$$\Psi_N = \frac{1}{\sqrt{\#(\text{states})}} = \frac{1}{\sqrt{2^{\#(\text{sites})\cdot 2}}} = \frac{1}{2^{\#(\text{sites})}}$$

$$\left|\Psi^{\mathrm{S}}(t=0)\right\rangle = \bigotimes_{l=1}^{\#(\text{states})} \frac{1}{2}\left(1 + \hat{\mathrm{h}}^{\dagger \mathrm{S}}_{l,\uparrow} + \hat{\mathrm{h}}^{\dagger \mathrm{S}}_{l,\downarrow} + \hat{\mathrm{h}}^{\dagger \mathrm{S}}_{l,\uparrow}\hat{\mathrm{h}}^{\dagger \mathrm{S}}_{l,\downarrow}\right)|0\rangle$$

$$
\begin{aligned}
E_0(N) - E_0(\widetilde{N}) &= U \sum_l n_{l,\downarrow} n_{l,\uparrow} - U \sum_l \widetilde{n}_{l,\downarrow} \widetilde{n}_{l,\uparrow} + \sum_{l,\sigma} \varepsilon_l n_{l,\sigma} - \sum_{l,\sigma} \varepsilon_l \widetilde{n}_{l,\sigma} \\
&= \varepsilon_i \left(2 n_{i,\sigma_i} - 1\right) + U \cdot \begin{cases} \sigma_i = \uparrow : & n_{i,\downarrow}(2 n_{i,\uparrow} - 1) \\ \sigma_i = \downarrow : & n_{i,\uparrow}(2 n_{i,\downarrow} - 1) \end{cases}
\end{aligned}
$$