$$\hat{\mathbf{V}}^{\mathrm{I}}(t) = \left\{ \hat{\mathbf{V}}^{\mathrm{S}} \right\}^{\mathrm{I}}(t) = -J \cdot \sum_{[l,m]} \left\{ \left( \hat{\mathbf{h}}_{l}^{\dagger \mathrm{S}} \hat{\mathbf{h}}_{m}^{\mathrm{S}} + \hat{\mathbf{d}}_{l}^{\dagger \mathrm{S}} \hat{\mathbf{d}}_{m}^{\mathrm{S}} \right) \right\}^{\mathrm{I}}(t)$$

$$= -J \cdot \sum_{[l,m]} \left( \hat{\mathbf{h}}_{l}^{\dagger \mathrm{I}}(t) \hat{\mathbf{h}}_{m}^{\mathrm{I}}(t) + \hat{\mathbf{d}}_{l}^{\dagger \mathrm{I}}(t) \hat{\mathbf{d}}_{m}^{\mathrm{I}}(t) \right)$$

 $= -J \cdot \sum \left[ \Lambda_{A}(l, m, t) \cdot \hat{F}_{A}(l, m) + \Lambda_{B}(l, m, t) \cdot \hat{F}_{B}(l, m) + \Lambda_{C}(l, m, t) \cdot \hat{F}_{C}(l, m) \right]$ 

$$egin{aligned} & \Lambda_{\mathrm{A}}\left(l,m,t
ight) \ = \ e^{i\cdot(arepsilon_{l}-arepsilon_{m})\cdot t} & \hat{\mathrm{F}}_{\mathrm{A}}\left(l,m
ight) \ = \ \sum_{\sigma\in\{\uparrow,\downarrow\}} \hat{\mathrm{h}}_{l,\sigma}^{\dagger\,\mathrm{S}} \hat{\mathrm{h}}_{m,\sigma}^{\mathrm{S}} \left(1 + 2\cdot\hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}} \hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}} - \hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}} - \hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}
ight) \ & \Lambda_{\mathrm{B}}\left(l,m,t
ight) \ = \ e^{i\cdot(arepsilon_{l}-arepsilon_{m}+U)\cdot t} & \hat{\mathrm{F}}_{\mathrm{B}}\left(l,m
ight) \ = \ \sum \ \hat{\mathrm{h}}_{l,\sigma}^{\dagger\,\mathrm{S}} \hat{\mathrm{h}}_{m,\sigma}^{\mathrm{S}} \left(\hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}} - \hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}} \hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}
ight) \end{aligned}$$

$$egin{aligned} & \Lambda_{\mathrm{B}}\left(l,m,t
ight) = e^{i\cdot(arepsilon_{l}-arepsilon_{m}-U)\cdot t} & \hat{\mathrm{F}}_{\mathrm{B}}\left(l,m
ight) = \sum_{\sigma\in\{\uparrow,\downarrow\}} \hat{\mathrm{h}}_{l,\sigma}^{\mathrm{S}}\hat{\mathrm{h}}_{m,\sigma}^{\mathrm{S}}\left(\hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}-\hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}\hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}}
ight) \\ & \Lambda_{\mathrm{C}}\left(l,m,t
ight) = e^{i\cdot(arepsilon_{l}-arepsilon_{m}-U)\cdot t} & \hat{\mathrm{F}}_{\mathrm{C}}\left(l,m
ight) = \sum_{\sigma\in\{\uparrow,\downarrow\}} \hat{\mathrm{h}}_{l,\sigma}^{\mathrm{S}}\hat{\mathrm{h}}_{m,\sigma}^{\mathrm{S}}\left(\hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}-\hat{\mathrm{n}}_{m,\overline{\sigma}}^{\mathrm{S}}\hat{\mathrm{n}}_{l,\overline{\sigma}}^{\mathrm{S}}\right) \end{aligned}$$