Foundations of Artificial Intelligence

Prof. Dr. J. Boedecker, Prof. Dr. W. Burgard, Prof. Dr. F. Hutter, Prof. Dr. B. Nebel,
Dr. rer. nat. M. Tangermann
T. Schulte, M. Krawez, R. Rajan, S. Adriaensen, K. Sirohi
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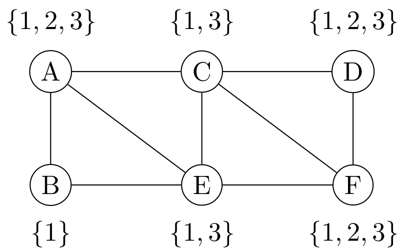
University of Freiburg
Department of Computer Science

Exercise Sheet 4

Due: Monday, June 08, 2020

Exercise 4.1 (Arc consistency)

Consider the constraint satisfaction problem given by the constraint graph below. The constraints are such that no two adjacent nodes have the same value. Establish arc-consistency.



Exercise 4.2 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

- (a) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$
- (b) $\mathit{Smoke} \Rightarrow \mathit{Fire}$
- (c) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Fire} \Rightarrow \neg \mathit{Smoke})$
- (d) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow ((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire})$
- (e) $\mathit{Spring} \Leftrightarrow \mathit{SunnyWeather}$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

- (a) $(A \wedge B) \vee (B \wedge C)$
- (b) $A \vee B$
- (c) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 4.3 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show whether $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Note: The exercise sheets may be worked on in groups of up to three students.