Introduction to Game Theory

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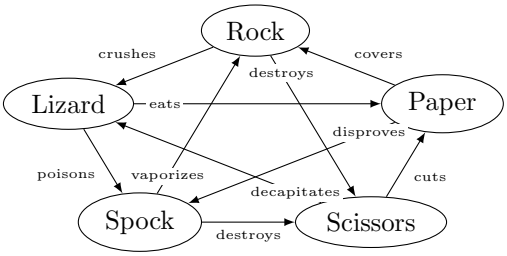
Exercise Sheet 1 — Solutions

Exercise 1.1 (Teamwork, 4 **additional** points)

We believe that (even in an ongoing pandemic) working in groups is a strictly dominant strategy for you. If you submit your solution as a group of two or three, you will automatically receive four additional points. You will get these points retroactively, when having formed a team by exercise sheet 3. Try using our forum to find team mates or send an email to `heinoldk@tf.uni-freiburg.de` with your name so that we can form groups of students. Please use as subject of the email: *[gt20] Teamwork*.

Exercise 1.2 (Strategic Games, 5 points)

Formalize the game "Rock, Paper, Scissors, Lizard, Spock" as a strategic game, i.e., specify a set of players, sets of actions for all players, and utility functions in terms of a payoff matrix. The winners of the possible pairings follow from the following graph.



Solution:

RPSLS is a strategic game $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with

- $N = \{1, 2\}$,
- $A_1 = A_2 = \{\text{Scissors}, \text{Rock}, \text{Paper}, \text{Lizard}, \text{Spock}\}$ und
- u_1, u_2 as given by the following utility-matrix

1 \ 2	Rock	Paper	Scissors	Lizard	Spock
Rock	0, 0	−1, 1	1, −1	1, −1	−1, 1
Paper	1, −1	0, 0	−1, 1	−1, 1	1, −1
Scissors	−1, 1	1, −1	0, 0	1, −1	−1, 1
Lizard	−1, 1	1, −1	−1, 1	0, 0	1, −1
Spock	1, −1	1, −1	−1, 1	−1, 1	0, 0

Exercise 1.3 (Elimination of strictly dominated strategies, 2+1 points)

Consider the game $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with $N = \{1, 2\}$, $A_i = \{a_i, b_i, c_i, d_i\}$, $i = 1, 2$, and the following payoff matrix.

1 \ 2	a_2	b_2	c_2	d_2
a_1	$6,2$	$2,7$	$1,4$	$0,3$
b_1	$1,0$	$3,2$	$2,1$	$1,1$
c_1	$7,0$	$2,2$	$1,5$	$6,1$
d_1	$8,4$	$1,2$	$0,2$	$3,9$

(a) Iteratively eliminate strictly dominated strategies for as many steps as possible. In each step, specify which strategy of which player was eliminated and by which strategy it was strictly dominated.

Solution:

In the following, strictly dominated actions are crossed out. The numbers give the elimination order and the action by which the eliminated action is strictly dominated. So $1, d_2$ below column a_2 means that action a_2 is strichtly dominated by action d_2.

	a_2	b_2	c_2	d_2	
~~a_1~~	~~$6,2$~~	~~$2,7$~~	~~$1,4$~~	~~$0,3$~~	$4, b_1$
b_1	$1,0$	$3,2$	$2,1$	$1,1$	
~~c_1~~	~~$7,0$~~	~~$2,2$~~	~~$1,5$~~	~~$6,1$~~	$5, b_1$
~~d_1~~	~~$8,4$~~	~~$1,2$~~	~~$0,2$~~	~~$3,9$~~	$2, c_1$
	$1, d_2$		$6, b_2$	$3, b_2$	