Introduction to Game Theory

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Exercise Sheet 8

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Exercise 8.1 (Voting procedures, 2 points)

In the following, we assume that ties are broken in favor of the candidate with the lower index. Consider the following preference relations:

20 voters have the preference $b \prec_i c \prec_i e \prec_i d \prec_i a$
10 voters have the preference $d \prec_i e \prec_i c \prec_i b \prec_i a$
15 voters have the preference $b \prec_i d \prec_i a \prec_i e \prec_i c$
12 voters have the preference $a \prec_i b \prec_i c \prec_i e \prec_i d$
13 voters have the preference $a \prec_i e \prec_i c \prec_i d \prec_i b$

(a) Determine the winner according to the Borda count method.

(b) Determine the winner according to the instant-runoff voting method.

(c) Determine the set of possible winners according to the Schulze-method[1].

Exercise 8.2 (Properties of voting procedures, 3 points)

Consider the following properties of voting procedures:

(a) **Majority criterion:**

If for more than half of the voters $i, b \prec_i a$ for all $b \in A \setminus \{a\}$, then $f(\prec_1, \ldots, \prec_n) = a$.

(b) **Reversal symmetry:**

If $f(\prec_1, \ldots, \prec_n) = a$ and $a \prec_i' b$ iff $b \prec_i a$ for all $i = 1, \ldots, n$ and $a, b \in A$, then $f(\prec_1', \ldots, \prec_n') \neq a$.

(c) **Incentive compatibility:**

$f(\prec_1, \ldots, \prec_i', \ldots, \prec_n) \preceq_i f(\prec_1, \ldots, \prec_i, \ldots, \prec_n)$ for all $\prec_1, \ldots, \prec_n, \prec_i' \in L$.

For each of the above properties, show that the Borda count method satisfies the property or give a counterexample. For simplicity, we assume that ties are broken in favor of the candidate with the lower index. Moreover, $|A| \geq 3$.

Exercise 8.3 (Social welfare functions: unanimity, 2 + 1 points)

A social welfare function $F : L^n \to L$ satisfies

- **total unanimity** if for all $\prec \in L, F(\prec, \ldots, \prec) = \prec$.
- **partial unanimity** if for all $\prec_1, \prec_2, \ldots, \prec_n \in L, a, b \in A$,

$$a \prec_i b \text{ for all } i = 1, \ldots, n \implies a \prec b, \text{ where } \prec := F(\prec_1, \ldots, \prec_n).$$

(a) Proof that *partial unanimity* implies *total unanimity.*

(b) Proof by counter-example that *total unanimity* does not imply *partial unanimity.*
Hint: specify a social welfare function F that satisfies *total unanimity* but does not satisfy *partial unanimity.*

[1] http://en.wikipedia.org/wiki/Schulze_method