



Mo Tu We Th Fr Sa Su

Memo No. _____

Date / /

3.4 Indistinguishability and Symmetry.

$$\exists \alpha_{p_1}^+ | 10 \rangle = | 10 \rangle, \alpha_{p_2}^+ | 10 \rangle = | 10 \rangle$$

$$\alpha_{p_1}^+ \alpha_{p_2}^+ \propto | 11 \rangle, \alpha_{p_2}^+ \alpha_{p_1}^+ \propto | 11 \rangle,$$

~~Want to know if these are equal.
They should be.~~

$$\Rightarrow \alpha_{p_1}^+ \alpha_{p_2}^+ = \lambda \alpha_{p_2}^+ \alpha_{p_1}^+$$

$$\text{Sup } \lambda = \pm 1.$$

$$\text{If } \lambda = 1 \Rightarrow \text{bosons} \Rightarrow \alpha_{p_2}^+ \alpha_{p_1}^+ = \alpha_{p_1}^+ \alpha_{p_2}^+$$

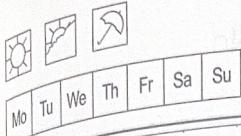
$$\Rightarrow [\alpha_i^+, \alpha_j^+] = \alpha_i^+ \alpha_j^+ - \alpha_j^+ \alpha_i^+ = 0. \text{ Also } [\alpha_i, \alpha_j] = 0$$

$$\text{also define } [\alpha_i, \alpha_j^+] = \delta_{ij}.$$

$$\Rightarrow | n_1, n_2, \dots \rangle = \frac{1}{\sqrt{\frac{1}{(n_{pm})!^{1/2}} \left(\alpha_{pm}^+ \right)^{n_{pm}}}} | 0 \rangle$$

$$\text{Then } \alpha_i^+ | n_1, \dots, n_i, \dots \rangle = \sqrt{n_i + 1} | n_1, \dots, n_i + 1, \dots \rangle$$

$$\alpha_i | n_1, \dots, n_i, \dots \rangle = \sqrt{n_i} | n_1, \dots, n_i - 1, \dots \rangle$$



Memo No. _____

Date / /

$\lambda = -1$. Fermions call the operators
 c_i^\dagger .

$$\{c_i^\dagger, c_j^\dagger\} = c_i^\dagger c_j^\dagger + c_j^\dagger c_i^\dagger = 0.$$

$$\Rightarrow c_i^\dagger c_j^\dagger = -c_j^\dagger c_i^\dagger$$

$$\Rightarrow c_i^\dagger c_i^\dagger = 0.$$

Pauli Exclusion Principle..

~~Also~~ $\{c_i, c_j\} = 0.$

$$\{c_i^\dagger, c_j^\dagger\} = \delta_{ij}.$$

$n_i = c_i^\dagger c_i$

$$\begin{aligned} n_1 |11\rangle &= c_1^\dagger c_1 |11\rangle \\ &= c_1^\dagger c_1 (c_1^\dagger c_2 |0\rangle) \\ &= c_1^\dagger (1 - c_1^\dagger c_1) c_2 |0\rangle \\ &= c_1^\dagger c_2^\dagger |0\rangle - c_1^\dagger c_1^\dagger c_1 c_2 |0\rangle \\ &= |11\rangle. \end{aligned}$$



Mo Tu We Th Fr Sa Su

Memo No. _____
Date / /

$$c_i^+ |n_1 \dots n_i \dots\rangle = (-1)^{\sum_i} \sqrt{1-n_i} |n_1 \dots n_{i+1} \dots\rangle$$

$$c_i^- |n_1 \dots n_i \dots\rangle = (-1)^{\sum_i} \sqrt{n_i} |n_1 \dots n_{i-1} \dots\rangle$$

$$(-1)^{\sum_i} = (-1)^{(n_1+n_2+\dots+n_{i-1})}$$

Ex

$$|110\rangle \rightarrow |01\rangle \rightarrow |101\rangle \rightarrow |110\rangle$$

$$a_1^\dagger a_3 a_2^\dagger a_1 a_3^\dagger a_2 |110\rangle = \pm |110\rangle.$$

Bosons

$$a_1^\dagger a_3 a_2^\dagger a_1 a_3^\dagger a_2 = a_3 a_1^\dagger a_2^\dagger a_3^\dagger a_1 a_2$$

$$= a_3 a_2^\dagger a_1^\dagger a_3^\dagger a_2 a_1$$

$$= a_3 a_2^\dagger a_3^\dagger a_1^\dagger a_2 a_1$$

$$= a_3 a_3^\dagger a_2^\dagger a_2 a_1^\dagger a_1$$

$$= a_3 a_3^\dagger n_2 n_1$$

$$\therefore a_3 a_3^\dagger n_2 n_1 |110\rangle = a_3 a_3^\dagger |110\rangle$$

$$= |110\rangle$$

Ans



Memo No. _____

Date / /

$$C_1^+ C_3 e_2^+ C_1 C_3 + C_2 = C_3 e_3^+ n_2 n_1$$

$$C_3 C_3^+ n_2 n_1 |110\rangle = \cancel{C_3 C_3^+} |n_2 n_1 |110\rangle$$

$$= -C_3 C_3^+ |110\rangle$$

$$= (C_3^+ C_3 - 1) |110\rangle$$

$$= (n_3 - 1) |110\rangle$$

$$= -|110\rangle.$$