Mo Tu We Th Fr Sa Su

Memo No.

Date / /

3.4 Indistinguishability and Symmetry.

Ex $a_{p_1}^\dagger|0\rangle = |10\rangle$, $a_{p_2}^\dagger|0\rangle = |01\rangle$

$a_{p_1}^\dagger a_{p_2}^\dagger \propto |11\rangle$, $a_{p_2}^\dagger a_{p_1}^\dagger \propto |11\rangle$.

~~Want to know if these are equal. they should be.~~

$\Rightarrow a_{p_1}^\dagger a_{p_2}^\dagger = \lambda a_{p_2}^\dagger a_{p_1}^\dagger$

Sup $\lambda = \pm 1$.

I: $\lambda = 1 \Rightarrow$ bosons $\Rightarrow a_{p_2}^\dagger a_{p_1}^\dagger = a_{p_1}^\dagger a_{p_2}^\dagger$

$\Rightarrow [a_i^\dagger, a_j^\dagger] = a_i^\dagger a_j^\dagger - a_j^\dagger a_i^\dagger = 0$. Also $[a_i, a_j] = 0$

also define $[a_i, a_j^\dagger] \equiv \delta_{ij}$.

$$\Rightarrow |n_1 n_2 \cdots\rangle = \prod_m \frac{1}{(n_{p_m}!)^{1/2}} (a_{p_m}^\dagger)^{n_{p_m}} |0\rangle$$

Then $a_i^\dagger |n_1 \cdots n_i \cdots\rangle = \sqrt{n_i+1}\, |n_1 \cdots n_i+1 \cdots\rangle$

$a_i |n_1 \cdots n_i \cdots\rangle = \sqrt{n_i}\, |n_1 \cdots n_i - 1 \cdots\rangle$

Memo No.
Date / /

II $\lambda = -1$. Fermions Call the operators $c_i^\dagger$.

$$\{c_i^\dagger, c_j^\dagger\} = c_i^\dagger c_j^\dagger + c_j^\dagger c_i^\dagger = 0.$$

$$\Rightarrow c_i^\dagger c_j^\dagger = -c_j^\dagger c_i^\dagger$$

$$\Rightarrow c_i^\dagger c_i^\dagger = 0.$$

Pauli Exclusion Principle.

Also $\{c_i, c_j\} = 0$.

$$\{c_i, c_j^\dagger\} = \delta_{ij}.$$

Ex $n_i = c_i^\dagger c_i$

$$\begin{aligned} n_1|11\rangle &= c_1^\dagger c_1 |11\rangle \\ &= c_1^\dagger c_1 (c_1^\dagger c_2^\dagger |0\rangle) \\ &= c_1^\dagger (1 - c_1^\dagger c_1) c_2^\dagger |0\rangle \\ &= c_1^\dagger c_2^\dagger |0\rangle - c_1^\dagger c_1^\dagger c_1 c_2 |0\rangle \\ &= |11\rangle. \end{aligned}$$

Memo No. ____

Date / /

$$c_i^\dagger |n_1 \cdots n_i \cdots\rangle = (-1)^{\Sigma_i} \sqrt{1-n_i}\, |n_1 \cdots n_i+1 \cdots\rangle$$

$$c_i |n_1 \cdots n_i \cdots\rangle = (-1)^{\Sigma_i} \sqrt{n_i}\, |n_1 \cdots n_i-1 \cdots\rangle$$

$$(-1)^{\Sigma_i} = (-1)^{(n_1+n_2+\cdots+n_{i-1})}$$

Ex

$$|110\rangle \rightarrow |101\rangle \rightarrow |011\rangle \rightarrow |110\rangle$$

$$a_1^\dagger a_3 a_2^\dagger a_1 a_3^\dagger a_2 |110\rangle = \pm |110\rangle$$

Bosons

$$a_1^\dagger a_3 a_2^\dagger a_1 a_3^\dagger a_2 = a_3 a_1^\dagger a_2^\dagger a_3^\dagger a_1 a_2$$

$$= a_3 a_2^\dagger a_1^\dagger a_3^\dagger a_2 a_1$$

$$= a_3 a_2^\dagger a_3^\dagger a_1^\dagger a_2 a_1$$

$$= a_3 a_3^\dagger a_2^\dagger a_2 a_1^\dagger a_1$$

$$= a_3 a_3^\dagger n_2 n_1$$

$$\therefore a_3 a_3^\dagger n_2 n_1 |110\rangle = a_3 a_3^\dagger |110\rangle$$

$$= |110\rangle$$

$$c_1^\dagger c_3 c_2^\dagger c_1 c_3^\dagger c_2 = c_3 c_3^\dagger n_2 n_1$$

$$c_3 c_3^\dagger n_2 n_1 |110\rangle = \cancel{tet}$$

$$c_3 c_3^\dagger n_2 |110\rangle$$

$$= -c_3 c_3^\dagger |110\rangle$$

$$= (c_3^\dagger c_3 - 1)|110\rangle$$

$$= (n_3 - 1)|110\rangle$$

$$= -|110\rangle.$$