Introduction to Game Theory

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Summer semester 2020

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Exercise Sheet 11 — Solutions

Exercise 11.1 (Greedy Mechanism for Single-Minded Bidders, 2 + 2 + 2 points)

Recall that the Greedy Mechanism for Single-Minded Bidders sorts bidders in descending order by $\frac{v_i^*}{\sqrt{|S_i^*|}}$. It has been shown that this mechanism has an approximation factor of $\sqrt{m}$, where m is the number of items. Consider a scenario with five bidder ($N = \{1, 2, 3, 4, 5\}$) and four 4 items ($G = \{1, 2, 3, 4\}$) where the bids be as follows:

- for all $i \in \{1, 2, 3, 4\}$: $S_i^* = \{i\}$ and $v_i^* = 1$, and
- $S_5^* = \{1, 2, 3, 4\}$ and $v_5^* = 2$.

(a) Apply the Greedy Mechanism for Single-Minded Bidders and report the winner set and the social welfare.

Solution:

i	$S_i^\star$	$v_i^\star$	$\frac{v_i^\star}{\sqrt{\lvert S_i^\star \rvert}}$	Assignment order
1	$\{1\}$	1	1	1
2	$\{2\}$	1	1	2
3	$\{3\}$	1	1	3
4	$\{4\}$	1	1	4
5	$\{1, 2, 3, 4\}$	2	1	5

$W = \{1, 2, 3, 4\}, U = \sum_{j \in W} v_j^\star = 4$

(b) Apply the Greedy Mechanism for Single-Minded Bidders but this time sort the bidders in descending order by their prices, i.e., v_i^*. Again, report the winner set and the social welfare.

Solution:

i	$S_i^\star$	$v_i^\star$	Assignment order
1	$\{1\}$	1	2
2	$\{2\}$	1	3
3	$\{3\}$	1	4
4	$\{4\}$	1	5
5	$\{1, 2, 3, 4\}$	2	1

$W = \{5\}, U = \sum_{j \in W} v_j^\star = 2$

(c) Show that the approximation factor of the Greedy Mechanism for Single-Minded Bidders is not "better" than m if you sort the bidders in descending order by their prices, i.e., v_i^*. *Hint*: Construct an example with m items where the optimal social welfare is m times better than the proposed solution of the modified Greedy Mechanism for Single-Minded Bidders.

Solution:

Consider an auction with m items and $m + 1$ bidders. Each bidder $i \in \{1, \ldots, m\}$ bids 1 for the one-item bundle that contains only item i. Bidder $m + 1$ bids $1 + \epsilon$ for the full bundle $\{1, \ldots, m\}$. In this variant of the algorithm, only bidder $m + 1$ wins, getting the full set of items while all other bidders get nothing. The social welfare is $U = 1 + \epsilon$. In the optimal solution, bidders $1, \ldots, m$ each win their desired item, with a social welfare of $U = m$.