Medical Image Segmentation

and other things I do at Rice University

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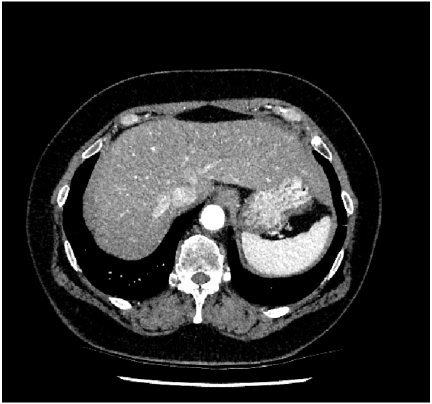
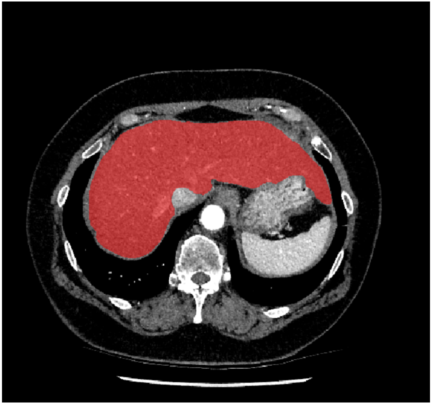
Medical Image Segmentation



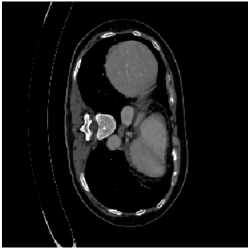
(a) abdominal CT scan

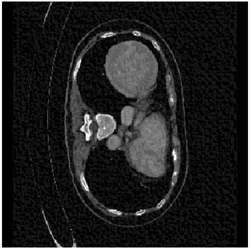


(b) segmentation of liver

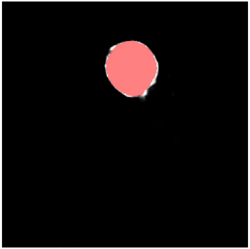
Goal.

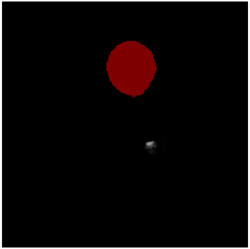
use **mathematics** to **improve clinician trust** in deep learning methods for image segmentation





(a) image (b) noisy image

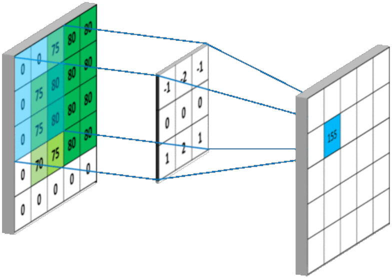




(c) original prediction (d) noisy prediction

Convolutions

$$K = \begin{bmatrix} k_{-1,-1} & k_{-1,0} & k_{-1,1} \\ k_{0,-1} & k_{0,0} & k_{0,1} \\ k_{1,-1} & k_{1,0} & k_{1,1} \end{bmatrix}$$



$$u_{i,j} \longmapsto \sum_{p=-1}^{1} \sum_{q=-1}^{1} K_{p,q} u_{i+p,j+q}$$

$$u \longmapsto K * u$$

$$\vec{u} \longmapsto A_{[K]} \vec{u}$$

Stability of Convolutional Layers

Stability $\sim$ small $\left\|A_{[K]}\right\|_2$ (largest singular value)

Reducing $\left\|A_{[K]}\right\|_2$ requires computing $\left\|A_{[K]}\right\|_2$

Convolution Linear Operators

$$T_i = \begin{bmatrix} k_{i,0} & k_{i,1} & & & \\ k_{i,-1} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & k_{i,1} \\ & & & k_{i,-1} & k_{i,0} \end{bmatrix}$$

$$A_{[K]} = \begin{bmatrix} T_0 & T_1 & & & \\ T_{-1} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & T_1 \\ & & & T_{-1} & T_0 \end{bmatrix}$$

Problem...

$A_{[K]}$ is very very **big** :

current DCNNs $\sim A_{[K]}$ are 100M $\times$ 100M

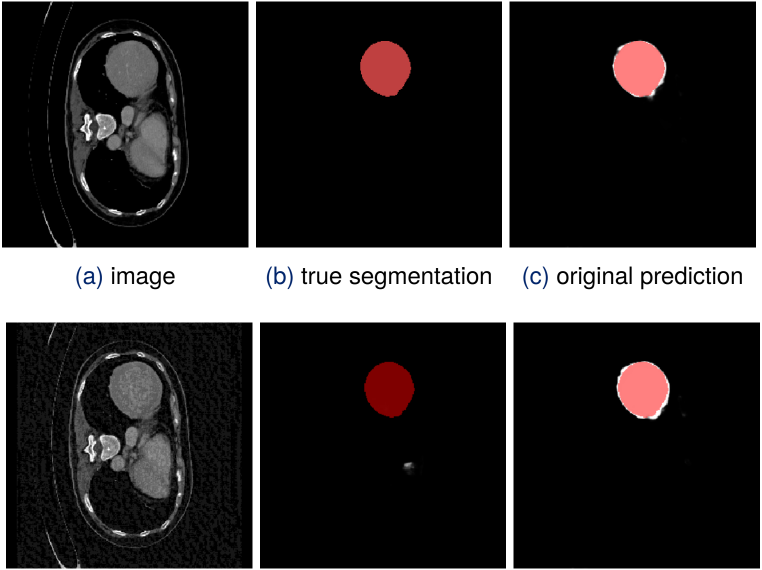
Approach: Hölder's Inequality

Bound $\left\|A_{[K]}\right\|_2$ instead of computing directly:

$$\left\|A_{[K]}\right\|_2 \leq \left\|A_{[K]}\right\|_1 \left\|A_{[K]}\right\|_\infty \leq \|K\|_{1,ent}$$

Train for robustness

- incorporate ℓ_1 regularization on kernels to training
- implement via proximal gradient algorithm $\to$ learn sparse kernels
- fast $\to$ linear in number of entries in K
- provable + empirical decrease in Lipschitz constant of DCNNs



(a) image (b) true segmentation (c) original prediction

(d) noisy image (e) noisy prediction (f) stable prediction

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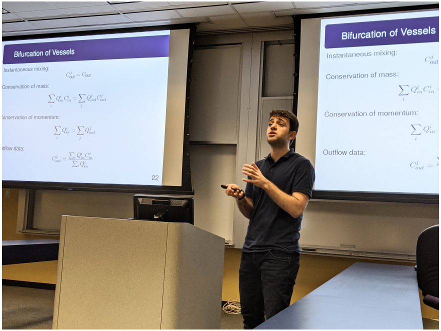
CAAM Graduate Seminar

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