AMTH 428 / E&EB 428 / EPS 428/528 / PHYS 428
Assignment #7

Due: 10:30 AM on November 18, 2020

1. 3-D site percolation with the renormalization group method.

   (a) (30%) We can generalize the 2-D site percolation model discussed in the class to a 3-D percolation model, by considering 3-D cubic arrays made up of cubic elements. The individual cubic elements are taken to be either permeable or impermeable, with the probability of p and $(1-p)$, respectively. The array is defined to be permeable if there is a continuous permeable path from top to bottom, through nearest-neighbor connections (in 3-D, there are six nearest neighbors). Assuming rescaling with $b = 2$ and the vertically-spanning rule, write down the renormalization group transformation $R_b(p)$.

2. (20%) It is vital to obtain the correct result in the above to proceed, so let's check the correctness of your answer numerically. Write a program to fill $2\times2\times2$ sites randomly with the occupation probability p and check whether it is permeable from top to bottom. For each value of $p = 0, 0.01, 0.02, ..., 0.99, 1$, randomly generate N instances of $2 \times 2 \times 2$ cubes and count the number of permeable cubes (call it M). The numerical estimate of $R_b(p)$ is then given by M/N. N has to be reasonably large (>100). Plot your answer for (1) with these numerical estimates. If you get a good match, proceed to (3). If not, go back to (1) or check your program.

   (b) (10%) Find both trivial and non-trivial fixed points of the transformation, and calculate the critical occupation probability as well as the critical exponent for correlation length. (You can use MATLAB's fzero, Mathematica's NSolve, Wolfram Alpha, or any other relevant function to find roots.)

   (c) (10%) Compare the critical occupation probability with those for the 1-D and 2-D cases, and explain the trend.

3. (optional: extra 40%) Describe a system with many degrees of freedom that has not been discussed in the course, is not the subject of your term project, and is something you are interested in understanding its behavior. Discuss how you might coarse-grain it.