$$
\begin{aligned}
\alpha &= \frac{\mathrm{P}(\tilde{N},t)}{\mathrm{P}(N,t)} = \frac{f(\tilde{N},t)}{f(N,t)} \overset{2.39}{=} \frac{\left|e^{\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)}\right|^2 \left|\Psi_{\tilde{N}}\right|^2}{\left|e^{\mathcal{H}_{\mathrm{eff}}(N,t)}\right|^2 \left|\Psi_{N}\right|^2} \\
&= \frac{\left|\Psi_{\tilde{N}}\right|^2}{\left|\Psi_{N}\right|^2} \frac{e^{\Re\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)\right)+i\Im\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)\right)+\Re\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)\right)-i\Im\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)\right)}}{e^{\Re(\mathcal{H}_{\mathrm{eff}}(N,t))+i\Im(\mathcal{H}_{\mathrm{eff}}(N,t))+\Re(\mathcal{H}_{\mathrm{eff}}(N,t))-i\Im(\mathcal{H}_{\mathrm{eff}}(N,t))}} \\
&= \frac{\left|\Psi_{\tilde{N}}\right|^2}{\left|\Psi_{N}\right|^2} e^{2\cdot\Re\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)\right)-2\cdot\Re(\mathcal{H}_{\mathrm{eff}}(N,t))} \\
&= \frac{\left|\Psi_{\tilde{N}}\right|^2}{\left|\Psi_{N}\right|^2} e^{2\cdot\Re\left(\mathcal{H}_{\mathrm{eff}}(\tilde{N},t)-\mathcal{H}_{\mathrm{eff}}(N,t)\right)}
\end{aligned}
$$