Introduction to Game Theory

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Exercise Sheet 3 Due: Friday, June 5, 2020

Exercise 3.1 (Support lemma, 3 Punkte)

Let α be a mixed strategy profile, $a_i \in supp(\alpha_i)$, $a_i \notin B_i(\alpha_{-i})$, $a_i' \in B_i(\alpha_{-i})$ and α_i' defined by $\alpha_i'(a_i) = 0$, $\alpha_i'(a_i') = \alpha_i(a_i') + \alpha_i(a_i)$ and $\alpha_i'(a_i'') = \alpha_i(a_i'')$ for all $a_i'' \in A_i \setminus \{a_i, a_i'\}$. Show formally that $U_i(\alpha_i', \alpha_{-i}) > U_i(\alpha_i, \alpha_{-i})$. Use the definition of the expected reward.

Exercise 3.2 (Mixed strategy Nash equilibria, 2 points)

Consider the following strategic game:

$$\begin{array}{c|cccc} & & \text{Player 2} \\ & X & Y \\ \hline \text{Player 1} & A & 1,1 & 3,2 \\ & 2,2 & 1,1 \\ \hline \end{array}$$

Determine and write down all mixed strategy Nash equilibria.

Exercise 3.3 (Linear Complementarity Problem, 2 + 1 points)

Consider the strategic game given by the following payoff matrix:

		Player 2		
		x	y	z
	a	0,0	3, 1	3,3
Player 1	b	1, 1	0,0	1,3
	c	1,1	1,1	0,0

- (a) For the following pair of support sets formulate the corresponding linear program: $(supp(\alpha), supp(\beta)) = (\{a, b, c\}, \{x, y, z\}).$
- (b) Solve the linear program and provide values for each $\alpha(a_1)$ and $\beta(a_2)$, $a_1 \in \{a, b, c\}, a_2 \in \{x, y, z\}$. What is the expected payoff (u, v) of the NE computed above?

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.