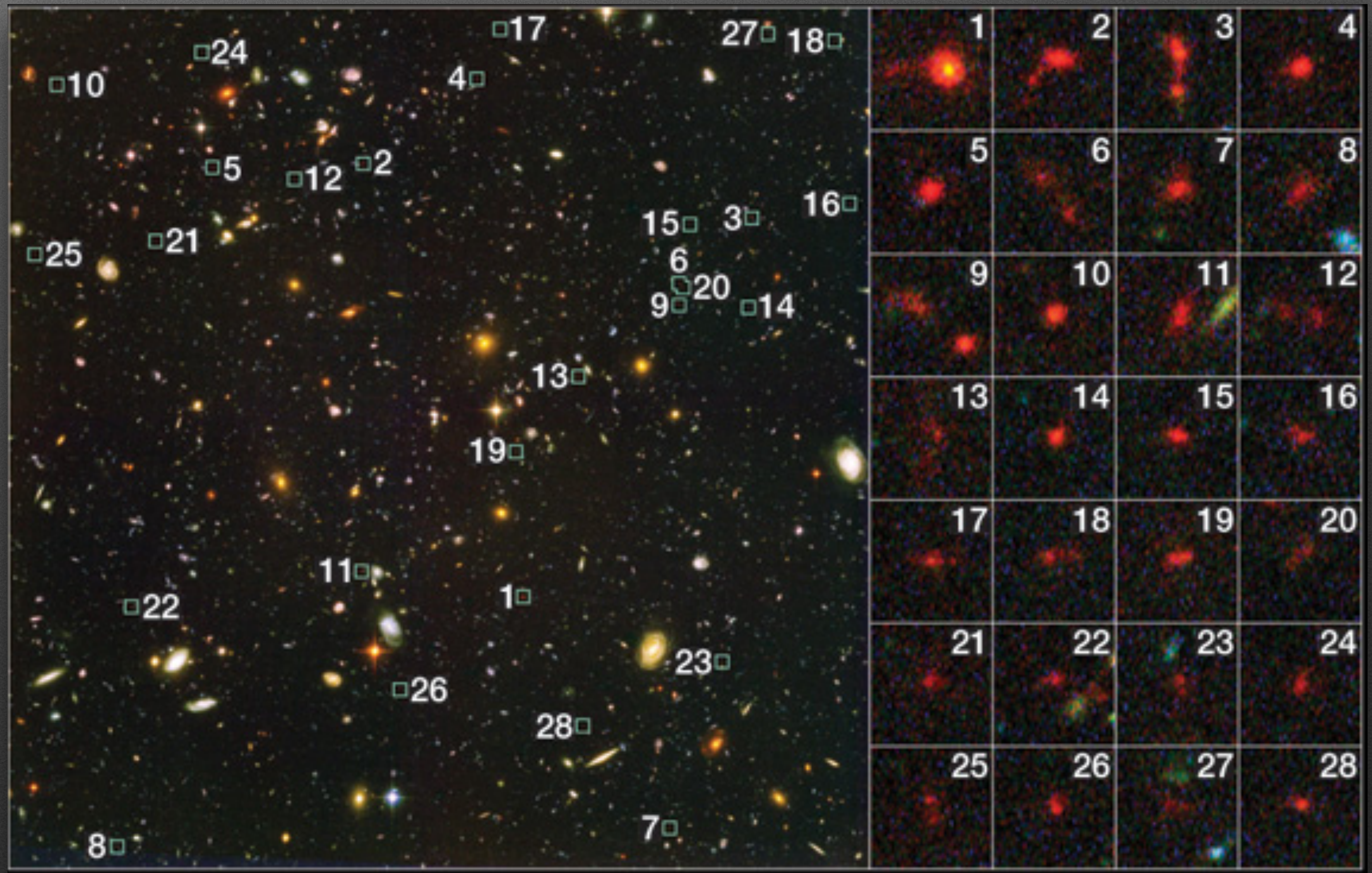
Simulating the Formation of Structure in the Universe

Oliver Clipsham

Context - Motivation

- Physical cosmology is the study of origins and evolution of the Universe
- Modern focus on obtaining confirmation that observations are matched by theory
- Current state of the art simulations have a sophistication which prohibits accessibility
- In order to test variations in cosmological parameters a simpler, more compact package is required

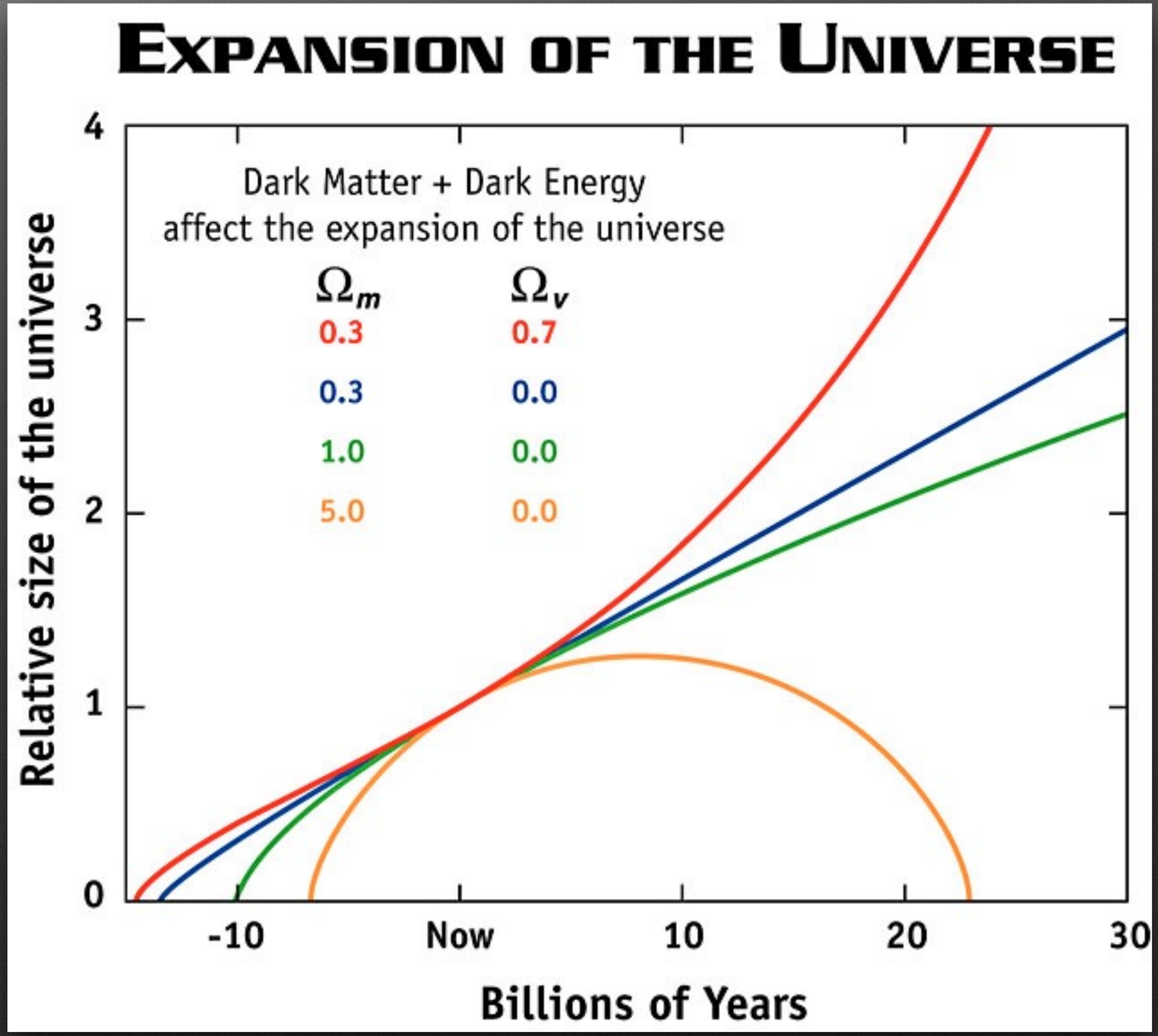


Distant galaxies in the Hubble Ultra Deep Field
Hubble Space Telescope • Advanced Camera for Surveys

Context - Evolution of the Universe

- Widely accepted that the Universe is expanding, characterised by scale factor a
- Evolution of a differs according to the assumed composition of the Universe
- Can express a in terms of the Hubble constant ($v = Hd$): $H(t) \equiv \dot{a}/a$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} = \frac{\Omega_{m,0}}{a^3} + (1 - \Omega_{m,0})$$

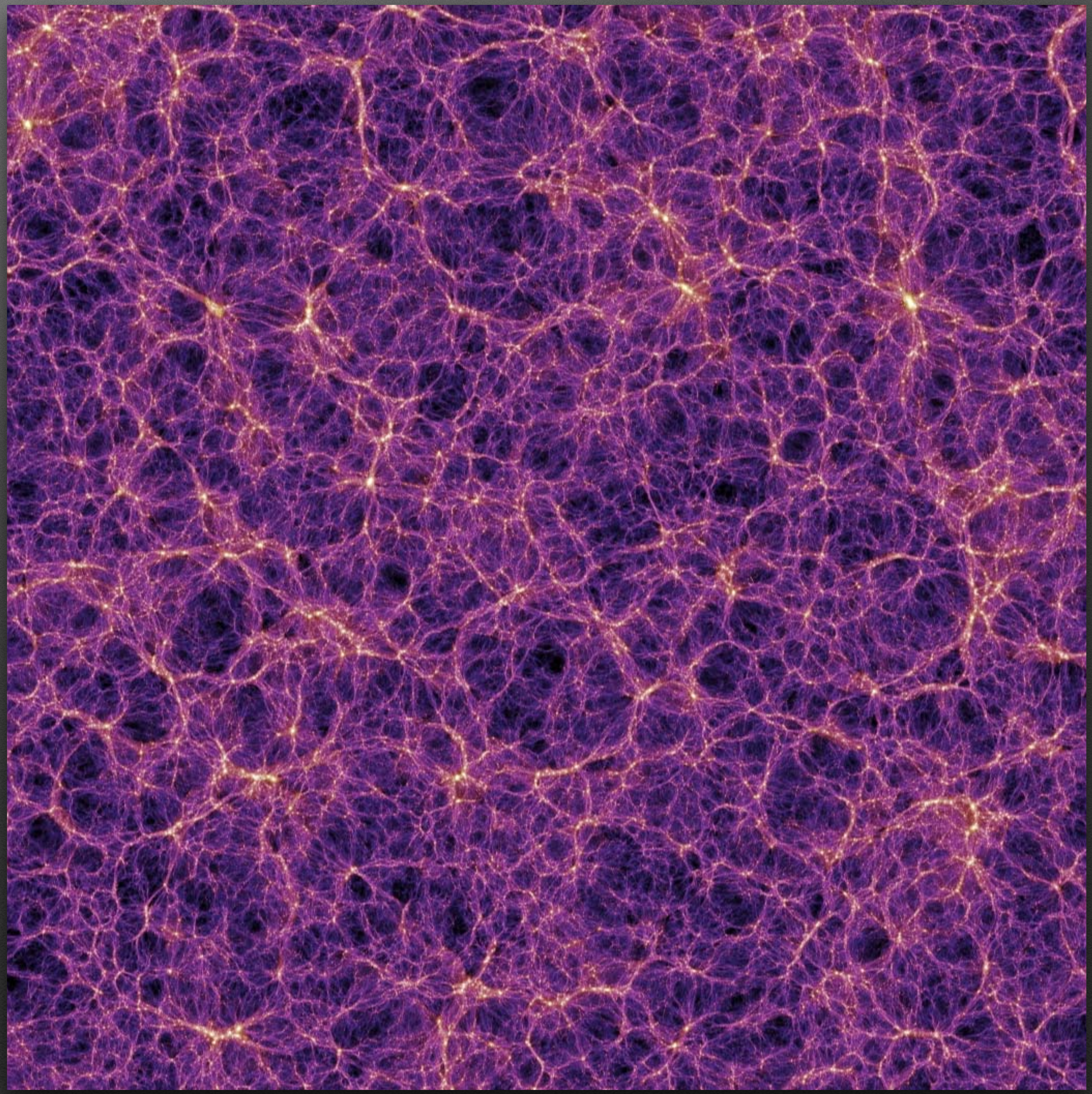


Context - Why are simulations required?

- Density contrast defined as difference between density of one area in space and the average density of all space
- As ratio of density contrast to average density exceeds one, evolution of density field becomes non-linear
- Non-linear solution prevents analytical solution, necessitating computational methods

Context - State of the Field

- Current state of the art is the “Millennium Simulation”
- Simulated a “Lambda-CDM” Universe, $\Omega_m = 0.25$, $\Omega_b = 0.045$, $\Omega_\Lambda = 0.75$
- 2160^3 individual particles, periodic box volume of size $500h^{-1}$ Mpc
- Starting at redshift $z = 127$, ran for 11,000 time steps
- Produced full particle data for 64 steps, nearly 20 terabytes of data
- Produces catalogue of 2×10^7 galaxies



Particle-Mesh Simulation - Solving Poisson Equation

- Gravitational Poisson equation describes gravitational potential at a point in space given the mass density of matter in space

- Given periodic boundary conditions, Fast Fourier Transforms (FFTs) are the most efficient way of solving this

- In Fourier space, the Poisson equation becomes: $\tilde{\phi}(\underline{k}) = G(\underline{k})\tilde{\delta}(\underline{k})$

- *G(k)* is the Greens function, which for the grid discretisation becomes:

$$G(\underline{k}) = -\frac{3\Omega_0}{8a}\left[\sin^2\left(\frac{k_x}{2}\right) + \sin^2\left(\frac{k_y}{2}\right) + \sin^2\left(\frac{k_z}{2}\right)\right]^{-1}$$

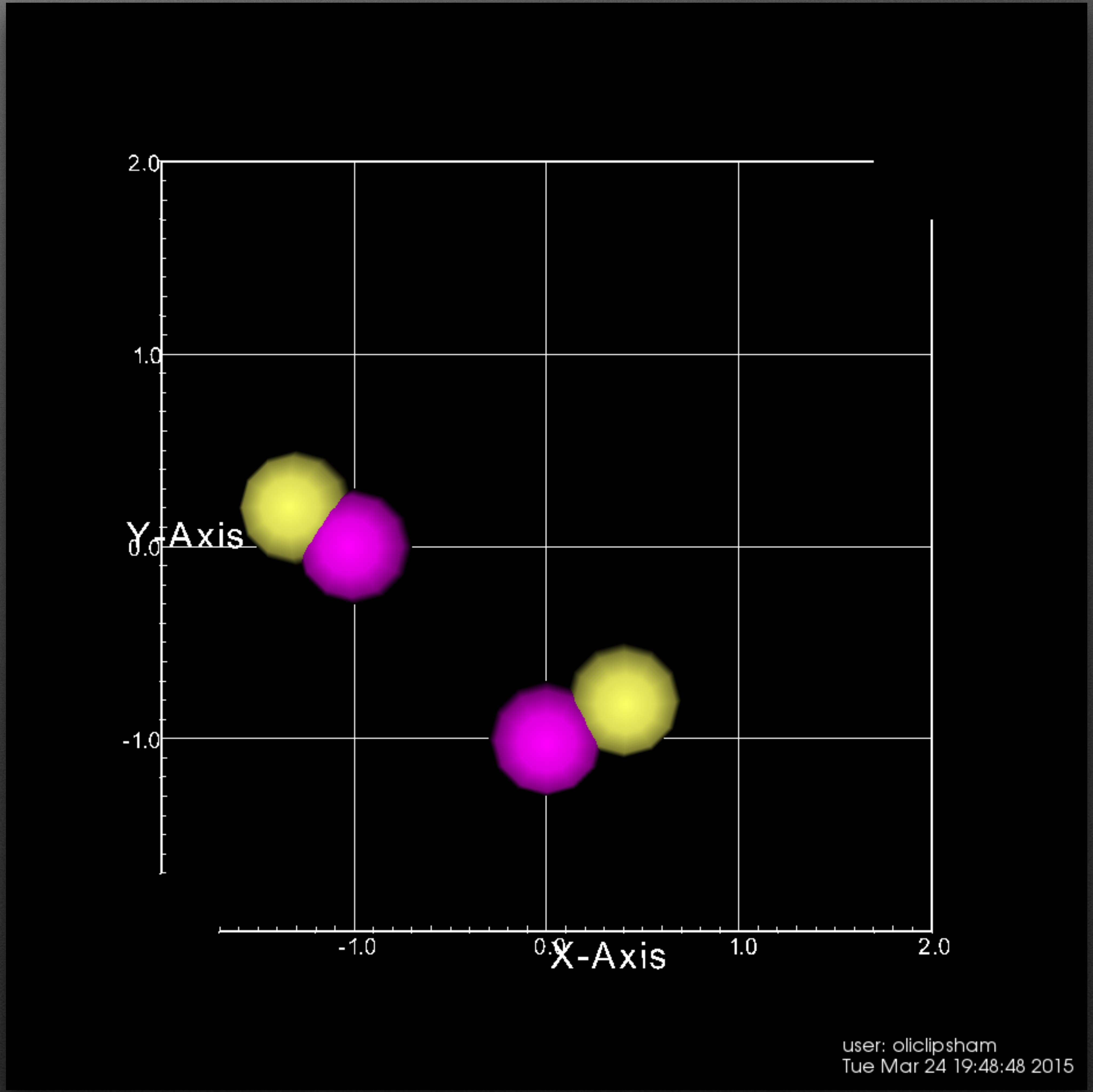
Particle-Mesh Simulation - Discretising Density Field

- Use of FFTs is limited to a discretised system, however simulation particles move with positions described continuously

- Use the “Nearest Grid Point” (NGP) density assignment scheme, treating particles as delta function in space

- Assigns the entire mass of each individual particle to the nearest point on the discrete grid

- Similar consideration made when calculating particle accelerations from potential field



Particle-Mesh Simulation - Use of Fast Fourier Transforms (FFTs)

- FFTs represent the most efficient computational method of solving a Laplacian equation
- Performance significantly increased over iterative methods
- “Fastest Fourier Transform in the West” (FFTW) package used with multi-threaded support, further increased the efficiency of the Fourier Transform method
- Special consideration given to packing of modes and normalisation of the inverse multi-dimensional Fourier Transform

Particle-Mesh Simulation - Momentum/Position Integrator

- Uses second-order leapfrog integrator, assuming constant step in *a*

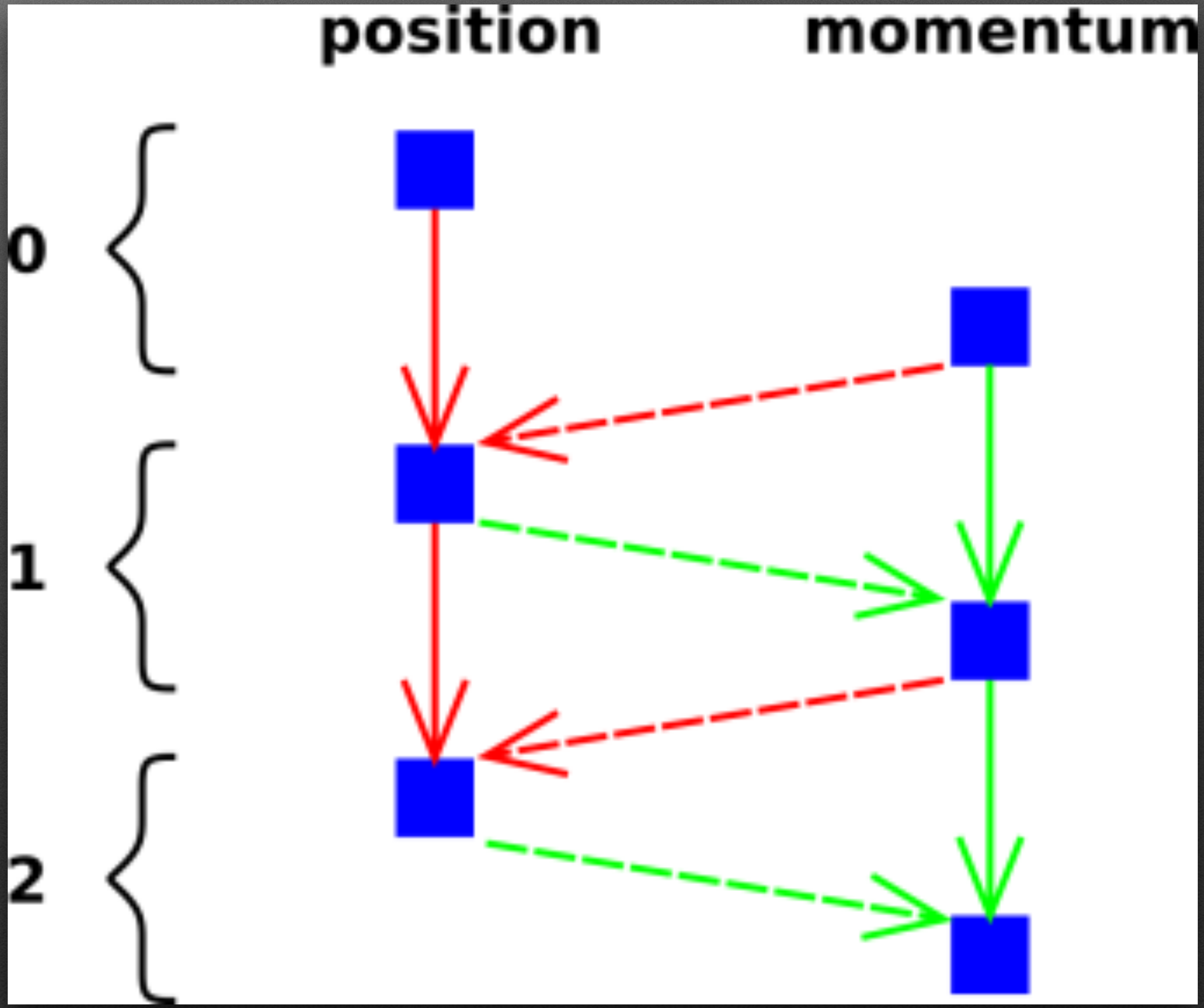
- Particle momenta are calculated using the acceleration of the particle given its position

- Represents the momentum halfway between time steps

$$\underline{\tilde{p}}_{n+1/2} = \underline{\tilde{p}}_{n-1/2} + f(a_n)\underline{\tilde{g}}_n \Delta a$$

- Particle positions are incremented using these momenta

$$\underline{\tilde{x}}_{n+1} = \underline{\tilde{x}}_n + a_{n+1/2}^{-2} f(a_{n+1/2})\underline{\tilde{p}}_{n+1/2} \Delta a$$

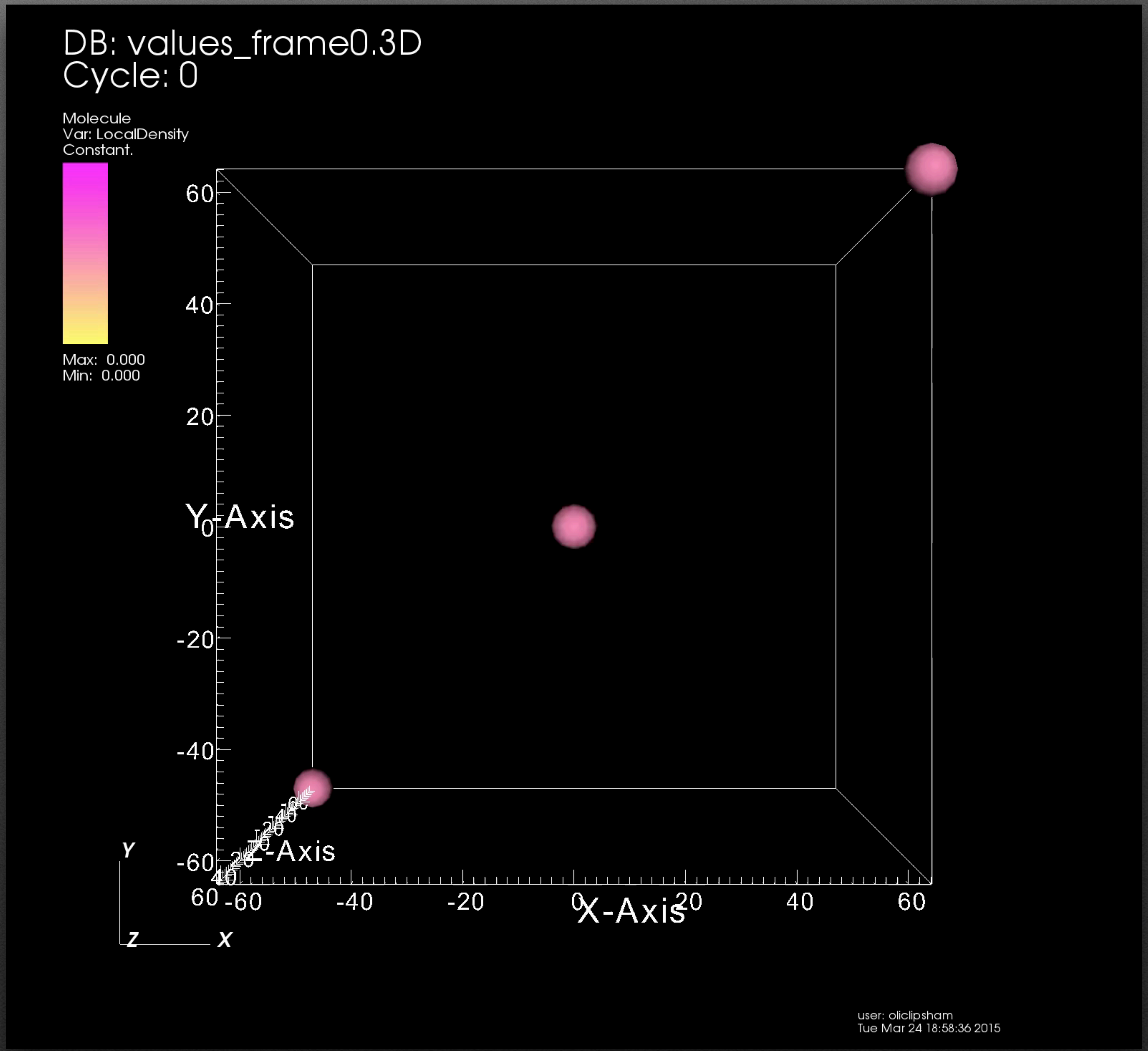


Results - Inclusion of Cosmology

- Take account of expanding Universe through use of co-moving coordinates
- Momentum and position are both integrated using a function of the scale factor

$$f(a) \equiv H_0 / \dot{a} = [a^{-1}(\Omega_{m,0} + \Omega_{k,0} a + \Omega_{\Lambda,0} a^3)]^{-1/2}$$

- Implementation demonstrated by movement of single particle in one dimension as time advances
- Choice of $\Omega_0 = 1$ has implications for the linear growth function $D_+(a) = a/a_0$ and the mass of each particle

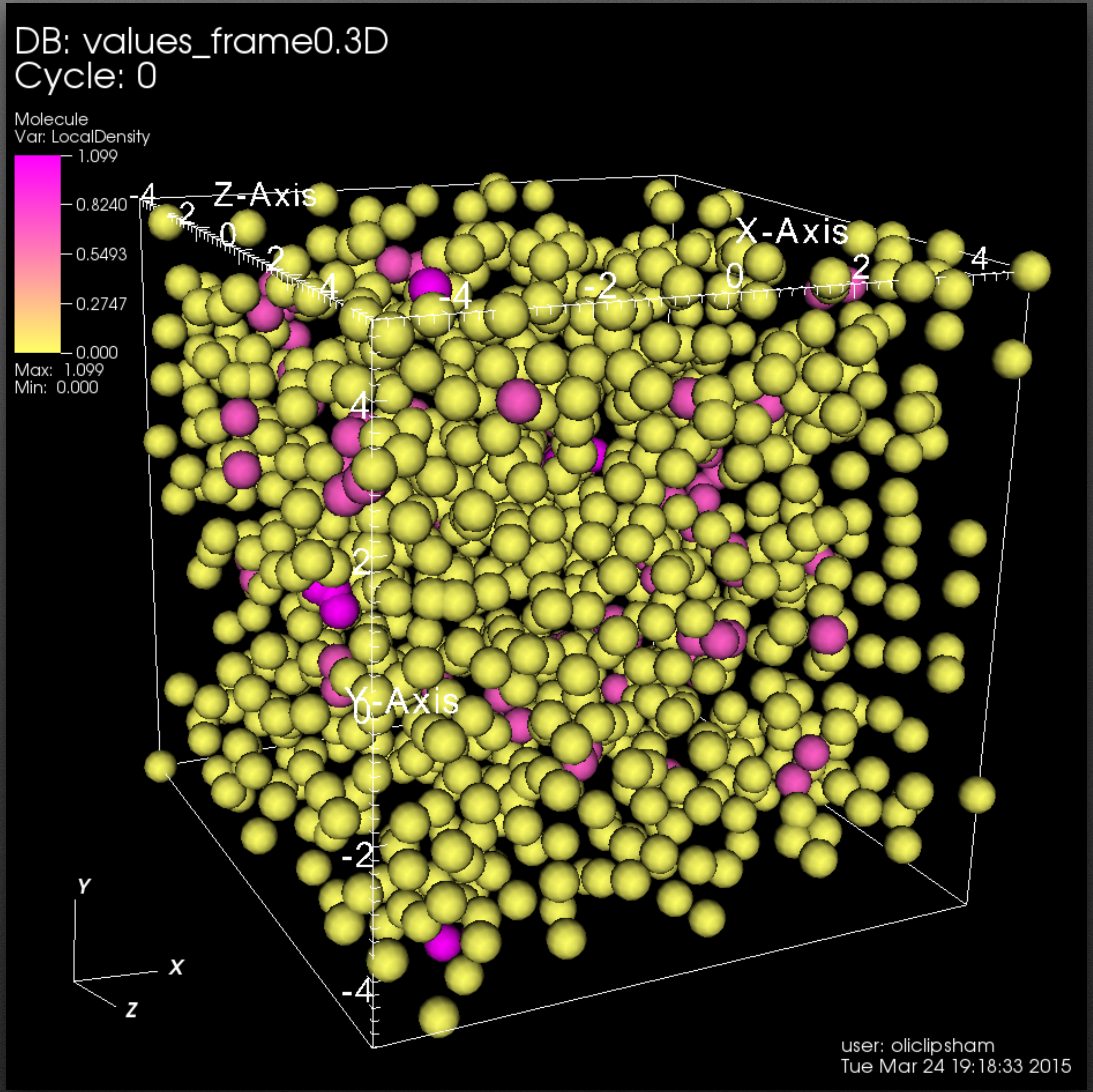


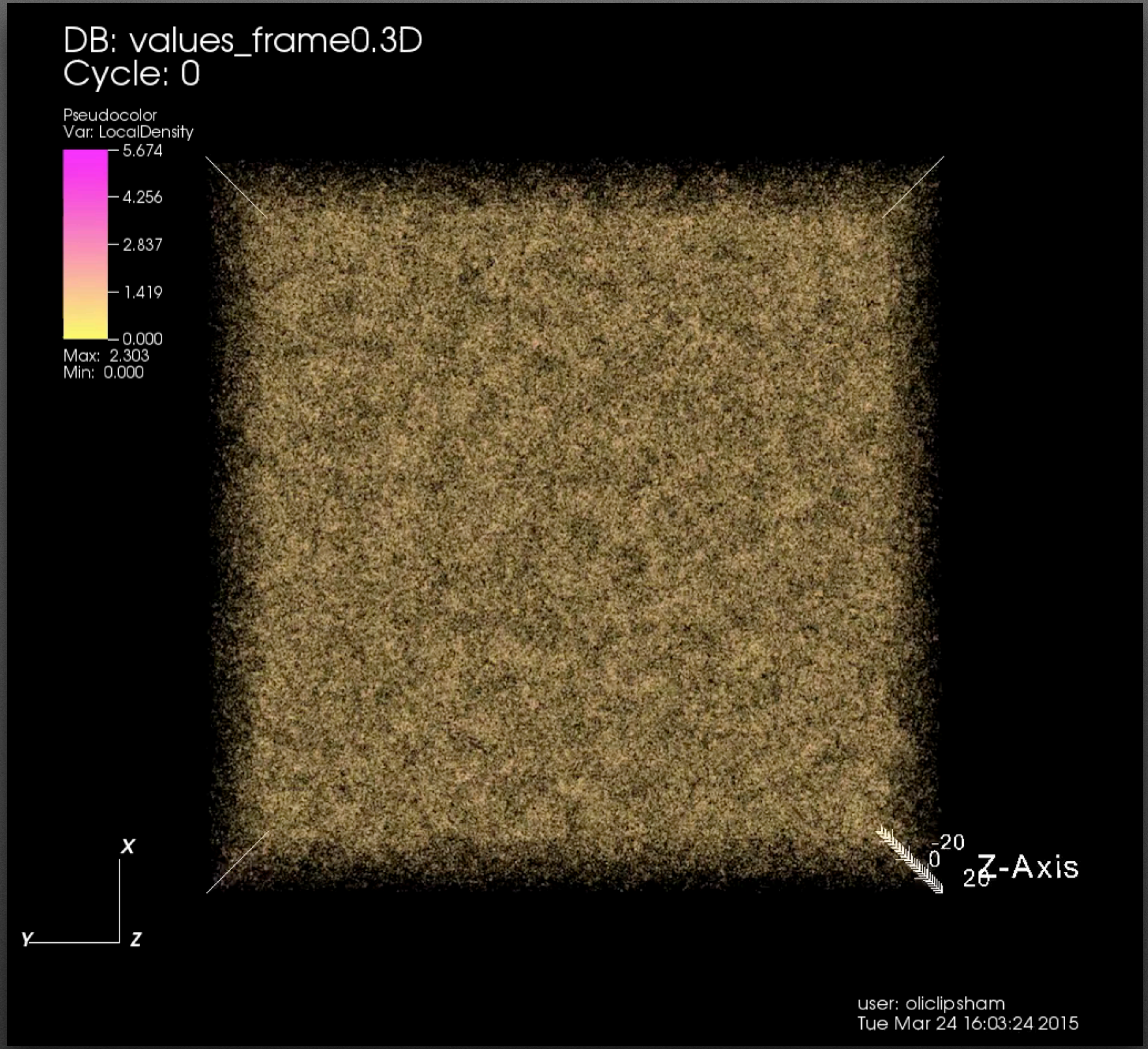
Results - Setting the Initial Conditions

- Utilise the Zeldovich approximation, a first-order Lagrangian collapse model
- Perturb particles from grid positions of uniform density

$$\underline{S}(\underline{q}) = \alpha \sum_{k_{x,y,z}=-k_{max}}^{k_{max}} i\underline{k}c_k \exp(i\underline{k}\cdot\underline{q})$$

- Real and imaginary parts of Fourier coefficients are independent Gaussian random numbers, mean zero, dispersion $\sigma^2 = P(k)/k^4$





Results - Example Simulation

Using a grid of particles 128x128x128, with physical dimensions 200x200x200 Mpc. Each particle represents 5.22×10^{11} solar masses.

Results - Potential Uses

- Can quantify the nature of structure formation using the density power spectrum
- Ultimately compare these spectra to those from observational evidence
- Agreements between simulated and observed spectra provide clues as to correct cosmology
- Used to develop our understanding of the standard cosmological model

Questions Please

References

- http://www.av8n.com/physics/img48/integrator-symplectic-stagger.png
- http://inspirehep.net/record/836873/files/lss800.png
- http://map.gsfc.nasa.gov/media/990350/990350b.jpg
- J. J. Dalcanton, *18 Years of Science with the Hubble Space Telescope*, Nature **457**, 41 (2009)