$$\hat{\mathrm{V}} = -J \cdot \sum_{\langle l,m \rangle} \left(\hat{\mathrm{c}}_l^{\dagger} \hat{\mathrm{c}}_m + \hat{\mathrm{c}}_m^{\dagger} \hat{\mathrm{c}}_l + \hat{\mathrm{d}}_l^{\dagger} \hat{\mathrm{d}}_m + \hat{\mathrm{d}}_m^{\dagger} \hat{\mathrm{d}}_l \right)$$

$$\hat{\mathrm{c}}_m^{\dagger \mathrm{I}}(t) = e^{i \cdot \varepsilon_m \cdot t} \left(1 + \left(e^{i \cdot U \cdot t} - 1\right) \hat{\mathrm{d}}_m^{\dagger \mathrm{S}} \hat{\mathrm{d}}_m^{\mathrm{S}}\right) \hat{\mathrm{c}}_m^{\dagger \mathrm{S}}$$

$$\hat{\mathrm{c}}_m^{\mathrm{I}}(t) = e^{-i \cdot \varepsilon_m \cdot t} \left(1 + \left(e^{-i \cdot U \cdot t} - 1\right) \hat{\mathrm{d}}_m^{\dagger \mathrm{S}} \hat{\mathrm{d}}_m^{\mathrm{S}}\right) \hat{\mathrm{c}}_m^{\mathrm{S}}$$

$$\hat{\mathrm{d}}_m^{\dagger \mathrm{I}}(t) = e^{i \cdot \varepsilon_m \cdot t} \left(1 + \left(e^{i \cdot U \cdot t} - 1\right) \hat{\mathrm{c}}_m^{\dagger \mathrm{S}} \hat{\mathrm{c}}_m^{\mathrm{S}}\right) \hat{\mathrm{d}}_m^{\dagger \mathrm{S}}$$

$$\hat{\mathrm{d}}_m^{\mathrm{I}}(t) = e^{-i \cdot \varepsilon_m \cdot t} \left(1 + \left(e^{-i \cdot U \cdot t} - 1\right) \hat{\mathrm{c}}_m^{\dagger \mathrm{S}} \hat{\mathrm{c}}_m^{\mathrm{S}}\right) \hat{\mathrm{d}}_m^{\mathrm{S}}$$

$$
\begin{aligned}
\hat{\mathrm{V}}^{\mathrm{I}}(t) &= \left\{\hat{\mathrm{V}}^{\mathrm{S}}\right\}(t) \overset{2.11}{=} -J \cdot \sum_{[l,m]} \left\{ \left(\hat{\mathrm{c}}_l^{\dagger \mathrm{S}} \hat{\mathrm{c}}_m^{\mathrm{S}} + \hat{\mathrm{d}}_l^{\dagger \mathrm{S}} \hat{\mathrm{d}}_m^{\mathrm{S}} \right) \right\}(t) \\
&= -J \cdot \sum_{[l,m]} \left(\hat{\mathrm{c}}_l^{\dagger \mathrm{I}}(t) \hat{\mathrm{c}}_m^{\mathrm{I}}(t) + \hat{\mathrm{d}}_l^{\dagger \mathrm{I}}(t) \hat{\mathrm{d}}_m^{\mathrm{I}}(t) \right) \\
&\overset{\mathrm{MM}}{=} -J \cdot \sum_{[l,m]} \Big[e^{i \cdot (\varepsilon_m - \varepsilon_l) \cdot t} \cdot \hat{\mathrm{V}}_{\text{Part A}}(l,m) + \\
&\qquad e^{i \cdot (\varepsilon_m - \varepsilon_l + U) \cdot t} \cdot \hat{\mathrm{V}}_{\text{Part B}}(l,m) + e^{i \cdot (\varepsilon_m - \varepsilon_l - U) \cdot t} \cdot \hat{\mathrm{V}}_{\text{Part C}}(l,m) \Big]
\end{aligned}
$$

$$\hat{\mathrm{V}}_{\mathrm{Part\ A}}(l,m) \overset{\mathrm{MM}}{=} \left(5\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\right) + \left(5\cdot \hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) +$$
$$+ \left(-3\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-3\cdot \hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-3\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\right) + \left(-3\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right)$$
$$\hat{\mathrm{V}}_{\mathrm{Part\ B}}(l,m) \overset{\mathrm{MM}}{=} \left(-2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\right) + \left(-2\cdot \hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) +$$
$$+ \left(1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(2\cdot \hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\right) + \left(2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right)$$
$$\hat{\mathrm{V}}_{\mathrm{Part\ C}}(l,m) \overset{\mathrm{MM}}{=} \left(-2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\right) + \left(-2\cdot \hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(-1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) +$$
$$+ \left(2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_l^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(1\cdot \hat{\mathrm{c}}_m^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right) + \left(2\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_l^{\mathrm{S}}\hat{\mathrm{d}}_l^{\dagger\mathrm{S}}\right) + \left(1\cdot \hat{\mathrm{c}}_l^{\mathrm{S}}\hat{\mathrm{c}}_m^{\dagger\mathrm{S}}\hat{\mathrm{d}}_m^{\mathrm{S}}\hat{\mathrm{d}}_m^{\dagger\mathrm{S}}\right)$$