

$$\hat{\mathcal{O}}_{\text{loc}}(N, t) = \sum_K \langle N | \hat{\mathcal{O}}_{\text{do}}(I) | K \rangle e^{\mathcal{H}_{\text{eff}}(K, t) - \mathcal{H}_{\text{eff}}(N, t)} \frac{\Psi_K}{\Psi_N} = n_{l, \uparrow} \cdot n_{l, \downarrow}$$

$$\hat{\mathcal{O}}_{\text{loc}}(N, t) = i \cdot \left[ n_{m, \sigma} \cdot (1 - n_{l, \sigma}) - n_{l, \sigma} \cdot (1 - n_{m, \sigma}) \right] \cdot \frac{\Psi_{\tilde{N}}}{\Psi_N} \cdot e^{\mathcal{H}_{\text{eff}}(\tilde{N}, t) - \mathcal{H}_{\text{eff}}(N, t)}$$