Patterns of performance degradation during sleep restriction of long distance truck drivers

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Presentation of the case study

We are analysing the effect of sleep deprivation on reaction time of long distance truck drivers. There are 18 subjects in the dataset and for each subject, the reaction time was measured for 10 days. The subjects were allowed only a limited amount of sleep for these 10 subsequent days. Each subject's reaction time was measured several times on each day of the trial and an average was taken. Reaction time is measured with a psychomotor vigilance task (PVT), which measures the speed with which subjects respond to a visual stimulus.

Our research questions is: Is there any relation between reaction time and the number of days of sleep deprivation?

Exploratory analysis

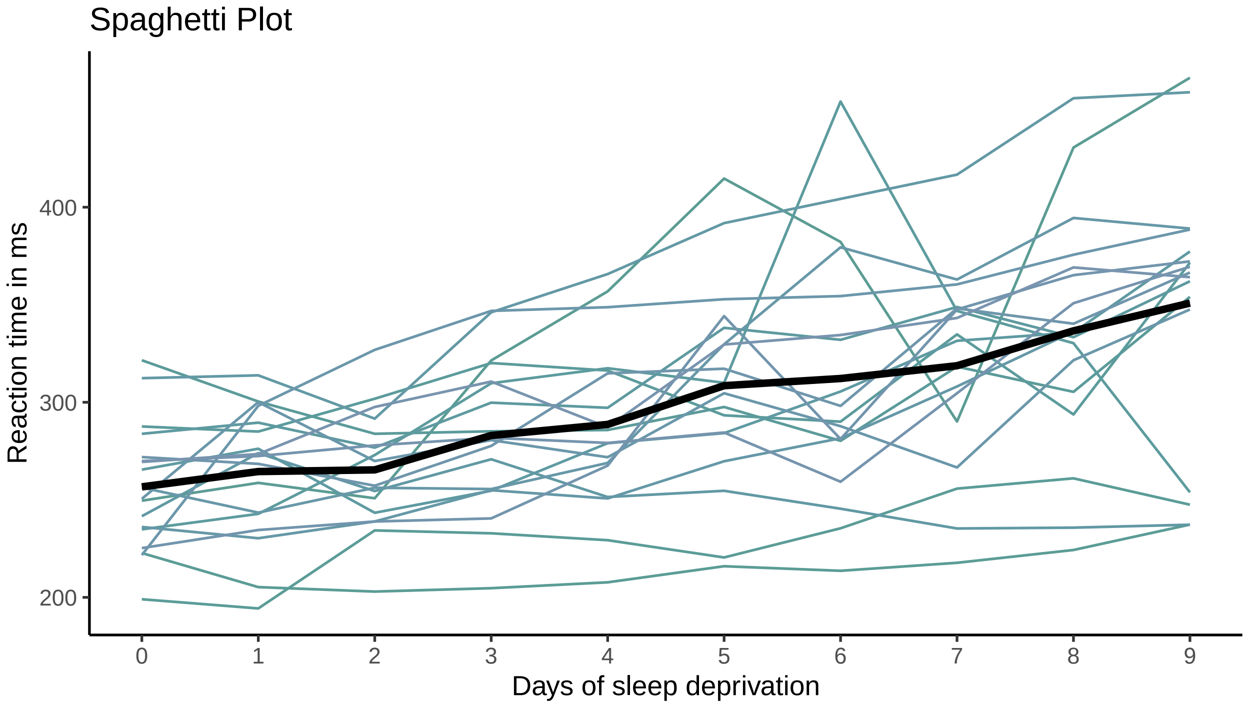
Reaction	Days	Subject
249.6	0	308
258.7	1	308
250.8	2	308
321.4	3	308
356.9	4	308
414.7	5	308
382.2	6	308
290.1	7	308
430.6	8	308
466.4	9	308
222.7	0	309
205.3	1	309

This dataset contains multiple measurements for each subject on consecutive days, with as response variable the continuous variable reaction time and explanatory variable days. Since there are 10 measurements for each subject, it is a longitudinal study. The dataset of 18 subjects is balanced and complete with an equal amount of measurements for each subject (i.e. no missing data).

Spaghetti Plot

To visualise the individual reaction times and how they compare to the mean, a spaghetti plot was created. This revealed that there was variation in intercepts or starting reaction times on day 0 between subjects. This variation between subjects increased with subsequent days.

For most subjects, the reaction time increased with the amount of days of sleep deprivation. This increase is also visible in the mean.



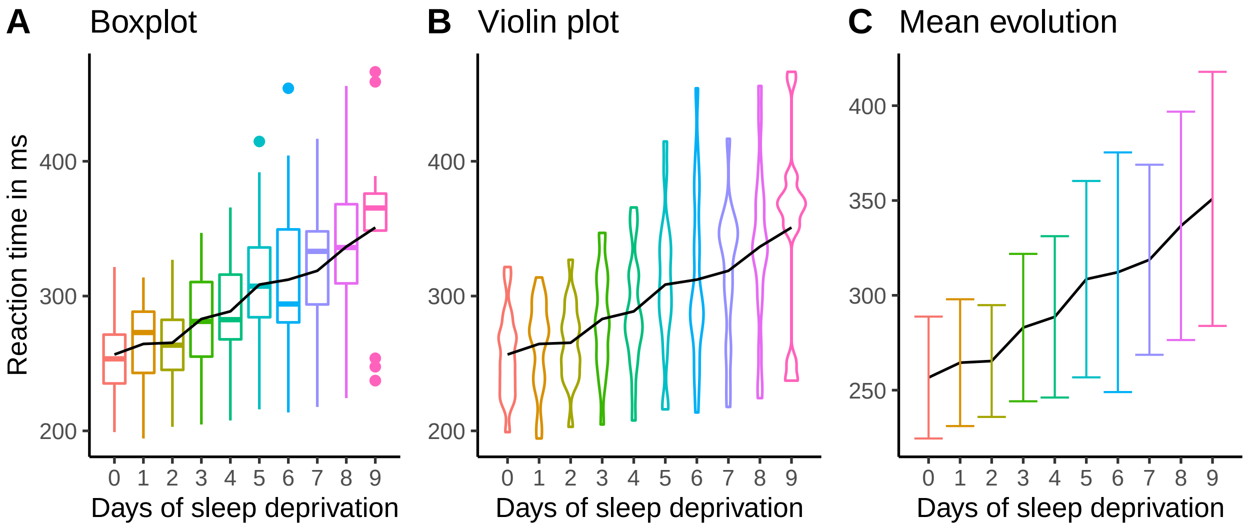
Boxplot, Violin plot and Mean evolution

The following boxplot (A) was created to get a quick summary of the dataset's characteristics. The mean and median seem to show a similar increase throughout the study. For day nr 5, 6 and 9, outliers are observed. We observe that the variance increases with an increase in days of sleep deprivation but the interquartile range appears to expand not as strongly as the minimum and maximum of the boxplot.

To put together, some subjects deviate more from the mean with an increase in days of sleep deprivation (see outliers on both sides) while most others stay around the mean (see slower increase in interquartile range).

The violin plot (B) supports the above observations of the distribution of the data around the mean with outliers.

To further support our previous findings, we looked at the mean evolution (C). Here, a trend of increasing reaction time with increasing number of days is also observed, together with an expanding standard deviation (see errorbars).

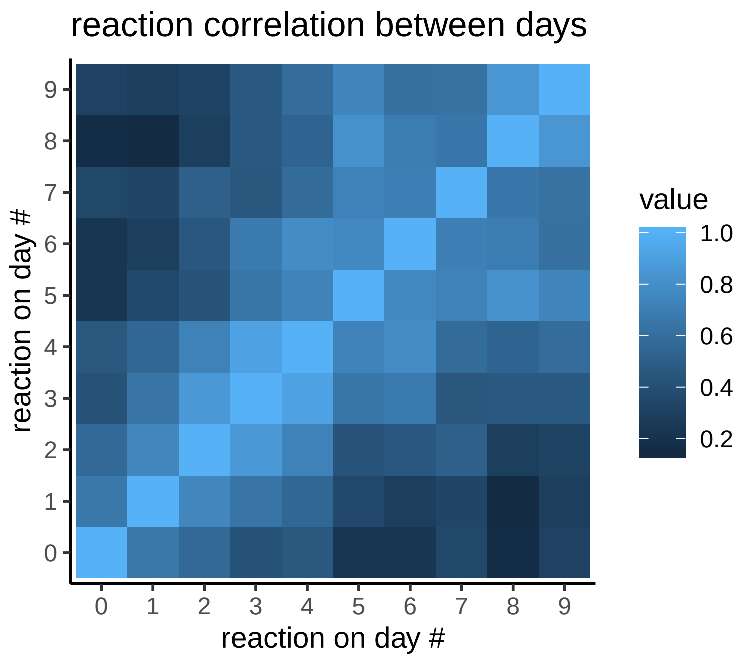


Descriptives

	Days	Mean	SD	Var	n
0	0	256.65	32.13	1032.30	18
1	1	264.50	33.43	1117.59	18
2	2	265.36	29.47	868.68	18
3	3	282.99	38.86	1509.92	18
4	4	288.65	42.54	1809.47	18
5	5	308.52	51.77	2680.09	18
6	6	312.18	63.17	3990.92	18
7	7	318.75	50.10	2510.41	18
8	8	336.63	60.20	3624.01	18
9	9	350.85	66.99	4487.15	18

The calculations of the mean, standard deviation and variance of the reaction time for each day of all subjects further support our exploratory plots: we observe an overall increase in the mean, variance and standard deviation with more days of sleep deprivation. (Note that for day 2 and 7 the variance and standard deviation decrease compared to the previous day. It continues to increase afterwards, however).

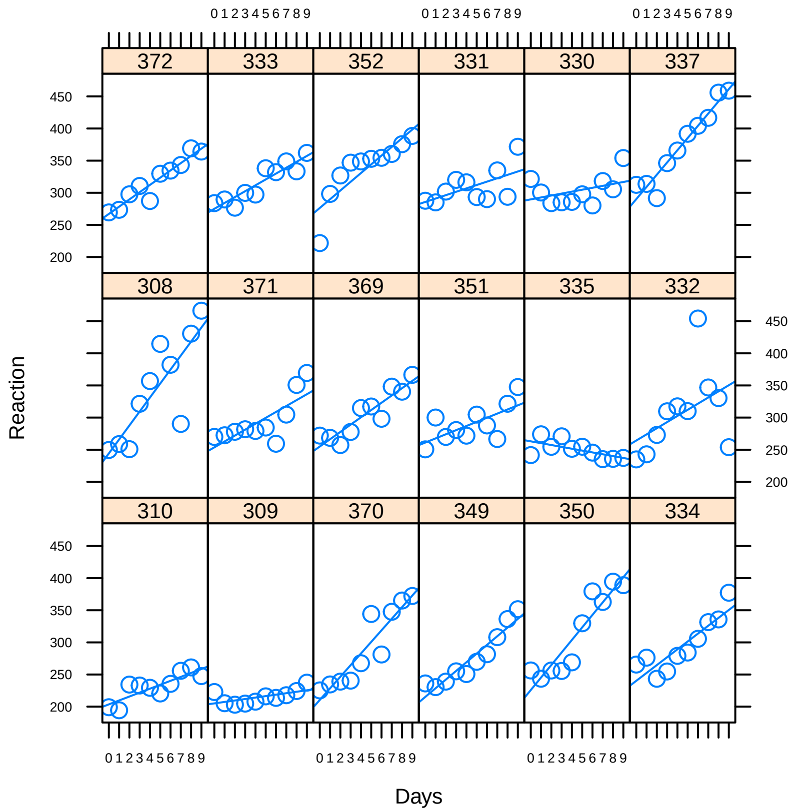
Correction



We used the Shapiro - Wilk test to check for the normality of the reaction times per day. The test revealed a non normal distribution of day 9. Thus, we performed the spearman correlation method instead of pearson to check for a correlation of the reaction times between days.

Looking at the correlation matrix, there is a correlation higher than 0.6 between subsequent days (e.g. between Day 3 and 4, between Day 8 and 9, etc). However, the further the days are apart, the lower the correlation (e.g. low correlation between Day 1 and Day 8).

Regression per person



We performed a linear regression model on each subject based on the function:

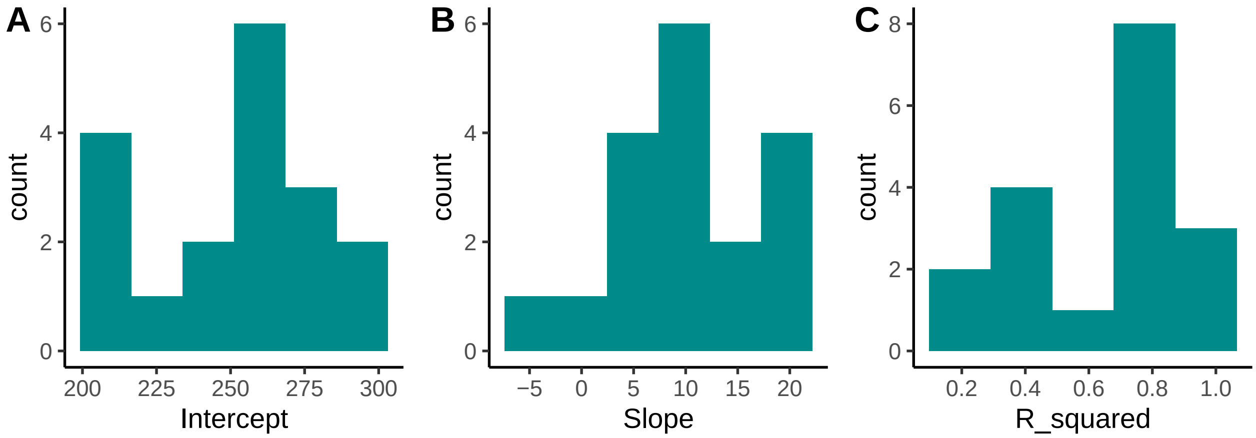
$$\text{reaction time (Reaction)} = b_0 + b_i * \text{Days}$$

We then created a trellis graph to visualise the intercepts and slopes of these subject-specific linear regression models.

The graph suggests that the slope and intercept of each subject's linear model are independent of each other as there is no observable trend between the height of the intercept and the steepness of the slope. This is further supported by plotting the intercept against the slope (see chapter "OLS vs. LMM", Fig. A (orange dots)). Overall, all subjects have a positive slope besides subject 335.

The linear regression lines fit the datapoints closely, suggesting that a linear model is appropriate to represent this dataset.

Between subject variability



The individual intercepts shown in the first histogram (A) correspond to the initial reaction time at day zero and are non normally distributed. Given the small data set, this is not surprising as it shows a variety of the initial reaction time. However, if this data came from a large dataset, it would be surprising that the initial data points are not normally distributed and could suggest a wrong data sample compared to the population.

Looking at the histogram of individual slopes (B), we see a normal distribution. As seen on the previous graph showcasing the individual linear regressions, only one slope is negative. This shows again that reaction time increases by days of sleep deprivation.

Finally, looking at the histogram of R squared (C), we see that the majority of subjects have a R squared of above 0.6. This shows that the linear model is appropriate for this data set. However, the individual linear model does not fit the specific data of some subjects, respectively 7 of the 18 subjects.

Fitting the model - with REML

Mathematical description

Level 1 model explains the evolution of Reaction time for each subject:

$$\begin{aligned} Y_{ij} &= \pi_{0i} + \pi_{1i} * \text{Days}_{ij} && \text{how do individuals evolve} \\ &+ \epsilon_{ij} && \text{how the individuals deviate from their own evolution} \end{aligned}$$

Level 2 model explains why the Subjects differ from each other:

$$\begin{aligned} \pi_{0i} &= \gamma_0 + b_{0i} && \text{explains the intercept} \\ \pi_{1i} &= \gamma_1 + b_{1i} && \text{explains the slope} \end{aligned}$$

σ_0^2 - Level 2 residual variance in true intercept π_{0i} across all individuals in the population

σ_1^2 - Level 2 residual variance in true slope π_{1i} across all individuals in the population

With the level 2 model we are trying to explain the variation between individuals using the intercept and at the slope while b_{0i} and b_{1i} describe the unexplained variability between subjects.

The full model describes the evolution observed in the spaghetti plot and other descriptive plots of the data:

$$\begin{aligned} Y_{ij} = &\gamma_0 + \gamma_1 * \text{Days}_{ij} && \text{fixed effects} \\ &+ b_{0i} + b_{1i} * \text{Days}_{ij} && \text{random effect} \\ &+ \epsilon_{ij} && \text{error} \end{aligned}$$

$$\begin{cases} Y_{ij} &= \pi_{0i} + \pi_{1i} * \text{Days}_{ij} + \epsilon_{ij} \\ \pi_{0i} &= \gamma_0 + b_{0i} \\ \pi_{1i} &= \gamma_1 + b_{1i} \end{cases}$$

Underneath is the average evolution of the whole population:

$$E(Y_{ij}) = \gamma_0 + \gamma_1 * \text{Days}_{ij}$$

The general linear mixed model is given by:

$$\begin{cases} Y_i = X_i\beta + Z_i b_i + \epsilon_i \\ b_i \sim N(0, D) \\ \epsilon_i \sim N(0, \Sigma_i) \end{cases}$$

$$Y_i \sim N(X_i\beta, Z_i D Z_i' + \Sigma_i)$$

R uses the marginal model and our calcuation are based on that.

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Reaction ~ 1 + Days + (1 + Days | Subject)
##    Data: sleep
##
## REML criterion at convergence: 1743.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9536 -0.4634  0.0231  0.4633  5.1793
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  Subject  (Intercept) 611.90   24.737
##           Days         35.08    5.923   0.07
##  Residual             654.94   25.592
## Number of obs: 180, groups:  Subject, 18
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  251.405      6.824  36.843
## Days          10.467      1.546   6.771
##
## Correlation of Fixed Effects:
##      (Intr)
## Days -0.138
```

Values of the REML model

Based on the above model, we find the following values:

$$
\begin{aligned}
\gamma_0 &= 251.405 \\
\gamma_1 &= 10.467 \\
\sigma^2_\epsilon &= 654.94 \\
\sigma^2_0 &= 611.90 \\
\sigma^2_1 &= 35.08 \\
corr(b_{0i}, b_{1i}) &= 0.07
\end{aligned}
$$

$$
\begin{cases}
Y_{ij} & = \pi_{0i} + \pi_{1i} * Days_{ij} + \epsilon_{ij} \\
\pi_{0i} & = 251.41 + b_{0i} \\
\pi_{1i} & = 10.47 + b_{1i}
\end{cases}
$$

$$\epsilon_{ij} \sim N(0, 25.59^2)$$

$$
\begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{pmatrix}\right)
$$

$$
\begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 611.9 & 9.61 \\ 9.61 & 35.08 \end{pmatrix}\right)
$$

Next step is to check if the values retrieved are actually significant. We therefore check if the number of days have a significant effect on the reaction time.

In the next chapter, we tested the fixed effects with Bootstrap and profile likelihood as the sample size was too small to use a Wald test. Next, we checked and compared different possible models using a likelihood ratio test.

Testing fixed effects - with bootstrap

Computing bootstrap confidence intervals ...

4 message(s): boundary (singular) fit: see ?isSingular 175 warning(s): Model failed to converge with max|grad| = 0.00201739 (tol = 0.002, component 1) (and others)

	2.5 %	97.5 %
sd__(Intercept)|Subject	13.15	35.7
cor__Days.(Intercept)|Subject	-0.4581	0.841
sd__Days|Subject	3.474	8.123
sigma	22.16	28.5
(Intercept)	240	265.4
Days	7.621	13.58

Computing profile confidence intervals ...

	2.5 %	97.5 %
sd__(Intercept)|Subject	14.38	37.72
cor__Days.(Intercept)|Subject	-0.4815	0.685
sd__Days|Subject	3.801	8.753
sigma	22.9	28.86
(Intercept)	237.7	265.1
Days	7.359	13.58

The confidence intervals of the intercept and Days do not include 0. Therefore, both have a significant effect on the Reaction time.

Likelihood ratio test with Anova

Table 5: Data: sleep (continued below)

	Df	AIC	BIC	logLik	deviance	Chisq	Chi Df
sleep.intercept	3	1917	1926	-955.3	1911	NA	NA
sleep.full	6	1764	1783	-876	1752	158.6	3

	Pr(>Chisq)
sleep.intercept	NA
sleep.full	3.672e-34

We compared an intercept-only model with a model that includes Days to find the best model using MLE.

Looking at the outcome of the likelihood ratio test with Anova, we can conclude that adding Days as covariate improves our model significantly. Days has a significant effect on the Reaction time with a p-value much smaller than 0.05.

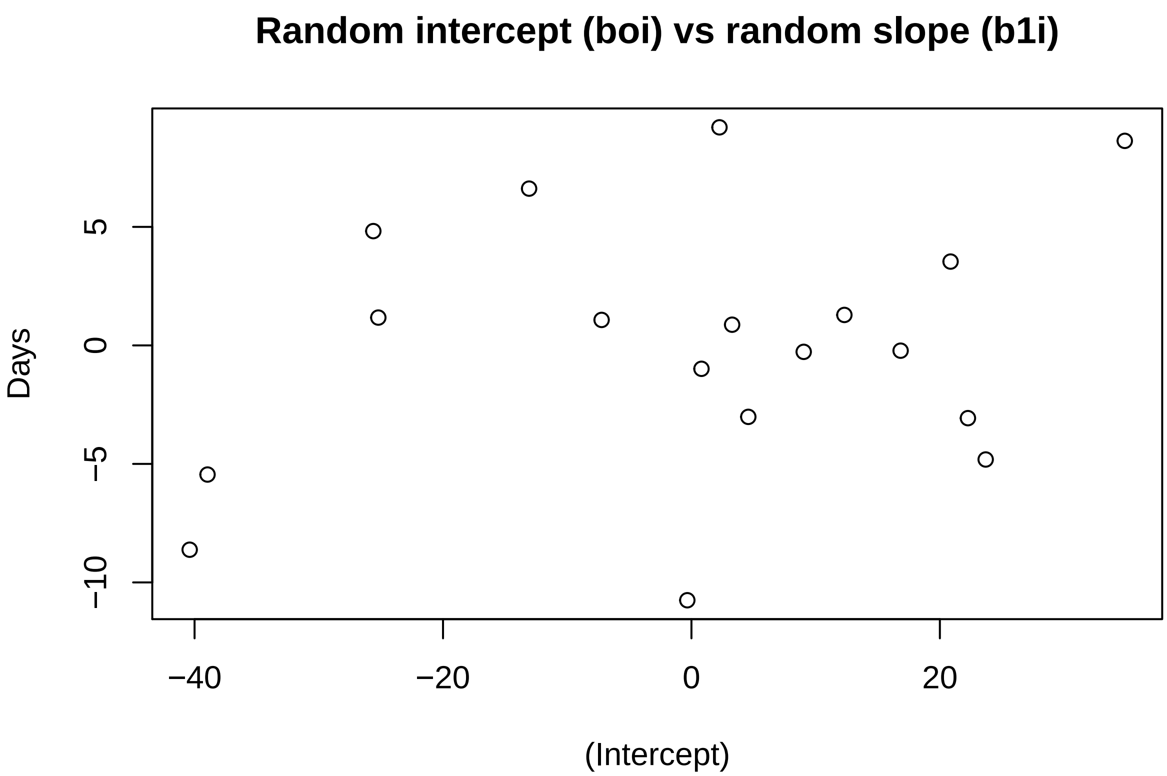
The decrease in AIC value also supports this conclusion.

Testing the random effects

	(Intercept)	Days
(Intercept)	611.9	9.614
Days	9.614	35.08

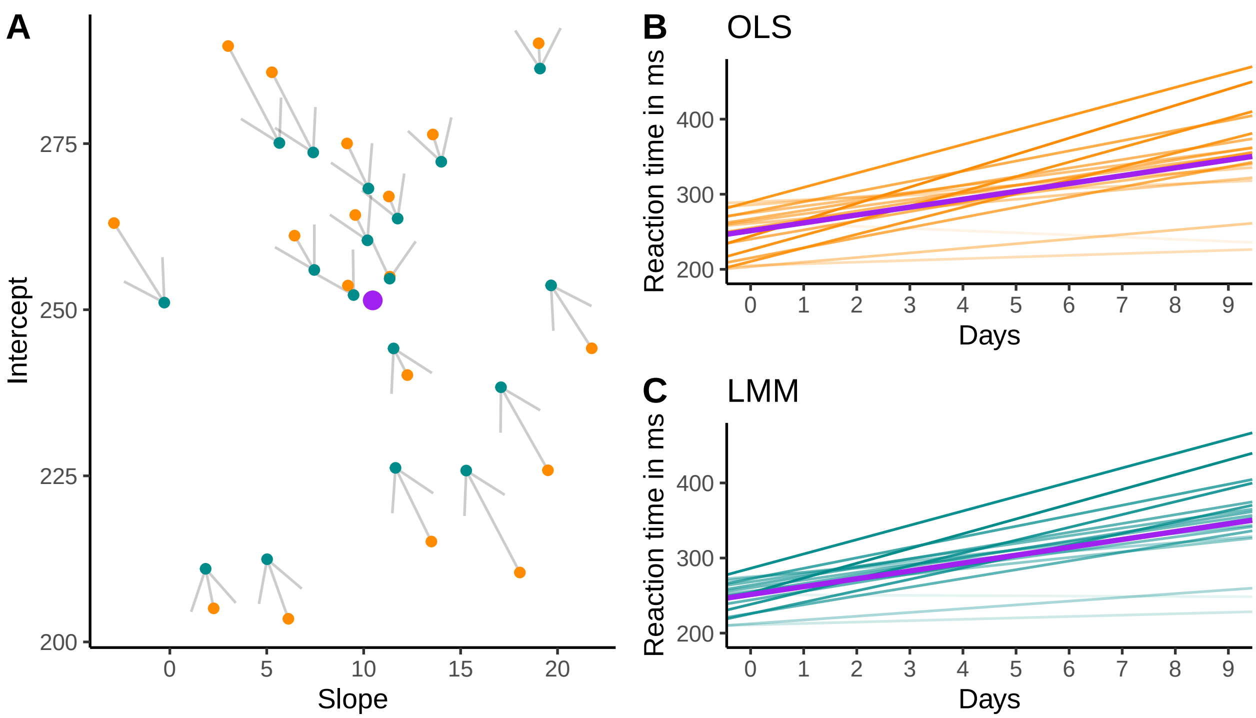
Firstly, we check the random effects covariance matrix.

The model is built on the assumption that the b's come from a normal distribution with mean 0. The residual variance in true intercept π_{0i} across all individuals in the population is 611.9, the residual variance in true slope π_{1i} across all individuals in the population is 35.08 and the residual covariance between the true intercept π_{0i} and the slope π_{1i} is 9.61, as seen on the matrix above.



Next, we plot the predicted random effects of the intercept b_{0i} compared to the random effects of the slope b_{1i}. These random effects reflect how the evolution of the ith Subject deviates from the expected evolution.

OLS vs LMM estimates



Having checked the fixed and random effects of the overall linear regression model, we now compare the original subject-specific regression model to the multilevel regression model we used for the full data set. (i.e. comparing the Ordinary Least Squares (OLS) model to the Linear Mixed Model (LMM))

Figure A displays the shrinkage effect of the LMM compared to OLS. If we fit a linear model to each individual subject separately (OLS model) without taking into account the data of the whole population (orange dots), intercepts and slopes vary largely. However, if we fit one linear model to all subjects (LMM, blue dots) the values for intercept and slope move more closely to the average population intercept and slope (purple dot).

Conclusion

From our analysis on the effect of sleep deprivation on the reaction time of long distance truck drivers, we can conclude that there is a linear relationship between the amount of days of sleep deprivation and the reaction time. More precisely, as the sleep deprivation proceeds, the time needed for a driver to respond to a visual stimulus is increasing by 10.47 ms (sd = 5.92) per day, on average. The reaction time of people before they were sleep deprived averages 251.41 ms (sd = 24.74).

Several groups of drivers with different conditions of restricted sleep deprivation or a control group would additionally help us draw a more concrete conclusion. From the existing literature, mathematical models predicting alertness from preceding sleep-wake history typically involve four factors: sleep homeostasis, circadian rhythm, sleep inertia and neuromodulatory changes. Thus, we can conclude that there is a relation between reaction time and sleep deprivation, but it is not the only factor that can fully describe the relationship of sleep deprivation and the reaction time.