

$$\hat{V}_{\text{Part A}}(l, m) \stackrel{\text{MM}}{=} \sum_{\sigma \in \{\uparrow, \downarrow\}} \hat{c}_{l, \sigma}^S \hat{c}_{m, \sigma}^{\dagger S} \left(1 + 2 \cdot \hat{n}_{l, \bar{\sigma}}^S \hat{n}_{m, \bar{\sigma}}^S - \hat{n}_{l, \bar{\sigma}}^S - \hat{n}_{m, \bar{\sigma}}^S \right)$$

$$\hat{V}_{\text{Part B}}(l, m) \stackrel{\text{MM}}{=} \sum_{\sigma \in \{\uparrow, \downarrow\}} \hat{c}_{l, \sigma}^S \hat{c}_{m, \sigma}^{\dagger S} \left(\hat{n}_{m, \bar{\sigma}}^S - \hat{n}_{l, \bar{\sigma}}^S \hat{n}_{m, \bar{\sigma}}^S \right)$$

$$\hat{V}_{\text{Part C}}(l, m) \stackrel{\text{MM}}{=} \sum_{\sigma \in \{\uparrow, \downarrow\}} \hat{c}_{l, \sigma}^S \hat{c}_{m, \sigma}^{\dagger S} \left(\hat{n}_{l, \bar{\sigma}}^S - \hat{n}_{l, \bar{\sigma}}^S \hat{n}_{m, \bar{\sigma}}^S \right)$$