

$$\frac{\delta F}{\delta f(x)} = \lim_{\epsilon \rightarrow 0} \frac{F[f(x) + \epsilon \delta(x' - x)] - F[f(x)]}{\epsilon}$$

$$S = \int dt dx \mathcal{L}(\varphi, \frac{\partial \varphi}{\partial t}, \frac{\partial \varphi}{\partial x})$$

$$\frac{\delta S}{\delta \varphi} = \lim_{\epsilon \rightarrow 0} \frac{S[\varphi(x, t) + \epsilon \delta^2((x' - x), (t' - t))] - S[\varphi(x, t)]}{\epsilon}$$

$$= \lim_{\epsilon \rightarrow 0} \int dt dx \mathcal{L}(\varphi + \epsilon \delta((x' - x), (t' - t)), //, //) - \int dt dx \mathcal{L}(\varphi, \frac{\partial \varphi}{\partial t}, \frac{\partial \varphi}{\partial x})$$

$$= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int dt dx (\epsilon \delta(x' - x) \frac{\partial \varphi}{\partial x} + \epsilon \delta(t' - t) \frac{\partial \varphi}{\partial t})$$

$$\frac{\delta S}{\delta \varphi} = \frac{\delta L}{\delta \varphi} - \frac{d}{dt} \frac{\delta L}{\delta \dot{\varphi}}$$

$$\begin{aligned} \frac{\delta L}{\delta \varphi} &= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int (\varphi[x+\epsilon\delta] - \varphi[x]) dx \\ &= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \int \epsilon \delta \frac{\partial \varphi}{\partial \varphi} dx \\ &= \frac{\partial \varphi}{\partial \varphi} \end{aligned}$$

$$\frac{\delta L}{\delta \dot{q}} = \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial t \frac{\partial q}{\partial t} - \frac{\partial q}{\partial t} \frac{\partial \mathcal{L}}{\partial t}}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} =$$

$$\frac{\delta L}{\delta \dot{q}} = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (\mathcal{L}(q, \dot{q} + \epsilon \delta) - \mathcal{L}(q, \dot{q}))$$

$$= \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \left(\epsilon \frac{\partial \mathcal{L}}{\partial \dot{q}} \right)$$

$$= \frac{\partial \mathcal{L}}{\partial \dot{q}}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} = \frac{d}{dx}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\frac{\partial q}{\partial t})} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial t \frac{\partial q}{\partial t} - \frac{\partial q}{\partial t} \frac{\partial \mathcal{L}}{\partial t}}$$

$$= \frac{d}{dt} \frac{(\frac{\partial \mathcal{L}}{\partial t}) \frac{\partial q}{\partial t}}{\partial t \frac{\partial q}{\partial t} - \frac{\partial q}{\partial t} \frac{\partial \mathcal{L}}{\partial t}}$$

$$= \frac{(\frac{\partial \mathcal{L}}{\partial t})^2 \frac{\partial q}{\partial t}}{\partial t \frac{\partial q}{\partial t} - \frac{\partial q}{\partial t} \frac{\partial \mathcal{L}}{\partial t}}$$

$$= \frac{2}{\partial t \frac{\partial q}{\partial t} - \frac{\partial q}{\partial t} \frac{\partial \mathcal{L}}{\partial t}}$$

Since $L = L(y, \partial y / \partial x, \partial y / \partial t)$,
 Euler-Lagrange $\rightarrow \frac{\delta S}{\delta y} = \frac{\delta L}{\delta y} - \frac{d}{dt} \frac{\delta L}{\delta (\partial y / \partial t)} - \frac{d}{dx} \frac{\delta L}{\delta (\partial y / \partial x)}$

$$\Rightarrow \frac{\delta S}{\delta y} = \frac{\partial \mathcal{L}}{\partial y} - \frac{d}{dx} \frac{\partial \mathcal{L}}{\partial (\partial y / \partial x)} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial (\partial y / \partial t)} = 0$$

$$\mathcal{L} = \frac{\rho}{2} \left(\frac{\partial y}{\partial t} \right)^2 - \frac{T}{2} \left(\frac{\partial y}{\partial x} \right)^2$$

$$\Rightarrow 0 - \frac{d}{dt} \left(\rho \frac{\partial y}{\partial t} \right) + \frac{d}{dx} T \left(\frac{\partial y}{\partial x} \right) = 0 \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow v = \sqrt{\frac{T}{\rho}}$$

$$v = \sqrt{\frac{\partial^2 y}{\partial x^2} \frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial}{\partial x} (0)$$

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\partial^2 \phi}{\partial t^2} \right)^2 \\ &= 2 \left(\frac{\partial^2 \phi}{\partial t^2} \right) \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial t^2} \\ &= 2 \frac{\partial^2 \phi}{\partial t^2} \frac{\partial^2 \phi}{\partial x \partial t^2} \\ &= 2 \frac{\partial^2 \phi}{\partial t^2} \frac{\partial}{\partial x} (1) \\ &= 0 \end{aligned}$$

$$\Rightarrow \frac{\partial}{\partial x} \left(\frac{\partial^2 \phi}{\partial t^2} \right)^2 = 0$$