Introduction to Game Theory

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Exercise Sheet 5

Due: Thursday, June 19, 2020

Exercise 5.1 (Uniqueness of SPE, 2 + 1 points)

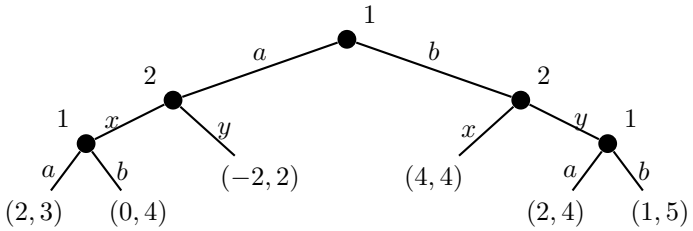
Let Γ be an extensive two-player game with s^* and r^* being subgame perfect equilibria of Γ. Show (for $i \in N$):

(a) If Γ is a ZSG, then $u_i(O(s^*)) = u_i(O(r^*))$.

(b) For general extensive games, $u_i(O(s^*)) = u_i(O(r^*))$ is not necessarily true.

Exercise 5.2 (Subgame perfect equilibria, 2 points)

Determine all subgame perfect equilibria of the extensive form game defined by the following game tree.



Exercise 5.3 (Extensive Games, 1 + 1 + 1 points)

The owner of a retail chain R operates stores in K cities. In each city k, $1 \leq k \leq K$, there is a potential competitor C_k who can decide to open up a store (O_k) or to stay out of business ($\neg O_k$). If competitor C_k opens a store, R can either start a price war (P_k) or ignore the competitor ($\neg P_k$). The competitors make their decisions sequentially, i.e. when C_k makes its decision, $C_1, \ldots, C_{k-1}$ have already made their decisions and C_k is aware of their choice and the reactions of R. In every city k competitor C_k gets payoff 0 if he chooses to stay out of business, payoff 2 if he opens a store and R is not starting a price war, and payoff -2 if he opens a store and R starts a price war. The retail chain owner R gets a payoff of $3K$ if no competitor opens a store. For every competitor opening a store R's payoff is reduced by 2. For every price war R decides to start the payoff is additionally reduced by 1. Regard the special case of $K = 2$.

(a) Model this situation as an extensive game with perfect information and specify the game tree.

(b) Specify each players set of strategies.

(c) Determine a subgame perfect equilibrium.