$$\frac{\delta F}{\delta f(x)} = \lim_{\epsilon \to 0} \frac{F[f(x) + \epsilon \delta(x' - x)] - F[f(x)]}{\epsilon}$$

$$S = \int dt\, L = \int dt\, dx\, \mathcal{L}\left(\phi, \frac{\partial \phi}{\partial t}, \frac{\partial \phi}{\partial x}\right)$$

$$\frac{\delta S}{\delta \phi} = \lim_{\epsilon \to 0} \frac{S[\phi(x,t) + \epsilon \delta^2((x'-x),(t'-t))] - S[\phi(x,t)]}{\epsilon}$$

$$= \lim_{\epsilon \to 0} \frac{\int dt\, dx\, \mathcal{L}(\phi + \epsilon \delta((x'-x),(t'-t)), //, ///) - \int dt\, dx\, \mathcal{L}\left(\phi, \frac{\partial \phi}{\partial t}, \frac{\partial}{\partial \ldots}\right.}{\epsilon}$$

$$= \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int dt\, dx \left(\epsilon \delta(x'-x) \frac{\partial \phi}{\partial x} + \epsilon \delta(t'-t) \frac{\partial \phi}{\partial t} \right.$$

$$\frac{\delta S}{\delta q} = \frac{\delta L}{\delta q} - \frac{d}{dt}\frac{\delta L}{\delta (\dot{q})}$$

$$\frac{\delta L}{\delta q} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int \left(\mathcal{L}[q + \epsilon \delta] - \mathcal{L}[q] \right) dx$$

$$= \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int \epsilon \delta \frac{\partial \mathcal{L}}{\partial q} \, dx$$

$$= \frac{\partial \mathcal{L}}{\partial q}$$

$$\frac{\delta L}{\partial \dot\psi} = \frac{\partial \mathcal{L}}{\partial \dot\psi} = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial\psi}{\partial t}\right)}$$

$$\partial\dot\psi = \frac{\partial t\,\partial^2\psi^2 - \partial\psi\,\partial^2 t^2}{(\partial t)^2}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial\left(\frac{\partial\psi}{\partial t}\right)} =$$

$$\frac{\delta L}{\delta\dot\psi} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int \mathcal{L}(\psi,\dot\psi+\epsilon\delta) - \mathcal{L}(\psi,\dot\psi)$$

$$= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int \epsilon\delta\frac{\partial\mathcal{L}}{\partial\dot\psi}$$

$$= \frac{\partial\mathcal{L}}{\partial\dot\psi}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\left(\frac{\partial\psi}{\partial t}\right)} = \frac{d}{dx}$$

$$\frac{d}{dt}\frac{\partial}{\partial\left(\frac{\partial\psi}{\partial t}\right)} = \frac{d}{dt}\frac{\partial}{\frac{\partial t\,\partial^2\psi - \partial\psi\,\partial^2 t}{(\partial t)^2}}$$

$$= \frac{d}{dt}\frac{(\partial t)^2\partial}{\partial t\,\partial^2\psi - \partial\psi\,\partial^2 t}$$

$$= \frac{(\partial t)^2\partial}{\partial_t\partial^2\psi - \partial\psi\,\partial^2 t}$$

$$= \frac{\partial}{\partial\psi - \partial\psi\,\partial t}$$

Since $\mathcal{L} = \mathcal{L}(\psi, \partial\psi/\partial x, \partial\psi/\partial t)$,

Euler-Lagrange $\longrightarrow \dfrac{\delta S}{\delta\psi} = \dfrac{\delta \mathcal{L}}{\delta\psi} - \dfrac{d}{dt}\dfrac{\delta\mathcal{L}}{\delta(\partial\psi/\partial t)} - \dfrac{d}{dx}\dfrac{\delta\mathcal{L}}{\delta(\partial\psi/\partial x)}$

$$\Longrightarrow \frac{\delta S}{\delta\psi} = \frac{\partial\mathcal{L}}{\partial\psi} - \frac{d}{dx}\frac{\partial\mathcal{L}}{\partial(\partial\psi/\partial x)} - \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial(\partial\psi/\partial t)} = 0$$

$$\mathcal{L} = \frac{\rho}{2}\left(\frac{\partial\psi}{\partial t}\right)^2 - \frac{T}{2}\left(\frac{\partial\psi}{\partial x}\right)^2$$

$$\Rightarrow 0 - \frac{d}{dt}\left(\rho\frac{\partial\psi}{\partial t}\right) + \frac{d}{dx}T\left(\frac{\partial\psi}{\partial x}\right) = 0 \Rightarrow \frac{\partial^2\psi}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2\psi}{\partial t^2}$$

$$\Rightarrow v = \sqrt{\frac{T}{\rho}}$$

for $\dfrac{\partial}{\partial\psi}\left(\dfrac{\partial\psi}{\partial t}\right)^2 = 0$

$\frac{\partial}{\partial y}(\partial$

$$\frac{\partial}{\partial y}\left(\frac{\partial y}{\partial t}\right)^2$$

$$= 2\left(\frac{\partial y}{\partial t}\right)\frac{\partial}{\partial y}\frac{\partial y}{\partial t}$$

$$= 2\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial y\,\partial t}$$

$$= 2\frac{\partial y}{\partial t}\frac{\partial}{\partial t}(1)$$

$$= 0$$

~ for $\frac{\partial}{\partial y}\left(\frac{\partial y}{\partial t}\right)^2$

$= \ldots$

$\Rightarrow$