AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
Assignment #8

Due: 10:30 AM on December 2, 2020

1. Implementing the original 2-D BTW sandpile model.

   (a) (20 points) Write a computer program to simulate the BTW model (as explained in the class with a flow chart). Write a function with the input parameters of model size $L \times L$ and the number of iterations $n_{\max}$ and with the output parameters including the time sequence of avalanche size and the total number of avalanche occurrence per each size. You may also want to add a flag as an input parameter to turn on an option of displaying intermediate model results (as done in the class demonstration).

   All boundaries are open; grains can fall off from the edges. The size of an avalanche is the total number of relaxed sites per iteration, so the size does not exceed L^2 (even if a site is relaxed more than once during one iteration, that site contributes only one to the size count).

   (b) (20 points) Run the model for L = 8, 16, 32, and 64, with $n_{\max} = 10^4$, and plot the probability $P(s; L)$ as the function of avalanche size s using the log-log scale. Plot all results in one figure (with different line colors) so that you can compare them. Describe and explain any trend you observe in the plot.

2. (20 points) Modify your BTW script to make it non-conservative. You may achieve this effect by adding a stochastic nature to the relaxation rule: i.e., instead of distributing 4 grains to the nearest neighbors, you may distribute $4p$ grains, where p is a given probability ($0 \le p \le 1$), so you lose $4(1-p)$ grains at every relaxation. Compare the cases of p=0.25, 0.5, 0.75, and 1 with L=64 and $n_{\max} = 10^4$. Explain your results, in reference to the lecture on the random-neighbor sandpile model.

   Note: the number of grains at each site should be integers all the time.