$$\hat{\sigma}_{l}^{x} = \hat{\mathbf{h}}_{l} + \hat{\mathbf{h}}_{l}^{\dagger}$$

$$\hat{\sigma}_{l}^{y} = i \cdot \left( \hat{\mathbf{h}}_{l} - \hat{\mathbf{h}}_{l}^{\dagger} \right)$$

$$\hat{\sigma}_{l}^{z} = 2 \cdot \hat{\mathbf{h}}_{l}^{\dagger} \hat{\mathbf{h}}_{l} - 1$$

$$\begin{aligned} |\downarrow\rangle &= \begin{pmatrix} 0\\1 \end{pmatrix} = |0\rangle \\ \hat{\sigma}^+ |\downarrow\rangle &= |\uparrow\rangle = \begin{pmatrix} 1\\0 \end{pmatrix} = |1\rangle = \hat{\mathbf{h}}^\dagger |0\rangle \end{aligned}$$