Controlling Vertical Landing of Rocket for Recovery

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I. *Abstract*— The problem to control vertical landing of a rocket was proposed and state space matrix of the system was formulated using linearized equations. LQR and LQG controllers were then developed and applied for regulatory control to land the rocket from its initial conditions of altitude = 1000m and horizontal displacement = 10m from the reference point with initial vertical orientation of rocket being -10°.

II. Introduction

There are multiple stages from vertical launch to vertical landing of a rocket like Falcon 9, which is the inspiration behind the project [1][2], but here we are concerned with just the recovery of the first stage rocket in 2D when rocket is few seconds away from the touch down. For actuators that enable vertical landing, there is a vertical thruster that slows down the free fall of rocket and this vertical thrust force is generated at the nozzle, which itself has a degree of freedom that needs yaw control. There are horizontal thrusters on the top of the first stage rocket that produce a net lateral thrust force to vertically orient the rocket. For sensors, we have altimeter, gyroscope, and GPS for positional tracking of the rocket.

We tackle the regulator control problem by using the mathematical model that was developed in a prior thesis [1]. We linearize the equations and formulate a state space matrix at the suitable equilibrium point. Next, with the realistic initial conditions, we first implement LQR control, which assumes all the states are available for feedback, and for it we tune the Q and R parameters to achieve successful regulation for vertical landing at reference point (0,0) in the Z-X axis plane. Next, we assume unavailability of the state feedback, which motivates us to develop Kalman filter for state estimation, which when coupled with LQR, performs regulation. This state estimation + regulation control is called LQG controller. To make problem more in line with the real world, the LQG controller will incorporate some random gaussian noise in the measurement as well as the actuator for more realism.

III. Mathematical Model

Figure 1 shows the free body diagram of the rocket which is used to formulate the mathematical model. Here the abbreviations mean the following:

F_E = *Vertical Thruster Force*
F_S = $F_L - F_R$ *(Net Lateral Thruster Force)*
θ = *Orientation of Rocket along Z-axis*
φ = *Orientation of Nozzle along Z axis*
l_1 = *Length between COG and Fe*
l_2 = *Length between COG &* F_R, F_L
l_n = *Length Fe to nozzle end*
m = *Dry mass of rocket + Fuel mass*
COG = *Center of Gravity*
J = *Moment of Inertia*

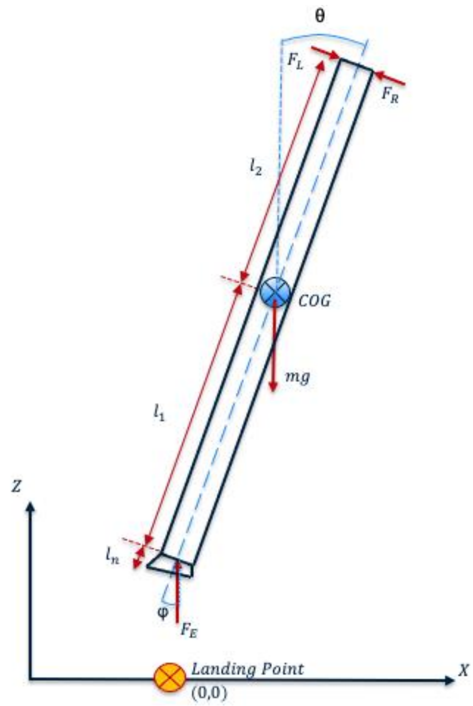

Figure 1: Free body diagram of First Stage Rocket

$$m\ddot{x} = F_E \sin(\theta + \varphi) + F_S \cos(\theta + \varphi)$$

$$\ddot{x} = \frac{F_E \cos(\varphi)\sin(\theta) + F_E sin(\varphi)\cos(\theta) + F_s\cos(\theta)}{m}$$

$$m\ddot{z} = F_E \cos(\theta + \varphi) - F_S \sin(\theta) - mg$$

$$\ddot{z} = \frac{F_E \cos(\varphi)\sin(\theta) - F_E sin(\varphi)\cos(\theta) + F_s \sin(\theta) - mg}{m}$$

$$\ddot{\theta} = \frac{-F_E \sin(\varphi)\,(l_1 + l_n\cos(\varphi)) + l_2 F_S}{J}$$

Taking small angle approximation, our equations reduce to:

$$\ddot{x} \approx \frac{F_E\theta + F_E\varphi + F_S}{m}$$

$$\ddot{z} \approx \frac{F_E - F_E\varphi\theta - F_S\theta - mg}{m}$$

$$\ddot{\theta} \approx \frac{-F_E\varphi(l_1 + l_n) + l_2 F_S}{J}$$

We still have some coupling of terms here like $F_E\varphi\theta$ in the equations, hence there is a need for linearization. Since, we are concerned with final moments of landing, we can linearize our system about the equilibrium point when the rocket is just about to touch down (in hovering state). From the equation above, we construct our state matrix as shown below. The state space matrix can be initialized at given equilibrium point $\bar{u} = (F_E = mg, F_S = 0, \varphi = 0°)$ to decouple terms.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & F_E/m & F_E/m \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Fs/m & -F_E\theta/m \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -F_E L/J \end{bmatrix} \qquad D = [\ 0]$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{(\theta+\varphi)}{m} & \frac{1}{m} & \frac{-F_E}{m} \\ 0 & 0 & 0 \\ \frac{(1-\theta\varphi)}{m} & -\frac{\theta}{m} & \frac{-F_E\theta}{m} \\ 0 & 0 & 0 \\ \frac{-\varphi L}{J} & \frac{L}{J} & -\frac{F_E L}{J} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The state space matrix is representative of the classical form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Here, the states are $x = [x, \dot{x}, z, \dot{z}, \theta, \dot{\theta}]$ and the inputs are $u = [F_E, F_S, \varphi]$. We take $l_1 + l_n \approx l_2 \approx L$. And the moment of inertia of the system is approximated as a rod.

Next, we setup the parameters and constraints for the mathematical model. Note: For the project, we have partially used the parameter values and physical constraints for actuators from the Falcon 9 datasheet to be close to realistic values [3]. These are as follows:

Parameters

m = Rocket Dry (25,600 Kg) + 5% Fuel (of 395,700 Kg)
g = 9.81 m/s^2
$l_1 + l_2$ ≈45m
J ≈ $m(l_1 + l_2)^2/12$

Constraints

$-2.5kN < F_S < 2.5kN$
$0 < F_E < 2535kN$ $(i.e.\ 3\ of\ 9\ engines)$
$-15° < \varphi < 15°)$

Initial Conditions

Horizontal Distance: $x_0 = 10$m
Altitude $z_0 = 1000$m
Initial Orientation: $\theta_0 = -10°$

IV. LQR Controller

LQR controller was designed and implemented for the state feedback control to enable vertical landing of the rocket. To make sure that the constraints were followed, the Q and R values were rigorously tuned to obtain feedback gain with Q = diag([0.01 0.01 150 30000 0.01 0.01]) and R = diag([0.00002 0.01 1250]). Here, the most important consideration was to make sure that the landing was as smooth as possible, which mean that the vertical velocity error had to be given the maximum weighting as shown in Q matrix, followed by the weighting to the error in vertical distance.

With the tuned weights for states (Q) and the input (R), we obtain the results shown below. From figure2, we see that the regulation problem was successfully achieved. The rocket lands in about 50s, and it is particularly worth noting how smooth the landing is from the dz/dt i.e., vertical velocity plot, where the rocket is at ~5m/s in its final moments, which is very desirable and realistic. The horizontal displacement and the orientation of rocket is corrected at around 25s mark, which is well before the landing as seen from landing trajectory in figure 3. From, the input plots in figure 3, we can see that we have accomplished regulator control while respecting the physical constraints of inputs/actuators as defined previously.

However, it is to be noted that there is a slight overshoot in the Vertical Thruster input plot, and this is because of the short comings of the LQR controller, which cannot factor input inequality constraints (like $0 < F_E < 2535kN$ here). MPC controller is therefore the go-to-choice for such constrained problems [4], but we could not implement the MPC controller for our project as it was out of the scope of the course.

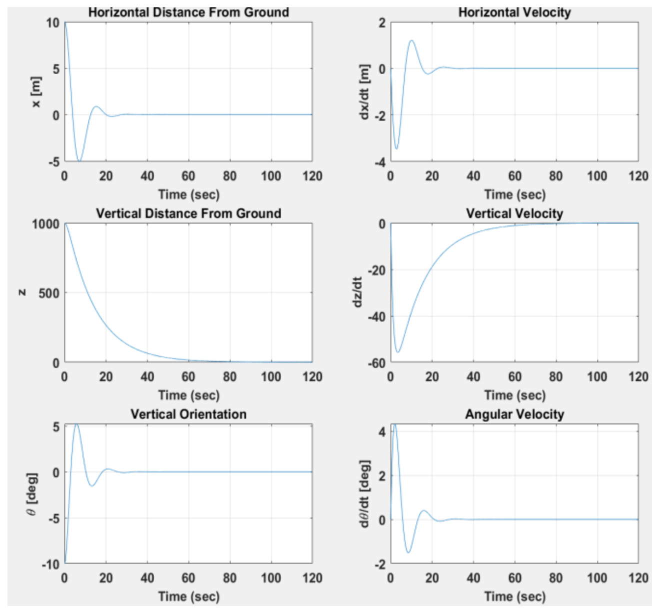

Figure 2: LQR State Plots

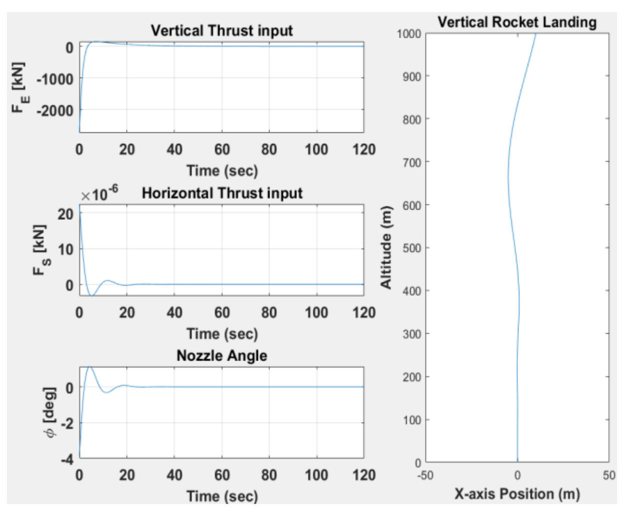

Figure 3: LQR Input Plots (L), Landing Trajectory (R)

V. LQG CONTROLLER AND KALMAN FILTER

The LQG controller was designed by adding a Kalman filter to the existing LQR controller. Since Kalman filters are usually implemented in the discrete domain, the LQR controller had to be converted from the continuous domain to the discrete domain with a sampling time of 1ms.

There are two noise/disturbance sources due to state estimation, which are the measurement disturbance/noise and the input disturbance or noise. Both noise sources are both estimated as gaussian noise. R_v is covariance of the measurement disturbance which is affected by accuracy of the sensors used to measure positions and angles while R_w is covariance of the input disturbance which primary source is the windy conditions [5]. An assumption that was made was that rocket landing are only schedule during good weather and the instruments and sensors on board the rocket are aerospace approved and have a certain degree of reliability in their measurements. Another assumption that was made was that the degree of noise for all three measured output are the same [5]. The values of R_v and R_w are detailed below:

$$R_v = 0.1^2 \qquad R_w = 0.3^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

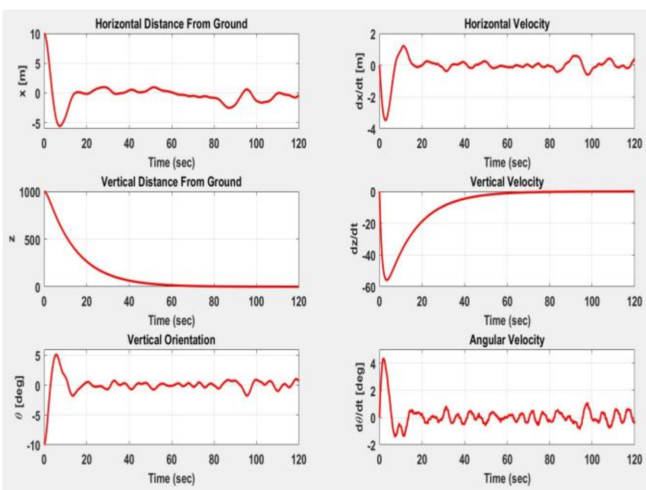

Figure 4: LQG State Plots

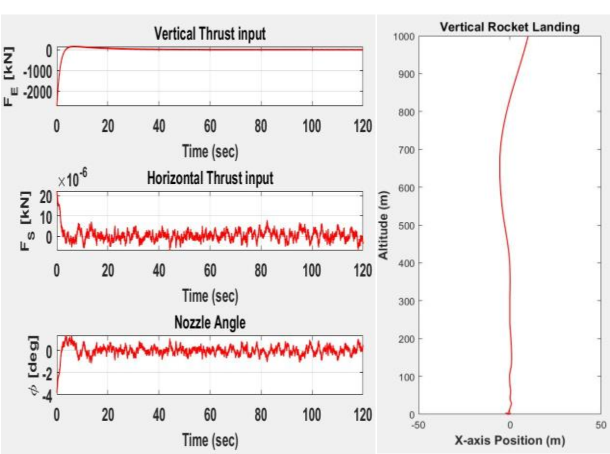

Figure 5: LQG Input Plots (L), Landing Trajectory (R)

With the Kalman filter implemented with the specified covariances, the results above are generated. Figure 4 shows that even with the Kalman filter estimation, the regulation problem was still successfully achieved. The LQG states over time are similar to the LQR states over time, which means that the Kalman filter was properly designed. The rocket still lands in about the same amount of time and the landing trajectory, figure 5 (Landing Trajectory), is maintained. Also, the inputs plots from figure 5 still respect the constraints.

VI. CONCLUSION

Overall, we were able to successfully model a rocket landing and were able to create LQR and LQG controllers that successfully achieved the desired behavior. It is important to note that the input constraints were respected to make the design and results more realistic. To summarize out results, the rocket can land in 50 seconds from an altitude of 1000m and a horizon displacement of 10m with a tilt of -10°. Adding a Kalman filter incorporating noise, had minimal effect on the trajectory and duration of landing.

A slight issue we ran into was having the upper bound of F_E as 0 and this is a shortcoming of the LQR controller. In the future work, MPC will replace the LQR controller because it is able to incorporate the inequality input constraints. The design and analysis could also be extended to landing any aerial vehicle or system or with a few changes could be used to not only control the landing but also the flight of the system. In this case, an integrator would have to be introduced to have a non-zero input.

Rocket landing are very important because they save on cost of having to constantly replace the entire rocket after every launch. This is especially true right now as companies such as SpaceX have led efforts to implement and improve rocket landing technology. If further manned space exploration is to be possible, a key cost that would need to decrease is the cost of replacing rockets.

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