Introduction to Game Theory

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Exercise Sheet 4 — Solutions

Exercise 4.1 (Correlated Equilibria, 1 + 3 points)

Consider the strategic game defined by the following payoff matrix:

Player 2

		A	B
Player 1	A	$-10, -10$	$1,\ 0$
	B	$0,\ 1$	$-1, -1$

(a) Compute the payoff profile for the mixed strategy Nash equilibrium α with the following support sets: $supp(\alpha_1) = supp(\alpha_2) = \{A, B\}$.

Solution:

First we need to compute the MSNE. It is $(\alpha_1, \alpha_2) = ((\frac{1}{6}, \frac{5}{6}), (\frac{1}{6}, \frac{5}{6}))$. We can now compute the payoffs using the definition of the expected utility:

$$U_1(\alpha_1, \alpha_2) = \frac{1}{6} \cdot \frac{1}{6} \cdot -10 + \frac{1}{6} \cdot \frac{5}{6} \cdot 1 + \frac{5}{6} \cdot \frac{1}{6} \cdot 0 + \frac{5}{6} \cdot \frac{5}{6} \cdot -1 = -\frac{5}{6}$$
$$U_2(\alpha_1, \alpha_2) = \frac{1}{6} \cdot \frac{1}{6} \cdot -10 + \frac{1}{6} \cdot \frac{5}{6} \cdot 0 + \frac{5}{6} \cdot \frac{1}{6} \cdot 1 + \frac{5}{6} \cdot \frac{5}{6} \cdot -1 = -\frac{5}{6}$$

The payoff profile for the MSNE α is $(-\frac{5}{6}, -\frac{5}{6})$.

(b) Construct a correlated equilibrium that yields a payoff profile such that both players have a higher payoff than in the mixed strategy Nash equilibrium computed above. Specify the probability space (Ω, π), the information partitions $\mathcal{P}_1$ and $\mathcal{P}_2$, and the strategies σ_1 and σ_2.

Solution:

Let $\Omega = \{red, green\}$ and $\pi(red) = \pi(green) = \frac{1}{2}$. Let $\mathcal{P}_1 = \mathcal{P}_2 = \{\{red\}, \{green\}\}$. Define the strategies as follows:
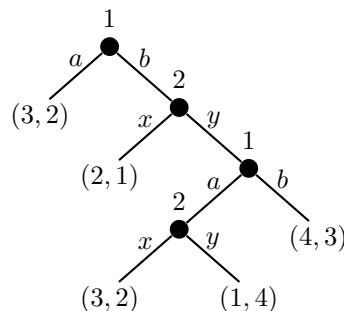
$$\sigma_1(red) = A, \ \sigma_1(green) = B$$
$$\sigma_2(red) = B, \ \sigma_2(green) = A$$

Both players play optimally and get a payoff profile of $(\frac{1}{2}, \frac{1}{2})$.

Exercise 4.2 (Induced Strategic Game, 2 + 2 points)

Consider the two player extensive form game defined by the following game tree.

(a) Specify the induced strategic game.

Solution:

| | player 2 | | | |
player 1	xx	xy	yx	yy
aa	3, 2	3, 2	3, 2	3, 2
ab	3, 2	3, 2	3, 2	3, 2
ba	2, 1	2, 1	3, 2	1, 4
bb	2, 1	2, 1	4, 3	4, 3

(b) Determine all Nash equilibria and decide for each one whether it is subgame perfect or not.

Solution:

NEs: $(aa, xx), (aa, xy), (ab, xx), (ab, xy), (bb, yx), (bb, yy)$
SPE: (bb, yy)