$$\frac{\left\langle \Psi^{\mathrm{S}}(t)\middle|\hat{\mathcal{O}}\middle|\Psi^{\mathrm{S}}(t)\right\rangle}{\left\langle \Psi^{\mathrm{S}}(t)\middle|\Psi^{\mathrm{S}}(t)\right\rangle} =$$

$$\sum_N \mathrm{P}(N,t) \underbrace{\sum_K \langle N|\hat{\mathcal{O}}|K\rangle \, e^{\mathcal{H}_{\mathrm{eff}}(K,t)-\mathcal{H}_{\mathrm{eff}}(N,t)} \frac{\Psi_K}{\Psi_N}}_{\hat{\mathcal{O}}_{\mathrm{loc}}(N,t)} = \sum_N \mathrm{P}(N,t)\hat{\mathcal{O}}_{\mathrm{loc}}(N,t)$$