$$\rho_{A} = \operatorname{Tr}_{B}\left(\rho_{AB}\right) = \sum_{k=0}^{M} \left(\mathbb{1}_{A} \otimes \left\langle X_{k} \right|\right) \rho_{AB} \left(\mathbb{1}_{A} \otimes \left| X_{k} \right\rangle\right)$$

$$ho_A(t) =
ho_{l,\,\sigma,\,m,\,\mu}(t) = rac{1}{4} \sum_{lpha,eta \in \{0,\mathrm{x},\mathrm{y},\mathrm{z}\}} \left\langle \hat{\sigma}^lpha_{l,\,\sigma} \hat{\sigma}^eta_{m,\,\mu}
ight
angle_{(t)} \left(\hat{\sigma}^lpha \otimes \hat{\sigma}^eta
ight)$$

$$\begin{split} \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{x}} \hat{\sigma}_{m,\mu}^{0} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{y}} \hat{\sigma}_{m,\mu}^{0} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{l,\sigma}) \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{z}} \hat{\sigma}_{m,\mu}^{0} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \cdot (2 \cdot n_{l,\sigma} - 1) \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{x}} \hat{\sigma}_{m,\mu}^{\mathsf{x}} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{y}} \hat{\sigma}_{m,\mu}^{\mathsf{y}} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \cdot (2 \cdot n_{l,\sigma} - 1) \cdot (1 - 2 \cdot n_{m,\mu}) \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{y}} \hat{\sigma}_{m,\mu}^{\mathsf{y}} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{m,\mu}) \\ \hat{\mathcal{O}} &= \hat{\sigma}_{l,\sigma}^{\mathsf{y}} \hat{\sigma}_{m,\mu}^{\mathsf{x}} : \quad \hat{\mathcal{O}}_{\mathrm{loc}}(N,t) = \frac{\Psi_{\tilde{N}}}{\Psi_{N}} \cdot e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t) - \mathcal{H}_{\mathrm{eff}}(N,t)} \cdot i \cdot (1 - 2 \cdot n_{l,\sigma}) \end{split}$$