Introduction to Game Theory

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Exercise Sheet 7 — Solutions

Exercise 7.1 (Repeated Games, 1 + 1 + 3 + 3 points)

Consider the following instance of the infinitely repeated prisoner's dilemma. The payoff matrix of the stage game is given below.

		Player 2	
		C	D
Player 1	C	$3, 3$	$0, 10$
	D	$10, 0$	$1, 1$

(a) Let t be the *tit-for-tat* strategy as defined in the lecture. Specify the unique run $O(t, t)$ that results from playing t against t.

Solution:

$$\begin{aligned} O(t,t) &= \langle (C,C), (C,C), \dots, (C,C) \rangle \\ &= \langle (C,C) \rangle_{k=1}^{\infty} \end{aligned}$$

(b) Compute the discounted payoff $v_1(O(t,t))$ of player 1 for the strategy profile (t,t) for general $0 < \delta < 1$ and for $\delta = \frac{1}{2}$ in particular.

Solution:

$$\begin{aligned} v_1(O(t,t)) &= 3 + 3 \cdot \delta + 3 \cdot \delta^2 + \dots \\ &= \frac{3}{1-\delta} \\ &= 6, \text{ for } \delta = {}^1\!/_2 \end{aligned}$$

(c) Under the discounting preference criterium, for which discount factor $0 < \delta < 1$ is $(\textsc{Grim}, \textsc{Grim})$ a Nash equilibrium? Justify your answer. (*Hint:* The $\textsc{Grim}$ strategy starts with playing C. After any play of D it plays D forever.)

Solution:

W.l.o.g. assume that player 1 deviates in the first round. After the first deviation player 1 can never get more than 1 utility, since player 2 will always defect.

$$\begin{aligned} v_1(O(s,g)) &= 10 + 1\delta + 1\delta^2 + 1\delta^3 + \dots \\ &= 10 + \sum_{i=0}^{N} \delta^i - 1 \\ &= 9 + \frac{1}{1-\delta} \\ v_1(O(g,g)) &= \frac{3}{1-\delta} \end{aligned}$$

A deviation is not profitable if

$$\begin{aligned} 9 + \frac{1}{1-\delta} &\leq \frac{3}{1-\delta} \\ \Leftrightarrow \delta &\geq \frac{7}{9} \end{aligned}$$

$(\textsc{Grim}, \textsc{Grim})$ is a NE for $\delta \geq \frac{7}{9}$.

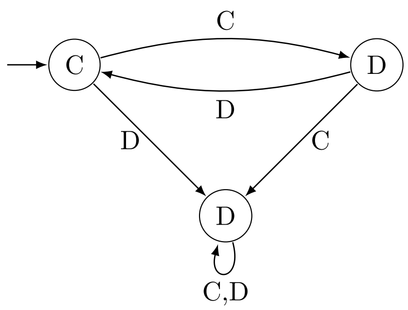
(i.e. if the players care about tomorrow at least $\frac{7}{9}$ as much as today.)

(d) Consider the following three payoff profiles under the limit-of-means preference criterium: 1. $(2,2)$, 2. $(10,10)$, and 3. $(3,0)$. For each payoff profile, either construct two automata that form a Nash equilibrium or argue that no Nash equilibrium with the given payoffs exists.

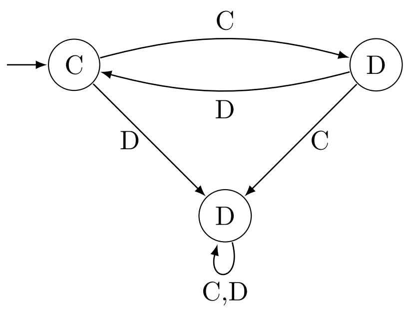
Solution:

- $(2,2)$: (s_1, s_2) is a NE with $v(O(s_1, s_2)) = (2,2)$, where

s_1 :



s_2 :



- $(10,10)$: not feasible
- $(3,0)$: not enforceable, not feasible