Diagnosis, Fault-tolerant and Robust Control for a Torsional Control System

Mandatory assignment in DTU course 31320

Part A

Dimitrios Papageorgiou

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Table 1: Revision history

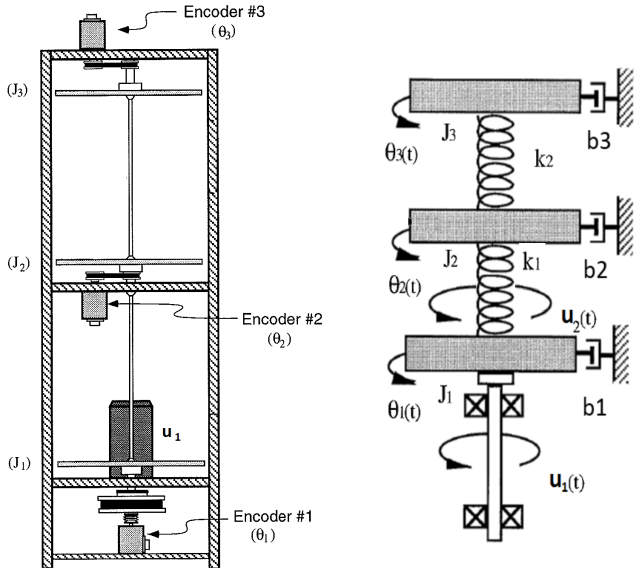
version	date	description	changes
1.a	10.02.2022	new	all pages new

Assignment part A - 2022

Introduction

This exercise presents a model of the ECP M205a system. It is a three-mass system that comprises three identical disks, connected in parallel via a flexible low-damped shaft. The system is equipped with three incremental position encoders (one for each disk) and two actuators, i.e. a motor applying torques to the bottom disk and one more for the middle disk.

The control goal is to ensure that either of the disks tracks a specified motion profile. The system considered is sketched in Figure 1. The variables are explained in Table 2.



The normal behaviours of the systems are described by the following con-

Table 2: **List of variables**

variable	**unit**	**description**
θ_1	rad	angular position of bottom disk
ω_1	rads^{-1}	angular velocity of bottom disk
θ_2	rad	angular position of middle disk
ω_2	rads^{-1}	angular velocity of middle disk
θ_3	rad	angular position of top disk
ω_3	rads^{-1}	angular velocity of top disk
u_1	Nm	torque command for the bottom disk
u_2	Nm	torque command for the middle disk
y_1	rad	measured angular position of bottom disk
y_2	rad	measured angular position of middle disk
y_3	rad	measured angular position of top disk

straints:

$$
\begin{aligned}
c_1 &: 0 = \dot{\theta}_1 - \omega_1 \\
c_2 &: 0 = J_1\dot{\omega}_1 - u_1 + b_1\omega_1 + k_1\left(\theta_1 - \theta_2\right) + d \\
c_3 &: 0 = \dot{\theta}_2 - \omega_2 \\
c_4 &: 0 = J_2\dot{\omega}_2 - u_2 + b_2\omega_2 + k_1\left(\theta_2 - \theta_1\right) + k_2\left(\theta_2 - \theta_3\right) \\
c_5 &: 0 = \dot{\theta}_3 - \omega_3 \\
c_6 &: 0 = J_3\dot{\omega}_3 + b_3\omega_3 + k_2\left(\theta_3 - \theta_2\right) \\
d_7 &: 0 = \dot{\theta}_1 - \frac{d\theta_1}{dt} \\
d_8 &: 0 = \dot{\omega}_1 - \frac{d\omega_1}{dt} \\
d_9 &: 0 = \dot{\theta}_2 - \frac{d\theta_2}{dt} \\
d_{10} &: 0 = \dot{\omega}_2 - \frac{d\omega_2}{dt} \\
d_{11} &: 0 = \dot{\theta}_3 - \frac{d\theta_3}{dt} \\
d_{12} &: 0 = \dot{\omega}_3 - \frac{d\omega_3}{dt} \\
m_{13} &: 0 = y_1 - \theta_1 \\
m_{14} &: 0 = y_2 - \theta_2 \\
m_{15} &: 0 = y_3 - \theta_3
\end{aligned}
$$

where $d \triangleq T_C(\omega_1)$ is a function of the bottom disk angular velocity and represents the *unknown* Coulomb friction torque on that disk. The control input saturates at 2 Nm, i.e. $u_1 \in [-2, 2]$ Nm. The parameters in the forgoing con-

straints are listed in Table 3.

Table 3: **List of parameters**

symbol	**value**	unit	**description**
J_1	0.0025	kgm^2	Bottom disk moment of inertia
J_2	0.0018	kgm^2	Middle disk moment of inertia
J_3	0.0018	kgm^2	Top disk moment of inertia
k_1	2.7	Nmrad^{-1}	Stiffness of the bottom shaft
k_2	2.6	Nmrad^{-1}	Stiffness of the middle shaft
b_1	0.0029	Nmsrad^{-1}	Damping/friction on the bottom disk
b_2	0.0002	Nmsrad^{-1}	Damping/friction on the middle disk
b_3	0.00015	Nmsrad^{-1}	Damping/friction on the top disk

A Simulink model and a parameter initialisation file have been uploaded to DTU Learn in the *File share/Matlab and Simulink files/Mandatory Assignment Part A* folder for your convenience.

Question 1

Make a structural analysis:

1. Determine a complete matching on the unknown variables.
2. Find the parity relations in symbolic form.
3. Investigate other properties you find relevant from a structural analysis.
4. Reformulate the parity relations to an analytic form, as functions only of known variables and the system parameters.

Practical notes

- The deadline for the report is **Wednesday March 16, 2022, at 23:55 hours.**
- Please note that max. 20 pages are allowed for part A, excluding front matter. Any pages in excess of 20 (excluding front matter) will be discarded.
- Layout must be reasonable: A4 paper, $20mm$ margins and font size $11pt$.
- The report need be delivered in electronic form, as a .pdf file, via the DTULearn assignment system. Only pdf files are accepted.
- Group-work is encouraged (max 2 persons in one group).

- Please do **write your name(s) and student number(s) at the front page and as running heading on each page of your report**. In addition, do not forget page numbers.