

$$\frac{\langle \psi^S(t) | \hat{\mathcal{O}} | \psi^S(t) \rangle}{\langle \psi^S(t) | \psi^S(t) \rangle} = \sum_N P(N, t) \cdot \hat{\mathcal{O}}_{\text{loc}}(N, t) \approx \frac{1}{|\{N\}_{\text{MC}}|} \sum_{\{N\}_{\text{MC}}} \hat{\mathcal{O}}_{\text{loc}}(N, t)$$

$$\begin{aligned}
\alpha &= \frac{P(\tilde{N}, t)}{P(N, t)} = \frac{f(\tilde{N}, t)}{f(N, t)} = \frac{|e^{\mathcal{H}_{\text{eff}}(\tilde{N}, t)}|^2 |\Psi_{\tilde{N}}|^2}{|e^{\mathcal{H}_{\text{eff}}(N, t)}|^2 |\Psi_N|^2} \\
&= \frac{|\Psi_{\tilde{N}}|^2}{|\Psi_N|^2} \frac{e^{\Re(\mathcal{H}_{\text{eff}}(\tilde{N}, t)) + i\Im(\mathcal{H}_{\text{eff}}(\tilde{N}, t)) + \Re(\mathcal{H}_{\text{eff}}(\tilde{N}, t)) - i\Im(\mathcal{H}_{\text{eff}}(\tilde{N}, t))}}{e^{\Re(\mathcal{H}_{\text{eff}}(N, t)) + i\Im(\mathcal{H}_{\text{eff}}(N, t)) + \Re(\mathcal{H}_{\text{eff}}(N, t)) - i\Im(\mathcal{H}_{\text{eff}}(N, t))}} \\
&= \frac{|\Psi_{\tilde{N}}|^2}{|\Psi_N|^2} e^{2 \cdot \Re(\mathcal{H}_{\text{eff}}(\tilde{N}, t)) - 2 \cdot \Re(\mathcal{H}_{\text{eff}}(N, t))} \\
&= \frac{|\Psi_{\tilde{N}}|^2}{|\Psi_N|^2} e^{2 \cdot \Re(\mathcal{H}_{\text{eff}}(\tilde{N}, t) - \mathcal{H}_{\text{eff}}(N, t))}
\end{aligned}$$