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2.2 Mass on a Spring

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{2}kx^2\right)\psi=E\psi$$

$$\psi_n(\xi)=\frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}H_n(\xi)e^{-\xi^2/2},$$

H_n is hermite, $\xi=\sqrt{m\omega/\hbar}\,x$

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega$$

This gives a vaugue physical idea and is complicated and confusion.

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$\hat{H}=\frac{\hat{p}^2}{2m}$

$K=m\omega^2$

but $\frac{1}{2}m\omega^2($

$=\frac{1}{2}m$

$=\frac{1}{2}m$

$\Longrightarrow \hat{H}-$

$\hat{a}=$

$\hat{a}^{\dagger}$

$[\hat{a},\hat{a}^{\dagger}$

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$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$\boxed{K = m\omega^2} \neq \frac{1}{2}m\omega^2\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$$

but $\frac{1}{2}m\omega^2\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$

$$= \frac{1}{2}m\omega^2\hat{x}^2 + \frac{\hat{p}^2}{2m} + \frac{i\omega}{2}[\hat{x}, \hat{p}]$$

$$= \frac{1}{2}m\omega^2\hat{x}^2 + \frac{\hat{p}^2}{2m} + \frac{i\omega}{2}(i\hbar)$$

$$\Longrightarrow \hat{H} - \frac{\hbar\omega}{2} = \frac{1}{2}m\omega^2\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right).$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$$

$$\hat{a}^{\dagger} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right).$$

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{m\omega}{2\hbar}\left(-\frac{i}{m\omega}[\hat{x}, \hat{p}] + \frac{i}{m\omega}[\hat{p}, \hat{x}]\right)$$

$$= \frac{m\omega}{2\hbar}\left(\frac{\hbar}{m\omega} + \frac{\hbar}{m\omega}\right)$$

$$= (.$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^{\dagger}\right)$$

$$\hat{p} = -i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a} - \hat{a}^{\dagger}\right)$$

$$\Longrightarrow \hat{H} = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$$

$$= \hbar\omega\left(\frac{m\omega}{2\hbar}\hat{x}^2 + \frac{\cancel{m\omega}\hat{p}^2}{2\hbar m^2\omega^2} + \frac{im\omega}{2\hbar m\omega}[\hat{x},\hat{p}]\right)$$

$$\cancel{= \frac{m\omega^2}{2}\hat{x}^2 + \frac{\hbar\hat{p}^2}{m^2\omega} - \frac{\hbar^2}{m}}$$

$$= \frac{1}{2}m\omega^2\hat{x}^2 \; \frac{\hat{p}^2}{2m} - \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega$$

$$= \frac{1}{2}m\omega^2\hat{x}^2 + \frac{\hat{p}^2}{2m}$$

$n = \langle n | a^\dagger a | n \rangle = \langle an | an \rangle = |\hat{a}|n\rangle|^2 \geq 0.$

$\hat{n} \equiv \hat{a}^\dagger \hat{a}, \quad \hat{n}|n\rangle = n|n\rangle \qquad a a^\dagger - a^\dagger a = 1$

$\Rightarrow \hat{H} = \hbar\omega(\hat{n} + \frac{1}{2})$

$\Rightarrow \hat{H}|n\rangle = (n + \frac{1}{2})\hbar\omega|n\rangle.$

$\Longrightarrow$ $|n\rangle$ eigenstate of $\hat{H}$.

Ex $\quad \hat{n}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger \hat{a} \hat{a}^\dagger |n\rangle$

$\Longrightarrow \hat{n}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger(a^\dagger a + 1)|n\rangle$

$\qquad = \hat{a}^\dagger(n+1)|n\rangle$

$\Longrightarrow \hat{n}\hat{a}^\dagger|n\rangle = (n+1)\,\hat{a}^\dagger|n\rangle$

$\Longrightarrow$ $\hat{a}^\dagger|n\rangle$ is eigenstate w/ eigenvalue $n+1$.

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Ex $\hat{n}$

$\rightarrow$ eigenst

$a|n\rangle =$

Ex

but

$\Longrightarrow k$

$a^\dagger|n\rangle$

$\langle n$

$\Longrightarrow$

Ex $\hat{n}\hat{a}|n\rangle = a^\dagger a a|n\rangle$

$= (aa^\dagger - 1)a|n\rangle$

$= aa^\dagger a|n\rangle - a|n\rangle$

$\cancel{= a|n+1\rangle}$

$= a\, n|n\rangle - a|n\rangle$

$= (n-1)a|n\rangle$

$\rightarrow$ eigenstate of $a|n\rangle$ is $n-1$.

$a|n\rangle = k|n-1\rangle$

Ex $\langle n|a^\dagger a|n\rangle = |k|^2\langle n-1|n-1\rangle = |k|^2$

but $|k|^2 = \langle n|a^\dagger a|n\rangle = \langle n|\hat{n}|n\rangle = n\langle n|n\rangle = n$

$\Longrightarrow k = \sqrt{n}$.

$a^\dagger|n\rangle = c|n+1\rangle$

$\langle n|aa^\dagger|n\rangle = |c|^2\langle n+1|n+1\rangle = |c|^2$

$|c|^2 = \langle n|aa^\dagger|n\rangle = \langle n|1+a^\dagger a|n\rangle$

$= \langle n|n\rangle + n\langle n|n\rangle$

$= 1+n$

$\Longrightarrow c = \sqrt{1+n}$

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$\Rightarrow a|n\rangle = \sqrt{n}\,|n-1\rangle$

$a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle.$

When we have $|0\rangle$,

$\hat{H}|0\rangle = \hbar\omega(\hat{n}+\frac{1}{2})|0\rangle = \frac{1}{2}\hbar\omega|0\rangle$

$\Rightarrow$ ground state energy is $\frac{1}{2}\hbar\omega$!

$a^{+}|0\rangle = |1\rangle$

$a^{+}|1\rangle = \sqrt{2}\,|2\rangle \Longrightarrow |2\rangle = \frac{(a^{+})^2}{\sqrt{2}}|0\rangle$

$a^{+}|2\rangle = \sqrt{3}\,|3\rangle \Rightarrow |3\rangle = \frac{(a^{+})^3}{\sqrt{3\cdot 2}}|0\rangle$

$\rightarrow |n\rangle = \frac{(a^{+})^n}{\sqrt{n!}}|0\rangle$