

$$|\Psi^S(t)\rangle$$

$$= \sum_N e^{\mathcal{H}_{\text{eff}}(N, t)} \Psi_N |N\rangle$$

$$H_{\text{eff}}(N, t) = -iE_0(N)t + H_N(t)$$

$$\begin{aligned}
E_0(N) &= \frac{\langle N | \mathcal{H}_0 | N \rangle}{\langle N | N \rangle} \\
&= \langle N | U \cdot \sum_l \hat{n}_{l,\uparrow} \hat{n}_{l,\downarrow} | N \rangle + \langle N | \sum_{l,\sigma} \varepsilon_l \hat{n}_{l,\sigma} | N \rangle \\
&= U \cdot \sum_l n_{l,\uparrow} n_{l,\downarrow} + \sum_{l,\sigma} \varepsilon_l n_{l,\sigma}
\end{aligned}$$