

AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
Assignment #4

Due: 10:30 AM on October 16, 2020

Lorenz equation, with $\sigma=10$ and $b=8/3$. You can use your own differential equation solver or something else (e.g., Matlab's `ode45`).

1. (15 points) Using $r=28$, integrate the equation starting with $(x, y, z) = (0, 1, 0)$, up to $t = 500$ (draw $x(t)$, $y(t)$, and $z(t)$ to see how the system evolves, but you don't have to submit those plots). Draw a corresponding Lorenz map, in which the x axis is z_n , and the y axis is z_{n+1} . Here z_n refers to the n th maximum in the time series of $z(t)$.
2. (15 points) We can write the above observed 'map' (in an approximated sense) as $z_{n+1} = f(z_n)$. Prove that, if $|f'(z)| > 1$ everywhere, any periodic solution is unstable. In terms of the Lorenz map, a general periodic solution with a period of p may be expressed as

$$z_n, z_{n+1}, \dots, z_{n+p} = z_n, z_{n+1}, \dots \tag{1}$$

3. (10 points) A more intuitive and graphical approach may be to draw a few more Lorenz maps for z_n vs. z_{n+2} , z_n vs. z_{n+3} , and so on. Draw such maps and discuss the maps in terms of the stability of periodic solutions.
4. (20 points) Again using $r=28$, compare the evolution of x , y , and z , up to $t = 100$ using the following two initial conditions: $(0, 1, 0)$ and $(10^{-6}, 1, 0)$. Also measure the distance between the two solutions, plot $\log(\text{distance})$ vs time, and provide a rough estimate on the Lyapunov exponent based on your result. If you use a differential equation solver with variable time steps, you need to interpolate these solutions at the same time instances to measure the distance. Explain what you observe in the plot.
5. (20 points) Repeat (1) and (3) with $r=100$. Discuss differences from the case with $r=28$.