Foundations of Artificial Intelligence

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Exercise Sheet 7 — Solutions

Exercise 7.1 (Planning)

Consider the following STRIPS-Task $\Pi = \langle \mathcal{S}, O, I, G \rangle$:

- $\mathcal{S}$: {X, Y, Z, G}
- $O : \{A, B, C, D, E, F\}$ where

$$\begin{array}{lll} A: & pre(A) = \{X\}, & eff(A) = \{Y, Z\} \\ B: & pre(B) = \{X\}, & eff(B) = \{\neg X, Z\} \\ C: & pre(C) = \{\neg Y\}, & eff(C) = \{Z\} \\ D: & pre(D) = \{\neg Z\}, & eff(D) = \{Y\} \\ E: & pre(E) = \{\neg X, Y\}, & eff(E) = \{\neg Y, G\} \\ F: & pre(F) = \{Z\}, & eff(F) = \{\neg Z, G\} \end{array}$$

- I: {X, Y}
- G: {G}

(a) State for each operator from O if it is applicable in I or not. For each applicable operator also give the resulting state after applying that operator in I.

Solution:

Operator	Applicable?	Resulting State
A	Yes	$\{X, Y, Z\}$
B	Yes	$\{Y, Z\}$
C	No	-
D	Yes	$\{X, Y\}$
E	No	-
F	No	-

(b) Give an applicable plan π that leads from I to G.

Solution:

$\pi = \langle B, E \rangle$, $\langle A, F \rangle$, $\langle B, F \rangle$, $\ldots$

Exercise 7.2 (Bayes' Rule)

In Freiburg 80% of all cars are red. You see a car at night that does *not* appear red to you. You know that you can correctly identify a red car only in 70% of the cases when the given car is red. And you can identify a non-red car correctly in 90% of the cases when the given car is non-red.

(a) List all conditional and non-conditional probabilities that you can determine directly from the task description. Note: Differentiate between the statement that a car *is red* and the statement that you have *seen a red car*.

(b) Compute the probability that the car is actually red, when you perceive a car as red in Freiburg at night.

Solution:

P(R): car is red
P(PR): car is perceived red
$P(R) = 0.8, P(\neg R) = 0.2$
$P(PR|R) = 0.7$
$P(\neg PR|\neg R) = 0.9$

$$P(R|PR) = \frac{P(PR|R) \cdot P(R)}{P(PR)}$$

$$= \frac{P(PR|R) \cdot P(R)}{P(PR|R) \cdot P(R) + P(PR|\neg R) \cdot P(\neg R)}$$

$$= \frac{0.7 \cdot 0.8}{0.7 \cdot 0.8 + 0.1 \cdot 0.2}$$

$$= \frac{0.56}{0.56 + 0.02} = \frac{0.56}{0.58} = \frac{28}{29}$$

Exercise 7.3 (Independence and Joint and Conditional Probabilities)

(a) A 6-sided die is rolled once. Which of the following events are independent? Show the probability values and reasoning.

- E : An even number is rolled
- O : An odd number is rolled
- T : A number ≥ 3 is rolled

Solution:

We know there are 6 possible outcomes for the roll of the die.

$P(E) = 0.5$	3 out of 6 possibilities are covered under the event
$P(O) = 0.5$	3 out of 6 possibilities are covered under the event
$P(T) = \frac{2}{3}$	4 out of 6 possibilities are covered under the event
$P(E \cap O) = 0 \neq P(E) * P(O) = 0.25$	E and O are disjoint events
$P(E \cap T) = \frac{1}{3} = P(E) * P(T) = \frac{1}{3}$	E and T cover 2 out of 6 possibilities
$P(O \cap T) = \frac{1}{3} = P(O) * P(T) = \frac{1}{3}$	O and T cover 2 out of 6 possibilities

By definition of independence, E and T are independent and O and T are independent.

(b) Make the joint probability distribution table for the events E and T.

Solution:

	$E = False$	$E = True$
T = *False*	0.167	0.167
T = *True*	0.333	0.333

(c) Calculate the conditional probability $P(\neg e \,|\, t)$.

Solution:

$P(\neg e \,|\, t) = P(\neg e \wedge t)/P(t) = 0.333/0.666 = 0.5$