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4.2 How to Second Quantize an operator.

We shall only focus on single particle operators for now!

$$\hat{A} = \sum_{\alpha\beta} |\alpha\rangle\langle\alpha|\hat{A}|\beta\rangle\langle\beta| = \sum_{\alpha\beta} A_{\alpha\beta} |\alpha\rangle\langle\beta|.$$

Single part states $\Rightarrow$ Hilbert Space

many Part States $\Rightarrow$ Fock Space.

Fock Space: $\boxed{\hat{A} = \sum_{\alpha\beta} A_{\alpha\beta} a^\dagger_\alpha a_\beta}$

Ex 4.4.

(i) $\hat{n} = \sum_{\alpha} a^{\dagger}_{\alpha} a_{\alpha}$

(ii) $\hat{p} = \sum_{p} p\, a^{\dagger}_{p} a_{p} = \sum_{p} p\, \hat{n}_{p}$

$\hat{A} = f(p) \Rightarrow \hat{A} = \sum_{p} f(p)\, a^{\dagger}_{p} a_{p} = \sum_{p} f(p)\, \hat{n}_{p}$

$\Rightarrow \hat{H} = \sum \frac{p^2}{2m} \hat{n}_{p}$ (free particle).

(iii) $\hat{V} = \int d^3x\, \hat{\psi}^{\dagger}(x)\, V(\hat{x})\, \hat{\psi}(x)$

$= \frac{1}{V} \int d^3x \sum_{p_1 p_2} a^{\dagger} e^{-ip_1 \cdot x}\, V(\hat{x})\, a_{p_2} e^{ip_2 \cdot x}$

$= \sum_{p_1 p_2} \tilde{V}_{p_1 - p_2}\, a^{\dagger}_{p_1} a_{p_2}$

$\tilde{V}_p = \frac{1}{V} \int d^3x\, V(x)\, e^{-ip \cdot x}$ is fourier trans of V

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Ex 4.5

$V = 0 =$

Do

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Ex 4.5

$$\hat{H} = E_0 \sum_p a_p^\dagger a_p - \frac{V}{2} \sum_{p_1 p_2} a_{p_1}^\dagger a_{p_2}$$

$V=0 \Rightarrow |100\rangle, |010\rangle, |001\rangle.$

Do this on board

Ex 4.6

$$\hat{\rho}(x) = \psi^\dagger(x)\psi(x)$$

$$= \frac{1}{V} \sum_{p_1 p_2} \left[e^{-i(p_1 - p_2)\cdot x} \right] a_{p_1}^\dagger a_{p_2}.$$

$$\Rightarrow \hat{V} = \int d^3x \, V(x) \hat{\rho}(x).$$