# AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
# Assignment #1

## Due: 10:30 AM on September 16, 2019

1-D dynamical system exercises.

1. (10 points) The insect outbreak model discussed in the lecture has the following governing equation:

$$\dot{x} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1+x^2}, \tag{1}$$

where $r$ and $k$ are control parameters. As we saw in the class, this model exhibits two saddle-node bifurcations, and you are here asked to plot the bifurcation diagram using MATLAB (or a similar software). Note that the following two identities hold at the saddle-node bifurcations:

$$rx\left(1 - \frac{x}{k}\right) = \frac{x^2}{1+x^2}, \tag{2}$$

and

$$\frac{d}{dx}\left[rx\left(1 - \frac{x}{k}\right)\right] = \frac{d}{dx}\left[\frac{x^2}{1+x^2}\right]. \tag{3}$$

The second identity corresponds to the tangential condition for saddle-node bifurcation. Use those relations to express $r$ and $k$ as a function of $x$, and plot $k(x)$ and $r(x)$ in the $k$-$r$ coordinates. Label different domains accordingly (i.e., "controlled", "outbreak", and "bistable"). Also, remember to label axes always.

2. (20 points) Code a lattice-based random-walker SIS epidemic spreading model, with the following input variables, the linear dimension $N$, the number of population $M$, the number of initial infected people $Q$, the days to recover $L$, and the number of total iterations $n$. At each iteration (= one day), each walker has five options to move: go left, right, up, down, or stay. Use the periodic boundary conditions. Plot how the number of infected people evolves with time, for the cases of $M$=500, 1000, 2000, 3000, 4000, with $N = 100$, $Q = 100$, $L = 14$, and $n = 500$. For each case, run 10 different simulations and use their average for your plot.

3. (20 points total) We can derive a difference equation corresponding to the above lattice-based SIS model, with the following steps. Assume that, at $n$th iteration, there are $I_n$ infected people and $S_n$ susceptible people ($I_n + S_n = M$), and we aim to derive an expression for $I_{n+1}$ or $dI = I_{n+1} - I_n$.

   (a) (1 point) What is the probability of finding a susceptible person in any lattice? Call this probability $p_s$.

   (b) (7 point) Focus on one infected person, and consider how many ways this person can infect a susceptible person in the next iteration. Calculate the probability of this person to spread infection at the next iteration (hint: you need to use $p_s$). Call this probability $p_i$. Note that some fraction of the currently infected people may recover at the next iteration, thereby being unable to infect others.

   (c) (2 point) What is the probability of each infected person to recover at the next iteration? Call this probability $p_r$.

   (d) (3 point) The increase in the number of infected people may then be described by $dI = I(p_i - p_r)$. By setting $dI = 0$, find fixed points.

   (e) (7 point) How well does this analysis explain your simulation results? If there is some discrepancy, discuss possible causes.