Optimizing Sustainable Architecture for Winter Living using Passive Solar Houses
Quantitative Engineering Analysis 3 (formerly 2)

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October 18, 2020

1 Background

This is the first project of QEA2 at Olin College of Engineering.

In order to solidify our understanding of differential equations and heat transfer, we are designing a passive solar house for Vermont. A passive solar house combines interior features, exterior landscaping, and architectural design choices to ultimately heat the house with minimal electricity and fossil fuels.

The basic principles of a passive solar house are an aperture (window) to let solar radiation in, an absorber and thermal mass to absorb and store the heat from the sun, and some components of heat transfer to evenly distribute the heat throughout the house. Additionally, there are often some control elements to adjust the amount of solar radiation coming in throughout the changing seasons, and thermal insulation can be useful to retain heat in colder climates.

For our house, we plan on starting with a simple house design to get a feel for how heat moves throughout the building. We plan on utilizing a south facing window with an overhang roof to let the sun in when we need it during the winter and block it out during the summer when it is naturally warmer. This is because in the northeast the sun takes a lower southern path during the winter, and a higher path during the summer. To absorb and store all of this solar radiation, we plan on using a large slab of tile as our floor. Additionally, Vermont can get quite cold during the winter, so we will be insulating all of our walls and our floor.

2 Modelling

Ideal orientation is true north but orientations of up to 20° west of north and 30° east of north still allow good passive sun control.

Some initial assumptions that we make about our model are:

- The house is on stilts and therefore the heat loss from all sides of the house are equal
- Heat capacity of our tile floor is much greater than the rest of the house, therefore we can ignore the rest.
- Our tile floor is suspended above our insulation (no conduction of heat)
- Air flow in and out of the house is negligible
- All solar radiation hitting the windows is absorbed by heat storage unit, and solar radiation on other parts of the house is negligible
- Heat storage unit is at a spatially uniform temperature

You can see a graphical representation of the heat flows here:

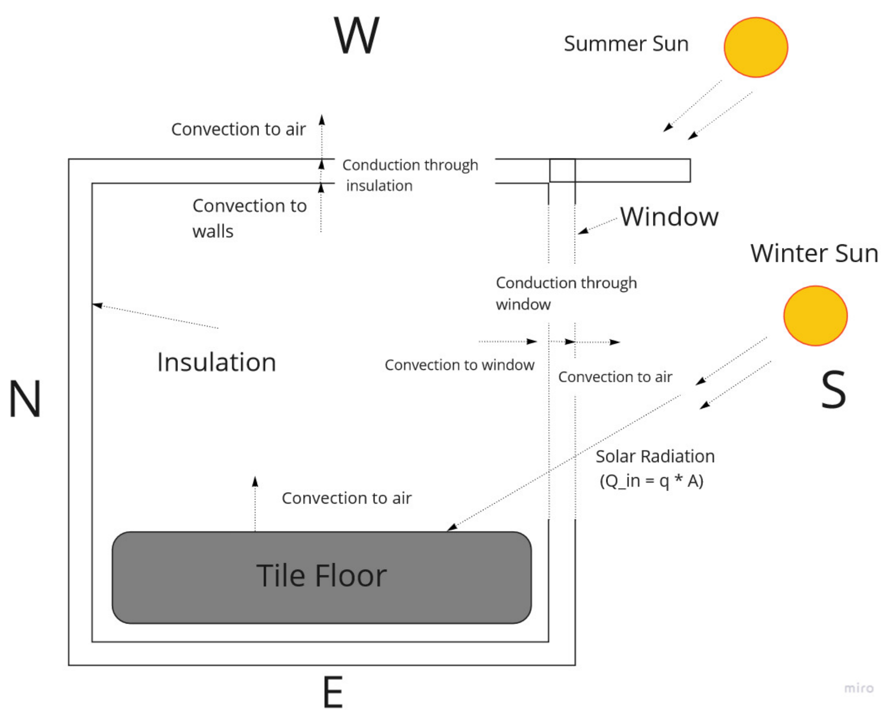

Figure 1: Graphical heat flow diagram

If we treat heat transfer by convection and conduction as electrical resistances, we get an equivalent resistance network like this:

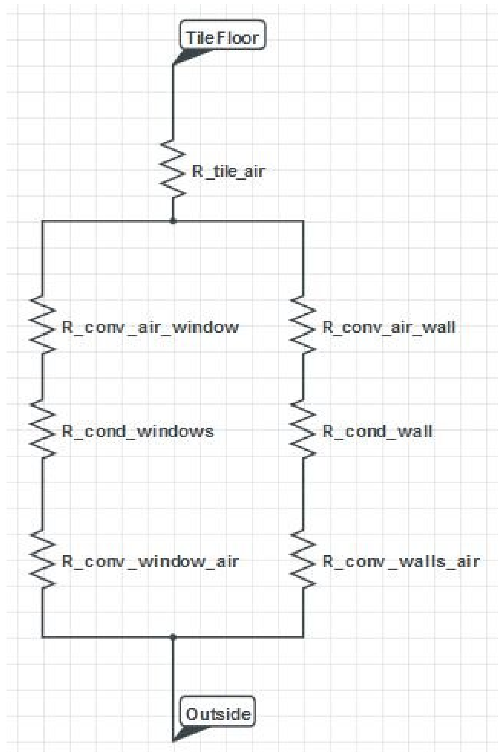

Figure 2: Equivalent resistive network by electrical analogue

Adding these following normal series/parallel resistor operations yields a total resistance of:

$$R_{tot} = \left(\frac{1}{h_{tile-air} \cdot A_{tile}}\right) + \left(\frac{1}{h_{air-wall} \cdot A_{tile}} + \frac{L_{wall}}{K_{wal}l \cdot A_{tile}} + \frac{1}{h_{wall-air} \cdot A_{tile}}\right)^{-1} + \left(\frac{1}{h_{air-window} \cdot A_{window}} + \frac{L_{window}}{K_{window} \cdot A_{window}} + \frac{1}{h_{window-air} \cdot A_{tile}}\right)^{-1}$$

where

- h is given in W/m^2-K
- A is given in m^2
- K is given in W/m-K
- L is given in m

We can then plug that into our governing differential equation:

$$q \cdot A - \frac{(T_{in} - T_{out})}{R_{tot}} = m \cdot C \cdot \frac{dT_{in}}{dt}$$

where

- q is given in W/m^2
- A is given in m^2
- T is given in °C
- R is given in K/W
- m is given in kg
- C is given in J/K

With these equations, we can determine the temperature of the house over time given some material properties, wall and window sizes, and solar flux through the windows. For this particular model we assume that solar flux and outside temperature are constant, however we will remove some of those simplifications later.

Give this stunningly simple model, we can the generate the periodic behavior of our house below:

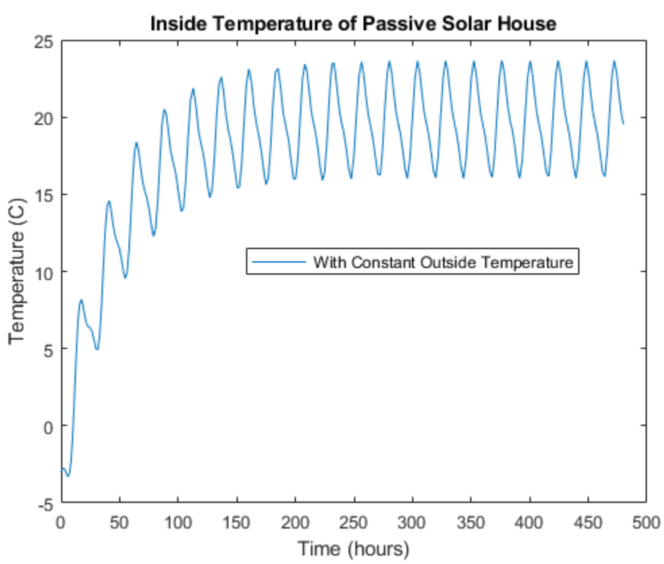

Figure 3: First pass model

Even though this is a simplified model, we get some pretty accurate behavior.

The temperature is within a "real" range, and the house takes a couple of weeks to reach a steady state, which is common of passively heated solar houses.

3 Optimization

Next, we can try to add some more complexity to our model to better simulate real conditions, and sweep some some parameters values to get a more comfortable house. At the moment, our dwelling's temperature range over the course of the day is a little large. Let's try and knock that down to about 17°- 23°

First, we modelled the outside air temperature as a variable function to better represent the changing weather conditions throughout a single day. You can see this compared to our original model below:

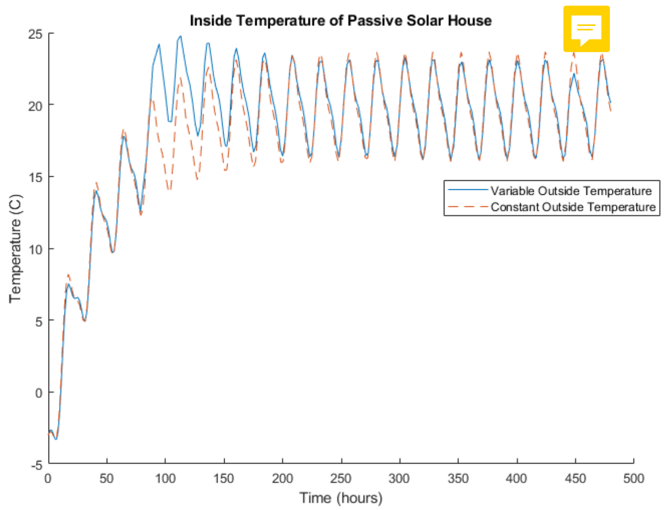

Figure 4: Added variable outside temperature to our model

Changing the temperature from constant to variable throughout the day did not have a significant effect on the temperature of our house. We noticed the variable temperature causes a spike in temperature as the house comes to equilibrium, but later settles to a similar temperature range as the constant temperature model.

Based on the temperature range from our initial model, our overall goal was to adjust our daily temperature range to something more comfortable (17-23 C). We started by sweeping different values that are "easily" changeable when building a house.

First, we tried changing insulation thickness:

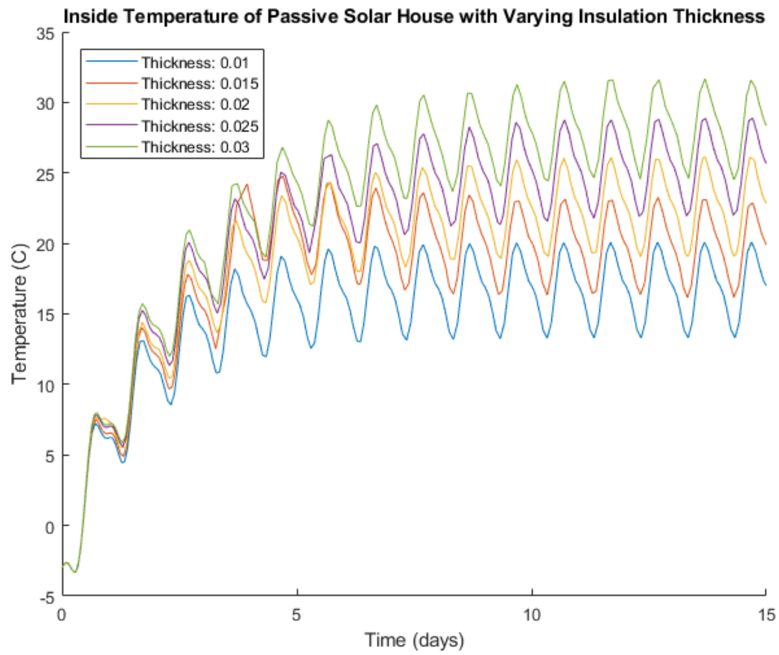

Figure 5: Sweeping insulation thickness values from 0.01 to 0.03 Meters

We found that this simply increased/decreased our overall behavior without affecting the range like we had hoped. This means that with a higher insulation thickness, the overall equilibrium temperature range is higher, and vice versa. This makes sense, because with a thicker insulation, the overall resistance in our resistance network goes up. Recalling our modelling equation:

$$q \cdot A - \frac{(T_{in} - T_{out})}{R_{tot}} = m \cdot c \cdot \frac{dT_{in}}{dt}$$

We can see that a higher resistance will reduce the fraction $\frac{(T_{in} - T_{out})}{R_{tot}}$ and - ceteris paribus - will result in a higher equilibrium temperature range.

We then swept different thicknesses of our tile, which turned out to achieve the desired behavior. This makes sense because a larger heat "capacitor" will take longer times to change temperature, reducing the frequency of our function. Furthermore, this behavior can be explained by re-arranging our original equation:

$$\frac{1}{m \cdot c} \cdot (q \cdot A - \frac{(T_{in} - T_{out})}{R_{tot}}) = \frac{dT_{in}}{dt}$$

Where a change in tile thickness changes the m term, which has an inverse relationship with $\frac{dT_{in}}{dt}$. A higher tile thickness will therefore result in a smaller overall change, reducing the range of temperatures. This can be seen below:

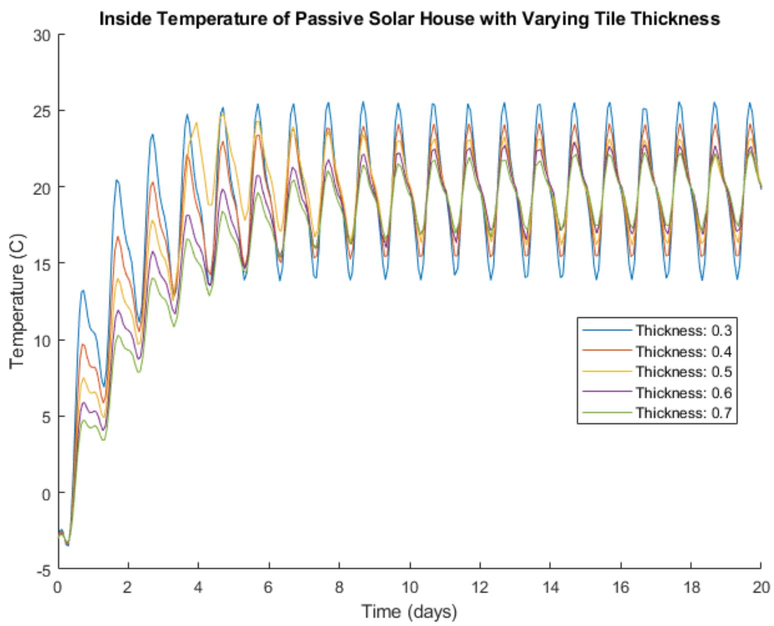

Figure 6: Sweeping tile thickness

You can see the effect more clearly below:

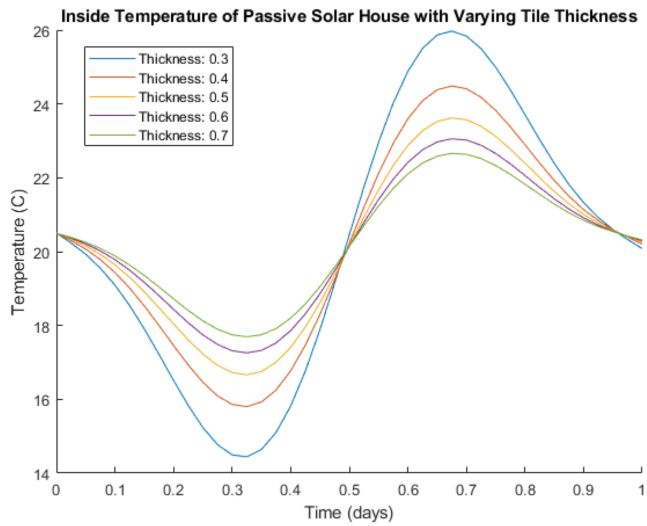

Figure 7: A closer look at tile thickness sweep

We now have two parameters that we can use to adjust our temperature range throughout the day and to move our overall temperature up and down. Next, to increase our model fidelity, we added heat storage in the air.

To do that, we can split up our original equation into two parts. For clarity, that is shown again below (with some modifications):

$$\frac{(T_{tile} - T_{air-in})}{R_{tot}} = m \cdot C_{tile} \cdot \frac{dT_{tile}}{dt}$$

To incorporate heat transfer through the air, we must create:

$$\frac{(T_{tile} - T_{air})}{R_{tile-air} \cdot m_{air} \cdot C_{air}} - \frac{(T_{air} - T_{outside})}{R_{remaining} \cdot m_{air} \cdot C_{air}} = m \cdot C_{tile} \cdot \frac{dT_{air-inside}}{dt}$$

where

$$R_{tile-air} = \left(\frac{1}{h_{tile-air} \cdot A_{tile}} \right)$$

$$R_{remaining} = \left(\frac{1}{h_{air-wall} \cdot A_{tile}} + \frac{L_{wall}}{K_{wall}l \cdot A_{tile}} + \frac{1}{h_{wall-air} \cdot A_{tile}} \right)^{-1} +$$

$$\left(\frac{1}{h_{air-window} \cdot A_{window}} + \frac{L_{window}}{K_{window} \cdot A_{window}} + \frac{1}{h_{window-air} \cdot A_{tile}}\right)^{-1}$$

We can see the results of this change below:

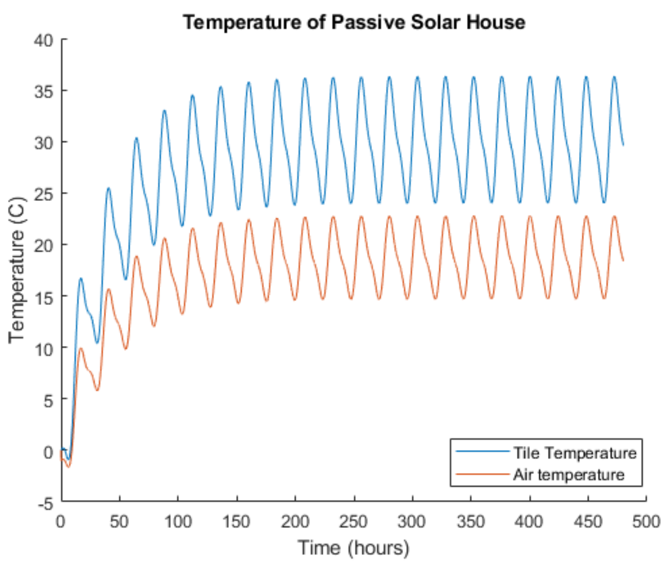

Figure 8: Temperature of tile and air in house over time with heat storage in air and tile

4 Results and Discussion

After determining the effect of tile thickness and insulation thickness on the temperature of our house, we used this insight to adjust the values until our house maintained a comfortable temperature range.

To achieve our ideal temperature range, it turns our we need a $0.7m$ thick tile. Realistically, this unfeasible for even the bravest contractors. By contrast, we need a measly $0.015m$ thick wall (insulation).

You can see the final behavior of our simple optimized function over a 24 hour period below:

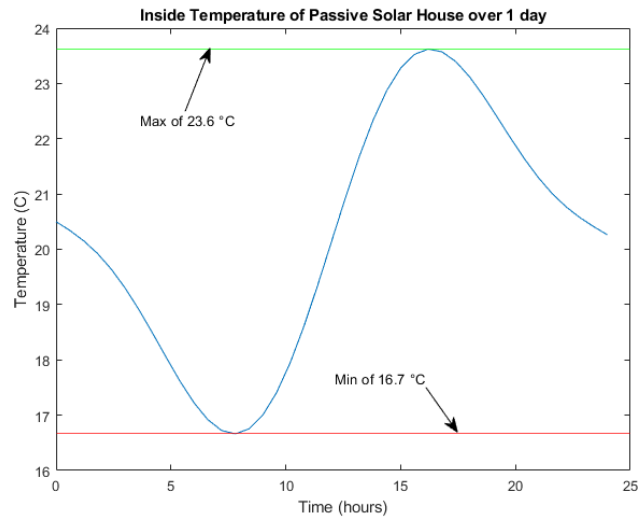

Figure 9: Temperature of inside house with a single thermal mass and insulation thickness of $0.015m$ and tile thickness of $0.7m$

Our higher fidelity model (adding in heat transfer through air) changes these values to something more realistic. We need to increase our wall insulation thickness to $0.0367m$, and decrease our tile thickness to $0.47m$. This makes sense, because if we treat air as having some heat capacitance, the capacitance of our "solar battery" does not need to be as high. You can see behavior of the high fidelity, optimized model over a 24 hour period below:

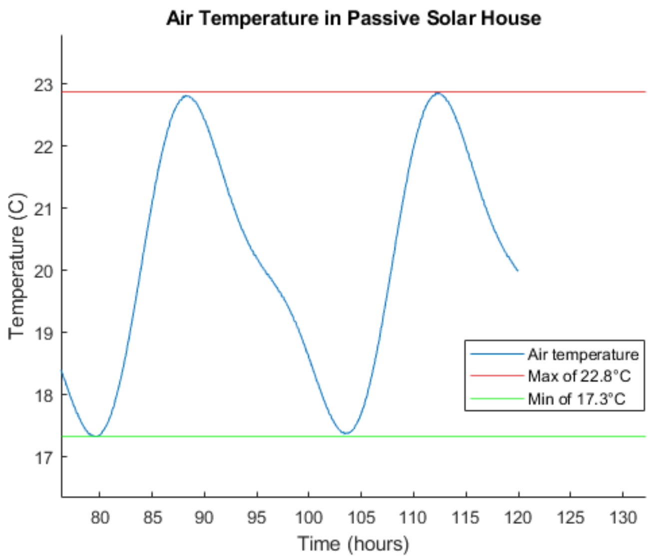

Figure 10: Temperature of the air in the house with heat storage in tile and air, and with insulation thickness of $0.0367m$ and tile thickness of $0.47m$

Our final model showed an exceedingly comfortable temperature range between 22.8 °C and 17.3 °C during the winter. This relatively comfortable for a human to live in (compared to the average house temperature of 20 °C - 24.4 °c). Another change we would like to make in the future is to include real walls in our house. At the moment, we simplified our walls to be fiberglass insulation, because that is the wall's most significant thermal property by volume. However in the future, we could easily include the thermal properties of wood, drywall, or whatever else we want to build our home out of. We think this would increase the overall resistance and add some heat storage, and would generally be a more accurate model.

The authors both feel that living in this house (temperature-wise at least). However, we noticed this house can become warm in the summer as temperatures rise. One way to combat this would be to include windows that allow airflow (i.e. posses an ability to open), which would decrease the overall resistance of the house and can let heat out. The eaves of house could also presumably be optimized further.

Play around with our model for yourself:
`https://github.com/jonaskaz/passive-solar-house`

References

[1] Reardon, Chris. “Passive Solar Heating.” YourHome, Oct. 2013, www.yourhome.gov.au/passive-design/passive-solar-heating.

[2] “Using Thermal Mass for Heating and Cooling - Smarter Homes Practical Advice on Smarter Home Essentials.” Smarter Homes, 29 July 2020, www.smarterhomes.org.nz/smart-guides/design/thermal-mass-for-heating-and-cooling/. Exploring possible thicknesses and materials for our thermal mass.

[3] Malley, Melinda, et al. Quantitative Engineering Analysis 2. 2nd ed., 2020.

[4] Young, Hugh D. “Thermal Conductivity.” HyperPhysics, hyperphysics.phy-astr.gsu.edu/hbase/Tables/thrcn.html. Used for thermal conductivity through ordinary glass

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