Memo No.
Date / /

Mo Tu We Th Fr Sa Su

1.4 Lagrangians and least action

$$\frac{\delta \bar{V}[x]}{\delta x(t)} = \frac{\bar{V}'[x(t)]}{\tau},$$

$$\frac{\delta \bar{T}[x]}{\delta x(t)} = -\frac{m\ddot{x}}{\tau}$$

Note that for the classical trajectory
$m\ddot{x} = -dV/dx$

$$\Rightarrow \frac{\delta \bar{V}[x]}{\delta x(t)} = \frac{\delta \bar{T}[x]}{\delta x(t)}. \quad [\text{classical}].$$

i.e. $\frac{\delta}{\delta x(t)}\left[\bar{T}[x] - \bar{V}[x]\right] = 0.$

$$L = T - V \quad [\text{Lagrangian}]$$

$$S = \int_0^\tau L\,dt, \quad [\text{Action}].$$

$$S = \int_0^\tau (T-V)\,dt = \tau(\bar{T}-\bar{V})$$

$$\Rightarrow \frac{\delta S}{\delta x(t)} = 0.$$

Hamilton's Principle of least Action

Mo Tu We Th Fr Sa Su

Memo No. ______

Date / /

Ex 1.3

$$\frac{\delta S}{\delta x(t)} = \int du \left[\frac{\delta L}{\delta x(u)} \frac{\delta x(u)}{\delta x(t)} + \frac{\delta L}{\delta \dot{x}(u)} \frac{\delta \dot{x}(u)}{\delta x(t)} \right.$$

$$= \int du \left[\frac{\delta L}{\delta x(u)} \delta(u-t) + \frac{\delta L}{\delta \dot{x}(u)} \frac{d}{dt} \delta(u-t) \right].$$

$$= \frac{\delta L}{\delta x(t)} - \frac{d}{dt} \frac{\delta L}{\delta \dot{x}(t)} = 0.$$

Euler-Lagrange Eqn.

$$L = \int dx\, \mathcal{L} \Rightarrow S = \int dt\, dx\, \mathcal{L}$$

Ex 1.4 $\rho = m/l$. $T = \frac{1}{2}\int_0^l dx\, \rho\, (\partial\psi/\partial t)^2$

$$V = \frac{1}{2}\int_0^l dx\, T (\partial\psi/\partial x)^2$$

$$S[\psi(x,t)] = \int dt (T-V) = \int dt\, dx\, \mathcal{L}\left(\psi, \frac{\partial\psi}{\partial t}, \frac{\partial\psi}{\partial x}\right)$$

$$\mathcal{L}\left(\psi, \frac{\partial\psi}{\partial t}, \frac{\partial\psi}{\partial x}\right) = \frac{\rho}{2}\left(\frac{\partial\psi}{\partial t}\right)^2 + \frac{T}{2}\left(\frac{\partial\psi}{\partial x}\right)^2$$

$$0 = \frac{\delta S}{\delta\psi} = \frac{\partial\mathcal{L}}{\partial\psi} -$$

(CHALK BOARD.

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu\phi)$$

$$\cancel{\frac{\partial S}{\partial\phi}}$$

$$\frac{\delta S}{\delta\phi} = \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) = 0$$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.
Date / /

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right)$$

$$= -\cancel{2}m^2\phi - \partial_\mu \partial^\mu \phi = 0$$

$$\Rightarrow (\partial^2 + m^2)\phi = 0.$$

KLEIN-GORDON. WOAH!