Advanced Algorithms — Problem set 4 (short)

Whenever you give an algorithm, also argue for its running time and correctness. Always feel free to ask in the forum or send a mail if you're stuck, need a clarification, or suspect a typo.

1. Calculate the probability of obtaining at least 55 heads when flipping a fair coin 100 times. Also calculate upper bounds on that probability using

   (a) Markov's inequality,

   (b) Chebyshev's inequality (use that the Binomial distribution $B(n, p)$ is symmetric around the mean and that its variance is np^2),

   (c) the Chernoff bound discussed in class.

   Repeat this for 550 heads in 1000 flips. This exercise shows the strength of the different bounds.

2. In the lecture on approximate DNF counting, the following was stated without proof: If $X_1, \ldots, X_m$ are independent and identically distributed indicator random variables with $\mu = \mathbf{E}[X_i]$, and we use $m \geq \frac{3\ln(2/\delta)}{\epsilon^2 \mu}$ samples, then the probability of the event

$$\left| \frac{1}{m} \sum_{i=1}^{m} X_i - \mu \right| \geq \epsilon\mu$$

   is upper-bounded by δ. Prove this using Chernoff bounds. (You may use the following corollary of the Chernoff bound discussed in class: For a sum $X = X_1 + \ldots + X_m$ of independent indicator random variables, with $\mu = \mathbf{E}[X]$ and $0 < \delta < 1$, the probability of the event

$$|X - \mu| \geq \delta\mu$$

   is bounded by $2e^{-\mu\delta^2/3}$.)