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4 Making Second Quantization work.

4.1 Field Operators

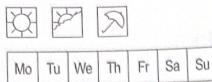
$$\langle x | a_p^\dagger | p \rangle = \sum_p \langle x | p \rangle \langle p | a_p^\dagger \rangle$$
$$= \sum_p$$

$$\langle x | a_p^\dagger = \sum_p \langle x | a_p^\dagger \rangle \langle p |$$

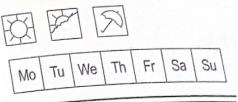
$$= \sum_p \langle x | p \rangle \langle p | a^\dagger$$

$$= \sqrt{\frac{1}{V}} \sum_p e^{ip \cdot x} \langle p | a^\dagger$$

$$\frac{1}{\sqrt{V}} \sum_p a_p^\dagger e^{-ip \cdot x} = \sum_p a_p^\dagger \langle p | x \rangle$$



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$$\hat{\psi}^+(x) = \frac{1}{\sqrt{V}} \sum_p a_p^+ e^{ip \cdot x}$$

Creates part Ψ \propto .

$$\hat{\psi}(x) = \frac{1}{\sqrt{V}} \sum_p a_p e^{ip \cdot x}$$

annihilates part Ψ \propto .

~~Ex 4.1~~

$$|\Psi\rangle = \hat{\psi}^+(x)|0\rangle$$

$$|\Psi\rangle = \hat{\psi}^+(x)|0\rangle = \frac{1}{\sqrt{V}} \sum_p e^{-ip \cdot x} a_p^+ |0\rangle$$

$$\sum_g |\Psi\rangle = \frac{1}{\sqrt{V}} \sum_{gP} a_g^+ a_g a_g^+ a_P^+ |0\rangle e^{-ip \cdot x}$$

$$a_g a_g^+ |0\rangle = S_{gp} |0\rangle$$

$$\Rightarrow \sum_g |\Psi\rangle = \frac{1}{\sqrt{V}} \sum_{gP} a_g^+ S_{gp} |0\rangle e^{-ip \cdot x}$$

$$= \frac{1}{\sqrt{V}} \sum_p a_p^+ |0\rangle e^{-ip \cdot x}$$

$$= |\Psi\rangle$$



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$$\langle y | \Psi \rangle = \langle y | \psi^*(x) | 0 \rangle$$

$$= \frac{1}{\sqrt{w}} \sum_p e^{-ip \cdot x} \langle y | p \rangle$$

$$= \frac{1}{\sqrt{w}} \sum_p e^{-ip(x-y)}$$

$$= \delta^{(3)}(x-y).$$

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Ex 4.2

$$[\psi(x), \psi^\dagger(y)] = \frac{1}{v} \sum_{pq} e^{i(pv - qy)} [a_p a_q^\dagger]$$
$$= \frac{1}{v} \sum_p e^{i(p \cdot (x-y))}$$
$$= \delta^{(3)}(x-y)$$

$$[\psi^\dagger(x), \psi^\dagger(y)] = [\psi(x), \psi(y)] = 0$$

$$\{\psi(x), \psi^\dagger(y)\} = \delta^{(3)}(x-y)$$

$$\{\psi^\dagger(x), \psi^\dagger(y)\} = \{\psi(x), \psi(y)\} = 0$$

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