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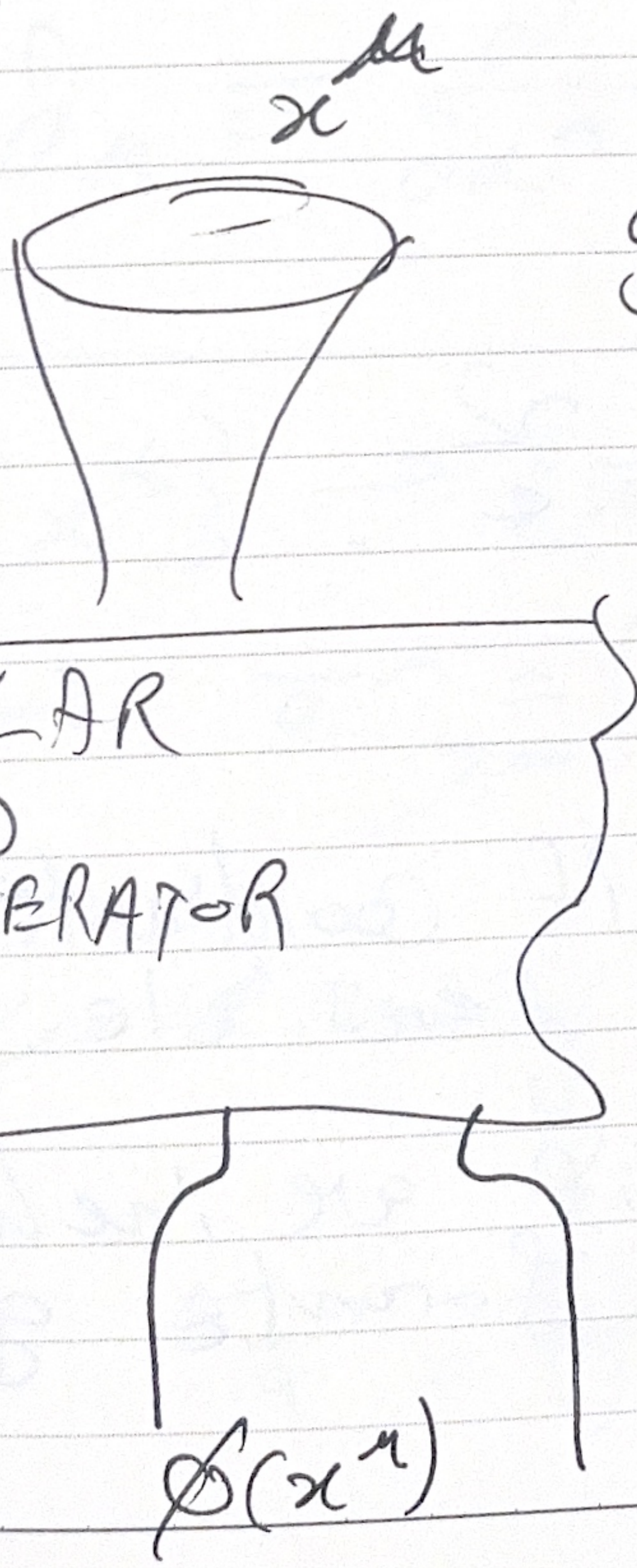
0.1 What is Quantum Field Theory?

"Every particle and every wave in the universe is simply an excitation of a quantum field that is defined over all space and time."

0.2 What is a Field?

Machine that takes pos and returns obj rep the amp of something.



Nice, book says grav waves are yet to be disc.

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~~Q.3 Who is this book for?~~

It is for me. :)

Q.4 Special Relativity:

$S, \bar{S}$ w/ $\bar{S}$ moving relative to S at speed v along x-axis.

$$\bar{t} = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\bar{x} = \gamma(x - vt)$$

$$\bar{y} = y$$

$$\bar{z} = z$$

$\gamma = (1-\beta^2)^{-1/2}$, $\beta = \frac{v}{c}$.

set $\hbar = c = 1$

Theory is covariant if coordinate transformations are sensible.

Lorentz covariant if we include transformations in Lorentz group.

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Scalars: Just a number that is same in every frame. Thus, it is Lorentz invariant.

Vectors: 4 vectors w/ x^μ, $\mu = 0,1,2,3$.

Ex 0.1

$$p = (E, \vec{p})$$

$$j = (\rho, \vec{j})$$

$$A = (V, \vec{A}).$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \nabla\right) = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\{x^\mu\} \to \{\bar{x}^\mu\}$$

$$\to \quad \bar{a}^\mu = \left(\frac{\partial \bar{x}^\mu}{\partial x^\nu}\right) a^\nu$$

of course using Einstein Sum.

$$\bar{a}^1 = \left(\frac{\partial \bar{x}^1}{\partial x^0}\right) a^0 + \left(\frac{\partial \bar{x}^1}{\partial x^1}\right) a^1 + \cdots$$

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$\partial_\mu \phi \equiv \partial\phi / \partial x^\mu$ transforms:

$$\frac{\partial \phi}{\partial x^\mu} = \left(\frac{\partial x^\nu}{\partial \bar{x}^\mu}\right)\frac{\partial \phi}{\partial x^\nu}$$

Something like a^μ is said to be contravariant and ~~ϕ~~ $\partial_\mu \phi$ is covariant.

$$\begin{pmatrix} \bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

or $\bar{x}^\mu = \Lambda^\mu{}_\nu x^\nu$, where

$$\Lambda^\mu{}_\nu \equiv (\partial \bar{x}^\mu / \partial x^\nu).$$

$$\bar{p}^\mu = \Lambda^\mu{}_\nu p^\nu$$

$$a_\mu = \Lambda_\mu{}^\nu a_\nu$$

$$\Lambda_\mu{}^\nu \equiv (\partial x^\nu / \partial x^\mu)$$

Note that $\eta^{\mu}{}_{\gamma}\,\eta_{\mu}{}^{\rho} = \delta^{\rho}_{\nu}$.

$$|x| = x\cdot x = (x^0)^2 - (x^1)^2 - \cdots$$

$$a\cdot b = a^0 b^0 - \vec{a}\cdot\vec{b}, \text{ or}$$

$$a\cdot b = g_{\mu\nu}a^{\mu}b^{\nu}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$a_{\mu} = g_{\mu\nu}a^{\nu}$$

$$a^0 = a_0, \quad a^i = -a_i$$

$$\Rightarrow a\cdot b = g_{\mu\nu}a^{\mu}b^{\nu} = a_{\mu}b^{\mu} = a^{\mu}b_{\mu}$$

$$a\cdot b = g^{\mu\nu}a_{\mu}b_{\nu}$$

$$g_{\mu\nu} = g^{\mu\nu}$$

$$\cancel{g_{\mu}{}^{\nu} = g \quad g_{\mu\nu} = g^{\mu\nu}}$$

$$g_{\mu}{}^{\nu} = g^{\gamma\nu}g_{\mu\gamma}, \quad \cancel{g^{\mu\nu} = g^{\mu\gamma}g_{\gamma}{}^{\nu}}$$

$$\cancel{= g^{\mu\gamma}g^{\gamma\nu}g_{\mu\gamma}}$$

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Exercise $g_{\mu\nu}g^{\nu\rho} = g_{\mu 0}g^{0\rho} + g_{\mu 1}g^{1\rho} + g_{\mu 2}g^{2\rho}$
$+ g_{\mu 3}g^{3\rho}$

Sup $\mu \neq \rho$ then one of g's in prod will be 0 for all $\Rightarrow g_{\mu\nu}g^{\nu\rho} = 0$, $\mu \neq \rho$.
if $\mu = \rho$, then $g_{\mu\nu}g^{\nu\rho} = 1$ obv.

Exercise 5 show $\Lambda_\mu{}^\nu = g_{\mu k}\Lambda^k{}_\rho g^{\rho\nu}$
$= \Lambda_{\mu\rho}g^{\rho\nu}$
$= \Lambda_\mu{}^\nu$.

~~$g_{\mu\nu} = g_\mu{}^k g_{k\nu} = g$~~

~~$g_\mu{}^\nu = g_\mu$~~
~~$g_\mu{}^\nu = g_k{}^\nu g_{\mu k}$~~

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$$g_{\mu\nu} = g_{\mu k}\, g^{k}{}_{\nu}$$

$$g^{k}{}_{\nu} = g^{k\gamma}\, g_{\gamma\nu}$$

$$g^{k}{}_{\nu} = \cancel{g_{\gamma\nu}\, g^{k\gamma}}$$

$$= g^{k\gamma}\, g_{\gamma\nu}$$

$$\Rightarrow g_{\mu\nu} = g_{\mu k}\, g^{k\gamma}\, g_{\gamma\nu}$$

$$\cancel{= \delta^{\gamma}_{\mu}\, g_{\gamma\nu}}$$

$$\cancel{= g}$$

$$= g_{\mu k}\, g^{k\gamma}\, g^{\gamma\nu}$$

$$= \delta^{\gamma}_{\mu}\, g^{\gamma\nu}$$

$$= g^{\mu\nu}$$

$$\Rightarrow g_{\mu\nu} = g^{\mu\nu}.$$

Ex 0.2

i) $P\cdot P = p_\mu p^\mu = (E,\vec{p})\cdot(E,\vec{p}) = E^2 - p^2 = m^2$

ii) $\cancel{\partial x}\ \partial_\mu x^\nu = \frac{\partial x^\nu}{\partial x^\mu} = \delta^\nu_\mu$

$\Rightarrow \partial_\mu x^\mu = 4.$

$\cancel{\partial^2 = \partial_\mu \partial}$

$\partial^2 = \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2.$

Finally,

$$\bar{T}^{i'\cdots k'}_{l'\cdots n'} = \frac{\partial \bar{x}^{i'}}{\partial x^i}\cdots\frac{\partial \bar{x}^{k'}}{\partial x^k}\frac{\partial x^l}{\partial x^{l'}} T^{i\cdots k}_{l\cdots n}$$

Ex 0.3 $\bar{\delta}^j_i = \frac{\partial \bar{x}^i}{\partial x^k}\frac{\partial x^l}{\partial \bar{x}^j}\delta^l_k = \frac{\partial \bar{x}^i}{\partial x^k}\frac{\partial x^k}{\partial \bar{x}^j} = \delta^i_j$

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0.5 Fourier Transforms

$$\tilde{f}(k) = \int d^4x\, e^{ik\cdot x} f(x)$$

$$\int d^4x = \int dx^0 dx^1 dx^2 dx^3$$

$$f(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \tilde{f}(k).$$

Also

$$\tilde{f}(\omega, \vec{k}) = \int d^3x\, dt\, e^{i(\omega t - \vec{k}\cdot\vec{x})} f(t, \vec{x})$$

Ex 0.4

$$\int d^d \delta^{(d)}(x) = 1$$

$$\int d^d x\, f(x)\, \delta^{(d)}(x) = f(0)$$

$$\tilde{\delta}^{(d)}(k) = \int d^d x\, e^{ik\cdot x} \delta^{(d)}(x) = 1$$

$$\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} = \delta^{(4)}(x).$$

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0-6 Electromagnetism

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}, \quad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Use $\varepsilon_0 = \mu_0 = 1$

$$\Rightarrow V(\vec{x}) = \frac{q}{4\pi|\vec{x}|}$$

$$\vec{\nabla}\cdot\vec{E} = \rho, \quad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$

$$\vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{B} = \frac{1}{c}\left(\vec{J} + \frac{\partial \vec{E}}{\partial t}\right).$$

$$\alpha = \frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}$$

$$\rightarrow \alpha = \frac{e^2}{4\pi}.$$

use $q = \cancel{Q}\,|e|$.