$$\hat{\mathbf{V}} = -J \cdot \sum_{\langle l, m \rangle} \left( \hat{\mathbf{h}}_l^{\dagger} \hat{\mathbf{h}}_m + \hat{\mathbf{h}}_m^{\dagger} \hat{\mathbf{h}}_l + \hat{\mathbf{d}}_l^{\dagger} \hat{\mathbf{d}}_m + \hat{\mathbf{d}}_m^{\dagger} \hat{\mathbf{d}}_l \right)$$

$$\begin{split} \hat{\mathbf{h}}_{m}^{\dagger \mathrm{I}}(t) &= e^{i \cdot \varepsilon_{m} \cdot t} \left( 1 + \left( e^{i \cdot U \cdot t} - 1 \right) \hat{\mathbf{d}}_{m}^{\dagger \mathrm{S}} \hat{\mathbf{d}}_{m}^{\mathrm{S}} \right) \hat{\mathbf{h}}_{m}^{\dagger \mathrm{S}} \\ \hat{\mathbf{h}}_{m}^{\mathrm{I}}(t) &= e^{-i \cdot \varepsilon_{m} \cdot t} \left( 1 + \left( e^{-i \cdot U \cdot t} - 1 \right) \hat{\mathbf{d}}_{m}^{\dagger \mathrm{S}} \hat{\mathbf{d}}_{m}^{\mathrm{S}} \right) \hat{\mathbf{h}}_{m}^{\mathrm{S}} \\ \hat{\mathbf{d}}_{m}^{\dagger \mathrm{I}}(t) &= e^{i \cdot \varepsilon_{m} \cdot t} \left( 1 + \left( e^{i \cdot U \cdot t} - 1 \right) \hat{\mathbf{h}}_{m}^{\dagger \mathrm{S}} \hat{\mathbf{h}}_{m}^{\mathrm{S}} \right) \hat{\mathbf{d}}_{m}^{\dagger \mathrm{S}} \\ \hat{\mathbf{d}}_{m}^{\mathrm{I}}(t) &= e^{-i \cdot \varepsilon_{m} \cdot t} \left( 1 + \left( e^{-i \cdot U \cdot t} - 1 \right) \hat{\mathbf{h}}_{m}^{\dagger \mathrm{S}} \hat{\mathbf{h}}_{m}^{\mathrm{S}} \right) \hat{\mathbf{d}}_{m}^{\mathrm{S}} \end{split}$$