Department of Mathematics	**Problem Set #3**
University of Toronto	**Deadline: Thursday, October 28, 6:10 p.m.**
MAT137Y, 2010-2011 Winter Session	Assignment Posted/Revised: October 25, 2010

Please be reminded of the following rules.

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number on your problem set will result in a zero on your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5×11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be staples in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to anybody who is in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severly by the Department of Mathematics and the Faculty of Arts & Science.

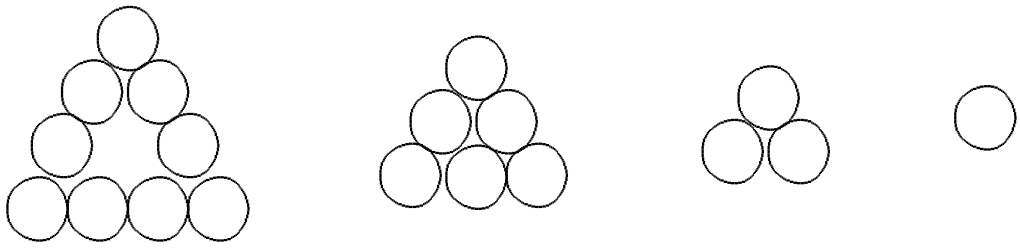
Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition).

1. SHE 1.2: 71, 73, 77
2. SHE 1.8: 1-13 (odd)
3. SHE 2.1: 3, 5, 9, 25, 31, 33, 43, 47
4. SHE 2.2: 5, 11, 13, 17, 23, 25, 35, 37, 39, 41, 45, 49, 53, 55

Required Problems. Hand in solutions to the problems listed below.

1. (a) Let n be an integer. Prove that if n^3 is even, then n is even.
   (b) Prove that $\sqrt[3]{2}$ is irrational. Mimic the proof that $\sqrt{2}$ is irrational and use part (a).
   (c) Suppose that two numbers, a and b, are irrational. Must the sum $a + b$ be irrational? Justify your answer with an appropriate proof or counterexample.
2. SHE 1.8: 4, 14

3. We can build a triangular pyramid out of balls by forming a triangle of balls for a base with n balls on each side of the triangle. Then in the "hollows" of that layer we can place balls forming a smaller triangle with $n - 1$ balls on a side. For instance, the layers of the pyramid in the case $n = 4$ are drawn below. We continue until we end up with just one ball in the top layer. Let P_n be the total number of balls in the pyramid whose base is a triangle with n balls on each side.
   (a) Calculate P_1, P_2, P_3, and P_4.
   (b) Find a formula in terms of n for P_n.
   (c) Use mathematical induction to prove that your formula is correct.



4. Solve the following limits algebraically.

(a) $\lim_{x\to -3} \dfrac{2x^2+7x+3}{x^2+4x+3}$

(b) $\lim_{x\to \frac{1}{2}} \dfrac{2x^2-5x+2}{2x^2+7x-4}$

(c) $\lim_{x\to 6} \dfrac{x^2-36}{\sqrt{2x+4}-4}$

The formal definition of limit is probably the hardest concept in calculus. We start with some exercises (5-8) which will help you to understand the definition, and then conclude with limits which you must prove using the formal definition. There are a number of resources available from the handouts page on the course website that you may find helpful while you're working through these problems.

5. How close to 3 do we have to take x so that $2x+4$ is within a distance of (a) 0.1 and (b) 0.01 from 10?

6. Sketch the graph of $y=\sqrt{x}$ for $x \geq 0$. Use the graph to find a number $\delta > 0$ such that

$$|\sqrt{x}-3| < \frac{1}{10} \quad \text{whenever} \quad |x-9| < \delta.$$

7. Find an upper bound M for $f(x) = \left|\dfrac{x+2}{x-8}\right|$ if $|x-7| < \frac{1}{2}$.

8. Suppose that $f(x) = 3x^2 - 2x + 4$, $a = 2$, and $L = 12$. Find a value for $\delta > 0$ so that $0 < |x-a| < \delta$ implies $|f(x) - L| < \frac{1}{100}$

9. Prove the following statements directly using the formal ϵ, δ definition of limit.

(a) $\lim_{x\to 2} -5x+7 = -3$

(b) $\lim_{x\to 2} \dfrac{2x-7}{x^2+3x+1} = -\dfrac{3}{11}$

(c) $\lim_{x\to 6} \sqrt{2x+4} = 4$

(d) $\lim_{x\to -1} \dfrac{-4x+6}{-3x+2} = 2$