Memo No.
Date / /

5.2 A Charged particle in an Electromagnetic field.

Free particle of mass m.

$$S = \int_{\tau_1}^{\tau_2} \gamma L \, d\tau$$

$\Rightarrow \gamma L$ is Lorentz invariant

$$\Rightarrow L = \frac{\text{constant}}{\gamma} = \frac{-mc^2}{\gamma}$$

$$\Rightarrow S = \int_{\tau_1}^{\tau_2} -mc^2 \, d\tau = -mc \int_a^b ds$$

Where $ds = \sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}$.

$$= c\, d\tau$$

$$\delta S = 0 \Rightarrow \delta \int_a^b ds = 0$$

$\Rightarrow$ particles move on straight lines in spacetime.

Ex 5.3

$$\vec{P} = \frac{\partial L}{\partial \vec{v}} = \frac{\partial}{\partial v}\left(-mc^2\left(1-v^2/c^2\right)^{1/2}\right)$$

$$= \gamma m \vec{v}$$

$$E = H = \vec{p}\cdot\vec{v} - L = \gamma m v^2 + \frac{mc^2}{\gamma} \cancel{= \gamma mc^2}$$

$$= \gamma mc^2\left(\frac{v^2}{c^2} + \frac{1}{\gamma^2}\right)$$

$$= \gamma mc^2\left(\frac{v^2}{c^2} + 1 - \frac{v^2}{c^2}\right)$$

$$= \gamma mc^2.$$

$$L = T - V$$

$$H = T - V + V + V$$

$$= T + V$$

$$= V - T + 2T$$

$$= 2T - (T - V)$$

$$= \vec{p}\cdot\vec{v} - L.$$

Memo No.
Date / /

Mo Tu We Th Fr Sa Su

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right), \quad p_\mu = \left(\frac{E}{c}, -\vec{p}\right).$$

Sup we have EM field:

$$A^\mu(x) = \left(\frac{V(x)}{c}, \vec{A}(x)\right)$$

pot energy: $-q A_\mu dx^\mu = -q(V - \vec{A}\cdot\vec{v})$

$$\Rightarrow S = \int -mc\,ds - q A_\mu dx^\mu$$

$$= \int_{t_1}^{t_2} \left(\frac{-mc^2}{\gamma} + q\vec{A}\cdot\vec{v} - qV\right) dt$$

$$\Rightarrow L = -\frac{mc^2}{\gamma} + q\vec{A}\cdot\vec{v} - qV$$

$$\Rightarrow \vec{p} = \frac{\partial L}{\partial \vec{v}} = \gamma m\vec{v} + q\vec{A}$$

$c = 1$

$$\vec{B} = \vec{\nabla}\times\vec{A}, \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla V$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & -B^3 & B^2 \\ -E^2 & B^3 & 0 & -B_1 \\ -E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

Lorentz Scalar!

$$F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$$

$$\Rightarrow L = -\frac{1}{4}\int d^3x \, F_{\mu\nu}F^{\mu\nu}$$

Cont eqn!

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = \partial_\mu J^\mu = 0.$$

$$J^\mu = (\rho, \vec{J}).$$