

Foundations of Artificial Intelligence

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Summer Term 2020

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Exercise Sheet 6

Due: Monday, June 22, 2020

Exercise 6.1 (Semantics of Predicate Logic)

Consider the Interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with

- $\mathcal{D} = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : \mathcal{D} \times \mathcal{D} \rightarrow \mathcal{D}, plus^{\mathcal{I}}(a, b) = (a + b) \bmod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.

Decide for the following formulae θ_i if \mathcal{I} is a model for θ_i under α , i.e. if $\mathcal{I}, \alpha \models \theta_i$.

Explain your answer by formally applying the semantics.

- (a) $\theta_1 = odd(y) \wedge even(two)$
- (b) $\theta_2 = \forall x (even(x) \vee odd(x))$
- (c) $\theta_3 = \forall x \exists y lessThan(x, y)$
- (d) $\theta_4 = \forall x (even(x) \Rightarrow \exists y lessThan(x, y))$
- (e) $\theta_5 = \forall x (odd(x) \Rightarrow even(plus(x, y)))$

Exercise 6.2 (Semantics of Predicate Logic)

Consider the Interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ and the variable assignment α :

- $\mathcal{D} = \{a, b, c\}$
- $P^{\mathcal{I}} = \{(a, a), (a, b), (b, a), (b, b), (b, c), (c, a)\}$
- $Q^{\mathcal{I}} = \{a, b\}$
- $R^{\mathcal{I}} = \{(a, a), (a, b), (a, c), (b, c), (c, b)\}$
- $\alpha = \{(v, a), (w, b)\}$

Decide for the following formulae θ_i if \mathcal{I} is a model for θ_i under α , i.e. if $\mathcal{I}, \alpha \models \theta_i$.

Explain your answer by formally applying the semantics.

(a) $\theta_1 = \forall x (P(x, w) \Rightarrow Q(x))$

(b) $\theta_2 = \exists x (R(v, x) \Rightarrow P(x, x))$

(c) $\theta_3 = \forall x \forall y (R(x, y) \iff Q(y))$

(d) $\theta_4 = \left[\neg \forall x \forall y (Q(y) \vee P(x, y)) \right] \wedge \left[\exists z (Q(z) \vee P(w, z)) \right]$

Note: The exercise sheets may be worked on in groups of up to three students.