Introduction to Game Theory

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Exercise Sheet 11 Due: Friday, July 31, 2020

Exercise 11.1 (Greedy Mechanism for Single-Minded Bidders, 2 + 2 + 2 points)

Recall that the Greedy Mechanism for Single-Minded Bidders sorts bidders in descending order by $\frac{v_i^*}{\sqrt{|S_i^*|}}$. It has been shown that this mechanism has an approximation factor of \sqrt{m} , where m is the number of items. Consider a scenario with five bidder $(N=\{1,2,3,4,5\})$ and four 4 items $(G=\{1,2,3,4\})$ where the bids be as follows:

- for all $i \in \{1, 2, 3, 4\}$: $S_i^* = \{i\}$ and $v_i^* = 1$, and
- $S_5^* = \{1, 2, 3, 4\}$ and $v_5^* = 2$.
- (a) Apply the Greedy Mechanism for Single-Minded Bidders and report the winner set and the social welfare.
- (b) Apply the Greedy Mechanism for Single-Minded Bidders but this time sort the bidders in descending order by their prices, i.e., v_i^* . Again, report the winner set and the social welfare.
- (c) Show that the approximation factor of the Greedy Mechanism for Single-Minded Bidders is not "better" than m if you sort the bidders in descending order by their prices, i.e., v_i^* . Hint: Construct an example with m items where the optimal social welfare is m times better than the proposed solution of the modified Greedy Mechanism for Single-Minded Bidders.