8,194,1.00 Derivatives Modeling in Python
University of St.Gallen
Spring Semester 2024
Mathis Mörke

Case Study

1 Learning Objectives

Pricing of an exotic structured product by means of Monte Carlo simulations.

2 Literature

Ammann, M., M. Arnold, and S. Straumann, 2022, Pricing, Demand, and Product Design of Innovative Securities: The Role of Investor Information, Working Paper, University of St. Gallen, `https://ssrn.com/abstract_id=2932067`.
Hull, J.C., 2018, Options, futures, and other derivatives, 10th edition, Chapter 26.

3 Procedure and Submission

This case study shall be solved in groups of two or three particiants. The groups are formed prior to the announcement of the case study. The assignment has to be handed in by email to Mathis Mörke (`mathis.moerke@unisg.ch`). The submission must contain the Python file and the two-page (!) memo. Submitting the assignment is a prerequisite for taking the final exam. It is your responsibility to make sure that the assignments arrive in the mailbox on time. Late assignments cannot be accepted.

4 Case Study Description

You have recently joined the structured products team of a major bank. Your team plans to issue a multi barrier reverse convertible. To develop a feeling for the pricing of this product, your team leader asks you to determine the price of a comparable product that has recently been issued by a competitor.

4.1 Multi Barrier Reverse Convertible

A multi barrier reverse convertible is a special type of a barrier reverse convertible that depends on multiple underlyings. A multi barrier reverse convertible typically comes with a *worst-of* feature: If no underlying hits its barrier during the product's lifetime, redemption at par occurs. If at least one barrier is hit but all underlying prices end up being above their corresponding strike prices at maturity, the nominal value is paid. If, on the other hand, at least one barrier is hit and at least one underlying's stock price is below its strike price at maturity, a pre-specified number of shares of the worst-performing underlying is delivered. A multi barrier reverse convertible can be replicated by combining a long position in a bond with a short position in a worst-of down-and-in put option.

4.2 Your Tasks

Your team leader wants you to value a multi barrier reverse convertible using Python. He urges you to make sure that your Python code is well-structured and easily understandable. Finally, he asks you to put together a two-page memo covering all tasks that are mentioned in the following:

- Carefully motivate the input values that you use in the valuation process.
- Report the issue premium and discuss how the input values affect the issue premium.
- Comment on the expectations that retail investors should have when purchasing a multi barrier reverse convertible. Do you consider these products to be suitable for retail investors? Why and/or why not?
- Read the paper by Ammann et al. (2022). Explain how issuers of structured products typically exploit retail investors by using their privileged access to information. Based on the valuation exercise, can you think of other features of structured products through which issuers could maximize the issue premium?

5 Appendix

5.1 Hints

- Value the multi barrier reverse convertible by means of Monte Carlo simulations. Price processes of three correlated stocks can be generated using the following equations:

$$S_{t+\Delta t}^{(1)} = S_t^{(1)} \cdot \exp\left[\left(r - 0.5\sigma^{(1)^2}\right)\Delta t + \sigma^{(1)}\sqrt{\Delta t}\varepsilon^{(1)}\right]$$

$$S_{t+\Delta t}^{(2)} = S_t^{(2)} \cdot \exp\left[\left(r - 0.5\sigma^{(2)^2}\right)\Delta t + \sigma^{(2)}\sqrt{\Delta t}\left(\rho_{12}\varepsilon^{(1)} + \sqrt{1-\rho_{12}^2}\varepsilon^{(2)}\right)\right]$$

$$S_{t+\Delta t}^{(3)} = S_t^{(3)} \cdot \exp\left[\left(r - 0.5\sigma^{(3)^2}\right)\Delta t + \sigma^{(3)}\sqrt{\Delta t}\left(\rho_{13}\varepsilon^{(1)} + \rho_{23}^*\varepsilon^{(2)} + \rho_{33}^*\varepsilon^{(3)}\right)\right]$$

$$\rho_{23}^* = \frac{\rho_{23} - \rho_{12}\rho_{13}}{\sqrt{1-\rho_{12}^2}}$$

$$\rho_{33}^* = \sqrt{\frac{1-\rho_{12}^2-\rho_{23}^2-\rho_{13}^2+2\rho_{12}\rho_{23}\rho_{13}}{1-\rho_{12}^2}}$$

  Implement a function `sim_correlated_paths` that generates three correlated stock price paths. Implement a function `worst_of_down_and_in_put` that determines the price of a worst-of down-and-in put option by means of Monte Carlo simulations.
- You are allowed to add input parameters to your functions.
- Use the functions to value the UBS-issued *8.50% p.a. USD Kick-In GOAL — Linked to worst of Adobe Systems, Apple, and Microsoft.* Your team leader provides you with some data that might be useful for the valuation of the product (see Section 5.2; more details in `CH0419059040_Termsheet_en.pdf`).
- You may compute the value of the bond directly in the Python file.
- You are not required to write separate functions to value the bond.
- You do not have to take taxes into account.

5.2 Information on multi barrier reverse convertible

8.50% p.a. USD Kick-In GOAL	
Issuer	UBS
Nominal (in USD)	1000
Underlying 1	Adobe
Initial underlying level (in USD)	237.68
Strike level (in USD) (in USD)	237.68
Kick-in level (in USD)	154.49
Conversion ratio	4.21
Underlying 2	Apple
Initial underlying level (in USD)	184.16
Strike level (in USD) (in USD)	184.16
Kick-in level (in USD)	119.70
Conversion ratio	5.43
Underlying 3	Microsoft
Initial underlying level (in USD)	97.54
Strike level (in USD) (in USD)	97.54
Kick-in level (in USD)	63.40
Conversion ratio	10.25
Coupon	0.0850
Coupon payment frequency	Semi-anually
Launch date	11/06/2018
Pricing date	27/06/2018
First SIX trading date	05/07/2018
Payment date	05/07/2018
Last trading date	20/06/2019
Expiration date	20/06/2019
Redemption date	27/06/2019

Table 1: Information on multi barrier reverse convertible CH0419059040. The table summaries the information in the term sheet for product CH0419059040.

5.3 Additional files

- Data
  - Stock prices (`stock_prices.csv`)
  - Option prices (`option_prices.csv`)
  - Interest rates (`interest_rates.csv`)
  - CDS spreads of UBS (`cds_spreads.csv`)
- Termsheet (`CH0419059040_Termsheet_en.pdf`)