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Analysis of assembly-time performance (ATP) in manufacturing operations with collaborative robots: a systems approach

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ABSTRACT

Reducing station processing times has a significant importance in manufacturing assembly systems. In recent years, there has been a growing interest in using collaborative robots to assist human operators in many manufacturing systems, which can not only improve ergonomics measures but also reduce processing time and increase throughput. In this paper, a system-theoretic approach is introduced to analyse the assembly-time performance (ATP) of assembly systems with collaborative robots, where ATP is defined as the probability to finish all the assembly operations in a station within a desired time interval. Specifically, the assembly operations are described by stochastic processes with both individual (human operator and robot) preparation tasks and joint collaboration tasks, characterised by general or arbitrary distributions of task times. Then an efficient algorithm is presented by using gamma distributions to approximate task times and aggregate multiple interacting tasks to calculate ATP. High accuracy in ATP evaluation is obtained through such an approximation method. In addition, system properties, such as monotonicity and sensitivity, i.e. bottlenecks, are investigated. Finally, a case study at an automotive powertrain assembly plant is introduced to illustrate the applicability of the method and the effectiveness for assembly time reduction through using collaborative robots.

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KEYWORDS
Assembly-time performance; collaborative robots; human operators; gamma distribution; coefficient of variation

1. Introduction

In recent years, Industry 4.0 has become prevalent in many manufacturing industries. More automation equipment and technology have been developed and implemented on the factory floor. Collaborative robots, often referred to as cobots, have been increasingly used to take advantages of the dexterity and cognitive skills of human operators and the repeatability and payload capability of the robots to achieve high productivity, flexibility and safety with reduced ergonomics risk and cost (Marvel 2014).

With cobots, the human workers and robots collaborate safely to carry out multiple tasks, such as assembly, in the same workspace. Extensive studies on cobot have been conducted, mainly focused on the technology, architecture, safety and control of cobots in manufacturing environment. Numerous efforts have been devoted to production planning and optimisation, such as task assignment, line balancing, and job scheduling to minimise makespan and improve ergonomics. Many of them consider sequential and dedicated or independent task allocation between robots and human operators.

Although manufacturing system research has been prevalent for many decades, the productivity analysis of collaborative assembly systems with cobots is still limited, particularly for human-cobot systems with joint collaboration tasks. In such systems, the human operator and the cobot can work both independently and jointly to finish assembly operations. For instance, a cobot can provide gravitation and force assistance in lifting a bulk component to a human operator. Then the operator assembles the component held and moved by the cobot into the place without damaging the part. Such a collaborative process can alleviate ergonomic stresses, reduce operation time and also improve assembly quality. Such scenarios can be widely observed in automotive, appliance, battery and equipment manufacturing industries. Developing an analytic model to study the collaborative assembly systems is of significant importance.

As the process time (or flow time, throughput) is a key performance index in manufacturing, an effective method to evaluate such a KPI is needed (Kang et al. 2016). To fully understand the impact of process time in collaborative assembly systems, in addition to

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calculating the mean process time and its associated variance, deriving the complete distribution of process or assembly time is necessary, as it indicates how often all the assembly operations can be carried out within a preferred time interval. Therefore, a new performance measure, assembly-time performance (ATP) of a collaborative assembly system, which is defined as the probability to finish the assembly process within a desired time interval, is introduced. However, the evaluation and analysis of ATP are not easy since the assembly system could involve both sequential and joint processes as well as parallel and merge operations. In addition, multiple steps are typically needed to finish an assembly process, and randomness exists in each task time, which could follow an unknown or arbitrary distribution. However, most of the manufacturing system research work assumes Markovian behaviour and focuses on mean time performance, with only a few addressing variances. To the best of our knowledge, evaluating and analysing the complete distribution of flow time in a complex multi-task collaborative assembly system with general distribution task times are not available, which are also extremely difficult.

To bridge this gap, in this paper, we introduce a system-theoretic approximation method to calculate ATP of collaborative assembly systems and analyse the system properties. Specifically, the collaborative assembly process is decomposed into sequential task sub-processes merged and connected together, where each task is described by a general random process approximated by gamma distribution using the mean and coefficient of variation (CV) of task time. Then the cumulative probability density (CDF) functions are derived for the sequential and joint sub-processes, and their aggregated process to calculate the ATP for a given time interval. Using such a method, system-theoretic properties, such as monotonicity and sensitivity, i.e. bottleneck identification and mitigation, are investigated. Furthermore, to illustrate the applicability of the method, a case study at an automotive powertrain assembly plant is presented, which also justifies the effectiveness of using cobots for process time reduction and productivity improvement. The main contribution of the paper lies in developing an accurate, efficient and effective method to evaluate ATP, analyse system properties and identify process bottlenecks in collaborative assembly systems.

The remainder of this paper is organised as follows: the related literature is reviewed in Section 2. The collaborative assembly systems are described in Section 3, and the performance evaluation method is presented in Section 4, while Section 5 investigates system properties. To illustrate the applicability of the method, Section 6 presents a case study in an automotive powertrain assembly line. Finally, conclusion is formulated in Section 7. All proofs are provided in Appendix.

2. Related literature

As more robots being used on the factory floor, increasing attentions have been paid to collaborative assembly systems using cobots. For example, a survey of technologies and methods for human–robot collaboration is provided by Bauer, Wollherr, and Buss (2008), where the intention estimation, action planning, joint action and machine learning methods as well as guidelines to hardware design are presented. Djuric, Urbanic, and Rickli (2016) review the safety and layout challenges and factory automation configurations, as well as a road map for education and research challenges, and describe a framework consisting of system, work-cell, machine and worker levels to facilitate design, development and integration of cobots. The safety, interfaces and applications of human–robot collaboration in industrial environments are reviewed by Villani et al. (2018), with a focus on physical and cognitive interactions. In addition, El Zaatari et al. (2019) summarise the collaborative industrial scenarios and cobot programming requirements in communication, optimisation and learning for effective implementation, identify the gaps between industry and research, and pinpoint future research directions to bridge the gaps. Moreover, Matheson et al. (2019) present the standards and modes of operations in human–robot collaboration for industrial applications and analyse future trends. Hentout et al. (2019) review the major works on human–robot interaction during 2008 to 2017, classify them into multiple categories and subcategories, and address the challenges and future research issues.

Manufacturing systems have been studied extensively for decades (see, for instance, monographs by Viswanadham and Narahari 1992; Buzacott and Shanthikumar 1993; Papadopolous, Heavey, and Browne 1993; Gershwin 1994; Li and Meerkov 2009 and reviews by Dallery and Gershwin 1992; Papadopoulos and Heavey 1996; Inman et al. 2003; Li et al. 2009; Inman et al. 2013; Papadopoulos, Li, and O'Kelly 2019). Most of the studies focus on modelling, analysis, improvement and control of system throughput, lead time and work-in-process, etc. Flow time (or cycle time, makespan, throughput, etc.), as an important measure of productivity, has been addressed intensively for different types of production systems (e.g. representative papers by Gershwin 1987; Dallery, David, and Xie 1988; Mascolo, David, and Dallery 1991; Jacobs and Meerkov 1995; Chiang et al. 2000; Helber 2000; Tempelmeier and Bürger 2001;

Tolio, Matta, and Gershwin 2002; Li 2005; Meerkov and Zhang 2008; Satyam and Krishnamurthy 2008; Tan and Gershwin 2009; Li 2013; Zhao, Li, and Huang 2015; Ju, Li, and Deng 2016; Feng et al. 2018).

Including robots and human–robot collaboration in manufacturing systems research, most efforts are devoted to job allocation, task assignment, line balancing, and part sequencing and scheduling issues. For example, a summary of major considerations and a productivity analysis procedure as well as computational techniques related to acquisition and deployment of cobots are introduced by Cohen et al. (2021). Tsarouchi et al. (2017) present a human–robot collaboration framework for allocation of sequential tasks to a robot and a human operator in hybrid assembly cells. The job schedules, task assignment and robots allocation problems in reconfigurable assembly lines with collaborated human operators and mobile robots are formulated and solved through a hybrid optimisation method by Maganha et al. (2019). A genetic algorithm for assembly line balancing in human–robot collaborative works is proposed by Dalle Mura and Dini (2019) to minimise assembly line cost, the number of skilled workers, and the energy load variance among workers according to the number of workers and equipment, as well as energy expenditures, physical capabilities and level of collaboration with robots. Faccio, Bottin, and Rosati (2019) compare the collaborative assembly systems over traditional manual or automated lines and discuss the preferred implementing conditions for better performance, and then present a set of system variables and a mathematical model of task allocation to maximise the collaborative system performance, such as throughput and production cost. In addition, a logic mathematical model with a genetic based revolutionary algorithm is introduced by Chen et al. (2013) to quantify the trade-off between assembly time cost and payment cost and allocate tasks to meet the required cost-effectiveness in human and robot collaborations. Mokhtarzadeh et al. (2020) use a constraint programming approach to minimise makespan in print circuit boards assembly through task allocation to humans and robots with different experimental instances. Recently, Zhang, Huang et al. (2021) develop a method to evaluate flow time in a collaborative assembly systems using phase-type distributions to describe both human and cobot preparation processes and their joint assembly process, where each task time is described by an exponential distribution.

As ergonomics performance is critical in manufacturing, Pearce et al. (2018) consider both makespan and strain index in the optimisation framework to assign tasks and schedules for a human–robot team. Zhang, Liu et al. (2021) integrate cycle time and strain index into one unified measure, throughput rate per work effort, to improve both productivity and ergonomics performances. In addition, the ergonomic constraints in repetitive works, such as lifting tasks and awkward postures in manufacturing environments, are considered by Sana et al. (2019) using a multi-objective optimisation model for job rotation with an improved non-dominated sorting genetic algorithm. A constraint optimisation model with ergonomic analysis for workforce ergonomic scheduling is developed by Savino, Riccio, and Menanno (2020) to assess the effect of ergonomic exposure on workforce allocation and production performance. Using a random forest method, Bettoni et al. (2020) propose a mutualistic and adaptive human–machine collaboration framework to continuously monitor workers' physiological parameters and dynamically assign tasks to humans or cobots based on workers' fatigue levels in an injection moulding manufacturing line.

Only limited research studies in manufacturing systems consider higher moments or complete distribution of general service times. For instance, the throughput, queue length and inter-departure time distributions are analysed by Krishnamurthy, Suri, and Vernon (2004) for a closed queuing network with inputs from servers with two-phase Coxian service distributions. The mean and variance of synchronisation and inter-arrival time at the assembly station are studied by De Boeck and Vandaele (2011) for a generic assembly system with two independent input streams having generally distributed inter-arrival times. In addition, Manitz and Tempelmeier (2012) propose an approximation method to evaluate the variance of inter-departure times in an assembly system with general service times, and Manitz (2015) studies both throughput and variance of inter-departure times in a multi-stage assembly/disassembly system with general service times using G/G/1/N queueing models for each two-station subsystem. Beyond variance, the due-time performance, i.e. the probability to ship the required number of parts in a fixed shipping period, has been studied by Jacobs and Meerkov (1995), Li and Meerkov (2001) and Li and Meerkov (2003) for Markovian production systems. Analogous work of response-, waiting- or discharge-time performances in healthcare systems have been carried out by Xie et al. (2013), Lee et al. (2017) and Chen et al. (2019) in healthcare delivery systems, respectively.

However, the above flow time analysis in manufacturing systems with collaborative robots mainly focuses on sequential and dedicated tasks for human operators and robots. Except in Zhang, Liu et al. (2021) and Zhang, Huang et al. (2021), the collaborative activity to jointly finish an operation is less studied. In addition, many

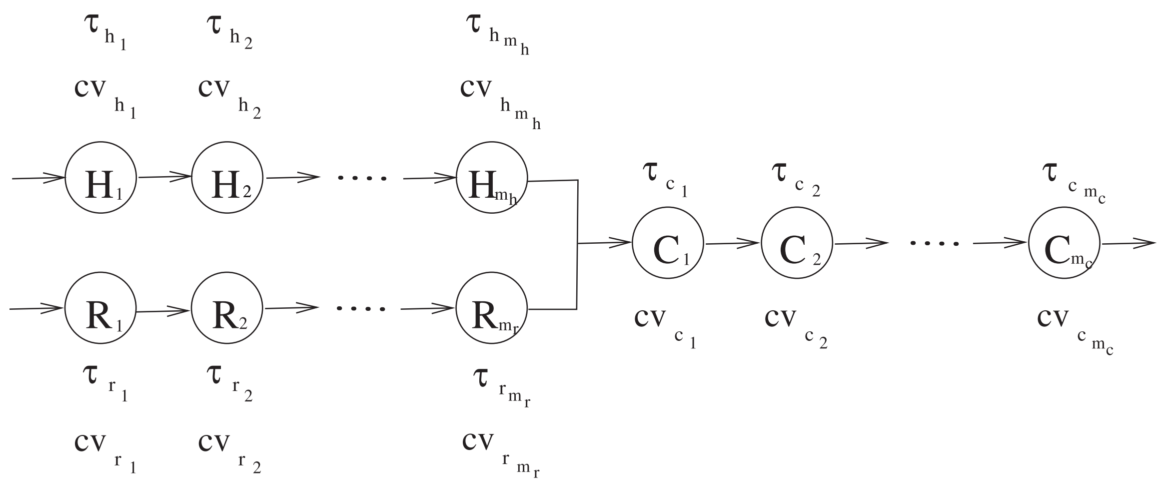

Figure 1. Collaborative assembly system.

references ignore the stochastic nature of tasks or assume specific distributions (such as exponential) for task time. As the task processing time is random due to variability in manual operations or breakdowns in robot manipulations, and can follow an arbitrary or general distribution, developing a collaborative assembly system model to include joint activities with general distribution task times is of importance.

3. System description

Consider a collaborative robot assembly system shown in Figure 1, which includes a human operator and a cobot carrying out both independent preparation tasks and joint collaboration tasks. The following assumptions define the stochastic process model describing the systems:

(i) The collaborative assembly system includes one human operator and one robot.
(ii) There are two parallel preparation sub-processes in the system by the operator and the robot independently, denoted as random processes h and r, respectively.
(iii) After both preparation sub-processes are finished, the collaboration sub-process c is carried out by the operator and the robot jointly.
(iv) In each random process i, $i = h, r, c$, there exist m_i sequential tasks, each being characterised by task time with mean $\tau_{i_j}, j = 1, \ldots, m_i$, and coefficient of variation cv_{i_j}.

Under assumptions (i)–(iv), denote the flow time to finish each sub-process i as t_i, $i = h, r, c$. Then the overall assembly time t of the collaborative assembly process can be defined as

$$t = \max(t_h, t_r) + t_c. \tag{1}$$

The mean value T and coefficient of variation CV of the assembly time characterise the average productivity performance and its variability of the system,

$$T = E(t), \tag{2}$$

$$CV = \frac{\sqrt{Var(t)}}{T}. \tag{3}$$

Note that, in addition to mean time, the variability of assembly time is also important to measure production performance. The variability can come from randomness in manual operations, breakdowns in cobots, process variations, safety adjustment, and interactions between human operator and cobot, etc. However, variance itself cannot directly characterise the variability as it is dependent on the mean value. Thus the CV is introduced. In addition, the complete distribution of assembly time needs to be evaluated. Therefore, introduce $S(T_d)$ as the assembly-time performance of the collaborative system, which is defined as the probability to finish the assembly process within a desired time period T_d. Then $S(T_d)$ represents the CDF of overall assembly time, i.e.

$$ATP = S(T_d) = P(t \leq T_d). \tag{4}$$

Then the problem to be studied in this paper is formulated as *Under assumptions (i)–(iv), develop a method to evaluate the assembly-time performance of the collaborative assembly system as functions of system parameters and investigate system properties.*

4. Performance evaluation

4.1. Approximation formulas

The collaborative assembly system (i)–(iv) includes three sequential sub-processes connected together. To study the assembly time of such a system, the performance of each sub-process needs to be evaluated first. However, direct calculation of the CDF of a sub-process is hardly possible since the distribution of the process is unknown or can be general or arbitrary. Thus an approximation method is pursued.

Specifically, since the processing time in many manufacturing activities has CV less than 1 (see Li and Meerkov 2005b), the resulting throughput is primarily dependent on the mean and CV rather than the complete distribution (Li and Meerkov 2005a; Ching, Meerkov, and Zhang 2008; Kang, Zheng, and Li 2015). Thus gamma distribution can be introduced to approximate the service or task time with an arbitrary distribution with varied shapes, as it has two parameters which can place the mean and CV with more freedom and it has a fairly flexible positive-skewed distribution with convenient mathematical properties. Examples of such approximations can be found in Kang, Zheng, and Li (2015), Lee et al. (2017) and Chen et al. (2019).

First, each sub-process is studied.

Lemma 4.1: *Under assumptions* (i)–(iv), *the CDF of flow time in manual preparation (h), robot preparation (r) and joint collaboration (c) sub-processes can be evaluated as*

$$G_i(x) = \prod_{j=1}^{m_i} \left(\frac{\beta_{i,\min}}{\beta_{i_j}} \right)^{\eta_{i_j}} \sum_{k=0}^{\infty} \frac{\delta_{i_k} \gamma(\rho_i + k, x/\beta_{i,\min})}{\Gamma(\rho_i + k)}, \quad i \in \{r, h, c\}, \tag{5}$$

where

$$\begin{aligned} \eta_{i_j} &= \frac{1}{cv_{i_j}^2}, \quad \rho_i = \sum_{j=1}^{m_i} \eta_{i_j}, \\ \beta_{i_j} &= cv_{i_j}^2 \tau_{i_j}, \quad \beta_{i,\min} = \min(\beta_{i_j}),\ j = 1, 2, \ldots, m_i, \\ v_{i_k} &= \frac{1}{k} \sum_{j=1}^{m_i} \eta_{i_j} (1 - \beta_{i,\min}/\beta_{i_j})^k, \quad k = 1, 2, \ldots, \\ \delta_{i_0} &= 1, \quad \delta_{i_k} = \frac{1}{k} \sum_{j=1}^{k} j v_{i_j} \delta_{i_{k-j}},\ k = 1, 2, \ldots, \\ \gamma(a, x) &= \int_0^x y^{a-1} e^{-y}\, \mathrm{d}y, \quad \Gamma(a) = \lim_{x \to \infty} \gamma(a, x). \end{aligned} \tag{6}$$

Proof: See the Appendix. ■

The results in Lemma 4.1 enable us to represent a serial manufacturing process using one stochastic model with each task time described by a gamma distribution. In other words, the multiple sequential tasks can be aggregated into one. Then, by merging the two preparation sub-processes and connect to the joint collaboration sub-process, the whole collaborative assembly process can be evaluated.

Proposition 4.2: *Under assumptions* (i)–(iv), *the CDF of assembly time, i.e. the assembly-time performance, in a collaborative assembly system can be evaluated as*

$$S(T_d) = \int_0^{T_d} G_h(T_d - x) G_r(T_d - x) g_c(x)\, \mathrm{d}x, \tag{7}$$

where $G_h(\cdot)$, $G_r(\cdot)$, and $G_c(\cdot)$ are derived in Lemma 4.1, *and $g_c(\cdot)$ is the probability density function (PDF) of joint collaboration sub-process and can be evaluated as*

$$g_c(x) = \prod_{j=1}^{m_c} \left(\frac{\beta_{c,\min}}{\beta_{c_j}} \right)^{\eta_{c_j}} \times \sum_{k=0}^{\infty} \frac{\delta_{c_k} (x/\beta_{c,\min})^{\rho_c + k - 1} e^{-x/\beta_{c,\min}}}{\beta_{c,\min} \Gamma(\rho_c + k)}, \tag{8}$$

here $\beta_{c,\min}$, β_{c_j}, η_{c_j}, δ_{c_k}, ρ_c and $\Gamma(\cdot)$ are defined in Lemma 4.1.

Proof: See the Appendix. ■

Using the CDF $S(T_d)$, the mean assembly time T and its variation CV can be derived.

Corollary 4.3: *Under assumptions* (i)–(iv), *the mean and coefficient of variation of assembly time in a collaborative assembly system can be calculated as*

$$T = \int_0^{\infty} t s(t)\, \mathrm{d}t, \tag{9}$$

$$CV = \frac{\sqrt{\int_0^{\infty} t^2 s(t)\, \mathrm{d}t - (\int_0^{\infty} t s(t)\, \mathrm{d}t)^2}}{\int_0^{\infty} t s(t)\, \mathrm{d}t}, \tag{10}$$

where $s(t)$ is the PDF of assembly time and

$$s(t) = \int_0^t \left[g_h(t - x) G_r(t - x) + G_h(t - x) g_r(t - x) \right] \times g_c(x)\, \mathrm{d}x, \tag{11}$$

with $g_h(\cdot)$ and $g_r(\cdot)$ are the PDFs of human and robot preparation sub-processes as derived in Proposition 4.2.

Proof: See Appendix. leibniz integral rule ■

Remark 4.1: In case there are more than two preparation sub-processes, the idea of gamma approximation is still applicable. However, Proposition 4.2 could be adapted by convolution of multiple branches. For instance, when there exist three branches of preparation processes, the formula for the assembly system can be written as

$$S(T_d) = \int_0^{T_d} G_1(T_d - x) G_2(T_d - x) \times G_3(T_d - x) g_c(x)\, \mathrm{d}x, \tag{12}$$

where $G_1(\cdot)$, $G_2(\cdot)$, $G_3(\cdot)$ and $g_c(\cdot)$ are derived as in Lemma 4.1 and Proposition 4.2, and $G_i(\cdot), i = 1, 2, 3$ represents the CDFs of the preparation sub-processes, either by robot or human operator.

Table 1. ATP comparison with simulations.

T_d	$\bar{\epsilon}_i(T_d)$	$\min_i(\epsilon_i(T_d))$	$\max_i(\epsilon_i(T_d))$
10	0.000000	0.000000	0.000000
20	0.000000	0.000000	0.000000
30	0.000001	0.000000	0.000007
40	0.000022	0.000000	0.000109
50	0.000312	0.000000	0.001722
60	0.001010	0.000000	0.004503
70	0.001560	0.000000	0.003573
80	0.002173	0.000003	0.004971
90	0.003606	0.000020	0.006774
100	0.003684	0.000022	0.009574
110	0.003080	0.000173	0.008892
120	0.002829	0.000002	0.006654
130	0.003198	0.000065	0.006664
140	0.003542	0.000134	0.007795
150	0.003434	0.000112	0.008275
160	0.003082	0.000330	0.008246
170	0.002672	0.000039	0.008674
180	0.002060	0.000068	0.007248
190	0.001552	0.000046	0.006189
200	0.001078	0.000140	0.003977

4.2. Validation

The approximation formula introduced in Proposition 4.2 needs to be validated, which can be carried out by comparing with simulations. Specifically, a discrete-event simulation model assuming randomly selected distributions of task times, such as Weibull, gamma, log-normal or a mixture of them, is developed and compared with the results from the gamma approximation model. The mean and CV of each task time are randomly generated and kept the same in both models. The ranges of these data are selected from the following sets:

$$\begin{aligned} m_i &\in [1, 4], \\ \tau_{i_j} &\in [5, 30], \\ cv_{i_j} &\in [0.1, 0.9]. \end{aligned} \tag{13}$$

As described in Li and Meerkov (2005b), since the probability to finish a task is increasing with respect to the time that has been spent on the task, the CVs of task times are less than 1.

The simulations are set up as follows: each simulation run consists of 1000 time units of warm up period and 100,000 time units of data collection period. 20 replications are carried out to ensure the confidence intervals smaller than 1% of the ATP values. The ATPs are simulated and compared for 20 datasets randomly generated from the sets in (13). Define the differences between approximation model and simulation as

$$\epsilon_i(T_d) = |ATP_i^{\mathrm{sim}}(T_d) - ATP_i^{\mathrm{appr}}(T_d)|, \quad i = 1, \ldots, 20,$$

where $ATP_i^{\mathrm{sim}}(T_d)$ and $ATP_i^{\mathrm{appr}}(T_d)$ represent the ATP values for a given T_d from simulation and gamma approximation using dataset i, respectively. Then the minimum, maximum and average of $\epsilon_i(T_d)$ of all i's for given T_d's are shown in Table 1, where $\bar{\epsilon}_i(T_d) = \sum_{i=1}^{20} \epsilon_i(T_d)/20$. For illustration purpose, four comparison examples of ATPs of gamma approximation and simulation are presented in Figure 2.

As one can see, all differences are very small, which could indicate the following: First, there exists a distribution-free property, i.e. the ATPs are primarily dependent on the first and second moments of task time (i.e. mean and CV). This justifies that gamma distribution can be used to characterise the processing time of each task. Second, the gamma approximation can achieve a good accuracy to estimate the ATP of a collaborative assembly system. Such properties have been observed in many manufacturing and service systems (e.g.Li, Enginarlar, and Meerkov 2004; Li and Meerkov 2005a, 2008; Ching, Meerkov, and Zhang 2008; Xie et al. 2013; Kang, Zheng, and Li 2015; Lee et al. 2017; Zeng et al. 2018; Chen et al. 2019). Since the mean assembly time T and coefficient of variation CV are derived from the assembly-time performance ATP, their estimation accuracies are similar to those of ATP's.

In addition, the computational times using gamma approximation (denoted as T^{appr}) and simulation (denoted as $T^{\sim}$) are compared and listed in Table 2. Both methods are coded in MATLAB on a personal computer with an Intel Core i5-8500 3.00 GHz CPU and 8 GB of RAM, and the computation times are counted in seconds. As shown in Table 2, the computation times of simulations are about 180 seconds while those using gamma approximation are less than 0.05 seconds. Thus the gamma approximation method can improve computation efficiency by at least 4000 times.

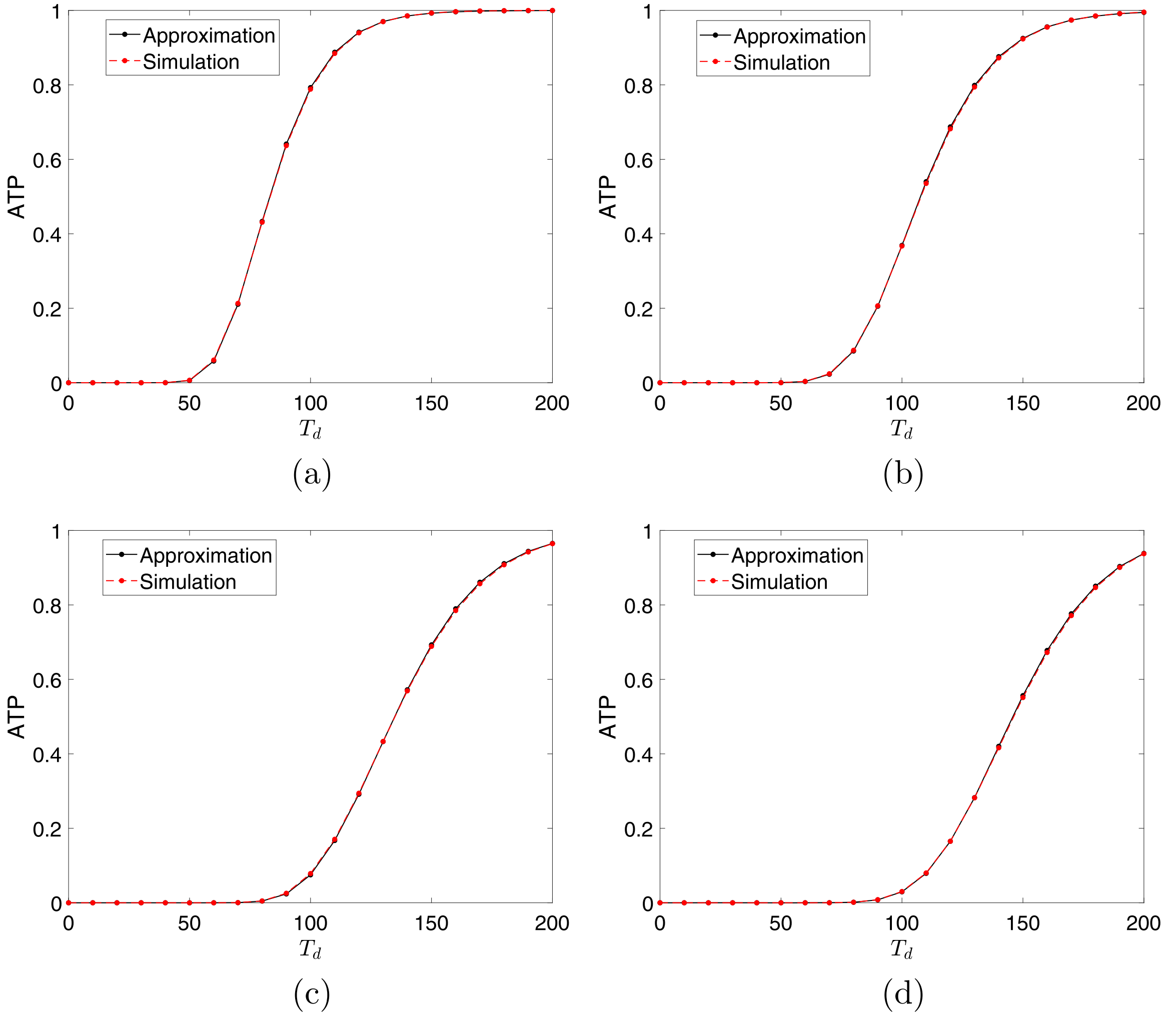

Figure 2. Comparison examples with simulation: (a) Example 1, (b) Example 2, (c) Example 3 and (d) Example 4.

Table 2. ATP computational times.

T_d	$T^{\sim}$	T^{appr}	$T^{\sim}/T^{\mathrm{appr}}$
10	179.9539	0.0162	11125
20	180.0761	0.0034	53740
30	178.6294	0.0051	34889
40	178.6397	0.0065	27586
50	178.4806	0.0085	20925
60	178.1687	0.0107	16699
70	178.4392	0.0123	14555
80	179.7572	0.0142	12634
90	181.5295	0.0161	11249
100	179.7399	0.0193	9298
110	177.9664	0.0205	8678
120	178.5236	0.0237	7545
130	178.3497	0.0260	6866
140	178.1074	0.0302	5903
150	177.1820	0.0311	5694
160	177.5501	0.0335	5305
170	178.0787	0.0363	4901
180	177.4195	0.0390	4554
190	178.0914	0.0419	4251
200	177.1352	0.0443	4001

5. System properties

Using the performance evaluation method introduced above, we investigate system properties and further seek to improve the performance of the whole collaborative assembly process.

5.1. Monotonic properties

First, monotonic properties with respect to mean task time and its CV are investigated, which can provide the direction of potential improvement.

5.1.1. ATP monotonicity

Proposition 5.1: *Under assumptions* (i)–(iv), *the assembly-time performance is monotonically decreasing with respect to the mean processing time of each task.*

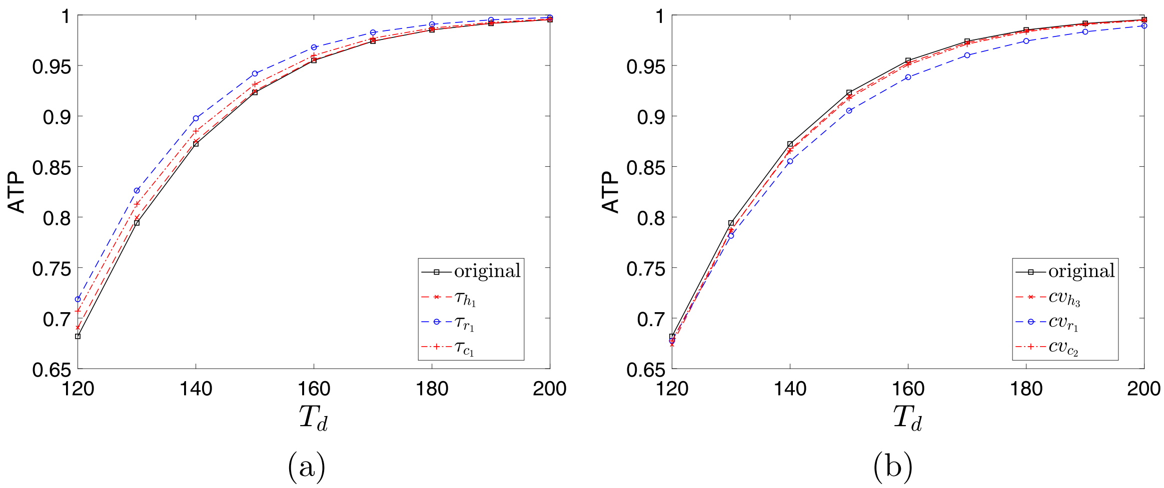

Figure 3. ATP monotonicity: (a) ATP monotonicity in τ_{i_j} and (b) ATP monotonicity in cv_{i_j}.

Proof: See Appendix. ∎

Such a property is also verified through extensive numerical experiments. Using a randomly generated dataset presented in (14) as an example, we investigate the ATP changes with respect to task times. As shown in Figure 3(a), when the mean processing time of three representative tasks in human preparation, robot preparation and joint collaboration sub-processes, is reduced by 10%, the ATPs are always increased, but with different extents. Such a result indicates that reducing mean task time can improve ATP. Then, the next question arises naturally, i.e. reducing which task time can lead to the largest improvement? This will be further discussed in bottleneck analysis.

$$
\begin{array}{llll}
\tau_{h_1} = 22, & \tau_{h_2} = 11, & \tau_{h_3} = 14, & \tau_{h_4} = 6, \\
\tau_{r_1} = 30, & \tau_{r_2} = 26, & \tau_{r_3} = 15, & \\
\tau_{c_1} = 18, & \tau_{c_2} = 18, & & \\
cv_{h_1} = 0.4749, & cv_{h_2} = 0.1386, & cv_{h_3} = 0.8133, & cv_{h_4} = 0.4086, \\
cv_{r_1} = 0.6993, & cv_{r_2} = 0.4626, & cv_{r_3} = 0.1317, & \\
cv_{c_1} = 0.4411, & cv_{c_2} = 0.5641. & &
\end{array}
\qquad (14)
$$

Usually, the upper proportion of ATP is the area a production manager concerns, namely, where T_d is larger than the mean cycle time T. Extensive numerical experiments show that ATP is monotonically decreasing with respect to the CV of each task time, cv_{i_j}, in this area. Figure 3(b) provides three examples of such an observation where the ATP curves of the scenarios that cv_{i_j} is increased by 20% are all below the ATP curve of the original one.

5.1.2. Mean time monotonicity

Similar to ATP monotonicity, the mean assembly time also exhibits monotonic property with respect to the mean processing time of each task.

Proposition 5.2: *Under assumptions* (i)–(iv), *the mean assembly time T is monotonically increasing with respect to the mean processing time of each task τ_{i_j}.*

Proof: See Appendix. ∎

Figure 4(a) illustrates the monotonocity of mean assembly time with respect to mean task time. When τ_{i_j} increases, the mean assembly time T is increased. In addition, it is observed that the mean task time in collaboration sub-process τ_{c_j} leads to the most increment in T. This is due to that the maximum time of preparation sub-processes is used for assembly time evaluation, while the collaboration sub-process is directly contributed to the overall assembly time. However, the mean assembly time is insensitive to the CV of task time. As observed in Figure 4(b), there exists almost no change even though the CV of task time varies significantly. This is due to that the mean time for each sub-process equals to the sum of mean task times in the sub-process and does not change with the CV of task time, thus the mean assembly time will not be sensitive to the CV of each task time.

5.1.3. CV monotonicity

Intuitively, the CV of assembly time is monotonically increasing with respect to the CV of each task time. However, a rigorous proof is not available due to the maximal function involved in calculation. Thus, by validating from extensive experiments, we formulate this as a numerical observation below.

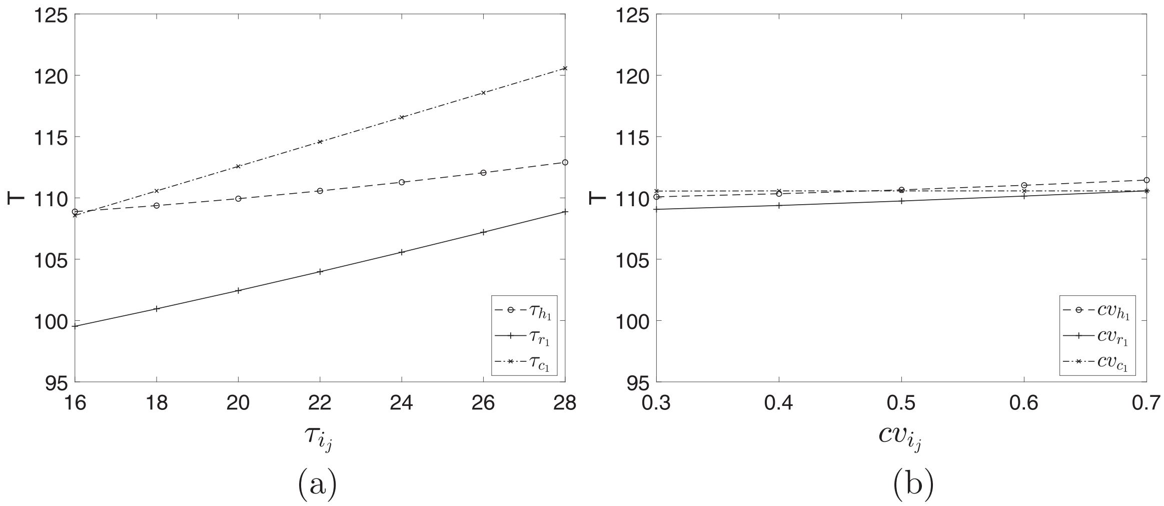

Figure 4. Mean assembly time monotonicity: (a) T vs mean task time and (b) T vs task time CV.

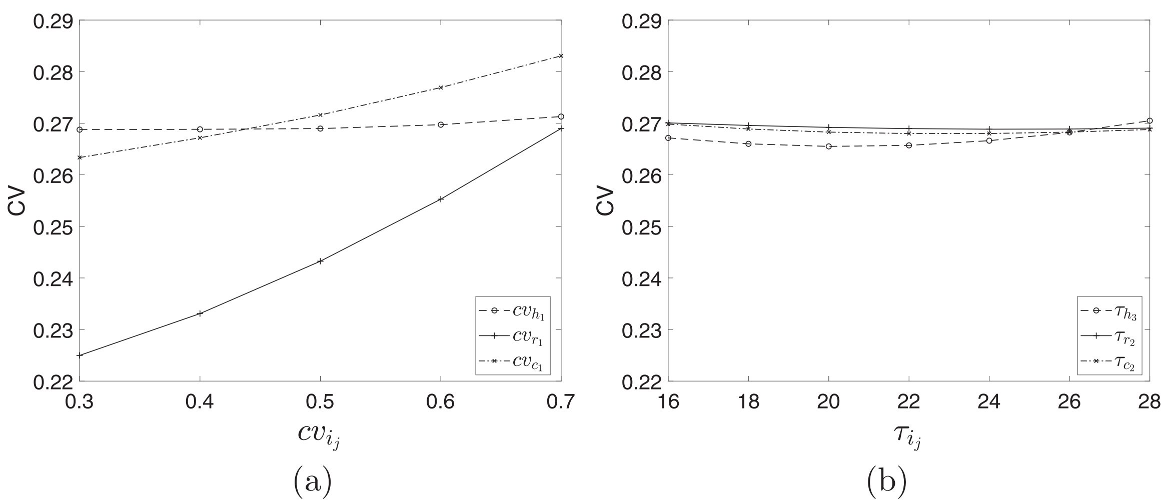

Figure 5. CV monotonicity: (a) CV vs task time CV and (b) CV vs mean task time.

Observation 5.1: Under assumptions (i)–(iv), the CV of assembly time is monotonically increasing with respect to the CV of task time cv_{i_j}.

Such a property is illustrated in Figure 5(a). When the CVs of task times in human and robot preparation sub-processes, and collaboration sub-process are increased from 0.3 to 0.7, the CV of assembly time also increases.

Again, the CV of assembly time does not exhibit a sensitive behaviour to the mean processing time of each task. As shown in Figure 5(b), changes in mean task time leads to very slight variation in assembly time CV, since the CV of each task time does not change with the mean task time, and the variance of each sub-process only depends on the sum of all task time variances in the sub-process. Thus the CV of assembly time will not be sensitive to the mean task time.

5.2. Bottleneck analysis

Bottleneck identification is one of the most effective ways in continuous improvement and has been widely applied in manufacturing system studies and practices (see, for instance, Kuo, Lim, and Meerkov 1996; Chiang, Kuo, and Meerkov 1998, 2000; Biller et al. 2008, 2009; Meerkov and Zhang 2011; Liu, Li, and Chiang 2012; Tu et al. 2019). In this paper, we intend to identify the most impeding task to assembly-time performance, mean assembly time and its variability.

5.2.1. ATP bottleneck

According to the monotonicity properties discussed above, reducing the mean time or CV of each task can lead to an increase in ATP in different extents. Then, bottleneck analysis is conducted to identify the most impeding task to system ATP. In other words, which task should be first trimmed to improve the performance of the whole assembly system most efficiently? Specifically, when the mean and CV of each task are reduced by a small proportion, i.e. $\delta\tau_{i_j}$ and δcv_{i_j}, respectively, where $0 < \delta \ll 1$, the task leading to the largest ATP increment will become the bottleneck. Let $S(T_d)|_x$ denote the *ATP* under condition x for a given time period T_d, where x refers to mean or CV of task time. Then, a bottleneck task can be defined as follows:

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Definition 5.3: Mean processing time τ_{i_j} is the ATP bottleneck with respect to mean task time, denoted as BN-*ATP*(τ), if $\forall\{k,l\} \neq \{i,j\}$, $i,k = h,r,c$, $j = 1,\dots,m_i$, $l = 1,\dots,m_k$,

$$S(T_d)|_{(\tau_{i_j}-\delta\tau_{i_j})} - S(T_d)|_{\tau_{i_j}} > S(T_d)|_{(\tau_{k_l}-\delta\tau_{k_l})} - S(T_d)|_{\tau_{k_l}}. \tag{15}$$

sandsynlighed for at fuldføre inden for tid forbedres mest ved task i_j

Definition 5.4: Coefficient of variation cv_{i_j} is the ATP bottleneck with respect to CV, denoted as BN-*ATP*(cv), if $\forall\{k,l\} \neq \{i,j\}$, $i,k = h,r,c$, $j = 1,\dots,m_i$, $l = 1,\dots,m_k$,

$$S(T_d)|_{(cv_{i_j}-\delta cv_{i_j})} - S(T_d)|_{cv_{i_j}} > S(T_d)|_{(cv_{k_l}-\delta cv_{k_l})} - S(T_d)|_{cv_{k_l}}. \tag{16}$$

Based on Definitions 5.3 and 5.4, using the dataset in (14) as an example, we illustrate the application of bottleneck analysis for performance improvement. The resulting ATP improvements to mean time and CV reductions are presented in Tables 3(a,b), respectively, where the bold numbers indicate the largest improvement in *ATP* for each given T_d.

As one can see, the reduction of mean or CV of the first task in robot preparation sub-process, τ_{r_1} and cv_{r_1}, can lead to the largest improvement in ATP (illustrated in bold numbers) due to its long task time and large CV. Following it are the second task in robot preparation sub-process and the second task in collaboration sub-process. Thus Task r_1 is both the BN-*ATP*(τ) and the BN-*ATP*(cv). The improvement of r_1 will substantially improve the ATP of the collaborative assembly system.

5.2.2. Bottlenecks for mean assembly time

Similarly, denote $T|_x$ as the mean time under condition x. The mean assembly time bottleneck can be defined and analysed as below.

Definition 5.5: Mean processing time τ_{i_j} is the mean assembly time bottleneck BN-$T(\tau)$ if $\forall\{k,l\} \neq \{i,j\}$, i, $k = h,r,c$, $j = 1,\dots,m_i$, $l = 1,\dots,m_k$,

$$T|_{\tau_{i_j}} - T|_{(\tau_{i_j}-\delta\tau_{i_j})} > T|_{\tau_{k_l}} - T|_{(\tau_{k_l}-\delta\tau_{k_l})}. \tag{17}$$

Table 3. ATP improvements.

T_d	τ_{h_1}	τ_{h_2}	τ_{h_3}	τ_{h_4}	τ_{r_1}
120	0.0086	0.0032	0.0075	0.0019	**0.0367**
130	0.0053	0.0019	0.0051	0.0011	**0.0320**
140	0.0029	0.0010	0.0030	0.0006	**0.0253**
150	0.0014	0.0005	0.0016	0.0003	**0.0187**
160	0.0007	0.0002	0.0008	0.0001	**0.0130**
T_d	τ_{r_2}	τ_{r_3}	τ_{c_1}	τ_{c_2}	
120	0.0291	0.0149	0.0249	0.0259	
130	0.0230	0.0113	0.0185	0.0199	
140	0.0165	0.0079	0.0126	0.0139	
150	0.0112	0.0052	0.0081	0.0091	
160	0.0071	0.0033	0.0050	0.0056	

(a) ATP improvement with respect to mean task time

T_d	cv_{h_1}	cv_{h_2}	cv_{h_3}	cv_{h_4}	cv_{r_1}
120	0.0030	0.0000	0.0040	0.0001	**0.0041**
130	0.0021	0.0000	0.0032	0.0001	**0.0084**
140	0.0013	0.0000	0.0022	0.0000	**0.0101**
150	0.0007	0.0000	0.0013	0.0000	**0.0097**
160	0.0003	0.0000	0.0007	0.0000	**0.0081**
T_d	cv_{r_2}	cv_{r_3}	cv_{c_1}	cv_{c_2}	
120	0.0037	0.0001	0.0019	0.0026	
130	0.0041	0.0001	0.0022	0.0034	
140	0.0036	0.0001	0.0018	0.0031	
150	0.0028	0.0001	0.0013	0.0024	
160	0.0020	0.0000	0.0009	0.0017	

(b) ATP improvement with respect to task time CV

The mean assembly time improvements are evaluated in Table 4(a) when the mean task times are reduced by $\delta = 10\%$, where $\Delta T_{i_j} = T|_{\tau_{i_j}} - T|_{(\tau_{i_j}-\delta\tau_{i_j})}$. It is observed that a 10% reduction of mean time of Task r_1 can lead to the largest improvement in mean assembly time, making r_1 to be the BN-$T(\tau)$. In addition, the reduction of mean time of Task r_2 can also lead to substantial improvement in mean assembly time due to the large task processing time.

Although the above approach enables bottleneck identification through numerical calculation, it still needs evaluation and comparison of all tasks. Since each sub-process only involves serial operations, through extensive numerical experiments, it is observed that in each preparation and collaboration sub-process, the task with the longest processing time in each sub-process always becomes the mean assembly time bottleneck BN-$T(\tau)$ of this sub-process. Then the bottleneck of the whole system can be identified by comparing the three bottlenecks of the preparation and collaboration sub-processes. Such an approach can save computation time significantly as it does not need to check every task.

To do this, the bottlenecks of preparation and collaboration sub-processes need to be identified first. Let $T_i|_x$ denote the mean time of sub-process i, $i = r, h, c$. Then the following property holds:

Table 4. Mean assembly time improvements with respect to mean task time.

	τ_{h_1}	τ_{h_2}	τ_{h_3}	τ_{h_4}	τ_{r_1}	τ_{r_2}	τ_{r_3}	τ_{c_1}	τ_{c_2}
ΔT_{i_j}	0.69	0.29	0.53	0.16	**2.54**	2.03	1.09	1.80	1.80

(a) Improvement evaluation using Definition 5.5

	τ_{h_1}	τ_{h_2}	τ_{h_3}	τ_{h_4}	τ_{r_1}	τ_{r_2}	τ_{r_3}	τ_{c_1}	τ_{c_2}
$I_{T_i,\tau}$	**22**	11	14	6	**30**	26	15	**18**	**18**

(b) Improvement evaluation using bottleneck indicator

For enhver sekventiel process. Bottlneck mean time er den delprocess med højest mean time

Proposition 5.6: *Under assumptions (i)–(iv), for any $i \in \{r, h, c\}$, and $j, k \in \{1, \ldots, m_i\}$,*

$$T_i|_{\tau_{i_j}} - T_i|_{(\tau_{i_j}-\delta\tau_{i_j})} > T_i|_{\tau_{i_k}} - T_i|_{(\tau_{i_k}-\delta\tau_{i_k})}, \quad \forall k \neq j, \tag{18}$$

if and only if

$$\tau_{i_j} > \tau_{i_k}. \tag{19}$$

Proof: See Appendix. ■

Using this result, the task with the longest processing time τ_{i_j} is the mean time bottleneck, BN-$T_i(\tau)$, for sub-process i under Definition 5.5. Then BN-$T_i(\tau)$ can be easily identified by an indicator $I_{T_i,\tau}$, defined as

Mean assembly time bottleneck Indicator of sub-process i:

$$I_{T_i,\tau} = \tau_{i_j}, \quad i = h, r, c. \tag{20}$$

Then the task with largest $I_{T_i,\tau}$, $i = h, r, c$, is the mean time bottleneck of sub-process i.

To illustrate, consider the example in Table 4. First, we calculate the bottleneck indicator of each task, as shown in Table 4(b). Tasks h_1, r_1, c_1 and c_2 are more likely to become the bottleneck of the whole system. Then we further check the mean assembly time improvements of these four tasks, which are 0.69, 2.54, 1.80 and 1.80, respectively, presented in Table 4(a). Again, Task r_1 is identified as the BN-$T(\tau)$, but calculations are significantly reduced.

5.2.3. Bottlenecks for assembly time CV

Analogously, denote $CV|_x$ as the CV of assembly time under condition x. Then the CV bottleneck can be defined and analysed as follows.

Definition 5.7: Coefficient of variation cv_{i_j} is the CV bottleneck BN-$CV(cv)$ if $\forall\{k, l\} \neq \{i, j\}$, $i, k = h, r, c$, $j = 1, \ldots, m_i$, $l = 1, \ldots, m_k$,

$$CV|_{cv_{i_j}} - CV|_{(cv_{i_j}-\delta cv_{i_j})} > CV|_{cv_{k_l}} - CV|_{(cv_{k_l}-\delta cv_{k_l})}. \tag{21}$$

The CV improvements are evaluated in Table 5(a) when the CVs of task times are reduced by $\delta = 10\%$, and $\Delta CV_{i_j} = CV|_{cv_{i_j}} - CV|_{(cv_{i_j}-\delta cv_{i_j})}$. As one can see, the reduction of cv_{r_1} can lead to largest improvement in CV, making Task r_1 to be the BN-$CV(cv)$.

Similarly, to identify CV bottleneck BN-$CV(cv)$, we can first identify the bottleneck for each sub-process and then compare the three bottlenecks to save computation effort. Let BN-$CV_i(cv)$ denote the CV bottleneck of sub-process i under Definition 5.7. Then we obtain

Proposition 5.8: *Under assumptions* (i)–(iv), *for any $i \in \{r, h, c\}$, and $j, k \in \{1, \ldots, m_i\}$,*

$$CV_i|_{cv_{i_j}} - CV_i|_{(cv_{i_j}-\delta cv_{i_j})} > CV_i|_{cv_{i_k}} - CV_i|_{(cv_{i_k}-\delta cv_{i_k})}, \quad \forall k \neq j, \tag{22}$$

if and only if

$$\sigma_{i_j} > \sigma_{i_k}, \tag{23}$$

where σ_{i_j} and σ_{i_k} are the standard deviations of processing times of tasks i_j and i_k, respectively.

Proof: See Appendix. ■

Using this result, the BN-$CV_i(cv)$ can be easily identified by the bottleneck CV indicator introduced below:

CV bottleneck Indicator of sub-process i:

$$I_{CV_i,cv} = \sigma_{i_j}, \quad i = h, r, c. \tag{24}$$

Then the task with the largest $I_{CV_i,cv}$ is the CV bottleneck of sub-process i.

In the above example, first we calculate the CV bottleneck indicator of each task described in Table 5(b). Tasks h_3, r_1 and c_2 are more likely to become the bottleneck of the whole system. Then the CV improvements of these 3 tasks are further checked, which are 0.0005, 0.0097, 0.0030, respectively, as in Table 5(a). Thus, Task r_1 is again identified as the BN-$CV(cv)$.

Remark 5.1: In the above example, the task with the largest $I_{T_i,\tau}$ and $I_{CV_i,cv}$ among the three candidate bottlenecks is finally the bottleneck of the whole system. However, this may not always hold since the results are dependent on the mean time of the sub-processes and

Table 5. CV improvements with respect to task time CV.

	cv_{h_1}	cv_{h_2}	cv_{h_3}	cv_{h_4}	cv_{r_1}	cv_{r_2}	cv_{r_3}	cv_{c_1}	cv_{c_2}
ΔCV_{i_j}	0.0001	0.0000	0.0005	0.0000	**0.0097**	0.0024	0.0000	0.0018	0.0030

(a) Improvement evaluation using Definition 5.7

	cv_{h_1}	cv_{h_2}	cv_{h_3}	cv_{h_4}	cv_{r_1}	cv_{r_2}	cv_{r_3}	cv_{c_1}	cv_{c_2}
$I_{CV_i,cv}$	10.45	1.52	**11.39**	2.45	**20.98**	12.03	1.97	7.94	**10.15**

(b) Improvement evaluation using bottleneck indicator

structure of the system. A counter example can be found in Section 6.3. Therefore, the bottleneck identification of the whole system still needs to check the mean time and CV improvement by Definitions 5.5 and 5.7, respectively. In addition, it is worthy to note that these two indicators may not always point to the same task.

Remark 5.2: The identification of BN-$T(\tau)$ and BN-$CV(cv)$ is substantially simplified by comparing only three candidates in the sub-processes after using bottleneck indicators. Then it is natural to ask whether such indicators hold or not in BN-$ATP(\tau)$ and BN-$ATP(cv)$ identification. Numerical experiments indicate that $I_{T_i,\tau}$ has an accuracy of 93% for BN-$ATP(\tau)$ identification and $I_{CV_i,cv}$ has an accuracy of 98% for BN-$ATP(cv)$ identification. The incorrect identification cases are often observed when the mean values (or CVs) are close but the differences in CVs (respectively, mean values) are large.

Kan være det skal undersøges så

Remark 5.3: Note that the results on mean time monotonicity and bottlenecks are similar to those introduced by Zhang, Huang et al. (2021) where the exponential distribution is assumed for all task times. However, the properties and bottlenecks related to CV and ATP are not investigated in Zhang, Huang et al. (2021). Due to the importance of variability in manufacturing systems, the results presented in this section can provide a comprehensive investigation on the behaviour of collaborative assembly systems.

6. An application study

6.1. System description

Consider the engine and transmission assembly section with three operators in a powertrain manufacturing line (see illustration in Figure 6(a)). The operations carried out by these operators are described below:

- Operator 1: The first operator walks to a feeding conveyor (typically an AGV) with a hoist, which is used to handle the heavy part. The operator needs to take the hoist and secure the gripper to load the engine, then walks back to the main line and drops it. Due to the weight and ergo requirement, the move is slow.

Table 6. Tasks in serial manual operations.

Operator	Tasks	Time (sec)	Deviation (sec)
1	Takes a hoist and walks to the feeder	17	1.155
	Secures engine and walks back to main line	23	1.732
	Takes a hoist and walks to the AGV	14	1.155
2	Grasps transmission and walks back to main line	19	1.732
	Allocates and aligns transmission onto engine	7	0.866
3	Tightens nuts	10	1.155
	Secures connectors and hoses	10	1.155

- Operator 2: At the same time, the second operator walks to another AGV and loads the matching transmission from it. A hoist is still needed. Then the operator grips the transmission and walks back. As the transmission is slightly less heavier, the move is relatively faster. In addition, the operator needs to allocate the transmission in the right position to the engine such that the locating holes and pins are perfectly aligned.
- Operator 3: The third operator is responsible to finish the assembly by tightening the nuts, and securing all electrical connectors and/or hoses.

Such a section can be represented by an assembly line model with decomposed manual tasks, as shown in Figure 6(b), where two operators load the parts (engine and transmission) and the third one assembles them together. The average task times and their standard deviations are presented in Table 6. Note that the data has been modified and is used for illustration purpose only to keep confidentiality. However, the nature of the data is still the same.

By introducing collaborative robots, these three manual stations are transformed into one collaborative assembly station with two robots and one operator, see Figure 7(a). In this station, the overall average time is 27 seconds. Specifically, the average processing time for both robots to pick up, transport and drop the matching parts is about 20 seconds, which is shorter than manual operation since the robot does not walk and can move faster. When the robots move the parts to the station vicinity,

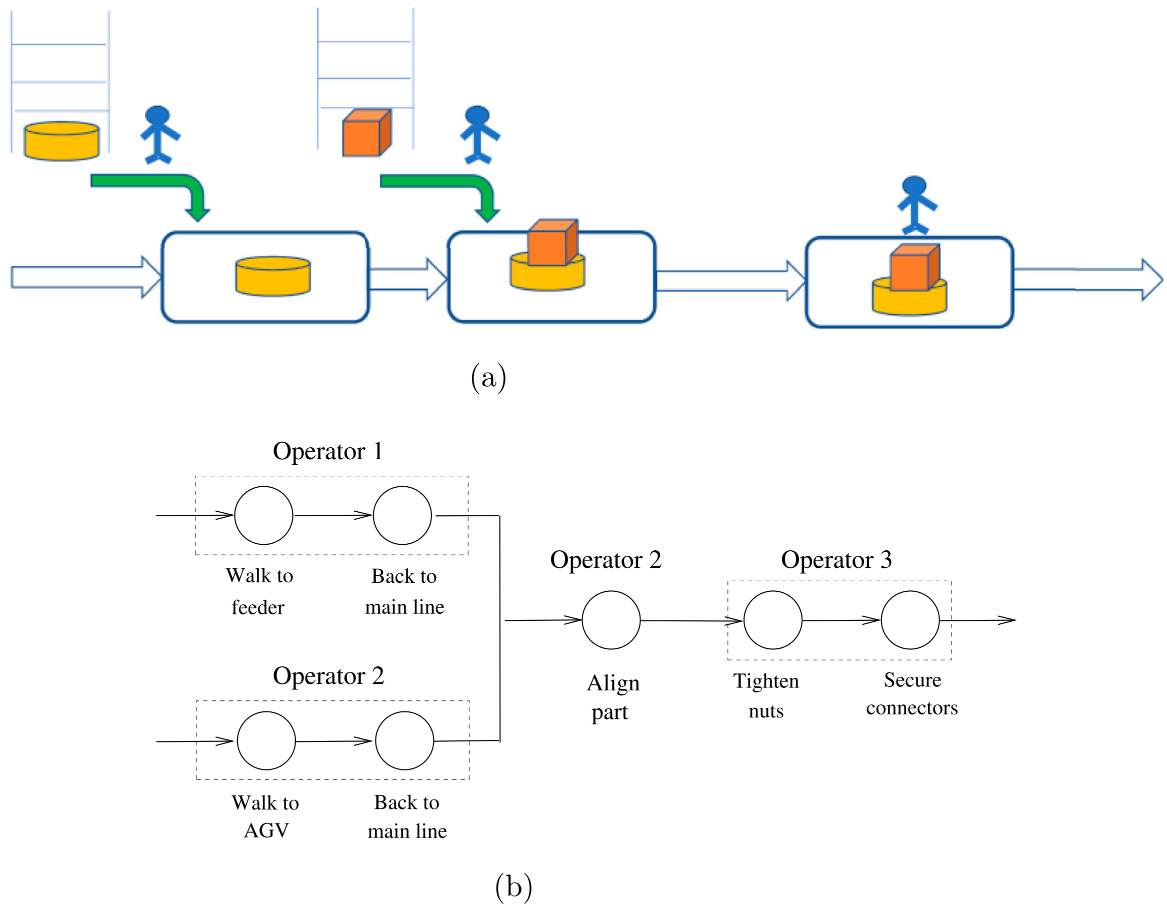

Figure 6. A powertrain manual assembly line: (a) Manual system illustration and (b) Manual system model.

an operator will guide the alignment, which takes about 7 seconds for this task. Then the robots are released to pick up the next parts, while the operator finishes assembly of the current parts. The average processing time is 20 seconds for this task. Therefore, among the 27 seconds overall time, the robots complete the pickup and drop tasks during the first 20 seconds, and the next 7 seconds are for guided alignment. Then during the next 20 seconds, the operator finishes assembly, while at the same time the robots complete the pick up and move task for the next parts. The timeline of such operations is illustrated in Figure 7(b).

6.2. System models

Such a station can be modelled as a collaborative assembly line similar to Figure 1. In other words, the robot independent work consists of pickup and transportation of parts. The joint task for operator to guide robot for alignment is the collaborative work. For the operator independent assembly work, although it happens after alignment for the current part, it is right before the joint alignment work for the next part. Thus the tasks in such a station can be represented by the model shown in Figure 7(c). The task times of the model are summarised in Table 7.

Table 7. Tasks in collaborative assembly operations.

	Task	Time (sec)	Deviation (sec)
Robots 1 & 2	Pick up parts	8	0.577
	Transport parts	12	0.577
Operator	Tighten nuts	10	1.155
	Secure connectors	10	1.155
Joint	Guide alignment	7	0.866

Using the gamma approximation method introduced above, the ATPs of the manual assembly line and collaborative assembly line can be evaluated as shown in Figure 8. The mean assembly time and CV of manual line and collaborative line are 67.01 seconds and 0.0414, 27.96 seconds and 0.0463, respectively. The introduction of collaborative assembly line substantially reduces the mean assembly time by 39.05 seconds (58.3%) and with similar CVs. Moreover, the ergonomics performance can be improved significantly due to removing the lifting and transporting tasks. The staffing level is reduced from 3 to 1 so that additional work can be done by the two operators released from the station.

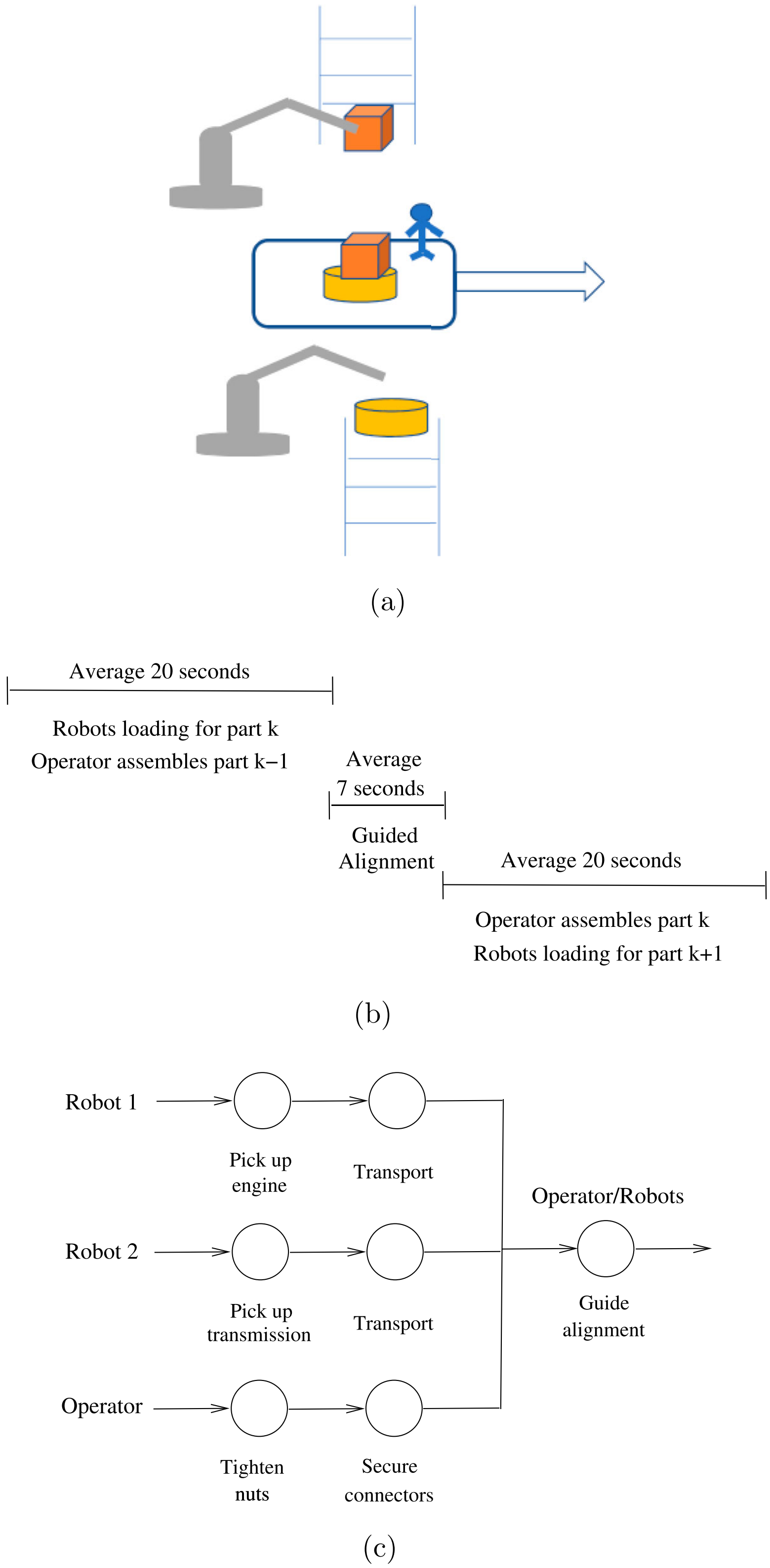

Figure 7. Collaborative assembly station: (a) Collaborative assembly system illustration, (b) Collaborative assembly system operation timeline and (c) Collaborative assembly system model.

6.3. System improvement

To seek further improvement of the process, bottleneck analysis is carried out for the new collaborative assembly system.

For the ATP bottleneck, the results of ATP improvements when the mean and CV of each task time are reduced by 10% are shown in Tables 8(a,b), respectively. As one can see, improving Task τ_{c_1}, namely,

Table 8. Improvements of collaborative system.

T_d	τ_{r1_1}	τ_{r1_2}	τ_{r2_1}	τ_{r2_2}	τ_{h_1}	τ_{h_2}	τ_{c_1}
29	0.0212	0.0228	0.0212	0.0228	0.0750	0.0750	**0**.1110
29.5	0.0102	0.0111	0.0102	0.0111	0.0568	0.0568	**0**.0695
30	0.0043	0.0046	0.0043	0.0046	0.0386	0.0386	**0**.0409

(a) ATP improvements with respect to mean task time

T_d	cv_{r1_1}	cv_{r1_2}	cv_{r2_1}	cv_{r2_2}	cv_{h_1}	cv_{h_2}	cv_{c_1}
29	0.0031	0.0029	0.0031	0.0029	0.0087	0.0087	**0**.0109
29.5	0.0019	0.0017	0.0019	0.0017	0.0082	0.0082	**0**.0090
30	0.0005	0.0012	0.0005	0.0012	**0**.0064	**0**.0064	**0**.0064

(b) ATP improvements with respect to task time CV

	τ_{r1_1}	τ_{r1_2}	τ_{r2_1}	τ_{r2_2}	τ_{h_1}	τ_{h_2}	τ_{c_1}
$I_{T_i,\tau}$		12		12	10	10	7
ΔT	0.17	0.20	0.17	0.20	0.31	0.31	**0**.70

(c) Mean assembly time improvements with respect to mean time

	cv_{r1_1}	cv_{r1_2}	cv_{r2_1}	cv_{r2_2}	cv_{h_1}	cv_{h_2}	cv_{c_1}
$I_{CV_i,cv}$	0.577	0.577	0.577	0.577	1.155	1.155	0.866
ΔCV	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	**0**.0020

(d) CV improvements with respect to task time CV

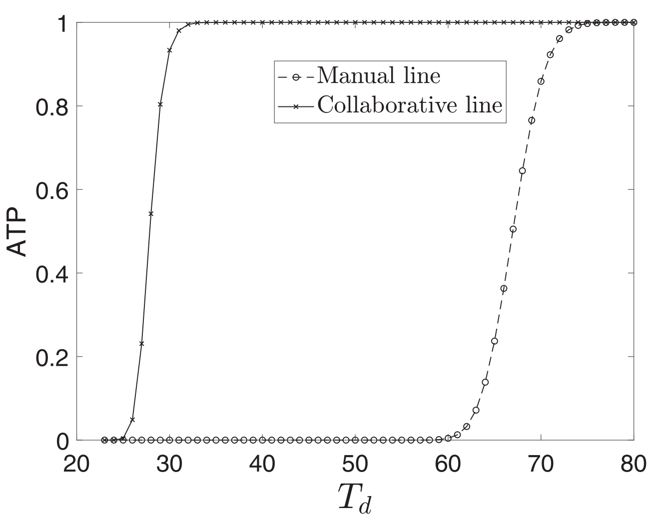

Figure 8. ATPs of manual and collaborative assembly lines.

the joint operation 'Guide alignment', can lead to the greatest improvement in ATP (as shown in bold numbers), making 'Guide alignment' both the ATP bottlenecks with respect to mean time, BN-$ATP(\tau)$, and CV, BN-$ATP(cv)$.

For the bottlenecks of mean assembly time and CV, using the Mean assembly time and CV bottleneck Indicators, $I_{T_i,\tau}$ and $I_{CV_i,cv}$, of sub-process i, respectively, Tasks $r1_2$ and $r2_2$ ('Transport parts', due to two identical robot operations), h_1 and h_2 ('Tighten nuts', 'Secure connectors', due to the same task time and CV), and c_1 are further compared. By reducing their mean time and CV of each task by 10%, the results of BN-$T(\tau)$ and BN-$CV(cv)$ are shown in Tables 8(c,d), respectively. For illustration purpose, results from robot Tasks $r1_1$ and $r2_1$ ('Pick up parts') improvements are also included in the tables. Again, the joint operation 'Guide alignment', i.e. Task c_1, can lead to the largest improvement in mean assembly time and CV (as shown in bold numbers), making 'Guide alignment' both the mean assembly time and CV bottlenecks. However, it is worthy to note that Task c_1 is not the largest Indicator in both cases.

Such a result suggests that although 'guided alignment' only takes a short time period, as it is in the collaboration sub-process, it is still the bottleneck of the assembly station. Moreover, tasks in human operation sub-process, both 'Tighten nuts' and 'Secure connectors', are the next bottlenecks due to the long mean time and large deviation. Particularly, in terms of ATP bottleneck with respect to CV, all operations involving operator activities become the bottlenecks when $T_d = 30$. Thus keeping consistent and short operator assembly time is critical to ensure a desired performance.

7. Conclusion

In this paper, a system-theoretic approach is introduced to model and analyse the assembly-time performance of collaborative assembly systems with human operators and robots. Gamma distribution is used to characterise the randomness in task times of operators and robots. An approximation method is presented to aggregate all the tasks to derive the general distribution of overall assembly time of the system. System performance measures, including the mean, CV and complete distribution of assembly time, are evaluated. Using them, system properties, such as monotonic properties, are investigated. Moreover, to improve system performance, bottleneck analysis is carried out identify and mitigate the task whose reduction can lead to the largest improvement in all performances. Finally, a case study at an automotive powertrain assembly line is introduced to illustrate the applicability of the method and the effectiveness and advantage of collaborative assembly systems.

Although such work presents a significant contribution in studying collaborative assembly systems, limitations still exist. For example, when finite buffers and multiple stages are introduced, the current approach is not suitable due to substantially explored computations. Probably an aggregation or decomposition method is used by using current model as a building block. robot reliability and strain index are introduced, how to represent their impact into task time is a critical issue. Moreover, scheduling and maintenance optimisation will become more difficult due to additional constraints. The dynamic control of human robot interactions and operator characterisation may need completely different methods to address. All these require substantial efforts. Due to space limitation and scope of the paper, those problems are impossible to be included in this paper and will be addressed in future work.

Therefore, the future work can be directed to studying the following topics:

- First, extending the work to study multiple stage assembly systems, where finite buffers are used to connect different stages, and blockage and starvation introduced due to assembly time variations can propagate up- and downstream to impact system throughput.
- Second, analysing and accommodating correlations between multiple activities and the associated task times in the model, and seeking analytical derivations under certain conditions in addition to numerical solutions.
- Third, introducing robot reliability and operator strain index in the model to address the task assignment and job scheduling issues with maintenance and ergonomic constraints, including multiple sequences in job shops.
- Fourth, on-line monitoring and control of human–robot interactions and dynamic allocation or reassignment of tasks, which are of significant importance to optimise system performance and ensure safety in real-time.
- Fifth, including operator characterisations and its in task times into the study to introduce operator-specific models.
- Finally, applying the model in the factory.

Such development will provide production engineers smart, quantitative and efficient tools and methods for factory automation management in the era of Industry 4.0.

Disclosure statement

No potential conflict of interest was reported by the authors.

Data availability statement

The data that support the findings of this study are available from the corresponding author, JL, upon reasonable request.

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Appendix. Proofs

Proof of Lemma 4.1.: The proof can be easily extended with the result of Lee et al. (2017) that if $\{X_i, i = 1, 2, \dots, n\}$ are independently distributed gamma random variable with mean τ_i and deviation σ_i, then

$$G(x) = \prod_{i=1}^{n} \left(\frac{\beta_{\min}}{\beta_i}\right)^{\eta_i} \sum_{k=0}^{\infty} \frac{\delta_k \gamma(\rho + k, x/\beta_{\min})}{\Gamma(\rho + k)},$$

where $\beta_{\min}$, β_i, η_i, δ_i, ρ, $\gamma(\cdot)$, and $\Gamma(\cdot)$ are defined as Lemma 4.1. In this paper, each random process has m_i sequential tasks, each being characterised by gamma variables with mean τ_{i_j} and coefficient of variation cv_{i_j}. ■

Proof of Proposition 4.2.: It has been shown in Lemma 4.1 that the cycle time CDFs of manual and robot preparation subprocesses can be evaluated as $G_h(x)$ and $G_r(x)$, respectively. The PDF of joint collaboration sub-process can be obtained by one step derivation of $G_c(x)$ as

$$\begin{aligned} g_c(x) &= G_c'(x) \\ &= \prod_{j=1}^{m_c} \left(\frac{\beta_{c,\min}}{\beta_{c_j}}\right)^{\eta_{c_j}} \sum_{k=0}^{\infty} \frac{\delta_{c_k} (x/\beta_{c,\min})^{\rho_c + k - 1} e^{-x/\beta_{c,\min}}}{\beta_{c,\min} \Gamma(\rho_c + k)}, \end{aligned}$$

where $\beta_{c,\min}$, β_{c_j}, η_{c_j}, δ_{c_k}, ρ_c, and $\Gamma(\cdot)$ are defined in Lemma 4.1. According to the system structure in Figure 1, the CDF of the whole process for a given time period T_d can be evaluated as

$$S(T_d) = \int_0^{T_d} G_h(T_d - x) G_r(T_d - x) g_c(x)\, \mathrm{d}x.$$

■

Proof of Corollary 4.3.: For the PDF of assembly time, $s(t)$, we obtain

$$\begin{aligned} s(t) = \frac{\partial S(t)}{\partial t} &= \int_0^t \frac{\partial G_h(t-x) G_r(t-x)}{\partial t} g_c(x)\, \mathrm{d}x \\ &= \int_0^t \Big[g_h(t-x) G_r(t-x) + G_h(t-x) g_r(t-x)\Big] g_c(x)\, \mathrm{d}x. \end{aligned}$$

Then mean assembly time and CV follow.

$$T = E(t) = \int_0^{\infty} t s(t)\, \mathrm{d}t,$$

$$CV = \frac{\sqrt{E(t^2) - (E(t))^2}}{E(t)} = \frac{\sqrt{\int_0^\infty t^2 s(t)\, \mathrm{d}t - (\int_0^\infty t s(t)\, \mathrm{d}t)^2}}{\int_0^\infty t s(t)\, \mathrm{d}t}.$$

■

Proof of Proposition 5.1.: First, it is easy to show that $\forall i \in \{h, r\}, j = 1, \ldots, m_i$,

$$\frac{\partial G_i(\cdot)}{\partial \tau_{i_j}} < 0,$$
$$\frac{\partial G_k(\cdot)}{\partial \tau_{i_j}} = 0, \quad k \in \{h, r\},\ k \neq i$$
$$\frac{\partial g_c(\cdot)}{\partial \tau_{i_j}} = 0.$$

Then, we have

$$\begin{aligned}\frac{\partial S(T_d)}{\partial \tau_{i_j}} &= \int_0^{T_d} \frac{\partial G_h(T_d - x)G_r(T_d - x)g_c(x)}{\partial \tau_{i_j}}\,\mathrm{d}x \\ &= \int_0^{T_d} \Bigg[\frac{\partial G_h(T_d - x)}{\partial \tau_{i_j}} G_r(T_d - x)g_c(x) \\ &\quad + \frac{\partial G_r(T_d - x)}{\partial \tau_{i_j}} G_h(T_d - x)g_c(x) \\ &\quad + \frac{\partial g_c(x)}{\partial \tau_{i_j}} G_h(T_d - x)G_r(T_d - x) \Bigg]\mathrm{d}x.\end{aligned}$$

Thus only one of the three partial derives, $\frac{\partial G_h(T_d-x)}{\partial \tau_{i_j}}$, $\frac{\partial G_r(T_d-x)}{\partial \tau_{i_j}}$ and $\frac{\partial g_c(x)}{\partial \tau_{i_j}}$, is less than 0 and the other two equal to 0. Therefore, we obtain

$$\frac{\partial S(T_d)}{\partial \tau_{i_j}} < 0.$$

As for $i = c$, the proof is similar provided that the *ATP* formula is equivalent to

$$S(T_d) = \int_0^{T_d} G_c(T_d - x)g_{hr}(x)\,\mathrm{d}x,$$

where g_{hr} is the PDF of the preparation process and it is not impacted by the partial derivative. Therefore, the argument follows.

Note that this proposition can also be proved through stochastic dominance approach (see, e.g. Levy 1992). ■

Proof of Proposition 5.2.: From Proposition 5.1, we can obtain

$$\begin{aligned}\frac{\partial T}{\partial \tau_{i_j}} &= \frac{\partial \int_0^{\infty}(1 - S(T_d))\,\mathrm{d}T_d}{\partial \tau_{i_j}} = \int_0^{\infty} \frac{\partial(1 - S(T_d))}{\partial \tau_{i_j}}\,\mathrm{d}T_d \\ &= -\int_0^{\infty} \frac{\partial S(T_d)}{\partial \tau_{i_j}}\,\mathrm{d}T_d > 0.\end{aligned}$$

Again note that stochastic dominance approach can also be used to prove this proposition. ■

Proof of Proposition 5.6.: Since

$$\begin{aligned}T_i|_{\tau_{i_j}} - T_i|_{(\tau_{i_j} - \delta\tau_{i_j})} &= \sum_{k=1}^{m_i} \tau_{i_k} - \left[\sum_{k=1,k\neq j}^{m_i} \tau_{i_k} + (1 - \delta)\tau_{i_j} \right] \\ &= \delta\tau_{i_j},\end{aligned}$$

the argument follows. ■

Proof of Proposition 5.8.: Since the mean time and CV of sub-process i satisfy

$$T_i|_{(cv_{i_j} - \delta cv_{i_j})} = T_i|_{(cv_{i_k} - \delta cv_{i_k})},$$
$$CV_i|_{cv_{i_j}} = CV_i|_{cv_{i_k}},$$

we obtain

$$\begin{aligned}&CV_i|_{cv_{i_j}} - CV_i|_{(cv_{i_j} - \delta cv_{i_j})} > CV_i|_{cv_{i_k}} - CV_i|_{(cv_{i_k} - \delta cv_{i_k})} \\ &\iff CV_i|_{(cv_{i_j} - \delta cv_{i_j})} < CV_i|_{(cv_{i_k} - \delta cv_{i_k})} \\ &\iff (T_i|_{(cv_{i_j} - \delta cv_{i_j})})^2 (CV_i|_{(cv_{i_j} - \delta cv_{i_j})})^2 \\ &< (T_i|_{(cv_{i_k} - \delta_{cv} cv_{i_k})})^2 (CV_i|_{(cv_{i_k} - \delta cv_{i_k})})^2 \\ &\iff Var_i|_{(cv_{i_j} - \delta cv_{i_j})} < Var_i|_{(cv_{i_k} - \delta cv_{i_k})} \\ &\iff \sum_{l=1,l\neq j}^{m_1} \tau_{i_l}^2 cv_{i_l}^2 + \tau_{i_j}^2 (1 - \delta)^2 cv_{i_j}^2 \\ &< \sum_{l=1,l\neq k}^{m_1} \tau_{i_l}^2 cv_{i_l}^2 + \tau_{i_k}^2 (1 - \delta)^2 cv_{i_k}^2 \\ &\iff \tau_{i_j}^2 cv_{i_j}^2 (-2\delta + \delta^2) < \tau_{i_k}^2 cv_{i_k}^2 (-2\delta + \delta^2) \\ &\iff \tau_{i_j}^2 cv_{i_j}^2 > \tau_{i_k}^2 cv_{i_k}^2 \iff \tau_{i_j}^2 \cdot \frac{\sigma_{i_j}^2}{\tau_{i_j}^2} > \tau_{i_k}^2 \cdot \frac{\sigma_{i_k}^2}{\tau_{i_k}^2} \\ &\iff \sigma_{i_j} > \sigma_{i_k}.\end{aligned}$$

■