Memo No.
Date / /

4.5 The Hubbard Model.

$$\hat{H} = \sum_{ij} (-t_{ij}) c_i^{+} c_j + \frac{1}{2} \sum_{ijkl} c_i^{+} c_j^{+} V_{ijkl} c_k c_l.$$

e^- can have a spin σ

$$\Rightarrow \hat{H} = \sum_{ij\sigma} (-t_{ij}) c_{i\sigma}^{+} c_{j\sigma} + \frac{1}{2} \sum_{ijkl\sigma\sigma'} c_{i\sigma}^{+} c_{j\sigma'}^{+} V_{ijkl} c_{k\sigma'} c_{l\sigma}$$

Hubbard's Model: e^- only have Coulomb interaction when occupying the same site. This interaction is a constant $U = V_{iiii}$.

Then $$\hat{H} = \sum_{ij\sigma} (-t_{ij}) c_{i\sigma}^{+} c_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

This looks simple, but solving is hard.

Memo No.
Date / /

Ex 4.11

Lattice of 2 sites.

Single $e^- \Rightarrow |\uparrow, 0\rangle$ or $|0, \uparrow\rangle$.

$$\Rightarrow |\psi\rangle = a|\uparrow,0\rangle + b|0,\uparrow\rangle, \quad a,b \in \mathbb{R}$$

$$\Rightarrow \hat{H} = \begin{pmatrix} 0 & -t \\ -t & 0 \end{pmatrix}$$

$$\Rightarrow |\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow,0\rangle + |0,\uparrow\rangle), \; E = -t$$

and $$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow,0\rangle - |0,\downarrow\rangle), \; E = t.$$

Two e^- w/ diff spin!

$$|\psi\rangle = a|\uparrow\downarrow,0\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\uparrow\rangle + d|0,\uparrow\downarrow\rangle.$$

and

$$H = \begin{pmatrix} u & -t & t & 0 \\ -t & 0 & 0 & -t \\ t & 0 & 0 & t \\ 0 & -t & t & u \end{pmatrix}$$

has gd state:

$$E = \frac{u}{2} - \frac{\sqrt{u^2+16t^2}}{2}$$

used Sympy to get eigenvects.