$$\rho_A = \mathrm{Tr}_B\left(\rho_{AB}\right) = \sum_{k=0}^{M} \left(\mathbb{1}_A \otimes \langle \chi_k|\right) \rho_{AB} \left(\mathbb{1}_A \otimes |\chi_k\rangle\right)$$

$$\rho_A(t) = \rho_{l,\sigma,m,\mu}(t) = \frac{1}{4} \sum_{\alpha,\beta\in\{0,x,y,z\}} \left\langle \hat{\sigma}^{\alpha}_{l,\sigma} \hat{\sigma}^{\beta}_{m,\mu} \right\rangle_{(t)} \left(\hat{\sigma}^{\alpha} \otimes \hat{\sigma}^{\beta} \right)$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{x}}_{l,\sigma}\hat{\sigma}^{0}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)-\mathcal{H}_{\mathrm{eff}}(N,t)}$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{y}}_{l,\sigma}\hat{\sigma}^{0}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{l,\sigma})$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{z}}_{l,\sigma}\hat{\sigma}^{0}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \cdot (2 \cdot n_{l,\sigma} - 1)$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{x}}_{l,\sigma}\hat{\sigma}^{\mathrm{x}}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{\tilde{N}}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{\tilde{N}},t)-\mathcal{H}_{\mathrm{eff}}(N,t)}$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{y}}_{l,\sigma}\hat{\sigma}^{\mathrm{y}}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{\tilde{N}}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{\tilde{N}},t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \cdot (2 \cdot n_{l,\sigma} - 1) \cdot (1 - 2 \cdot n_{m,\mu})$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{x}}_{l,\sigma}\hat{\sigma}^{\mathrm{y}}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{\tilde{N}}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{\tilde{N}},t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{m,\mu})$$

$$\hat{\mathcal{O}} = \hat{\sigma}^{\mathrm{y}}_{l,\sigma}\hat{\sigma}^{\mathrm{x}}_{m,\mu} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{\tilde{N}}}}{\Psi_N} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{\tilde{N}},t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{l,\sigma})$$