AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
Assignment #3

Due: 10:30 AM on October 7, 2020

Biological neuron model exercises.

1. (40 points) For the $I_{\mathrm{Na,p}} + I_{\mathrm{K}}$ model, write a program to draw a phase portrait with the following information: (1) nullclines and (2) fixed points (red circle for unstable types and blue circle for stable types). You need to conduct linear stability analysis numerically (e.g., use finite difference approximation when calculating Jacobian). Note that you don't have to draw the flow field (doing so with MATLAB is a headache because MATLAB's `quiver` is poorly implemented; it's maybe easier to do so with Python).

2. (20 points) Use the program to demonstrate that the model exhibits subcritical Hopf bifurcation when varying I from 43 to 47 with the following parameter values: C=1, g_{L}=1, E_{L}=-78, g_{Na}=4, E_{Na}=60, $V_{1/2}$=-30 and k=7 for $m_\infty(V)$, g_{K}=4, E_{K}=-90, $V_{1/2}$=-45 and k=5 for $n_\infty(V)$, and $\tau(V)$=1. Draw multiple (3-4) phase trajectories (with different colors) so that the presence of stable and unstable limit cycles are evident.

3. (optional; 20 points extra) Use the program to demonstrate that the model exhibits saddle-node bifurcation when varying I from 3 to 6 with the following parameter values: C=1, g_{L}=8, E_{L}=-80, g_{Na}=20, E_{Na}=60, $V_{1/2}$=-20 and k=15 for $m_\infty(V)$, g_{K}=10, E_{K}=-90, $V_{1/2}$=-25 and k=5 for $n_\infty(V)$, and $\tau(V)$=0.14. Draw multiple phase trajectories (with different colors) so that the presence of stable limit cycles are evident. This bifurcation may look similar to SNIC bifurcation, but they are not the same. Describe the difference.