TheoryExercise.pdf

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March 2021

1 Introduction

Ray equation:

$$\overrightarrow{r}(t) = o + t\overrightarrow{d} \tag{1}$$

All the points of the infinite cylinder $(c, \overrightarrow{a})$ are at a distance r of the line of axis defined by $c + k\overrightarrow{a}$ for all k. To find the points of intersection between the ray line and the cylinder, we are looking for points on the line at a distance r of the axis of the cylinder.

$$\frac{||\overrightarrow{cx} \times \overrightarrow{a}||}{||\overrightarrow{a}||} - r = 0 \tag{2}$$

By replacing x by (1), we obtain:

$$\frac{||((\overrightarrow{o} + t\overrightarrow{d}) - \overrightarrow{c}) \times \overrightarrow{a}||}{||\overrightarrow{a}||} - r = 0 \tag{3}$$

$$||\overrightarrow{co} \times \overrightarrow{a} + t(\overrightarrow{d} \times \overrightarrow{a})|| = ||\overrightarrow{a}||r \tag{4}$$

Let $\overrightarrow{v} = \overrightarrow{d} \times \overrightarrow{a}$ and $\overrightarrow{w} = \overrightarrow{co} \times \overrightarrow{a}$

$$||\overrightarrow{w} + t\overrightarrow{v}|| = ||\overrightarrow{a}||r \tag{5}$$

$$||\overrightarrow{w} + t\overrightarrow{v}||^2 = ||\overrightarrow{a}||^2 r^2 \tag{6}$$

$$||\overrightarrow{w}||^2 + 2\langle\overrightarrow{v}, \overrightarrow{w}\rangle t + ||\overrightarrow{v}||^2 t^2 = ||\overrightarrow{a}||^2 r^2 \tag{7}$$

$$||\overrightarrow{v}||^2 t^2 + 2\langle\overrightarrow{v}, \overrightarrow{w}\rangle t + ||\overrightarrow{w}||^2 - ||\overrightarrow{a}||^2 r^2 = 0 \tag{8}$$

We use the provided function *solve_quadratic* to solve the equation above.

Remark: This equation uses the norm of the axis vector $||\overrightarrow{a}||$, which would not be needed here since $\overrightarrow{a}$ is normalized. However, we decided to leave it, in order to derive a general equation.

Now that we have the intersections points between the ray line and the infinite cylinder, we need to take care of the length of the cylinder. The distance between the center and the farest point of the cylinder is given by:

$$l = \sqrt{(\frac{h}{2})^2 + r^2} \tag{9}$$

Let's take y a previously found intersection point, to be part of the cylinder y must be at most at a distance l of the center c.

$$||y - c|| - l <= 0 \tag{10}$$

Once we solved equations (8) and (10) and found the right value for t (the one which is closest to the viewer and in front of him), we still had to compute the normal vector pointing towards the viewer. For that, we first computed the intersection point y using equation (1) with the given t. Then, we computed the orthogonal projection $\overrightarrow{p}$ of this point on the axis $\overrightarrow{a}$ with the following formula:

$$\overrightarrow{p} = \frac{\langle \overrightarrow{cy}, \overrightarrow{a} \rangle}{||\overrightarrow{a}||^2} \overrightarrow{a} \tag{11}$$

Then our normal $\overrightarrow{n}$ is given by:

$$\overrightarrow{n} = \overrightarrow{cy} - \overrightarrow{p} \tag{12}$$

Finally, we had to make sure to take the normal pointing towards the viewer. For that, we computed the dot product between the ray_direction $\overrightarrow{d}$ and our normal $\overrightarrow{n}$. If the result was postive (meaning that they point in the same direction), we took -$\overrightarrow{n}$ as normal and otherwsise just $\overrightarrow{n}$.