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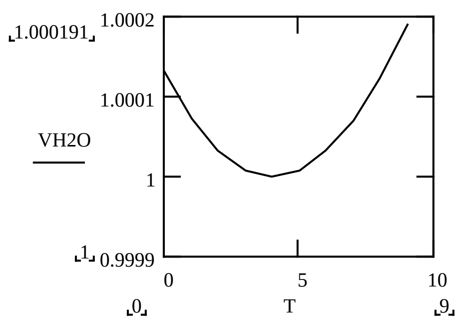
1.2 (a) Water is inappropriate as a thermometric fluid between 0°C and 10°C, since the volume is not a unique function of temperature in this range, i.e., two temperatures will correspond to the same specific volume,

$\hat{V}(T=1°\mathrm{C}) \sim V(T=7°\mathrm{C})$; $\hat{V}(T=2°\mathrm{C}) \sim V(T=6°\mathrm{C})$; etc.



$\left[T \text{ in } {}^{\mathrm{o}}\mathrm{C} \text{ and } \hat{V} \text{ in cc/g}\right]$

Consequently, while T uniquely determines, $\hat{V}$, $\hat{V}$ does not uniquely determine T.

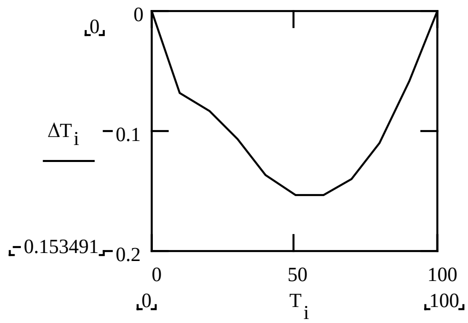
(b) Assuming that a mercury thermometer is calibrated at 0°C and 100°C, and that the specific volume of mercury varies linearly between these two temperatures yields

$$\hat{V}(T) = \hat{V}(0^{\mathrm{o}}\mathrm{C}) + \frac{\hat{V}(T=100^{\mathrm{o}}\mathrm{C}) - \hat{V}(T=0^{\mathrm{o}}\mathrm{C})}{100^{\mathrm{o}}\mathrm{C} - 0^{\mathrm{o}}\mathrm{C}}\left(T_{\mathrm{s}} - 0^{\mathrm{o}}\mathrm{C}\right)$$
$$= 0.0735560 + 0.000013421\, T_{\mathrm{s}} \qquad (*)$$

where T is the actual temperature, and T_{s} is the temperature read on the thermometer scale. At 10°C, $\hat{V}_{\mathrm{exp}}(T=10°\mathrm{C}) = 0.0736893$ cc/g. However, the scale temperature for this specific volume is, from eqn. (*) above

$$T_{\mathrm{s}} = \frac{\hat{V}_{\mathrm{exp}}(T) - 0.0735560}{1.3421\times10^{-5}} = \frac{0.0736893 - 0.0735560}{1.3421\times10^{-5}} = 9.932°\mathrm{C}$$

Thus, $T - T_{\mathrm{s}}$ at 10°C $= -0.068°\mathrm{C}$. Repeating calculation at other temperatures yields figure below.



The temperature error plotted here results from the nonlinear dependence of the volume of mercury on temperature. In a real thermometer there will also be an error associated with the imperfect bore of the capillary tube.

(c) When we use a fluid-filled thermometer to measure ΔT we really measure ΔL, where

$$\Delta L = \frac{\Delta V}{A} = \frac{M\left(\partial \hat{V}/\partial T\right)\Delta T}{A}$$

A small area *A* and a large mass of fluid *M* magnifies ΔL obtained for a given ΔT. Thus, we use a capillary tube (small *A*) and bulb (large *M*) to get an accurate thermometer, since $\left(\partial \hat{V}/\partial T\right)$ is so small.