Estimation and Hypothesis Testing

Agenda

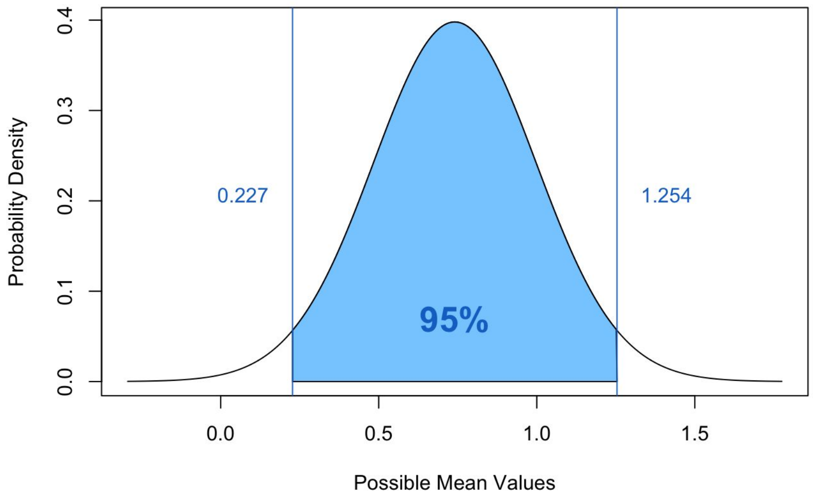
1. Sampling & Central Limit Theorem
2. Estimation
3. Null and Alternative hypothesis
4. Hypothesis testing steps
5. Hypothesis formulation
6. One-tailed and two-tailed tests
7. Type I and Type II errors

Pop Quiz

1. Why do we need sampling?
2. What makes the central limit theorem so important?
3. What do you mean by interval estimation ?
4. What is null and alternative hypothesis?
5. What are the steps to perform a hypothesis test?
6. What is the way to formulate a correct hypothesis?
7. What is the difference between one-tailed and two-tailed tests?
8. What are type I and type II errors?

Sampling Distributions

- **Need for sampling**
  - Given the limited resources and time, it is not always possible to study the population. That’s why we choose a sample out of the population to make inference about the population
- **Sampling Distributions**
  - It is a distribution of a particular sample statistic obtained from all possible samples drawn from a specific population



Central Limit Theorem

The sampling distribution of the sample means will approach normal distribution as the sample size gets bigger, no matter what the shape of the population distribution is.

Assumptions

Data must be **randomly sampled**

Sample values must be **independent** of each other

Samples should come from the **same distribution**

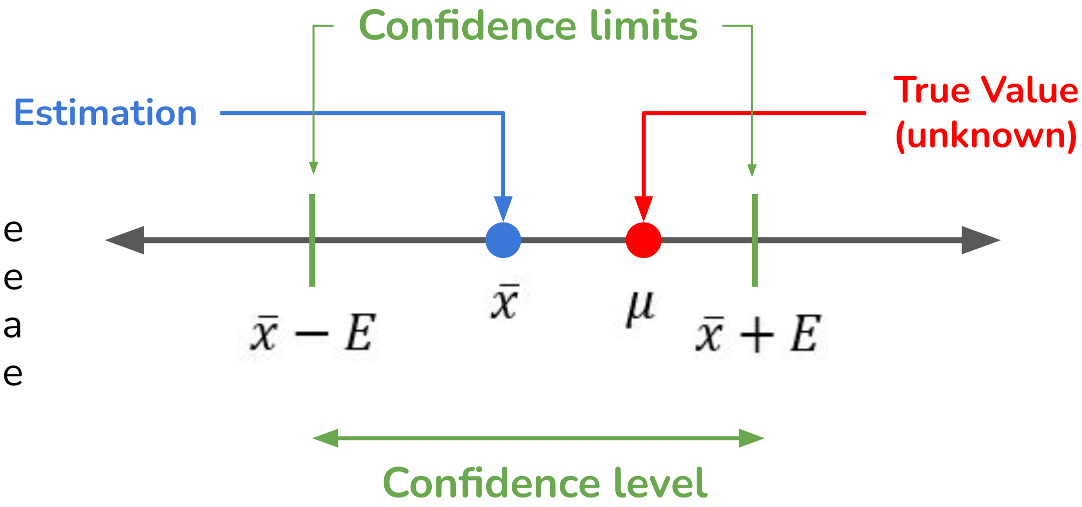
Sample size must be **sufficiently large (≥30)**

Let's see CLT in action by simulation - Link to external site

Confidence interval

Confidence interval provides an interval, or a range of values, which is expected to cover the true unknown parameter.

The upper and lower limits of the interval are determined using the distribution of the sample mean and a multiplier which specifies the 'confidence'



Introduction to Hypothesis Testing

Question of Interest

e.g. has the new online Ad increased the conversion rates for an E-commerce website?

Hypotheses about the population parameter(s)

Null Hypothesis (H_0)

The status quo

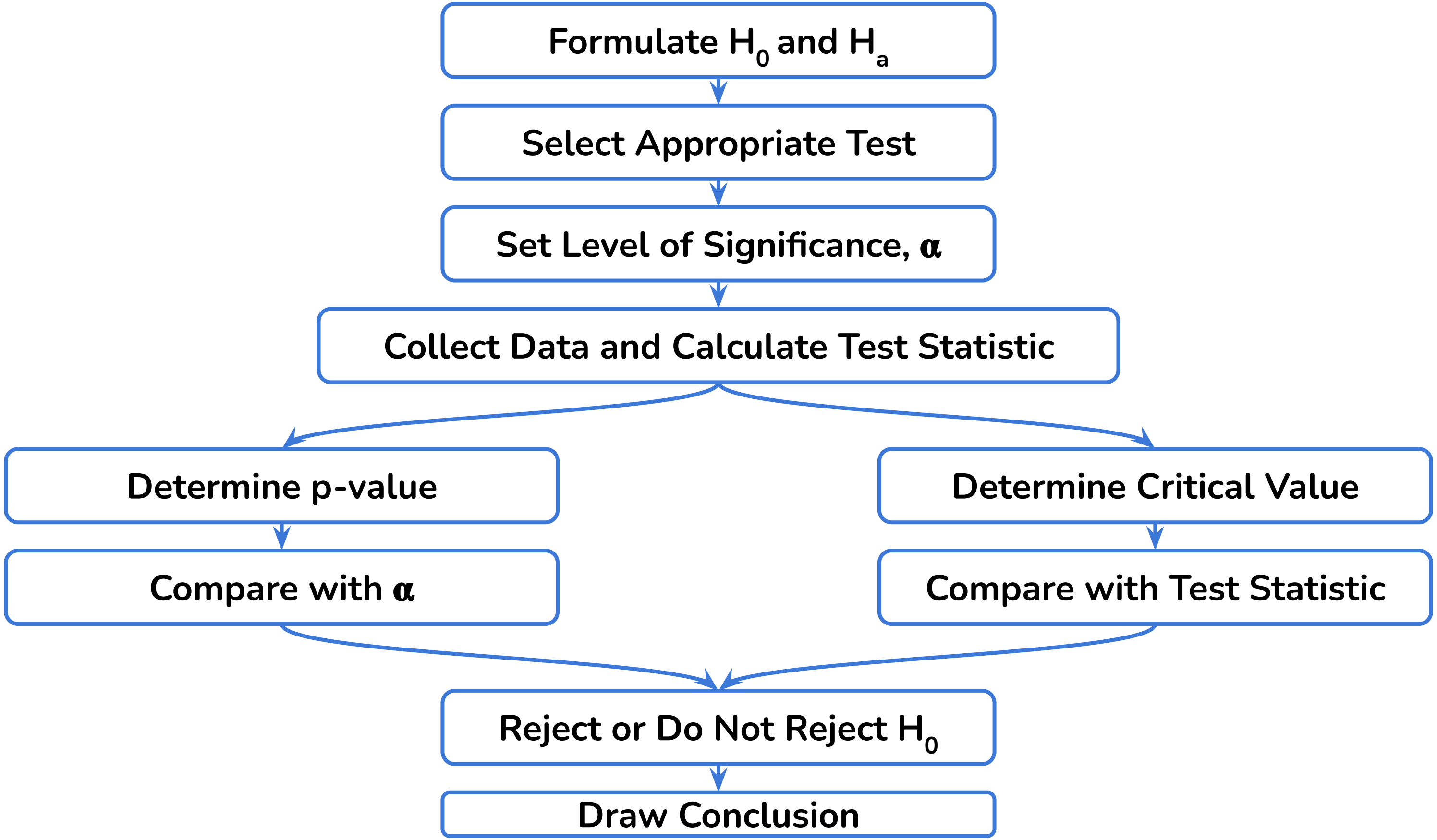
e.g. The new Ad has not increased the conversion rate.

Alternative Hypothesis (H_a)

The research hypothesis

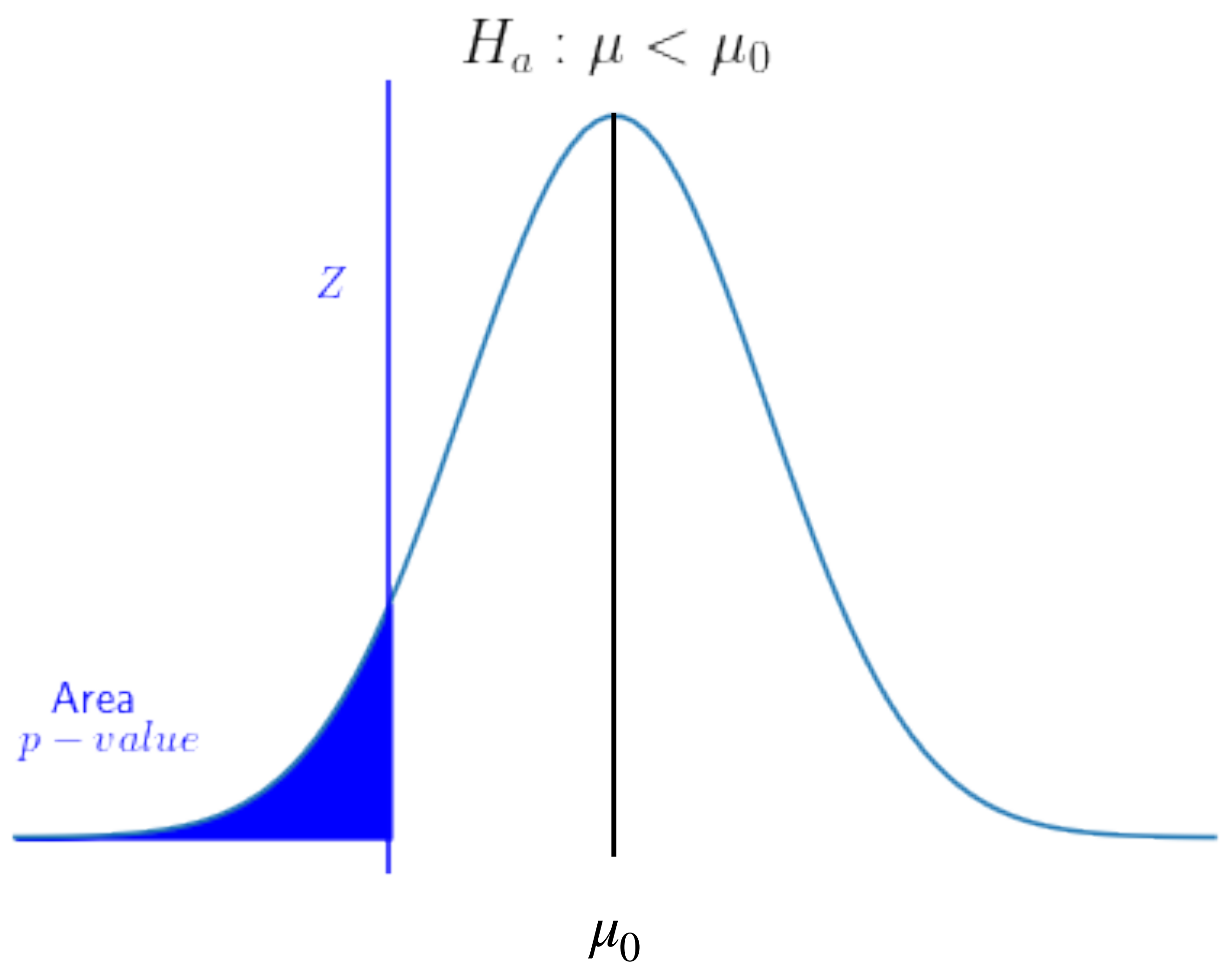
e.g. The new Ad has increased the conversion rate.

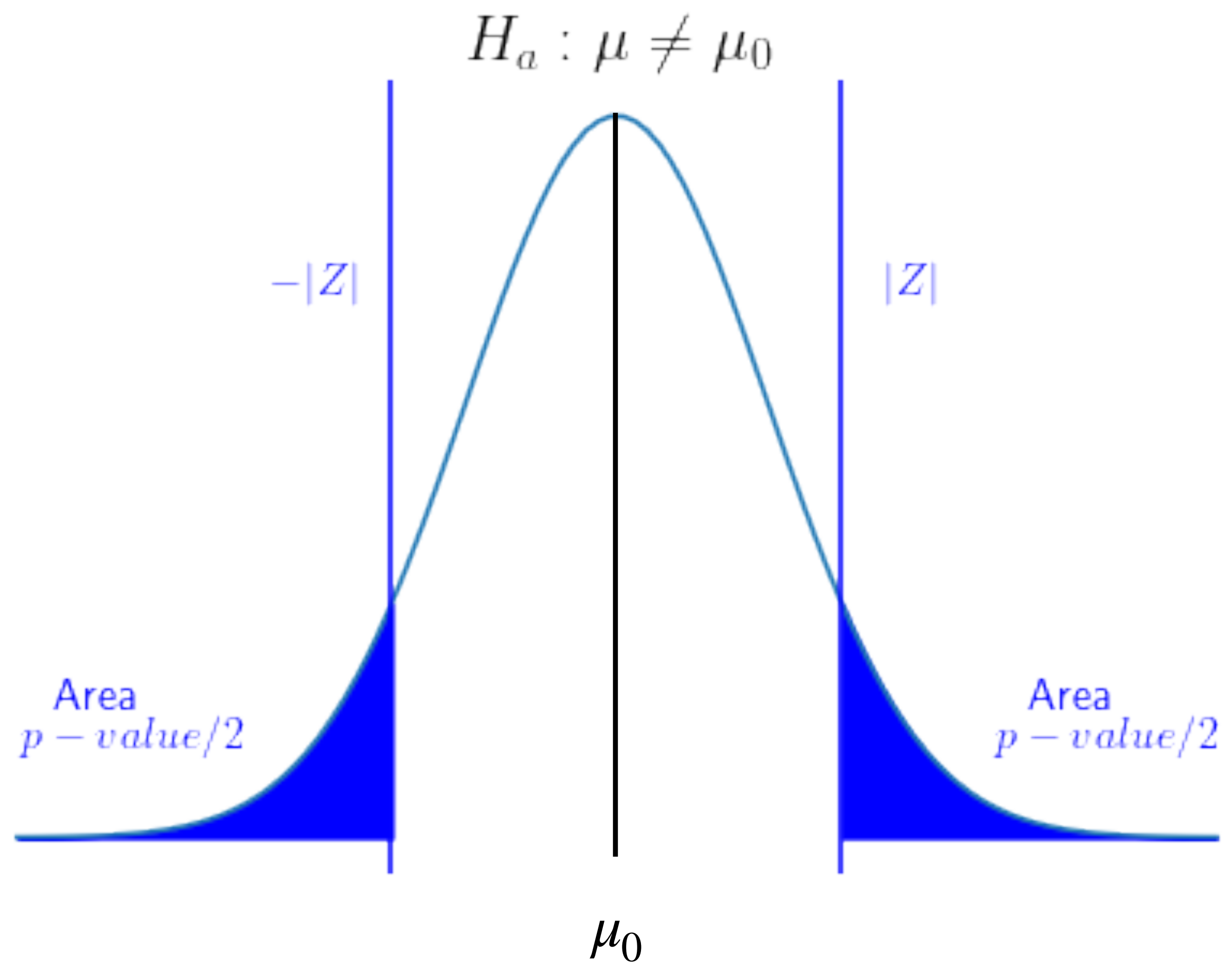
Hypothesis Testing Steps

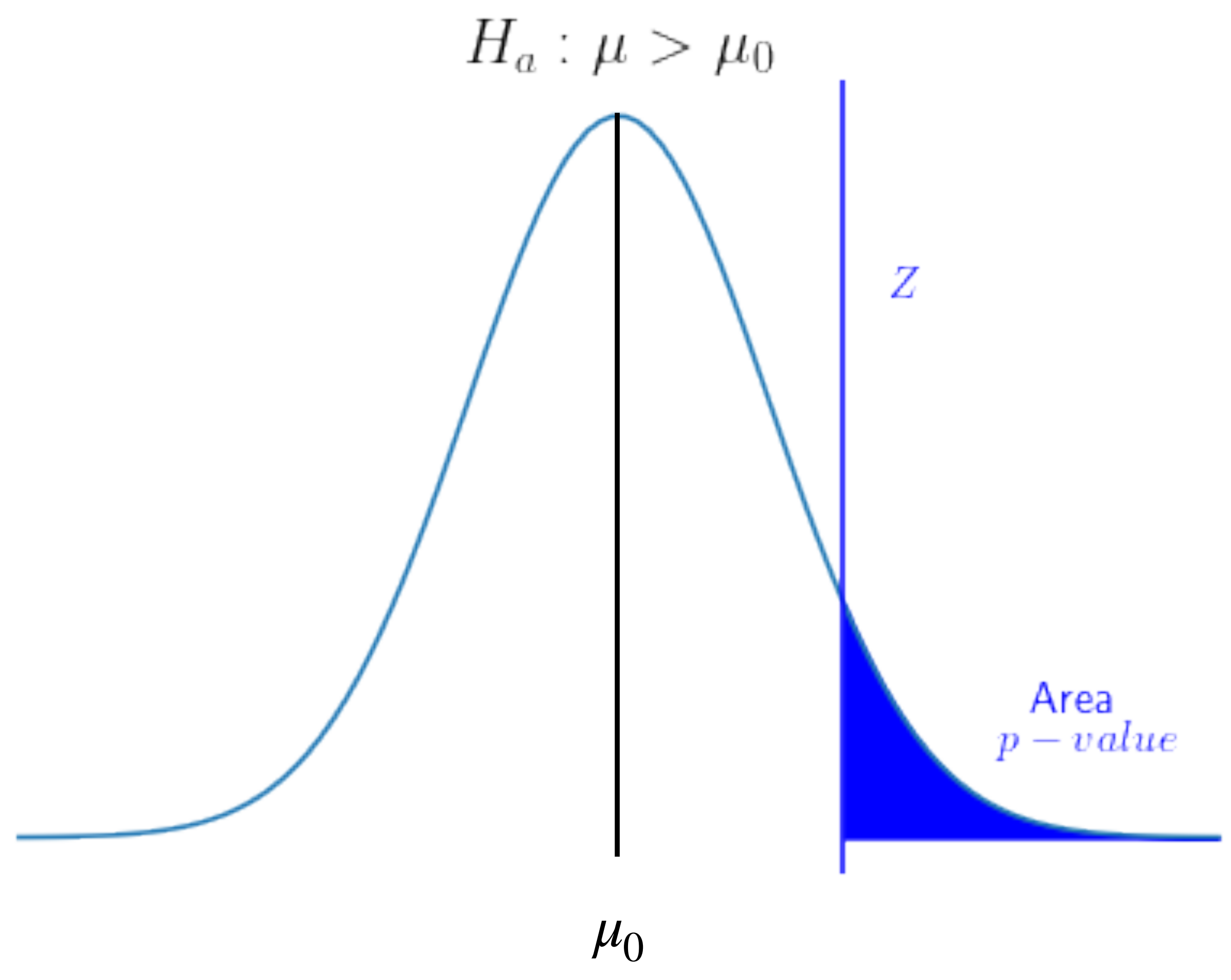


Hypothesis testing: p-value approach

- In the **p-value approach**, we calculate the likelihood (*p-value*) of the test statistic Z *given the assumption of the null hypothesis* H_0
- Low *p-values* are obtained for *extreme* test statistics with respect to H_0
- The area used to compute the *p-value* depends on the alternative hypothesis H_a (in blue below)
- **Decision of the test**: reject the H_0 when $p - value < \alpha$

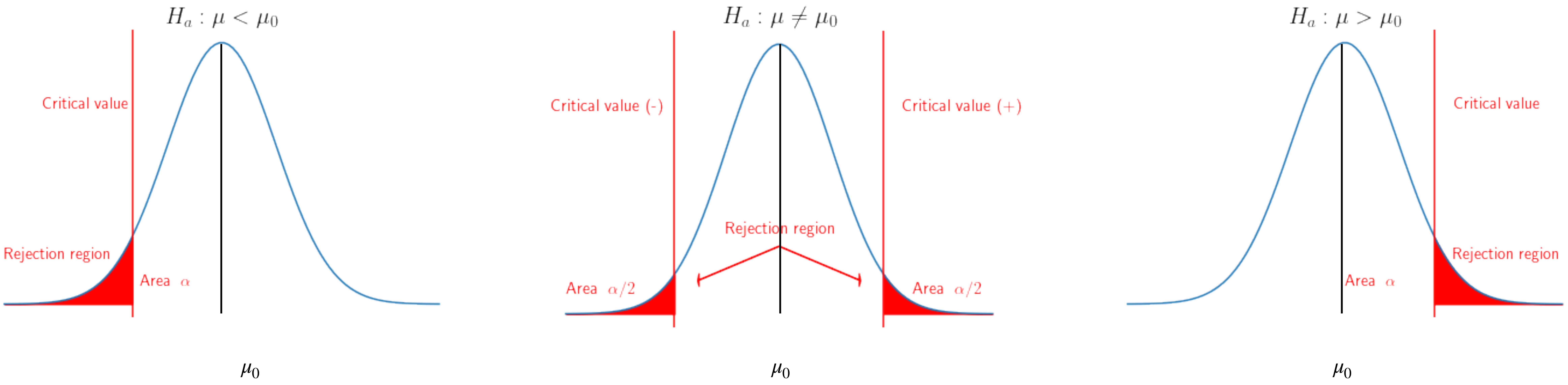






Hypothesis testing: rejection region approach

- In the **rejection region approach**, we define a region whose total area is equal to the significance level α (in red below)
- The location of the rejection region depends on the alternative hypothesis H_a
- **Decision of the test**: reject the null hypothesis H_0 when the test statistic lies in the rejection region



Hypothesis Formulation

Problem Statement 1:

The store manager believes that the average waiting time for the customers of the smart supermarkets at checkouts has become worse than 15 minutes. Formulate the hypothesis.

$$H_0: \mu = 15$$

$$H_a: \mu > 15$$

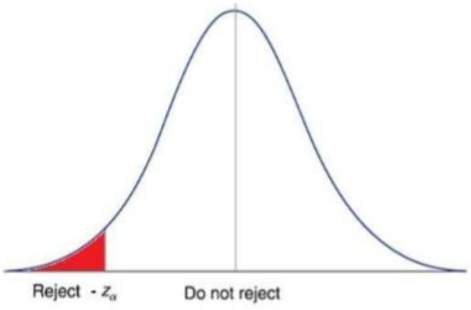
Problem Statement 2:

A pharmaceutical company has developed an improved drug. The company claims that it takes less than 12 minutes for the drug to enter in the patient's bloodstream. Formulate the hypothesis to convince the FDA to approve the claim.

$$H_0: \mu = 12$$

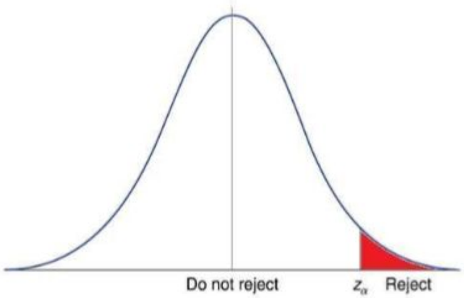
$$H_a: \mu < 12$$

One-tailed vs Two-tailed Test



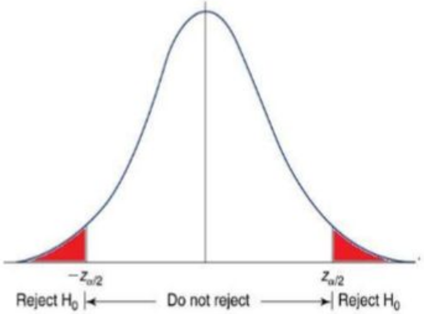
- Lower tail test.
- H_1: μ < ……

Reject H_0 if the value of test statistic is too small



- Upper tail test.
- H_1: μ > ……

Reject H_0 if the value of test statistic is too large



- Two tail test.
- H_1: μ ≠ ……

Reject H_0 if the value of test statistic is either too small or too large

Type I and Type II errors

Level of significance = α
Confidence Interval = (1 - α)

	H_0 True	H_0 False
Reject H_0	Type I Error (α)	Correct decision
Fail to reject H_0	Correct decision	Type II Error (β)

Power of the test = (1 - β)

Example

Problem Statement: The store manager believes that the average waiting time for the customers of the smart supermarkets at checkouts has become worse than 15 minutes. Formulate the hypothesis.

Null Hypothesis (H_0): The average waiting time at checkouts is less than equal to 15 minutes.

Alternate Hypothesis (H_a): The average waiting time at checkouts is more than 15 minutes.

Type I error (false positive): Reject Null hypothesis when it is indeed true. “The fact is that the average waiting time at checkout is less than equal to 15 minutes but the store manager has identified that it is more than 15 minutes”.

Type II error (false negative): Fail to reject Null hypothesis when it is indeed false. “The fact is that the average waiting time at checkout is more than 15 minutes but the store manager has identified that it is less than equal to 15 minutes”.

Let’s discuss the case study!

greatlearning
Power Ahead

Happy Learning !