Advanced Algorithms — Problem set 6

Whenever you give an algorithm, also argue for its running time and correctness. Always feel free to ask in the forum or send a mail if you're stuck, need a clarification, or suspect a typo.

1. Experiments with TSP approximations:

   - Describe a DP algorithm for finding a TSP in a general graph G. The algorithm should run in $O(2^n p(n))$ time for some polynomial p, where n is the number of vertices in G. Argue for the correctness of this algorithm and implement it.
   - Implement the 2-approximation algorithm for Metric TSP discussed in the lecture. Your implementation may use inefficient data structures when implementing the MST algorithm.
   - Compare the output of the 2-approximation algorithm and the DP: As input, use the lat/lon coordinates of Danish cities on the next page. Use the haversine formula to compute distances.
   - For a bonus, figure out how to plot the two tours of the approximation algorithm and the TSP on a map.

2. Experiments with Vertex Cover and LPs:

   - Implement the 2-approximation algorithm for Vertex Cover based on *maximal* matchings. (This was already done in class.)
   - Implement the LP-based rounding algorithm for Vertex Cover. You may use an LP solver of your choice. (For example, in Python: `scipy.optimize.linprog`.)
   - If your LP solver outputs an integral solution for the LP relaxation of an ILP, what can you conclude?
   - Show that, for the Vertex Cover ILP and its LP relaxation, the ratio between the ILP optimum and the LP optimum can be arbitrarily close to 2.
   - Exploratory exercise: Find inputs on which the LP-based rounding algorithm outperforms the matching-based algorithm, and vice versa. (The results here will depend on the internals of the LP solver. It may be able to favor integral solutions.)

3. (Started in class.) In a Set Cover instance with universe $U = \{x_1, \ldots, x_n\}$ and sets $S_1, \ldots, S_m$, the frequency of element x_i is the number of sets among $S_1, \ldots, S_m$ that contain x_i. Let f be the maximum frequency over all elements in U.

   (a) Give the ILP formulation of Set Cover and describe how to obtain an LP relaxation of this ILP.

   (b) Describe an LP-based rounding algorithm for Set Cover that results in an f-approximation.

Some Danish cities

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Copenhagen, 55.676 / 12.566
Aarhus, 56.157 / 10.211
Odense, 55.396 / 10.388
Aalborg, 57.048 / 9.919
Esbjerg, 55.47 / 8.452
Horsens, 55.861 / 9.85
Randers, 56.461 / 10.036
Kolding, 55.49 / 9.472
Vejle, 55.709 / 9.536
Greve, 55.583 / 12.3
Svendborg, 55.060337 / 10.611613
Thisted, 56.956957 / 8.686066
Holstebro, 56.358404 / 8.613281
Aabenraa, 55.045335 / 9.419403
Faaborg, 55.098016 / 10.244751
Grenaa, 56.413142 / 10.879211
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