$$\hat{\sigma}_l^{\mathrm{x}} = \hat{\mathrm{h}}_l + \hat{\mathrm{h}}_l^{\dagger}$$

$$\hat{\sigma}_l^{\mathrm{y}} = i \cdot \left(\hat{\mathrm{h}}_l - \hat{\mathrm{h}}_l^{\dagger} \right)$$

$$\hat{\sigma}_l^{\mathrm{z}} = 2 \cdot \hat{\mathrm{h}}_l^{\dagger} \hat{\mathrm{h}}_l - 1$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle$$

$$\hat{\sigma}^{+} |\downarrow\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |1\rangle = \hat{\mathrm{h}}^{\dagger} |0\rangle$$