### **MECH 541 PROJECT III:**

# Simulation of Contouring Performance in

## **Coordinated Two Axis Motion**

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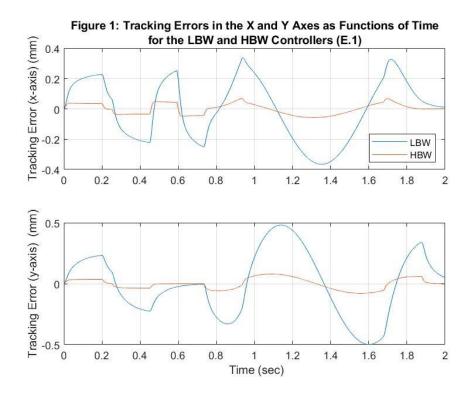
#### **Abstract**

The objective of this lab is to learn how to design a 2 axes machine controller from beginning to the end. Specifically, I learnt how to generate trajectories for different toolpaths, how different controllers affect toolpath response and accuracy, how federate affects toolpath accuracy and how the experimental and simulated results differ.

#### Introduction

Multi-axis trajectory design is important in the industry to produce the desired toolpath. It is most useful in the machining industry and with CNC machines to create precise and accurate movements. The report will first present the results and analysis, which is broken down into four sections: The effect of bandwidth, the effect of maximum federate, the experimental vs simulated results and finally the pole placement controller. Next, there will a conclusion of the report and lastly the appendix containing the Matlab script and Simulink diagrams.

Part E: Effect of Bandwidth



The higher the bandwidth, the lower the tracking error.

Figure 2: Tool Motion for Reference Toolpath and Simulations using Three Cases of Control System (E.2)

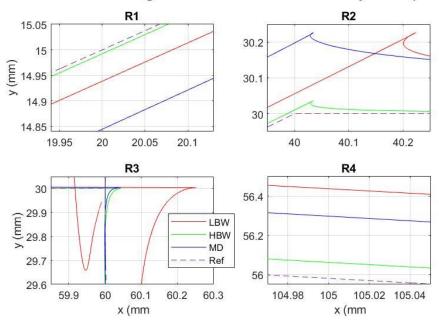


Table 1: Approximate Largest Value of the Contour Error at Different Locations for Different Controllers (E.2)

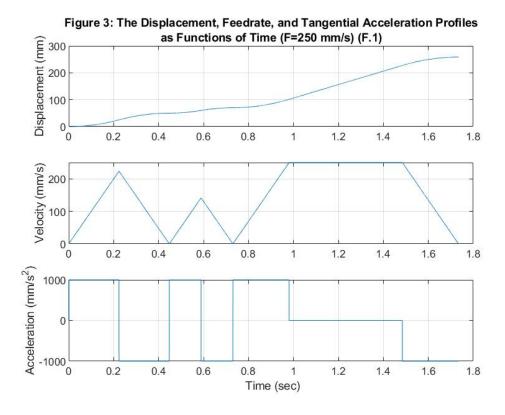
Controllers	Max Contouring Error (mm)						
	R1	R2	R3	R4	Max		
Low Bandwidth	0.0505	0.1198	0.2398	0.3971	0.3971		
High Bandwidth	0.0071	0.0196	0.0366	0.0634	0.0634		
Mismatched Dynamics	0.14	0.1701	0.0905	0.3718	0.3718		

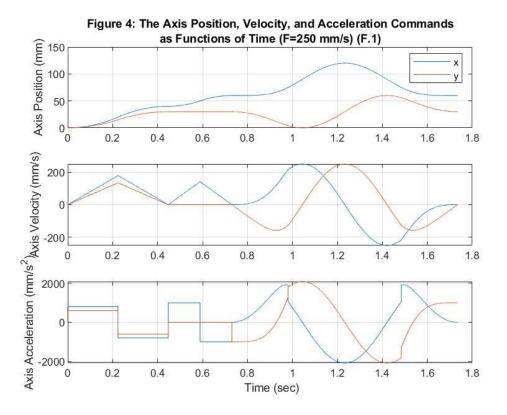
The higher the bandwidth, the lower the contouring error.

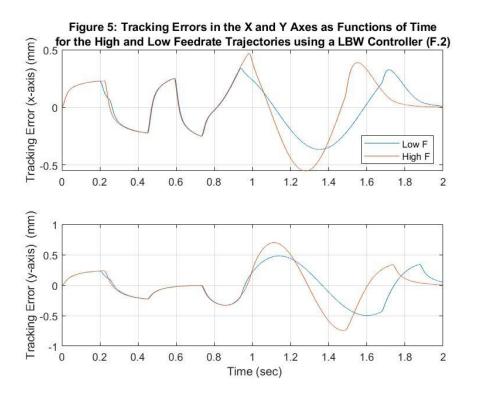
For midpoint of the first line (region R1) and Case 1 (LBW) controller, the tracking error is 0.0505mm. It is not always possible to calculate the contouring error analytically, especially when the trajectory profile isn't linear.

Mismatched Dynamics controller has a worse contouring performance than LBW controller in critical regions, R1 and R2 but it has a better contouring performance than LBW controller in critical regions, R3, R4, and overall. It still performs worse than HBW controller in all regions. Although the Mismatched Dynamics controller seems to perform worse in some region, overall, it performs better than the LBW controller, and so it is desirable to have mismatched controller. A reason Mismatched Dynamics controller is desirable is due to the fact that the dynamics of the physical system is sometimes different on different axes, as is our case, and so each designed controllers should take into account the individual physical dynamics of the different axes.

#### Part F: Effect of Maximum Feedrate

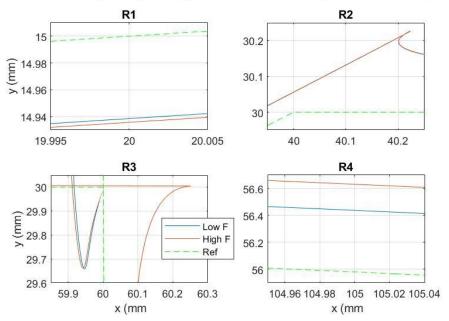






Overall, The higher the maximum feedrate, the higher the tracking errors.

Figure 6: Tool Motion for Reference Toolpath and Simulations (LBW) using The Low and High Feedrates (F.3)



The higher the maximum feedrate, the higher the contouring errors. The contouring errors of the low and high feedrates is relatively low for R1,R2 and R3, but is much higher for R4, which is a region of the circle, and this means that the axis are not linear.

#### Part G: Experiment vs Simulation

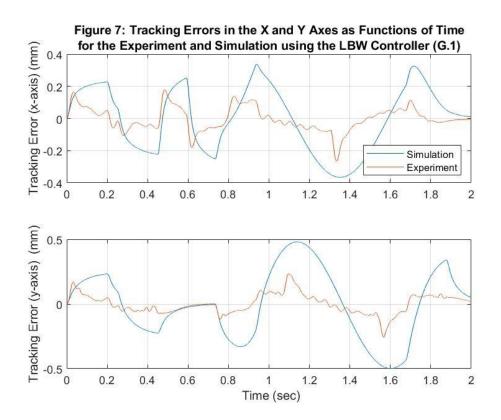
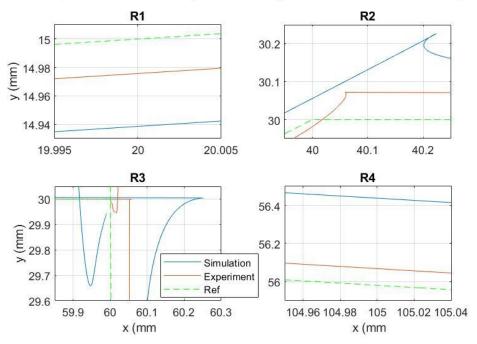
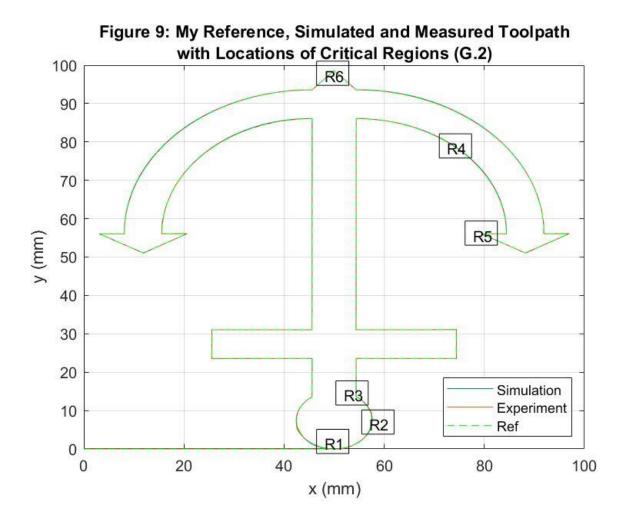


Figure 8: Tool Motion for Reference, Simulated and Experimental Toolpaths using the LBW Controller (G.1)

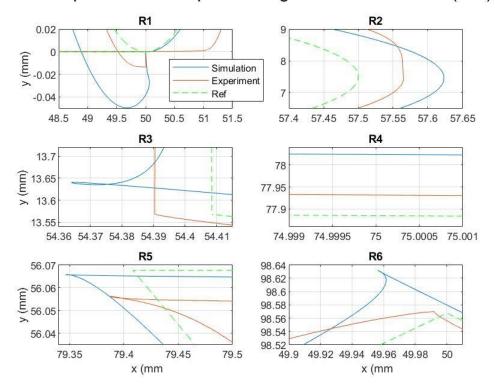


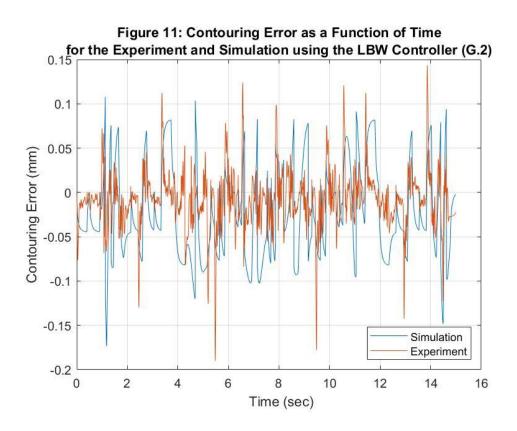
Overall, the experimental result seems to have a smaller tracking and contouring error compared to the simulated result. Possible reasons for the discrepancy between the simulation and the experimental include simulation model inaccuracies because of slightly different inertia and viscous damping constant. In addition, the modelled gain might not be the same as the actual gain.



<sup>\*</sup> See the zoomed plot for the critical regions in figure 10 below

Figure 10: Tool Motion of My Trajectory for Reference, Simulated and Experimental Toolpaths using the LBW Controller (G.2)





#### **Part H: Pole Placement Controller**

Table 2: Poles, Zeros, Bandwidth, Overshoot, and Rise Time of the Closed Loop Systems (Both Continuous (s) and Discrete (z) Domains) for LBW and PP Controllers. (H)

Axes	Controllers	BW [rad/s]	P.O. [%]	Rise Time [s]				
Continuous (s) Domain								
X Axis	LBW	206.1182	22.1096	0.0091				
	PP	126.2081	4.5811	0.0170				
Y Axis	LBW	202.5180	17.2006	0.0095				
	PP	126.2089	4.5750	0.0170				
	Di	iscrete (z) Don	nain					
X Axis	LBW	207.3230	22.3750	0.0090				
	pp	126.7781	4.5988	0.0169				
Y Axis	LBW	203.8057	17.4198	0.0095				
	pp	126.7781	4.5988	0.0169				

The pole placement controller has a lower bandwidth and percent overshoot and a higher rise time compared to the low bandwidth controller.

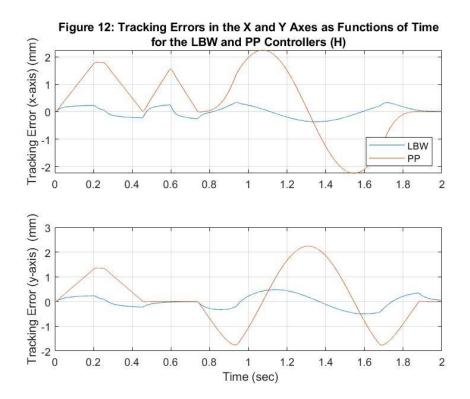
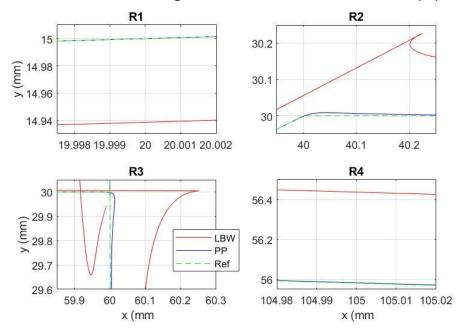
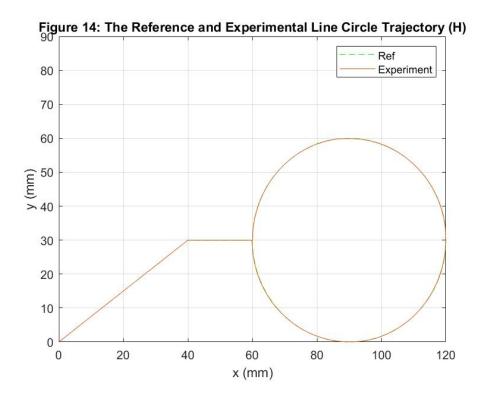
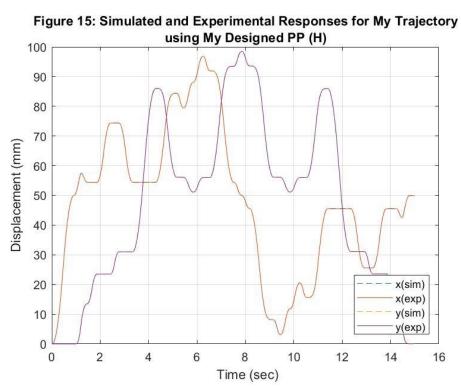


Figure 13: Tool Motion for Reference Toolpath and Simulations using the LBW and PP Controllers (H)



The tracking error of the pole placement controller is worse than the low bandwidth controller, but the pole placement controller has a much lower contouring error.





Using my designed pole placement controller on my generated trajectory, the simulated and experimental result are practically equal as show in the above figure.

#### Conclusion

In this lab, I learnt how to generate trajectories for a trapezoidal acceleration profile. I was able to understand how to evaluate the tracking and contouring error on the toolpath accuracy. Furthermore, I was able to get more practice designing different types of controllers, and I was able to learn the importance of considering mixed dynamics when designing controllers. I learnt how different controllers affect the contouring and tracking error, and how the maximum federate affects the performance/accuracy of the toolpath. Finally, I was able to learn how the simulation differed from the experimental results. Ultimately, I was able to learn that contouring error is the most important consideration in the industry, although tracking error has its use.

#### Appendix: MATLAB Script (Lab only) and Simulink Block Diagrams

Figure 16: Lead-Lag-Integrator controller simulation

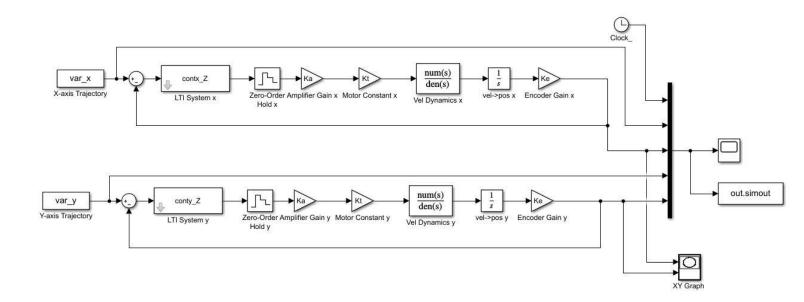
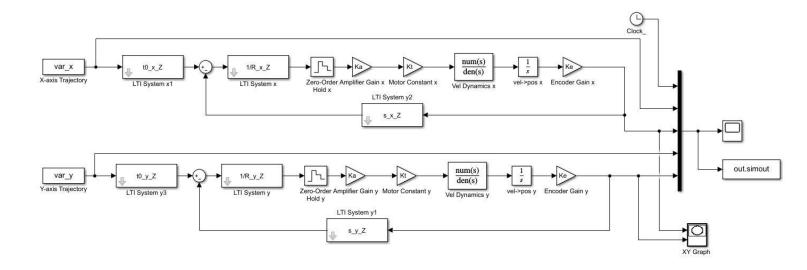


Figure 17: Pole Placement controller simulation



```
%% Matlab Code for Project 3
% LAB
% MECH 541
% University of British Columbia
% Jonas Chianu
% 31298391
% jchianu@student.ubc.ca
% December 2020
close all; bdclose all;
Project3 Prelab;
close all; bdclose all;
%% Part E - Effect of Bandwidth
%% Part E.1
xe_L=xr-xa_L; ye_L=yr-ya_L;
xe H=xr-xa H; ye H=yr-ya H;
xe_M=xr-xa_M; ye_M=yr-ya_M;
figure(1);
subplot(2,1,1);
plot(tr,xe L,tr,xe H);
title({ 'Figure 1: Tracking Errors in the X and Y Axes as Functions of Time'
    ,'for the LBW and HBW Controllers (E.1)'});
ylabel('Tracking Error (x-axis) (mm)');
legend('LBW','HBW','Location','SouthEast');
grid();
subplot(2,1,2);
plot(tr,ye L,tr,ye H);
xlabel(' Time (sec)'); ylabel('Tracking Error (y-axis) (mm)');
grid();
%% Part E.2
figure(2);
sqtitle({ 'Figure 2: Tool Motion for Reference Toolpath and',
    'Simulations using Three Cases of Control System (E.2)'});
subplot(2,2,1);
plot(xa L, ya L, 'r', xa H, ya H, 'g', xa M, ya M, 'b', xr, yr, '--');
title('R1')
ylabel('y (mm)');
axis([19.94 20.13 14.84 15.05])
grid;
subplot(2,2,2);
plot(xa L, ya L, 'r', xa H, ya H, 'g', xa M, ya M, 'b', xr, yr, '--');
title('R2')
axis([39.95 40.25 29.95 30.25])
grid;
```

```
subplot(2,2,3);
plot(xa L, ya L, 'r', xa H, ya H, 'g', xa M, ya M, 'b', xr, yr, '--');
title('R3')
xlabel(' x (mm'); ylabel('y (mm)');
legend('LBW','HBW','MD','Ref');
axis([59.85 60.3 29.6 30.05])
grid;
subplot(2,2,4);
plot(xa L, ya L, 'r', xa H, ya H, 'g', xa M, ya M, 'b', xr, yr, '--');
title('R4')
xlabel(' x (mm');
axis([104.97 105.05 55.95 56.5])
grid;
%% Part E.3
i = find(xr > 19, 1, 'first');
iend= find(xr > 20, 1, 'first');
Ce max=0;
for k=i:iend
    leR1 = sqrt(xe_L(k)^2+ye_L(k)^2);
    alpha = atan(ye L(k)/xe L(k));
    theta = atan(3/4);
    beta = alpha - theta;
    Ce = leR1*sin(beta);
    if Ce>Ce max
        Ce max=Ce;
    end
end
Ce max;
%% Part F - Effect of Maximum Feedrate
%% Part F.1
%The Displacement, Feedrate, and Tangential Acceleration Profiles
figure(3)
subplot(3,1,1)
plot(t fast, l fast)
title({ 'Figure 3 : The Displacement, Feedrate and Tangential Acceleration',
    'as Functions of Time (F=250 mm/s) (F.1)'})
ylabel('Displacement (mm)');
grid();
subplot(3,1,2)
plot(t fast, f fast)
ylabel('Velocity (mm/s)');
grid();
```

```
subplot(3,1,3)
plot(t fast,a fast)
xlabel(' Time (sec)'); ylabel('Acceleration (mm/s^2)');
grid();
%The Axis Position, Velocity, and Acceleration Profiles
figure (4)
subplot(3,1,1)
plot(t fast,x fast,t fast,y fast)
title({ 'Figure 4: The Axis Position, Velocity, and Acceleration Commands',
    'as Functions of Time (F=250 mm/s) (F.1)'})
ylabel('Axis Position (mm)');
legend('x','y','Location','NorthEast')
grid();
subplot(3,1,2)
plot(t fast,dx fast,t fast,dy fast)
ylabel('Axis Velocity (mm/s)');
grid();
subplot(3,1,3)
plot(t_fast,dxdx_fast,t_fast,dydy_fast)
xlabel(' Time (sec)'); ylabel('Axis Acceleration (mm/s^2)');
grid();
%% Part F.2
var_x=[transpose(t_fast) transpose(x_fast)];
var y=[transpose(t fast) transpose(y fast)];
contx Z=LLI L x Z;
conty_Z=LLI_L_y_Z;
stop time=2;
out L=sim('PartC1.slx');
bdclose all;
xr Fast = out L.simout.signals.values(:,2);
yr Fast = out L.simout.signals.values(:,4);
xa L Fast = out L.simout.signals.values(:,3);
ya L Fast = out L.simout.signals.values(:,5);
xe L Fast=xr Fast-xa L Fast; ye L Fast=yr Fast-ya L Fast;
figure(5);
subplot(2,1,1);
plot(tr,xe L,tr,xe L Fast);
title({ 'Figure 5: Tracking Errors in the X and Y Axes as Functions of Time'
,'for High and Low Feedrate Trajectories using a LBW Controller (F.2)'});
ylabel('Tracking Error (x-axis) (mm)');
legend('Low F', 'High F', 'Location', 'SouthEast');
grid();
```

```
subplot(2,1,2);
plot(tr,ye L,tr,ye L Fast);
xlabel(' Time (sec)'); ylabel('Tracking Error (y-axis) (mm)');
grid();
%% Part F.3
figure(6);
sgtitle({ 'Figure 6: Tool Motion for Reference Toolpath and',
    'Simulations (LBW) using The Low and High Feedrates (F.3)'});
subplot(2,2,1);
plot(xa L,ya L,xa L Fast,ya L Fast,xr,yr,'--g');
title('R1')
ylabel('y (mm)');
axis([19.995 20.005 14.93 15.01])
grid;
subplot(2,2,2);
plot(xa L,ya L,xa L_Fast,ya_L_Fast,xr,yr,'--g');
title('R2')
axis([39.95 40.25 29.95 30.25])
grid;
subplot(2,2,3);
plot(xa_L,ya_L,xa_L_Fast,ya_L_Fast,xr,yr,'--g');
title('R3')
xlabel(' x (mm'); ylabel('y (mm)');
legend('Low F', 'High F', 'Ref');
axis([59.85 60.3 29.6 30.05])
grid;
subplot(2,2,4);
plot(xa L, ya L, xa L Fast, ya L Fast, xr, yr, '--g');
title('R4')
xlabel(' x (mm');
axis([104.95 105.04 55.9 56.7])
grid;
%% Part G - Experiment vs Simulation
%% Part G.1
%%% Tracking Error
load('lli', 'lli');
t exp=lli.X.Data;
xe exp=lli.Y(3).Data-lli.Y(1).Data; ye exp=lli.Y(4).Data-lli.Y(2).Data;
figure(7);
subplot(2,1,1);
plot(tr,xe_L,t_exp(8616:28615)-0.8614,xe_exp(8616:28615));
title({ 'Figure 7: Tracking Errors in the X and Y Axes as Functions of Time'
```

```
,'for the Experiment and Simulation using the LBW Controller (G.1)'});
ylabel('Tracking Error (x-axis) (mm)');
legend('Simulation', 'Experiment', 'Location', 'SouthEast');
grid();
subplot(2,1,2);
plot(tr, ye L, t exp(8616:28615) - 0.8614, ye exp(8616:28615));
xlabel(' Time (sec)'); ylabel('Tracking Error (y-axis) (mm)');
grid();
%%% Toolpaths
figure(8);
sgtitle({'Figure 8: Tool Motion for Reference, Simulated',
    ' and Experimental Toolpaths using the LBW Controller (G.1)'});
subplot(2,2,1);
plot(xa L, ya L, lli.Y(1).Data, lli.Y(2).Data, xr, yr, '--g');
title('R1')
ylabel('y (mm)');
axis([19.995 20.005 14.93 15.01])
grid;
subplot(2,2,2);
plot(xa L, ya L, lli.Y(1).Data, lli.Y(2).Data, xr, yr, '--g');
title('R2')
axis([39.95 40.25 29.95 30.25])
grid;
subplot(2,2,3);
plot(xa L, ya L, lli.Y(1).Data, lli.Y(2).Data, xr, yr, '--g');
title('R3')
xlabel(' x (mm'); ylabel('y (mm)');
legend('Simulation', 'Experiment', 'Ref');
axis([59.85 60.3 29.6 30.05])
grid;
subplot(2,2,4);
plot(xa L, ya L, lli.Y(1).Data, lli.Y(2).Data, xr, yr, '--g');
title('R4')
xlabel(' x (mm');
axis([104.95 105.04 55.9 56.5])
grid;
%% Part G.2
var x=[transpose(traj.t) transpose(traj.x)];
var y=[transpose(traj.t) transpose(traj.y)];
contx Z=LLI L x Z;
conty Z=LLI L y Z;
stop time=15;
out L s=sim('PartC1.slx');
```

```
bdclose all;
tr s = out L s.simout.time;
xr s = out L s.simout.signals.values(:,2);
yr s = out L s.simout.signals.values(:,4);
xa s = out L s.simout.signals.values(:,3);
ya s = out L s.simout.signals.values(:,5);
load('lli mytraj.mat', 'lli');
%%% Toolpaths
figure (9)
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
'with Locations of Critical Regions (G.2)'});
legend('Simulation','Experiment','Ref','Location','SouthEast');
axis([0 100 0 100])
xlabel('x (mm)'); ylabel('y (mm)');
grid;
dim = [.49 .86 .05 .05]; str = 'R6';
annotation('textbox',dim,'String',str)
dim = [.72 .53 .05 .05]; str = 'R5';
annotation('textbox',dim,'String',str)
dim = [.68 .71 .05 .05]; str = 'R4';
annotation('textbox', dim, 'String', str)
dim = [.52 .2 .05 .05]; str = 'R3';
annotation('textbox',dim,'String',str)
dim = [.56 .14 .05 .05]; str = 'R2';
annotation('textbox',dim,'String',str)
dim = [.49 .1 .05 .05]; str = 'R1';
annotation('textbox',dim,'String',str)
%%% Toolpaths (Zoomed views of critical regions)
figure(10);
sqtitle({'Figure 10: Tool Motion of My Trajectory for Reference, Simulated'
    ,' and Experimental Toolpaths using the LBW Controller (G.2)'});
subplot(3,2,1);
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
title('R1')
ylabel('y (mm)');
legend('Simulation', 'Experiment', 'Ref');
axis([48.5 51.5 -0.05 0.02])
grid;
subplot(3,2,2);
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
title('R2')
axis([57.4 57.65 6.5 9])
grid;
subplot(3,2,3);
```

```
plot(xa s, ya s, lli.Y(1).Data, lli.Y(2).Data, xr s, yr s, '--g');
title('R3')
ylabel('y (mm)');
axis([54.36 54.415 13.54 13.72])
grid;
subplot(3,2,4);
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
title('R4')
axis([74.999 75.001 77.86 78.04])
grid;
subplot(3,2,5);
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
title('R5')
xlabel(' x (mm'); ylabel('y (mm)');
axis([79.34 79.5 56.035 56.07])
grid;
subplot(3,2,6);
plot(xa s,ya s,lli.Y(1).Data,lli.Y(2).Data,xr s,yr s,'--g');
title('R6')
xlabel(' x (mm');
axis([49.9 50.01 98.52 98.64])
grid;
%%Contouring error plot
xe s=xr s-xa s; ye s=yr s-ya s;
t exp=lli.X.Data;
xe exp=lli.Y(3).Data-lli.Y(1).Data; ye exp=lli.Y(4).Data-lli.Y(2).Data;
theta = atan(3/4);
for k=1:length(xe s)
    leR1 = sqrt((xe_s(k))^2+(ye_s(k))^2);
    alpha = atan(ye s(k)/xe s(k));
    beta = alpha - theta;
    Ce s(k) = leR1*sin(beta);
end
for k=1:length(xe exp)
    leR1 = sqrt((xe exp(k))^2+(ye exp(k))^2);
    alpha = atan(ye exp(k)/xe exp(k));
    beta = alpha - theta;
    Ce exp(k) = leR1*sin(beta);
end
figure(11)
plot(tr_s,Ce_s,t_exp(17138:17138+length(xe_s))-1.7136, ...
    Ce \exp(17138:17138+length(xe s)))
```

```
title({ 'Figure 11: Contouring Error as a Function of Time',
    'for the Experiment and Simulation using the LBW Controller (G.2)'});
xlabel(' Time (sec)'); ylabel('Contouring Error (mm)');
legend('Simulation','Experiment','Location','SouthEast');
grid();
%% Part H - Pole Placement Controller
%%%Simulated Bandwidth, Rise Time, and Overshoot
stepinfo(Gcl P x);
bandwidth(Gcl P x);
stepinfo(Gcl P x Z);
bandwidth(Gcl P x Z);
stepinfo(Gcl P y);
bandwidth(Gcl P y);
stepinfo(Gcl P y Z);
bandwidth (Gcl P y Z);
%%%Tracking Errors
xe P=xr-xa P; ye P=yr-ya P;
figure(12);
subplot(2,1,1);
plot(tr,xe L,tr,xe P);
title({ 'Figure 12: Tracking Errors in the X and Y Axes as Functions of Time'
    ,'for the LBW and PP Controllers (H)'});
ylabel('Tracking Error (x-axis) (mm)');
legend('LBW', 'PP', 'Location', 'SouthEast');
grid();
subplot(2,1,2);
plot(tr,ye L,tr,ye P);
xlabel(' Time (sec)'); ylabel('Tracking Error (y-axis) (mm)');
grid();
%%%Contouring Performance
figure (13);
sgtitle({ 'Figure 13: Tool Motion for Reference Toolpath and',
    'Simulations using the LBW and PP Controllers (H)'});
subplot(2,2,1);
plot(xa L, ya L, 'r', xa P, ya P, 'b', xr, yr, 'g--');
title('R1')
ylabel('y (mm)');
axis([19.9975 20.002 14.93 15.01])
grid;
subplot(2,2,2);
```

```
plot(xa L, ya L, 'r', xa P, ya P, 'b', xr, yr, 'g--');
title('R2')
axis([39.95 40.25 29.95 30.25])
grid;
subplot(2,2,3);
plot(xa L, ya L, 'r', xa P, ya P, 'b', xr, yr, 'g--');
title('R3')
xlabel(' x (mm'); ylabel('y (mm)');
legend('LBW', 'PP', 'Ref');
axis([59.85 60.3 29.6 30.05])
grid;
subplot(2,2,4);
plot(xa L, ya L, 'r', xa P, ya P, 'b', xr, yr, 'g--');
title('R4')
xlabel(' x (mm');
axis([104.98 105.02 55.95 56.5])
grid;
%%%Experimental Line Circle Trajectory
load('pp', 'pp');
t exp=pp.X.Data;
xa exp=pp.Y(1).Data; ya exp=pp.Y(2).Data;
figure (14)
plot(xr,yr,'--g',xa exp,ya exp);
title('Figure14: The Reference and Experimental Line Circle Trajectory(H)');
legend('Ref','Experiment','Location','NorthEast');
ylim([0 90])
xlabel('x (mm)'); ylabel('y (mm)');
grid;
%%% Experimental vs Simulated Responses for my Designed PP Controller on my
%%% Generated Trajectory
var x=[transpose(traj.t) transpose(traj.x)];
var y=[transpose(traj.t) transpose(traj.y)];
t0 y Z=t0 y Z mytraj;
R_y_Z=R_y_Z_mytraj;
s_y_Z=s_y_Z_mytraj;
stop time=15;
out P s=sim('PartH.slx');
bdclose all;
tr s = out P s.simout.time;
xr s = out P s.simout.signals.values(:,2);
yr s = out P s.simout.signals.values(:,4);
xa P s = out P s.simout.signals.values(:,3);
ya P s = out P s.simout.signals.values(:,5);
```