

Advanced Algorithms — Problem set 8

Whenever you give an algorithm, also argue for its running time and correctness. Always feel free to ask in the forum or send a mail if you're stuck, need a clarification, or suspect a typo.

1. Implement an algorithm that outputs, on input $m, n \in \mathbb{N}$, a nice path decomposition of minimal width for an $m \times n$ square grid. A nice path decomposition is a nice tree decomposition in which the underlying tree is a path. (This implies the absence of join nodes.) Show the output of your algorithm for the 3×3 grid.
2. Implement an algorithm that determines the *number* of independent sets in $n \times n$ square grids, for as large n as possible. To this end, modify the DP algorithm for finding a maximum independent set in bounded-treewidth graphs from the lecture so it doesn't find a maximum-weight independent set, but instead counts them.
 - The vertex weights are irrelevant in this setting.
 - You may import support for large integers.
 - The tables on the linked website list the numbers of independent sets in $m \times n$ square grids:
<https://oeis.org/A089934/table>