

$$\hat{V} = -J \cdot \sum_{\langle l,m \rangle} \left( \hat{c}_l^\dagger \hat{c}_m + \hat{c}_m^\dagger \hat{c}_l + \hat{d}_l^\dagger \hat{d}_m + \hat{d}_m^\dagger \hat{d}_l \right)$$

$$\hat{\mathbf{c}}_m^{\dagger\text{I}}(t) = e^{i\cdot\epsilon_m\cdot t} \left( 1 + \left( e^{i\cdot U\cdot t} - 1 \right) \hat{\mathbf{d}}_m^{\dagger\text{S}} \hat{\mathbf{d}}_m^{\text{S}} \right) \hat{\mathbf{c}}_m^{\dagger\text{S}}$$

$$\hat{\mathbf{c}}_m^{\text{I}}(t) = e^{-i\cdot\epsilon_m\cdot t} \left( 1 + \left( e^{-i\cdot U\cdot t} - 1 \right) \hat{\mathbf{d}}_m^{\text{S}} \hat{\mathbf{d}}_m^{\text{S}} \right) \hat{\mathbf{c}}_m^{\text{S}}$$

$$\hat{\mathbf{d}}_m^{\dagger\text{I}}(t) = e^{i\cdot\epsilon_m\cdot t} \left( 1 + \left( e^{i\cdot U\cdot t} - 1 \right) \hat{\mathbf{c}}_m^{\dagger\text{S}} \hat{\mathbf{c}}_m^{\text{S}} \right) \hat{\mathbf{d}}_m^{\dagger\text{S}}$$

$$\hat{\mathbf{d}}_m^{\text{I}}(t) = e^{-i\cdot\epsilon_m\cdot t} \left( 1 + \left( e^{-i\cdot U\cdot t} - 1 \right) \hat{\mathbf{c}}_m^{\text{S}} \hat{\mathbf{c}}_m^{\text{S}} \right) \hat{\mathbf{d}}_m^{\text{S}}$$

$$\begin{aligned}
\hat{V}^{\text{I}}(t) &= \left\{ \hat{V}^{\text{S}} \right\} (t) \stackrel{2.11}{=} -J \cdot \sum_{[l,m]} \left\{ \left( \hat{c}_l^{\dagger \text{S}} \hat{c}_m^{\text{S}} + \hat{d}_l^{\dagger \text{S}} \hat{d}_m^{\text{S}} \right) \right\} (t) \\
&= -J \cdot \sum_{[l,m]} \left( \hat{c}_l^{\dagger \text{I}}(t) \hat{c}_m^{\text{I}}(t) + \hat{d}_l^{\dagger \text{I}}(t) \hat{d}_m^{\text{I}}(t) \right) \\
&\stackrel{\text{MM}}{=} -J \cdot \sum_{[l,m]} \left[ e^{i \cdot (\varepsilon_m - \varepsilon_l) \cdot t} \cdot \hat{V}_{\text{Part A}}(l, m) + \right. \\
&\quad \left. e^{i \cdot (\varepsilon_m - \varepsilon_l + U) \cdot t} \cdot \hat{V}_{\text{Part B}}(l, m) + e^{i \cdot (\varepsilon_m - \varepsilon_l - U) \cdot t} \cdot \hat{V}_{\text{Part C}}(l, m) \right]
\end{aligned}$$

$$\begin{aligned}
\hat{V}_{\text{Part A}}(l, m) &\stackrel{\text{MM}}{=} \left(5 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S}\right) + \left(5 \cdot \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(2 \cdot \hat{c}_l^S \hat{c}_m^S \hat{c}_l^{\dagger S} \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(2 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^S \hat{d}_l^{\dagger S} \hat{d}_m^{\dagger S}\right) + \\
&\quad + \left(-3 \cdot \hat{c}_l^S \hat{c}_l^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-3 \cdot \hat{c}_m^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-3 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_l^{\dagger S}\right) + \left(-3 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_m^S \hat{d}_m^{\dagger S}\right) \\
\hat{V}_{\text{Part B}}(l, m) &\stackrel{\text{MM}}{=} \left(-2 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S}\right) + \left(-2 \cdot \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-1 \cdot \hat{c}_l^S \hat{c}_m^S \hat{c}_l^{\dagger S} \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-1 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^S \hat{d}_l^{\dagger S} \hat{d}_m^{\dagger S}\right) + \\
&\quad + \left(1 \cdot \hat{c}_l^S \hat{c}_l^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(2 \cdot \hat{c}_m^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(1 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_l^{\dagger S}\right) + \left(2 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_m^S \hat{d}_m^{\dagger S}\right) \\
\hat{V}_{\text{Part C}}(l, m) &\stackrel{\text{MM}}{=} \left(-2 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S}\right) + \left(-2 \cdot \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-1 \cdot \hat{c}_l^S \hat{c}_m^S \hat{c}_l^{\dagger S} \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(-1 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^S \hat{d}_l^{\dagger S} \hat{d}_m^{\dagger S}\right) + \\
&\quad + \left(2 \cdot \hat{c}_l^S \hat{c}_l^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(1 \cdot \hat{c}_m^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_m^{\dagger S}\right) + \left(2 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_l^S \hat{d}_l^{\dagger S}\right) + \left(1 \cdot \hat{c}_l^S \hat{c}_m^{\dagger S} \hat{d}_m^S \hat{d}_m^{\dagger S}\right)
\end{aligned}$$