



Mo Tu We Th Fr Sa Su

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1.4 Lagrangians and least action.

$$\cancel{\frac{\delta \bar{T}[x]}{\delta x(t)}} = \frac{\bar{V}'[x(t)]}{\bar{T}},$$

$$\frac{\delta \bar{T}[x]}{\delta x(t)} = - \frac{m \ddot{x}}{\bar{T}}$$

Note that for the classical trajectory
 $m \ddot{x} = -dV/dx$

$$\Rightarrow \frac{\delta \bar{V}[x]}{\delta x(t)} = \frac{\delta \bar{T}[x]}{\delta x(t)}. \text{ [classical].}$$

$$\text{i.e. } \frac{\delta}{\delta x(t)} [\bar{T}[x] - \bar{V}[x]] = 0.$$

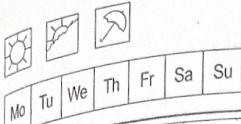
$$L = T - V \quad \text{[Lagrangian]}$$

$$S = \int_0^T L dt, \quad \text{[Act. m].}$$

$$S = \int_0^T (T - V) dt = T(\bar{T} - \bar{V}) \cancel{\text{H.O.}}$$

$$\Rightarrow \frac{\delta S}{\delta x(t)} = 0.$$

Hamilton's Principle of least Action



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Ex 1.3

$$\frac{\delta S}{\delta x(t)} = \int du \left[\frac{\delta L}{\delta x(u)} \frac{\delta x(u)}{\delta x(t)} + \frac{\delta L}{\delta \dot{x}(u)} \frac{\delta \dot{x}(u)}{\delta x(t)} \right]$$

$$= \int du \left[\frac{\delta L}{\delta x(u)} \delta(x-u) + \frac{\delta L}{\delta \dot{x}(u)} \frac{d}{dt} \delta(x-u) \right].$$

$$= \frac{\delta L}{\delta x(t)} - \frac{d}{dt} \frac{\delta L}{\delta \dot{x}(t)} = 0.$$

Euler-Lagrange Eqn.

$$L = \int dx \mathcal{L} \Rightarrow S = \int dt dx \mathcal{L}$$



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Ex 1.4 $f = m/l$. $T = \frac{1}{2} \int_0^l dx f (\partial \psi / \partial t)^2$

 $v = \frac{1}{2} \int_0^l dx \cancel{f} (\partial \psi / \partial x)^2$

$$S[\psi(x,t)] = \int dt (T - v) = \int dt dx \mathcal{L}(\psi, \frac{\partial \psi}{\partial t}, \frac{\partial \psi}{\partial x})$$
 $\mathcal{L}(\psi, \frac{\partial \psi}{\partial t}, \frac{\partial \psi}{\partial x}) = \frac{\rho}{2} \left(\frac{\partial \psi}{\partial t} \right)^2 + \frac{T}{2} \left(\frac{\partial \psi}{\partial x} \right)^2$

$$\mathcal{O} = \frac{\delta S}{\delta \psi} = \frac{\partial \mathcal{L}}{\partial \psi} - \quad \left| \begin{array}{l} \text{CHALK BOARD} \\ \text{K} \end{array} \right.$$

$$S = \int dx \mathcal{L}(\phi, \partial_\mu \phi)$$

~~2S
Sφ~~

$$\frac{\delta S}{\delta \phi} = \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) \approx$$



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$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2$$

$$\Rightarrow \frac{\partial L}{\partial \phi} - \partial_\mu \left(\frac{\partial L}{\partial \partial_\mu \phi} \right)$$

$$= -\cancel{2}m^2\phi - \partial_\mu \partial^\mu \phi = 0$$

$$\Rightarrow (\partial^2 + m^2)\phi = 0.$$

Klein-Gordon. woah!