Equations

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$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\frac{1}{\rho_0}\nabla p + \nu \Delta \boldsymbol{u}. \tag{1}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \tag{2}$$

$$\Delta p = -\rho_0 (\nabla \boldsymbol{u}) : (\nabla \boldsymbol{u})^T. \tag{3}$$

$$\boldsymbol{u}(\boldsymbol{x}, t) \tag{4}$$

$$p(\boldsymbol{x}, t) \tag{5}$$

$$\nu \tag{6}$$

$$\vec{u}^* = \frac{\vec{u}}{U} \tag{7}$$

$$p^* = \frac{p}{\rho_0 U^2} \tag{8}$$

$$\partial_t^* = \frac{L}{V}\partial_t \tag{9}$$

$$\vec{\nabla}^* = L\vec{\nabla} \tag{10}$$

$$Re = \frac{UL}{\nu} \tag{11}$$

$$\partial_t^* \boldsymbol{u^*} + (\boldsymbol{u^*} \cdot \boldsymbol{\nabla^*})\boldsymbol{u^*} = -\nabla p^* + \frac{\nu}{UL}\Delta \boldsymbol{u^*}. \tag{12}$$

$$f_i^{\mathrm{in}}(\boldsymbol{x}, t) \tag{13}$$

$$f_i^{\mathrm{out}}(\boldsymbol{x}, t) \tag{14}$$

$$f^{\mathrm{in}} \tag{15}$$

$$f^{\mathrm{out}} \tag{16}$$

$$\rho(\boldsymbol{x},t) = \sum_{i=0}^{8} f_i^{\text{in}}(\boldsymbol{x},t) \tag{17}$$

$$\boldsymbol{u}(\boldsymbol{x},t) = \frac{1}{\rho(\boldsymbol{x},t)} \frac{\delta x}{\delta t} \sum_{i=0}^{8} \boldsymbol{v}_i f_i^{\text{in}}(\boldsymbol{x},t) \tag{18}$$

$$n_i^{\text{out}}(\boldsymbol{x},t) = n_i^{\text{in}}(\boldsymbol{x},t) + \Omega_i(\boldsymbol{x},t) \tag{19}$$

$$f_i^{\text{in}}(\boldsymbol{x},t) = f_i^{\text{out}}(\boldsymbol{x} - \boldsymbol{v}_i \delta x, t - \delta t) \tag{20}$$

$$n_i^{\text{in}}(\boldsymbol{x},t) = n_i^{\text{out}}(\boldsymbol{x} - \boldsymbol{v}_i \delta x, t - \delta t) \tag{21}$$

$$p = c_s^2 \rho \tag{22}$$

$$c_s^2 = \frac{1}{3} \frac{\delta x^2}{\delta t^2} \tag{23}$$

$$f_i^{\text{out}} - f_i^{\text{in}} = -\omega \left(f_i^{\text{in}} - E(i, \rho, \vec{u}) \right) \tag{24}$$

$$E(i, \rho, \boldsymbol{u}) = \rho\, t_i \left(1 + \frac{\frac{\delta x}{\delta t} \boldsymbol{v}_i \cdot \boldsymbol{u}}{c_s^2} + \frac{1}{2\, c_s^4} \left(\tfrac{\delta x}{\delta t} \boldsymbol{v}_i \cdot \boldsymbol{u} \right)^2 - \frac{1}{2\, c_s^2} \left| \boldsymbol{u} \right|^2 \right) \tag{25}$$

$$\nu = \delta t\, c_s^2 \left(\frac{1}{\omega} - \frac{1}{2} \right) \tag{26}$$

$$f_8^{\text{in}} \tag{27}$$

$$f_7^{\text{in}} \tag{28}$$

$$f_6^{\text{in}} \tag{29}$$

$$f_5^{\text{in}} \tag{30}$$

$$f_4^{\text{in}} \tag{31}$$

$$f_3^{\text{in}} \tag{32}$$

$$f_2^{\text{in}} \tag{33}$$

$$f_1^{\text{in}} \tag{34}$$

$$f_0^{\text{in}} \tag{35}$$

$$f_8^{\mathrm{out}} \tag{36}$$

$$f_7^{\mathrm{out}} \tag{37}$$

$$f_6^{\mathrm{out}} \tag{38}$$

$$f_5^{\mathrm{out}} \tag{39}$$

$$f_4^{\mathrm{out}} \tag{40}$$

$$f_3^{\mathrm{out}} \tag{41}$$

$$f_2^{\mathrm{out}} \tag{42}$$

$$f_1^{\mathrm{out}} \tag{43}$$

$$f_0^{\mathrm{out}} \tag{44}$$

$$f_i^{\mathrm{in}}(\boldsymbol{x}, t+1) = f_j^{\mathrm{out}}(\boldsymbol{x}, t) \tag{45}$$

$$v_i = -v_j \tag{46}$$

$$f_0^{\mathrm{in}} = E(0, \rho\,\boldsymbol{u}) + (f_8^{\mathrm{in}} - E(8, \rho, \boldsymbol{u})) \tag{47}$$

$$f_1^{\mathrm{in}} = E(1, \rho\,\boldsymbol{u}) + (f_7^{\mathrm{in}} - E(7, \rho, \boldsymbol{u})) \tag{48}$$

$$f_2^{\mathrm{in}} = E(2, \rho\,\boldsymbol{u}) + (f_6^{\mathrm{in}} - E(6, \rho, \boldsymbol{u})) \tag{49}$$