AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
Assignment #6

Due: 10:30 AM on November 6, 2020

1. (10 points) In the class, I discussed a very small web made of six pages with the following Markov matrix:

$$M_1 = \begin{pmatrix} 1/2 & 1/4 & 1/3 & 1/4 & 1/4 & 1/3 \\ 0 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 1/4 & 0 & 0 \\ 0 & 1/4 & 1/3 & 1/4 & 1/4 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 1/4 & 1/3 \\ 0 & 0 & 0 & 0 & 1/4 & 1/3 \end{pmatrix}. \tag{1}$$

   Now assume that page 1 doesn't link to anything. How do you need to modify the above matrix? Call the new matrix M_2 and calculate its stationary distribution vector. Now further assume that during 10 % of your total net-surfing time, you randomly pick a page to visit, instead of navigating through links. How do you need to modify M_2 to reflect this? Call it M_3 and calculate its stationary distribution vector. Explain the differences among the stationary distribution vectors of M_1 through M_3.

2. (50 points) Write an MCMC simulation code for 2-D square-lattice Ising model of $N \times N$. According to theory, a continuous phase transition is expected to take place at the reduced coupling coefficient J/kT of $[\ln(1+\sqrt{2})]/2 = 0.4407....$ Run your code with $N = 20$ for 10^6 iterations, calculate the mean and standard deviation of the average spin using the last half of the iterations, for a range of the reduced coupling coefficient (e.g., 0, 0.1, 0.2, .. 1.0), and compare with the analytical solution.