Aufgabensammlung 1

Aufgabe 3:

$A(n) = \log(n)$ $\quad B(n) = 2^{\sqrt{\log n}}$ $\quad C(n) = (\log n)^{\log n}$

$D(n) = \log(n!)$ $\quad E(n) = (\log n)^{\sqrt{n}}$ $\quad F(n) = (\log(\log n))^n$

$G(n) = 42\ n$

$$2^{\log(x)} = x$$

$A(n) = 2^{\log(\log(n))}$ $\quad B(n) = 2^{\sqrt{\log n}}$

$C(n) = 2^{\log((\log(n))^{\log(n)})} = 2^{\log(n)\cdot\log(\log(n))}$

$D(n) = \log(n!) \approx n\cdot\log(n) = 2^{\log(n)+\log(\log(n))}$

$E(n) = 2^{\sqrt{n}\cdot\log(\log(n))}$ $\quad F(n) = \log(\log(n)) \geq 2^{\frac{n}{}}$ für $n \geq 16$

$G(n) = 42\cdot 2^{\log(n)}$

$A < B < G(n) < C < E$

$A < B$: $\log(\log(n)) \leq \sqrt{\log(n)}$ für x groß genug

$B < G$: $\sqrt{\log(n)} \leq \log(n)$ $\quad G < D$

$G < C$: $\log(n) \leq \log(n)\cdot\log(\log(n))$ $\quad D < C$

$C < E$: $\log(n)\cdot\log(\log(n)) \leq \sqrt{n}\cdot\log(\log(n))$

$E < F$: $\sqrt{n}\cdot\log(\log(n)) < \sqrt{n}\cdot\sqrt{n} = n$

Aufgabensammlung 2

Aufgabe 1:

e) $T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n \quad , \quad T(2) = 1 \qquad n = 2^{2^k}$

$$T(n) = T(2^{2^k}) = (2^{2^k})^{\frac{1}{2}} \cdot T\left((2^{2^k})^{\frac{1}{2}}\right) + 2^{2^k}$$

$$= (2^{2^k})^{\frac{1}{2}} \cdot \left[(2^{2^k})^{\frac{1}{4}} \cdot T\left((2^{2^k})^{\frac{1}{4}}\right) + (2^{2^k})^{\frac{1}{2}} \right] + 2^{2^k}$$

$$= (2^{2^k})^{\frac{1}{2}} \cdot \left[(2^{2^k})^{\frac{1}{4}} \cdot \left\{ (2^{2^k})^{\frac{1}{8}} \cdot T\left((2^{2^k})^{\frac{1}{8}}\right) + (2^{2^k})^{\frac{1}{4}} \right\} + (2^{2^k})^{\frac{1}{2}} \right] + 2^{2^k}$$

$$= (2^{2^k})^{\frac{1}{2}} \cdot \left[(2^{2^k})^{3/8} \cdot T\left((2^{2^k})^{\frac{1}{8}}\right) + (2^{2^k})^{\frac{1}{2}} + (2^{2^k})^{\frac{1}{2}} \right] + 2^{2^k}$$

$$= (2^{2^k})^{7/8} \, T\left((2^{2^k})^{\frac{1}{8}}\right) + 2^{2^k} + 2^{2^k} + 2^{2^k}$$

$$\vdots$$

$$= (2^{2^k})^{1-\frac{1}{2^k}} \cdot 1 + k \cdot 2^{2^k}$$

$$= \frac{2^{2^k}}{(2^{2^k})^{\frac{1}{2^k}}} \cdot 1 + k \cdot 2^{2^k}$$

$$= \frac{2^{2^k}}{2} + k \cdot 2^{2^k}$$

$$= \left(k + \frac{1}{2}\right) \cdot 2^{2^k}$$

$$T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$$

$$T(n) = T(2^{2^k}) = (k + 1/2) \cdot 2^{2^k}$$

IA: $k = 0$ $\quad (0 + 1/2) \cdot 2^{2^0} = 1/2 \cdot 2^1 = 1$ ✓

IS: $k \to k+1$

$$T(2^{2^{k+1}}) = 2^{2^k} \cdot T(2^{2^k}) + 2^{2^{k+1}}$$

$$\overset{IA}{=} 2^{2^k} \cdot \left[(k + \tfrac{1}{2}) \cdot 2^{2^k} \right] + 2^{2^{k+1}}$$

$$= (k + \tfrac{1}{2}) \cdot 2^{2^{k+1}} + 2^{2^{k+1}}$$

$$= (k + \tfrac{1}{2} + 1) \cdot 2^{2^{k+1}}$$ ✓