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3.5 The Continuum Limit

Continuous case, $[a_{\vec{p}}, a_{\vec{q}}^{\dagger}] = \delta^3(\vec{p}-\vec{q})$

$$\hat{H} = \int d^3p\, E_{\vec{p}}\, a_{\vec{p}}^{\dagger} a_{\vec{p}}$$

$$a_p a_{p'}^{\dagger}, \quad a_p a_{p'}^{\dagger} - a_{p'}^{\dagger} a_p = \delta^{(3)}(p-p')$$

$$a_p a_{p'}^{\dagger} = \delta^{(3)}(p-p') + a_{p'}^{\dagger} a_p.$$

$$\langle \vec{p}, \vec{p}' \rangle = \delta^{(3)}(\vec{p}-\vec{p}').$$

$$|x\rangle = \int d^3q\, \phi_q^{*}(x)|q\rangle$$

$$= \int d^3q\, |q\rangle\langle q|x\rangle$$

$$= \int d^3q\, \phi_q^{*}(x)\, |q\rangle$$

$$\rightarrow \langle x|p\rangle = \int d^3q \langle x|q\rangle\langle q|p\rangle$$

$$= \int d^3q\, \phi_q(x)\, \delta^{(3)}(\vec{q}-\vec{p})$$

$$= \phi_p(x).$$

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$$\cancel{\langle p' q' | q p \rangle = \langle 0 | a_{p'} a_{q'} a_p^\dagger a_q^\dagger | 0 \rangle}$$

$$\langle p' q' | q p \rangle = \langle 0 | a_{p'} a_{q'} a_q^\dagger a_p^\dagger | 0 \rangle$$

$$\cancel{= \langle 0 | a_{p'} a_p^\dagger a_{q'}}$$

$$= \langle 0 | a_{p'} \left(\delta^{(3)}(q'-q) \pm a_q^\dagger a_{q'} \right) a_p^\dagger | 0 \rangle.$$

$$= \langle 0 | a_{p'} \delta^{(3)}(q'-q) a_p^\dagger | 0 \rangle \pm \langle 0 | a_{p'} a_q^\dagger a_{q'} a_p^\dagger | 0 \rangle$$

$$= \langle 0 | \delta^{(3)}(q'-q) \left(\delta^{(3)}(p'-p) \pm a_p^\dagger a_{p'} \right) | 0 \rangle$$

$$\pm \langle 0 | a_{p'} a_q^\dagger \left(\delta^{(3)}(q'-p) \pm a_p^\dagger a_{q'} \right) | 0 \rangle$$

$$= \delta^{(3)}(q'-q) \delta^{(3)}(p'-p) \pm \langle 0 | \delta^{(3)}(q'-p) \left(\delta^{(3)}(p'-q) \pm \cdots \right)$$

$$= \delta^{(3)}(q'-q) \delta^{(3)}(p'-p) \pm \delta^{(3)}(q'-p) \delta^{(3)}(p'-q).$$

$$\cancel{|xy\rangle = \int d^3p' \, d^3q' \, \phi^{*}_{p'}}$$

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$$\langle xy | pq \rangle = \frac{1}{\sqrt{2}} \int d^3p' d^3q' \, \phi_{p'}(x) \phi_{q'}(y) \langle p'q' | pq \rangle$$

$$= \frac{1}{\sqrt{2}} \left[\phi_p(x)\phi_q(y) \pm \phi_q(x)\phi_p(y) \right]$$

Wich is expected.