partial q}{\partial t}, \frac{\partial q}{\partial x}) - \int dt dx \mathcal{L}(q, \frac{\partial q}{\partial t}, \frac{\partial q}{\partial x}) \\ &= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int dt dx (\epsilon \delta(x-t) \frac{\partial q}{\partial t} + \epsilon \delta(t-t) \frac{\partial q}{\partial x}) \end{aligned}$$

$$\frac{SS}{S^4} = \frac{SL}{S^4} - \frac{d}{dt} \frac{SL}{S^4}$$

$$\frac{SL}{S^4} = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int (g^{[4+\epsilon S]} - g^{[4]}) dx.$$

$$= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int \epsilon S \frac{\partial g}{\partial \epsilon} dx$$

$$= \frac{\partial g}{\partial \epsilon}$$

$$\frac{\delta L}{\delta \dot{q}} = \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})}$$

$$\partial \dot{q} = \frac{\partial t \partial^2 q - \partial q \partial t^2}{(\partial t)^2}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} =$$

$$\begin{aligned}\frac{\delta L}{\delta \dot{q}} &= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \left\{ \mathcal{L}(q, \dot{q} + \epsilon \dot{q}_\epsilon) - \mathcal{L}(q, \dot{q}) \right\} \\ &= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int \frac{\partial \mathcal{L}}{\partial \dot{q}} d\dot{q}\end{aligned}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} = \frac{d}{dx}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} = \frac{d}{dt} \frac{\partial}{\partial \frac{\partial^2 q - \partial q \partial t^2}{\partial t^2}}$$

$$= \frac{d}{dt} \frac{(\partial q)^2}{\partial t^2}$$

$$= - \frac{(\partial q)^2}{\partial t^2 \partial^2 q - \partial q \partial t^2}$$

$$= \frac{\partial}{\partial \dot{q} - \partial q \partial t}$$

Since $L = L(y, \frac{dy}{dx}, \frac{d^2y}{dt^2})$,
Euler-Lagrange $\rightarrow \frac{\delta S}{\delta y} = \frac{\delta L}{\delta y} - \frac{d}{dt} \frac{\delta L}{\delta (\frac{dy}{dt})} - \frac{d}{dx} \frac{\delta L}{\delta (\frac{d^2y}{dt^2})}$

$$\Rightarrow \frac{\delta S}{\delta y} = \frac{\partial y}{\partial t} - \frac{d}{dx} \frac{\partial y}{\partial (\frac{dy}{dt})} - \frac{d}{dt} \frac{\partial y}{\partial (\frac{d^2y}{dt^2})} = 0$$

$$L = \frac{1}{2} \left(\frac{dy}{dt} \right)^2 - \frac{1}{2} \left(\frac{d^2y}{dt^2} \right)^2$$

$$\Rightarrow 0 - \frac{d}{dt} \left(\frac{1}{2} \frac{dy}{dt} \right) + \frac{d}{dx} \nabla \left(\frac{1}{2} \frac{dy}{dt} \right) = 0 \Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{1}{2} \frac{\partial^2 y}{\partial x^2}$$

$$\Rightarrow v = \sqrt{\int p^2}$$

$$\text{for } \frac{\partial^2 y}{\partial t^2} \left(\frac{\partial y}{\partial t} \right)^2$$

$$\frac{\partial^2}{\partial t^2}(\dots)$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^4}{\partial t^4}\right)^2$$

$$= 2\left(\frac{\partial^4}{\partial t^4}\right) \frac{\partial^2}{\partial t^2} \frac{\partial^4}{\partial t^4}$$

$$= 2 \frac{\partial^4}{\partial t^4} \frac{\partial^2}{\partial t^2}$$

$$= 2 \frac{\partial^4}{\partial t^4} \frac{\partial^2}{\partial t^2}$$

$$= 2 \frac{\partial^4}{\partial t^4} \frac{\partial^2}{\partial t^2}(1)$$

\Rightarrow

$$\text{for } \frac{\partial^2}{\partial t^2}\left(\frac{\partial^4}{\partial t^4}\right)^2$$

= 0.