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4.3 The Kinetic Energy and the tight binding Hamiltonian

Electrons Don't like being confined

$E_n = \frac{1}{2m}\left(\frac{n\pi}{L}\right)^2 \Rightarrow L \to 0, E_n \to \infty.$

Lattice pts. labeled w/ i. t_{ij} is KE saved by going from $j \to i$.

$$\Rightarrow \hat{H} = \sum_{ij} (-t_{ij}) c_i^\dagger c_j$$

$t_{ij} = t$ for nearest neighbors and $t_{ij} = 0$ otherwise.

$$\Rightarrow \hat{H} = -t \sum_{i} \hat{c}_i^\dagger \hat{c}_{i+1}$$

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Ex 4.7

Want to diagonalize the tight bounding Hamiltonian.

$$\hat{c}_i = \frac{1}{\sqrt{V}} \sum_k e^{ik\cdot r_i} c_k$$

$$c_i^{\dagger} = \frac{1}{\sqrt{V}} \sum_q e^{-iq\cdot r_i} c_q^{\dagger}$$

$$\Rightarrow \hat{H} = -\frac{t}{V} \sum_{i\tau} \sum_{kq} e^{-i(q-k)\cdot r_i + ik\cdot r_\tau} c_q^{\dagger} c_k$$

Note $\frac{1}{V} \sum_i e^{-i(q-k)r_i} = \delta_{qk}$

Then $\hat{H} = -t \sum_\tau \sum_k e^{ik\cdot r_\tau} c_k^{\dagger} c_k$

Which is diagonal.

$$\hat{H} = \sum_k E_k c_k^{\dagger} c_k,$$

$$E_k = -\sum_\tau t e^{ik\cdot r_\tau}.$$

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Ex4.8 For 2D square lattice w/ t coupling over $(a,0)$, $(-a,0)$, $(0,a)$, $(0,-a)$

$$\Rightarrow E_k = -2t(\cos k_x a + \cos k_y a).$$