Patterns of performance degradation during sleep restriction of long distance truck drivers

Patterns of performance degradation during sleep restriction of long distance truck drivers

Presentation of the case study

We are analysing the effect of sleep deprivation on reaction time of long distance truck drivers. There are 18 subjects in the dataset and for each subject, the reaction time was measured for 10 days. The subjects were allowed only a limited amount of sleep for these 10 subsequent days. Each subject's reaction time was measured several times on each day of the trial and an average was taken.

Reaction time is measured with a psychomotor vigilance task (PVT), which measures the speed with which subjects respond to a visual stimulus.

Is there any relation between reaction time and the number of days of sleep deprivation?

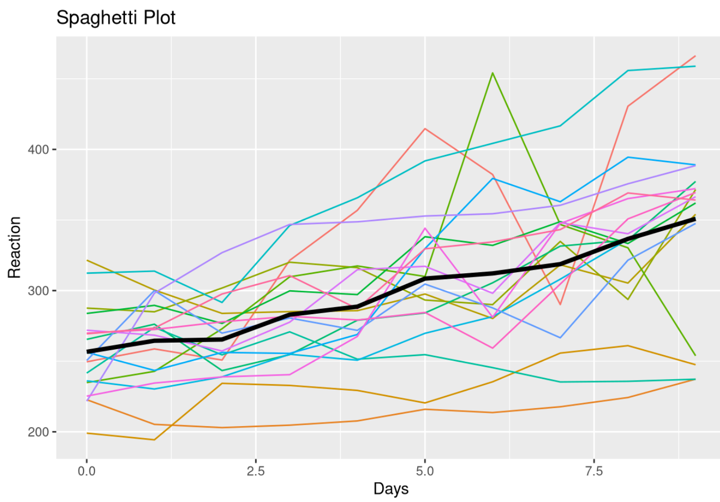
Exploratory analysis

This dataset contains multiple measurements for each subject on consecutive days, with as response variable the continuous variable reaction time and explanatory variable Days. Since there are 10 measurements for each subject, it is a longitudinal study. The dataset of 18 subjects is balanced with an equal amount of measurements for each subject.

Spaghetti Plot

To visualise the individual reaction times and how they compare to the mean, a spaghetti plot was created. This revealed that there was a variation in intercepts or starting reaction times on day 0 between subjects. This variation increased with subsequent days.

For most subjects, the reaction time increased with the amount of days of sleep deprivation. This increase is also visible in the mean.



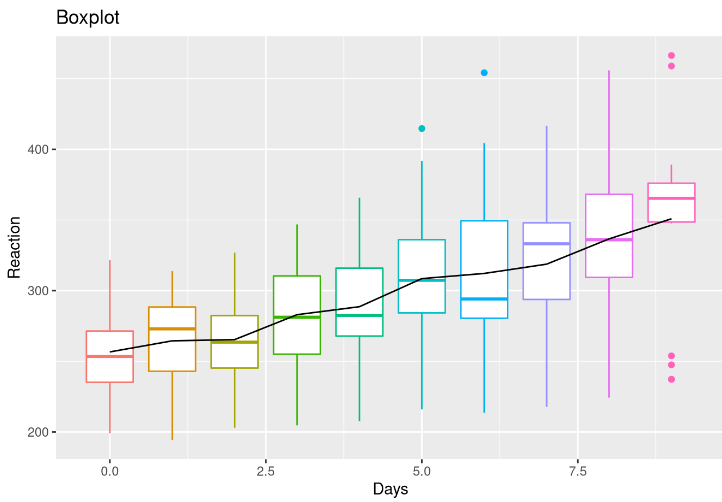
Already from this plot you can assume that the reaction time is increasing with increasing number of days of sleep deprivation.

Boxplot

The following boxplot was created to get a quick summary of the dataset's characteristics. The mean and median seem to show a similar increase throughout the study. For day nr 6, 7 and 10, outliers are observed.
The variance increases with an increase in days of sleep deprivation but the interquartile range appears to expand not as strongly as the minimum and maximum of the boxplot.

To put together, some subjects deviate more from the mean with an increase in days of sleep deprivation (see outliers on both sides) while most others stay around the mean (see slower increase in interquartile range).

The violin plot supports the above observations of the distribution of the data around the mean with outliers.

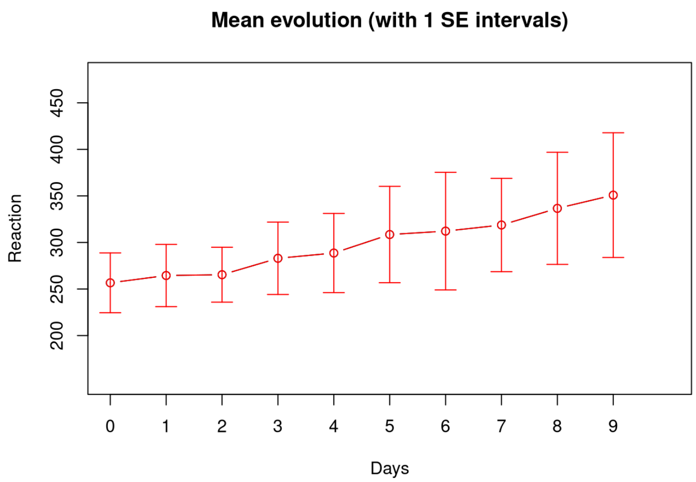


on time. Also the variance of the reaction time seems to increase with increasing days of sleep deprivation. (at Day 9: 5 outliers)

Summary

	Days	Mean	SD	Var	
0	0	256.65	32.13	1032.30	
1	1	264.50	33.43	1117.59	
2	2	265.36	29.47	868.68	
3	3	282.99	38.86	1509.92	
4	4	288.65	42.54	1809.47	
5	5	308.52	51.77	2680.09	
6	6	312.18	63.17	3990.92	
7	7	318.75	50.10	2510.41	
8	8	336.63	60.20	3624.01	
9	9	350.85	66.99	4487.15	
The c	alculat	ions of t	he mean,	standard	deviation and variance of the reaction time for each day o

Mean evolution



To further support our previous findings, we looked at the mean evolution. Here, an increasing trend of reaction time with increasing number of days is also observed, together with expanding standard deviations (see errorbars).

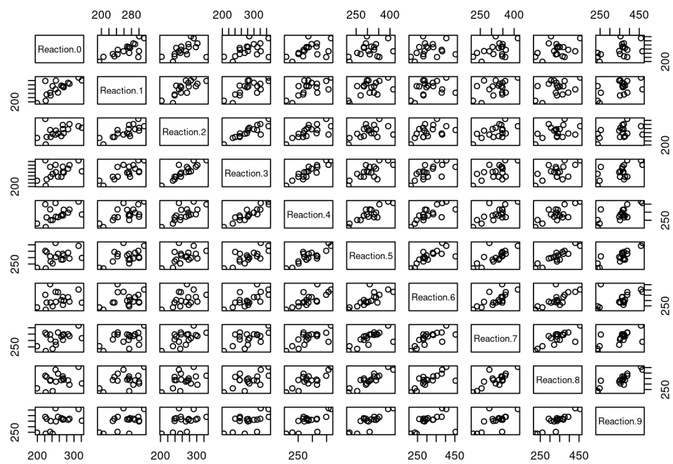
Correlation

	Subject	Reaction.0	Reaction.1	Reaction.2	Reaction.3	Reaction.4	Reaction.5	Reaction.6
1	308	249.5600	258.7047	250.8006	321.4398	356.8519	414.6901	382.2038
11	309	222.7339	205.2658	202.9778	204.7070	207.7161	215.9618	213.6303
21	310	199.0539	194.3322	234.3200	232.8416	229.3074	220.4579	235.4208
31	330	321.5426	300.4002	283.8565	285.1330	285.7973	297.5855	280.2396
41	331	287.6079	285.0000	301.8206	320.1153	316.2773	293.3187	290.0750
51	332	234.8606	242.8118	272.9613	309.7688	317.4629	309.9976	454.1619
61	333	283.8424	289.5550	276.7693	299.8097	297.1710	338.1665	332.0265
71	334	265.4731	276.2012	243.3647	254.6723	279.0244	284.1912	305.5248
81	335	241.6083	273.9472	254.4907	270.8021	251.4519	254.6362	245.4523
91	337	312.3666	313.8058	291.6112	346.1222	365.7324	391.8385	404.2601
101	349	236.1032	230.3167	238.9256	254.9220	250.7103	269.7744	281.5648
111	350	256.2968	243.4543	256.2046	255.5271	268.9165	329.7247	379.4445
121	351	250.5265	300.0576	269.8939	280.5891	271.8274	304.6336	287.7466
131	352	221.6771	298.1939	326.8785	346.8555	348.7402	352.8287	354.4266
141	369	271.9235	268.4369	257.2424	277.6566	314.8222	317.2135	298.1353
151	370	225.2640	234.5235	238.9008	240.4730	267.5373	344.1937	281.1481
161	371	269.8804	272.4428	277.8989	281.7895	279.1705	284.5120	259.2658
171	372	269.4117	273.4740	297.5968	310.6316	287.1726	329.6076	334.4818

```
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.97667, p-value = 0.9093
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.94756, p-value = 0.388
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.98688, p-value = 0.9936
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.97738, p-value = 0.919
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.97247, p-value = 0.8427
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.978, p-value = 0.9271
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.95912, p-value = 0.5847
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
```

```
## W = 0.94648, p-value = 0.3724
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.97112, p-value = 0.8186
##
##
##  Shapiro-Wilk normality test
##
## data:  sleep.resh[, i]
## W = 0.86251, p-value = 0.01342
```

```
##            Reaction.0 Reaction.1 Reaction.2 Reaction.3 Reaction.4 Reaction.5
## Reaction.0  1.0000000  0.6594427  0.5686275  0.4179567  0.4571723  0.2239422
## Reaction.1  0.6594427  1.0000000  0.7461300  0.6367389  0.5562436  0.3581011
## Reaction.2  0.5686275  0.7461300  1.0000000  0.8534572  0.7234262  0.4344685
## Reaction.3  0.4179567  0.6367389  0.8534572  1.0000000  0.9133127  0.6553148
## Reaction.4  0.4571723  0.5562436  0.7234262  0.9133127  1.0000000  0.7296182
## Reaction.5  0.2239422  0.3581011  0.4344685  0.6553148  0.7296182  1.0000000
## Reaction.6  0.2218782  0.2920537  0.4551084  0.6759546  0.7812178  0.7667699
## Reaction.7  0.3457172  0.3312693  0.5087719  0.4509804  0.5789474  0.7254902
## Reaction.8  0.1640867  0.1496388  0.2899897  0.4654283  0.5376677  0.8121775
## Reaction.9  0.3106295  0.2899897  0.3168215  0.4633643  0.5933953  0.7378741
##            Reaction.6 Reaction.7 Reaction.8 Reaction.9
## Reaction.0  0.2218782  0.3457172  0.1640867  0.3106295
## Reaction.1  0.2920537  0.3312693  0.1496388  0.2899897
## Reaction.2  0.4551084  0.5087719  0.2899897  0.3168215
## Reaction.3  0.6759546  0.4509804  0.4654283  0.4633643
## Reaction.4  0.7812178  0.5789474  0.5376677  0.5933953
## Reaction.5  0.7667699  0.7254902  0.8121775  0.7378741
## Reaction.6  1.0000000  0.7110423  0.6904025  0.6181631
## Reaction.7  0.7110423  1.0000000  0.6573787  0.6243550
## Reaction.8  0.6904025  0.6573787  1.0000000  0.8452012
## Reaction.9  0.6181631  0.6243550  0.8452012  1.0000000
```



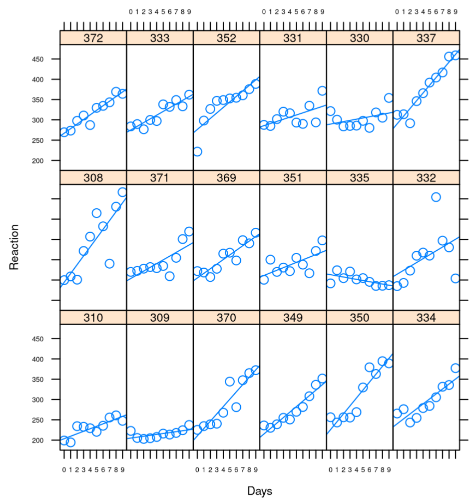
We used the Shapiro-Wilk test to check for normality of the reaction times per day.

The test revealed a non normal distribution of day 9. Thus, we performed the spearman correlation method instead of pearson to check for a correlation of the reaction times between days.

Looking at the correlation matrix, there is a correlation higher then 0.6 between subsequent days (e.g. between Day 8 and 9, between Day 3 and 4, …). However, the further the days are apart, the lower the correlation (e.g. low correlation between Day 1 and Day 8).

Aligning nicely with our previous results, there is a linear trend between the number of Days and reaction time.

Regression per person



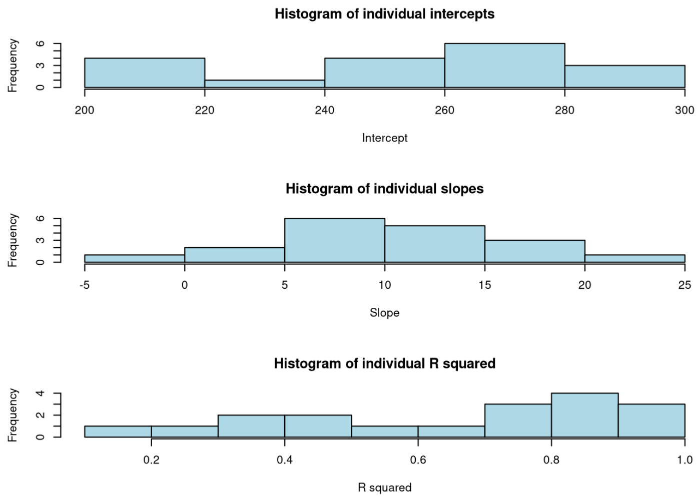
We performed a linear regression model on each subject based on the function: reaction time = b0 + bi* Days.

We then created a trellis graph to visualise the intercepts and slopes of these subject-specific linear regression models.

The graph suggests that the slope and intercept of each subject's linear model are independent of each other as there is no observable trend between the height of the intercept and the steepness of the slope. This is further supported by plotting the intercept against the slope. Overall, all subjects have a positive slope besides subject 335.

The linear regression lines fit the datapoints closely, suggesting that a linear model is appropriate to represent this dataset.

Between subject variability



The individual intercepts shown in the first histogram correspond to the initial reaction time and are non normally distributed. Given the small data set, this is not surprising as it shows a variety of their initial reaction time. However, if this data came from a large dataset, it would be surprising that even the initial data points are not normally distributed and could suggest a wrong data sample compared to the population.

Looking at the histogram of individual slopes, we see a normal distribution. As seen on the previous graph showcasing the individual linear regressions, very little slopes are negative. This shows again that reaction time increases by days of sleep deprivation.

Finally, looking at the histogram of R squared, we see that the majority of subjects have a R squared of above 0.6. This shows that the linear model is appropriate for this data set. However, sometimes the individual linear model does not fit the specific data of some subjects, specifically 7 of the 18 subjects.

Fitting the model - with REML

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Reaction ~ 1 + Days + (1 + Days | Subject)
##    Data: sleep
##
## REML criterion at convergence: 1743.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9536 -0.4634  0.0231  0.4633  5.1793
```

```
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  Subject  (Intercept) 611.90   24.737
##           Days         35.08    5.923   0.07
##  Residual             654.94   25.592
## Number of obs: 180, groups:  Subject, 18
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  251.405      6.824  36.843
## Days          10.467      1.546   6.771
##
## Correlation of Fixed Effects:
##      (Intr)
## Days -0.138
```

Values

\[\begin{align} \gamma_{0} &= 251.405 \\ \gamma_{1} &= 10.467 \\ \\ \sigma_{\epsilon}^{2} &= 654.94 \\ \sigma_{0}^{2} &= 611.90 \\ \sigma_{1}^{2} &= 35.08 \\ corr(b_{0i}, b_{1i}) &= 0.07 \end{align}\]

##Mathematical description

Level 1 model explains the evolution of Reaction time for each subject \(Y_{ij}\)= \(\pi_{0i}\) + \(\pi_{1i}\)* \(Days_{ij}\) -> how do individuals evolve + \(\epsilon_{ij}\) -> how the individuals deviate from their own evolution

\[\begin{align} Y_{ij}&= \pi_{0i} + \pi_{1i}* \text{Days}_{ij} && \text{how do individuals evolve} \\ &+ \epsilon_{ij} &&\text{how the individuals deviate from their own evolution} \end{align}\]

Level 2 model tries to explain why the Subjects differ from each other \(\pi_{0i}\) = \(\gamma_{0}\) + \(b_{0i}\) - model for explaining the intercept \(\pi_{1i}\) = \(\gamma_{1}\) + \(b_{1i}\) - model for explaining the slope

\[\begin{align} _{0i} &= _{0} + b_{0i} &&\text{model for explaining the intercept} \\ _{1i} &= _{1} + b_{1i} &&\text{model for explaining the slope} \end{align}\]

\(\sigma_{0}^{2}\) - Level 2 residual variance in true intercept \(\pi_{0i}\) across all individuals in the population \(\sigma_{1}^{2}\) - Level 2 residual variance in true slope \(\pi_{1i}\) across all individuals in the population

With level 2 model we are trying to see why there is variation between individuals by looking at the intercept and at the slope. \(b_{0i}\) and \(b_{1i}\)

are the unexplained variability

The full model describes the evolution observed in spaghetti plot: \(Y_{ij}\) = \(\gamma_{0}\) + \(b_{0i}\) + \(\gamma_{1}\)* \(Days_{ij}\) - fixed effects + \(b_{0i}\) + \(b_{1i}\)*\(Days_{ij}\) - random effects + \(\epsilon_{ij}\) - error

\[\begin{align} Y_{ij} =& _{0} + b_{0i} + _{1}* \text{Days}_{ij} &&\text{fixed effects}\\ &+ b_{0i} + b_{1i}*\text{Days}_{ij} &&\text{random effect}\\ &+ \epsilon_{ij} &&\text{error} \end{align}\]

\[\begin{cases} Y_{ij}&= _{0i} + _{1i}* \text{Days}_{ij} + _{ij} \\ _{0i} &= _{0} + b_{0i} &&\text{intercept} \\ <!-- _{1i} &= _{1} + b_{1i} &&\text{slope} --> \end{cases} \]

Following is the average evolution of the population: E(\(Y_{ij}\)) = \(\gamma_{0}\) + \(\gamma_{1}\)* \(Days_{ij}\)

The general liniar mixed model is given by:

\(Y_{i}\) = \(X_{i}\beta\) + \(Z_{i}b_{i}\) + \(\epsilon_{i}\) \(b_{i}\) ~ N(0,D) , \(\epsilon_{i}\) ~ N(0,\(\Sigma{i}\)) \(b_{ 1}...,b_{N}\),\(\epsilon_{1}...,\epsilon_{N}\) independent

\[Y_i \tilde{} N(X_i\beta, Z_iDZ_i'+\Sigma_i)\] ###todo:write the model with values given by R

\[\begin{align} Y_i = X_i\beta + Z_i b_i + _i \\ b_i \sim N(0,D)\text{, } _i \sim N(0,Σ_i) \\ b_1\cdots,b_N, _1\cdots, _N \text{independent} \end{align}\]

R uses the marginal model and our calcuation are based on that. Next step is to check if the values we retrieved are actually significant so does the number of days have a significant effect on the reaction time? We tested this with Bootstrap and Likelihood Ratio tests because th sample zise wasn't large enough(excluding Wald test).

Testing fixed effects - with bootstrap

```
## Computing bootstrap confidence intervals ...
```

```
##
## 5 message(s): boundary (singular) fit: see ?isSingular
## 181 warning(s): Model failed to converge with max|grad| = 0.00200037 (tol = 0.002, compor
```

```
##                                  2.5 %      97.5 %
## sd_(Intercept)|Subject        12.7491323  36.0232739
## cor_Days.(Intercept)|Subject  -0.4755649   0.9537855
## sd_Days|Subject                3.1536667   8.4768127
## sigma                         22.6776027  28.5583139
## (Intercept)                  238.5741931 265.2626365
```

```
## Days                              7.5194160  13.6058339
```

Confidence interval of the intercept and days does not include 0 therefore both of them have a significant effect on reaction time.

likelihood ratio test with anova

We compared an intercept only model with a model that includes days as well and we concluded that adding days as covariate it improves our model significantly. Days have have a significant effect on the reaction time with the p-value smaller than 0.05. The decrease in AIC value also supports this conclusion.

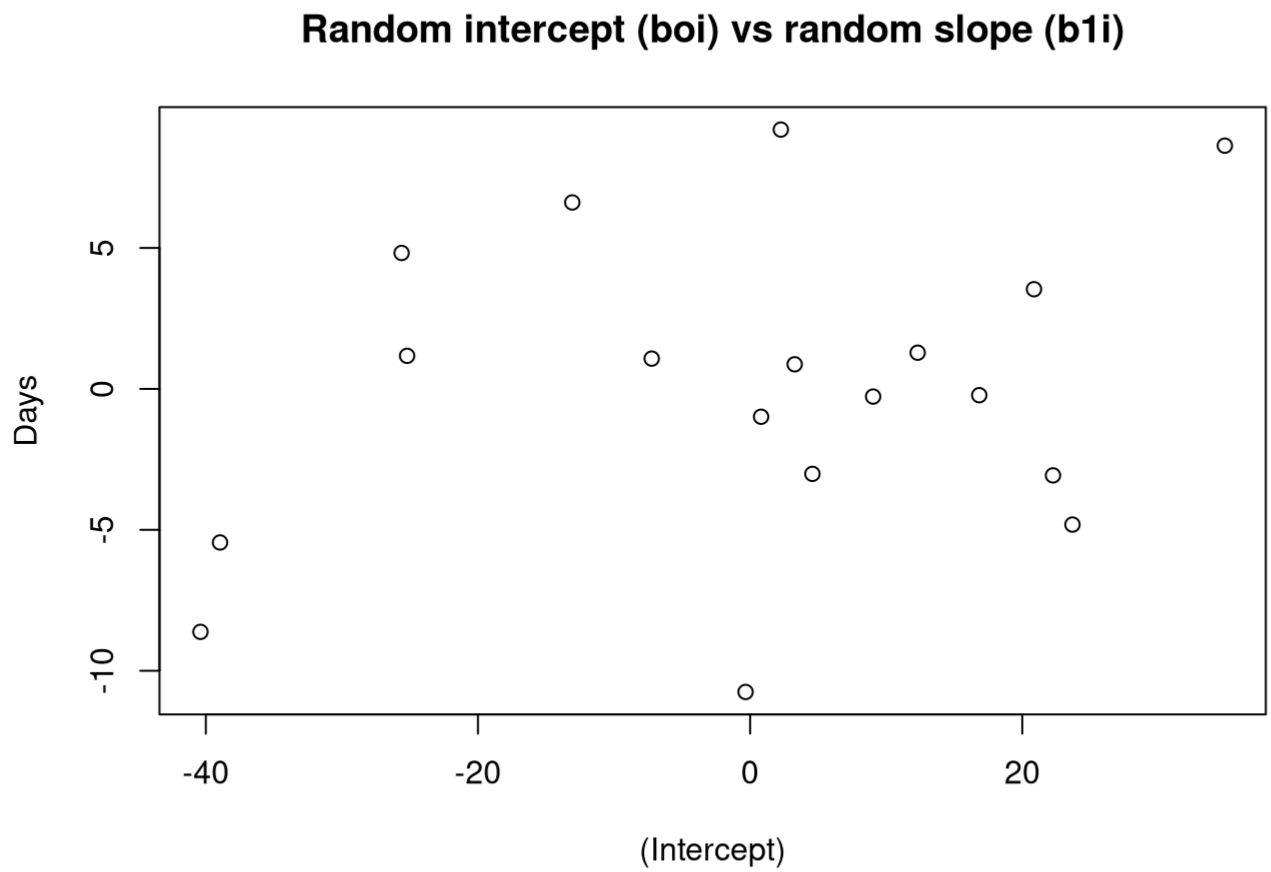
OLS vs LMM estimates

#plot random intercept and random slope

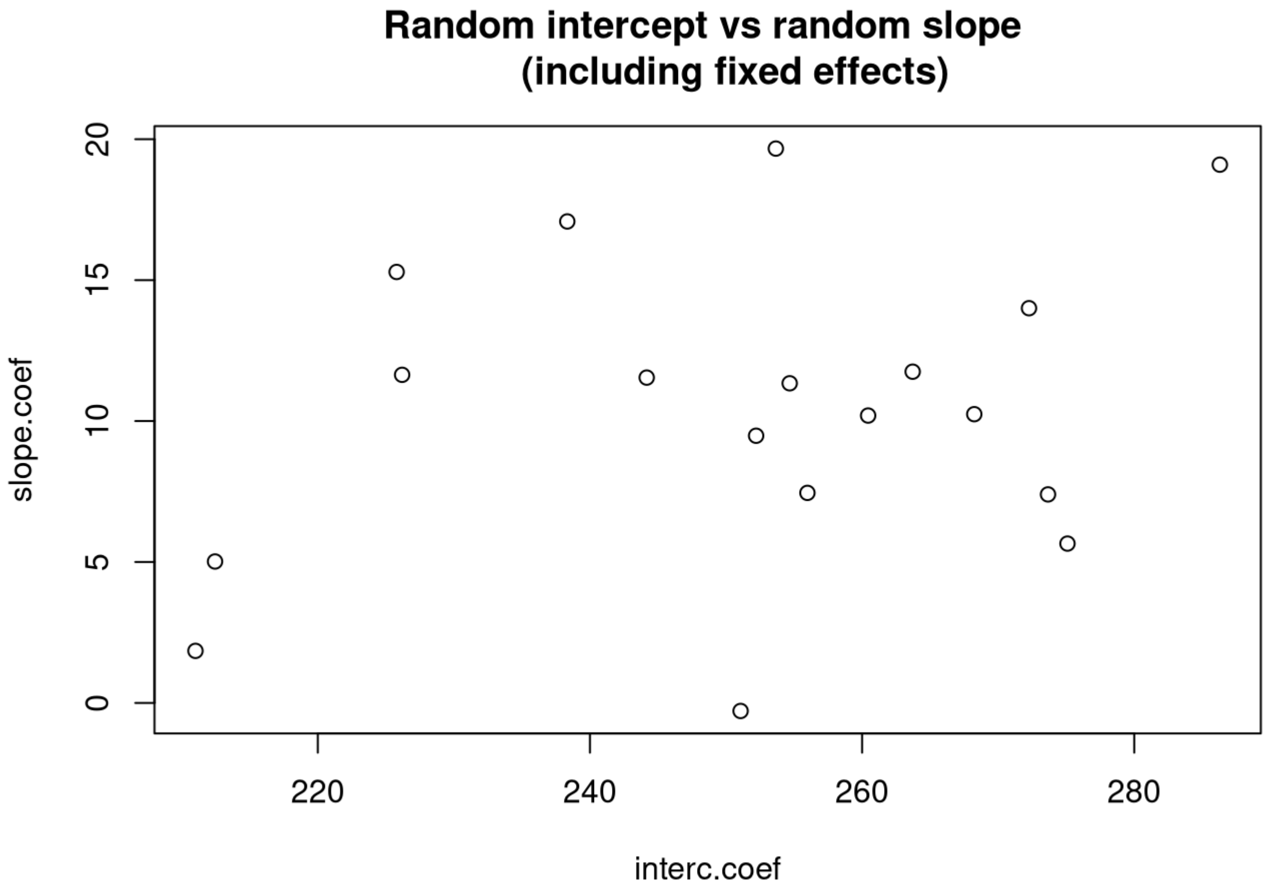
```
##             (Intercept)      Days
## (Intercept)  611.897607  9.613886
## Days           9.613886 35.081069
## attr(,"stddev")
## (Intercept)        Days
##   24.736564    5.922927
## attr(,"correlation")
##             (Intercept)       Days
## (Intercept)  1.00000000 0.06561803
## Days         0.06561803 1.00000000
```

\[\begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim N\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{0}^2 & \sigma_{01} \\ \sigma_{01} & \sigma_{1}^2 \end{pmatrix} \end{pmatrix}\\ \begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim N\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 611.9 & 9.61 \\ 9.61 & 35.08 \end{pmatrix} \end{pmatrix} \] D: Random effects covariance matrix The model is built on the assumption that the b's come from a normal distribution with mean 0 and the residual variance in true intercept \(\pi_{0i}\) across all individuals in the population of 611.9, the residual variance in true slope for all individuals is 35.08 and residual covariance between the intercept and the slope of 9.61.

This table preditcs the random effect for each subject.Almost all values lay within two standard devation from the mean.



Random intercept (boi) vs random slope (b1i)
Days
5
0
-5
-10
-40
-20
0
20
(Intercept)



to do: compare the two models by creating the mean!

Discussion ???

From our analysis on the effect of sleep deprivation on the reaction time of long distance truck drivers, we concluded that there is a significant positive correlation between them. More precisely, as the sleep deprivation proceeds, the time needed for a driver to respond to a visual stimulus is increasing. Several groups of drivers with different conditions of restricted sleep deprivation or a control group would additionally help us draw a more concrete conclusion. From the existing literature, mathematical models predicting alertness from preceding sleep-wake history typically involve four factors, sleep homeostasis, circadian rhythm, sleep inertia and neuromodulatory changes. Thus, we can conclude that there is a relation between reaction time and sleep deprivation, but it is not the only factor that can fully describe the relationship of sleep deprivation and the reaction time.