



A new approach to the lattice Boltzmann method for graphics processing units

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ABSTRACT

Emerging many-core processors, like CUDA capable nVidia GPUs, are promising platforms for regular parallel algorithms such as the Lattice Boltzmann Method (LBM). Since the global memory for graphic devices shows high latency and LBM is data intensive, the memory access pattern is an important issue for achieving good performances. Whenever possible, global memory loads and stores should be coalescent and aligned, but the propagation phase in LBM can lead to frequent misaligned memory accesses. Most previous CUDA implementations of 3D LBM addressed this problem by using low latency on chip shared memory. Instead of this, our CUDA implementation of LBM follows carefully chosen data transfer schemes in global memory. For the 3D lid-driven cavity test case, we obtained up to 86% of the global memory maximal throughput on nVidia's GT200. We show that as a consequence highly efficient implementations of LBM on GPUs are possible, even for complex models.

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1. Introduction

During the last decade, the computational power of commodity graphics hardware has dramatically increased, as shown in Fig. 1, nearing 1 GFlop/s with nVidia's latest GT200. Yet, one should be aware that this performance is attainable only for single-precision computations, which are not fully IEEE-754 compliant. Nonetheless, due to their low cost, GPUs are becoming more and more popular for scientific computations (see [1,2]).

The lattice Boltzmann method, which originates from the lattice gas automata methods, is an efficient alternative to the numerical solving of Navier–Stokes equations for simulations of complex fluid systems. Besides its numerical stability and accuracy, one of the major advantages of LBM is its data parallel nature. Nevertheless, using LBM for practical purposes requires large computational power. Thus, several attempts to implement LBM on GPUs have been made recently.

In this paper, we intend to present some optimisation principles for CUDA programming. These principles led us to a GPU implementation of 3D LBM which appears to be a good alternative to previously published ones.

2. CUDA

The Compute Unified Device Architecture (CUDA), released by nVidia in early 2007, has been up to now the leading technology for general purpose GPU programming (see [3]). It consists of hardware specifications, a specific programming model, and a programming environment (API and SDK).

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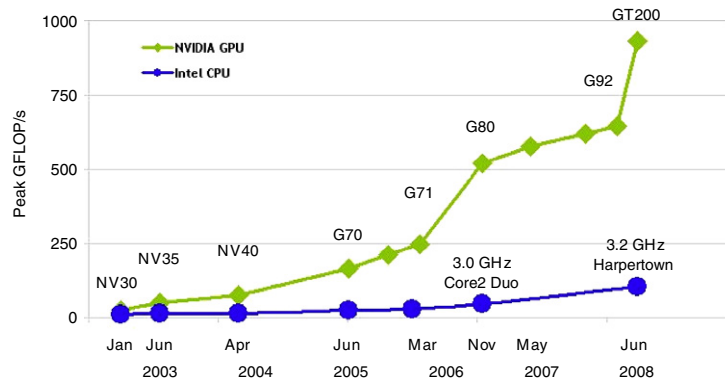


Fig. 1. Peak performances GPU vs CPU (source nVidia).

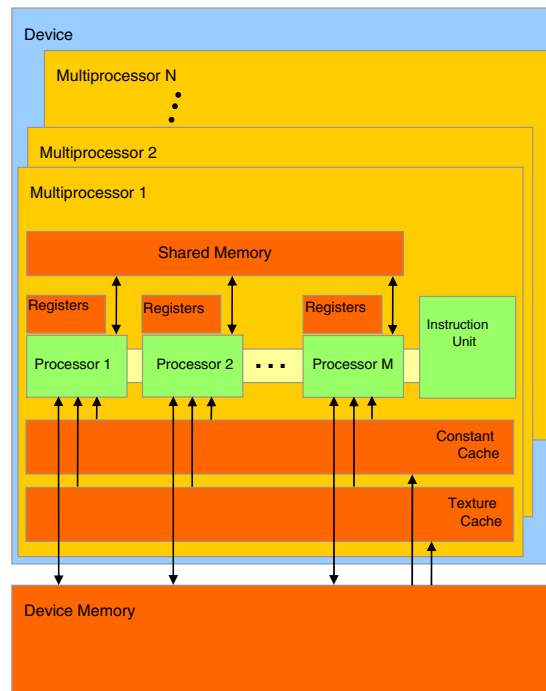


Fig. 2. CUDA hardware (source nVidia).

2.1. Architecture

General purpose GPU programming usually requires one to take some architectural aspects into consideration. CUDA hardware specifications make the optimisation process easier by providing a general model for the nVidia GPUs architecture from the G80 generation on.

Fig. 2 shows the main aspects of the CUDA hardware specifications. A GPU consists in several *Streaming Multiprocessors* (SMs). Each SM contains *Scalar Processors* (SPs), an instruction unit, and a shared memory, concurrently accessible by the SPs through 16 memory banks. Two cached, read-only memories for constants and textures are also available. The device memory, usually named the *global memory*, is accessible by both the GPU and the CPU. Table 1 specifies some of the features of the GT200 processor on which our implementations were tested.

SPs are only able to perform single-precision computations. From compute capability 1.3 on, CUDA supports double precision. On this kind of hardware, each SM is linked to a double-precision computation unit. Both single- and double-precision calculations are mostly IEEE-754 compliant. The main divergences from the standard are:

- No denormalised numbers. Numbers with null exponent are considered as zero.
- Partial support of rounding modes.
- No floating point exception mechanism.

Table 1
Features of the GT200.

Number of SMs	30
Number of SPs per SM	8
Registers per SM	16,384
Shared memory	16 KB
Constant cache	8 KB
Texture cache	8 KB
Global memory	up to 4 GB

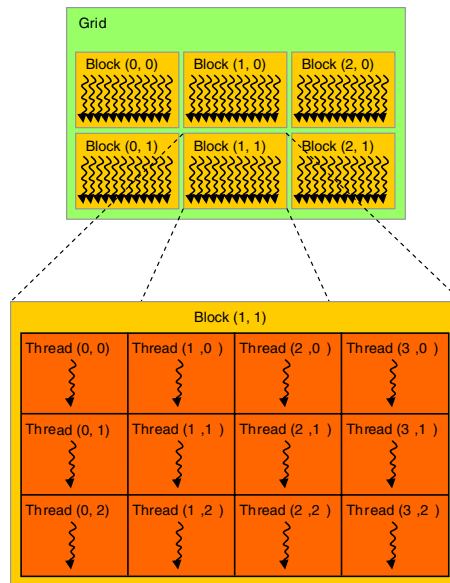


Fig. 3. CUDA programming model (source nVidia).

- Multiply-add operations with truncated intermediate results.
- Non-compliant implementations of some operations like division and square root.

2.2. Programming

The CUDA programming model (see [4]) relies on the concept of a kernel. A kernel is a function that is executed in concurrent threads on the GPU. Threads are grouped into blocks which in turn form the execution grid (see Fig. 3).

The CUDA technology makes use of a slightly modified version of the C (or C++) language as a programming language. The code of a CUDA application consists in functions which can be classified in four categories:

1. Sequential functions run by the CPU.
2. Launching functions allowing the CPU to start a kernel.
3. Kernels run by the GPU.
4. Auxiliary functions which are inlined into the kernels at compile time.

The execution grid's layout is specified at run time. A grid may have one or two dimensions. The blocks of threads within a grid must be identical and may have up to three dimensions. A thread is identified with respect to the grid using the two structures `threadIdx` and `blockIdx`, containing the three fields `x`, `y`, and `z`.

A block may only be executed on a single SM, which yields an upper bound of the number of threads within a block.¹ Scheduling is carried out at hardware level and may not be adjusted. It is still possible to place synchronisation barriers, but their scope is limited to blocks. The only way to ensure global synchronisation is to use a kernel for each step.

The local variables of a kernel are stored in the registers of the SMs. Their scope is limited to threads and they cannot be used for communication purposes. Data exchanges between threads require the use of shared memory. The management of these exchanges is left to the programmer. It is worth noting that no protection mechanism is available; hence concurrent writes at the same memory location yield unpredictable results. The shared memory's scope is limited to blocks. Communication between threads belonging to different blocks requires the use of global memory.

¹ As of compute capability 1.3, this maximum is 1024.

Table 2

Cost of floating point operations.

Operation	Cycles
Add, multiply, multiply-add	4
Reciprocal, logarithm	16
Sine, cosine, exponential	32
Divide	36

3. Optimisation principles

3.1. The computational aspect

Generally speaking, the occupancy rate of the SPs, i.e. the ratio between the number of threads run and the maximum number of executable threads, is an important aspect to take into consideration for the optimisation of a CUDA kernel. Even though a block may only be run on a single SM, it is possible to execute several blocks concurrently on the same SM. Hence tuning the execution grid's layout allows one to increase the occupancy rate. Nevertheless, reaching the maximal occupancy is usually not possible: the threads executed in parallel on one SM have to share the available registers. On compute capability 1.3 architectures, for instance, maximal occupancy is achieved only for kernels using at most 16 registers, that is to say only the simplest ones. It should be noted that shared memory, which is rather scarce too, may also be a limiting factor for the occupancy.

The rather elementary optimisation technique consisting in common sub-expression elimination should be used with care. As a matter of fact, this method implies storing the values of these sub-expressions in temporary variables, hence increasing the use of registers, which in turn may lead to lower occupancy. In some cases, this common sense technique has negative effects, and it may be better to recompute some values than to store them. Anyway, general principles regarding this topic are not relevant. Since the compiler performs aggressive optimisation, the number of registers needed for a given kernel is scarcely predictable.

The hardware scheduler groups threads in warps of 32 threads. Though not mandatory, the number of threads in a block should be a multiple of the warp size. Whenever a warp is running, all the corresponding threads are executed concurrently by the SPs, except when conditional branching occurs. Divergent branches are executed sequentially by the SM. Even though serialisation only happens at warp level, conditional structures should be avoided as much as possible, being likely to have a major impact on actual parallelism.

As regards optimisation, the cost of arithmetic operations (in clock cycles) must also be taken in consideration. Table 2 displays the time needed for a warp to perform the most common single-precision floating point operations:

It should be noted that, since addition and multiplication are merged in one single *axpy* operation whenever possible, evaluating the actual algorithmic complexity of a computation is not straightforward. It's also worth noting that division is rather expensive and should be used parsimoniously.

3.2. The data transfer aspect

For many applications, memory transaction optimisation appears to be even more important than computation optimisation. Registers do not give rise to any specific problem apart from their limited amount. Shared memory is, in terms of speed, similar to a register but is accessed by the SPs through 16 memory banks. For efficient accesses, each thread in a half-warp must use a different bank. When this condition is not met, the transaction is repeated as much as necessary.

Global memory, being the only one accessible by both the CPU and the GPU, is a critical path for CUDA applications. Unlike registers and shared memory, global memory suffers high latency ranging from 400 to 600 clock cycles. Nonetheless, this latency can be mostly hidden by the scheduler which stalls inactive warps until data are available. Furthermore, global memory throughput is significantly less than register throughput. For data intensive applications like LBM, this aspect is generally the limiting factor.

Global memory accesses are performed by half-warp on segments of 32, 64, or 128 bytes whose start addresses are multiples of the segment's size. To optimise global memory transactions, memory accesses should be coalesced and aligned whenever possible. To achieve coalescence, threads within a half-warp must access contiguous memory locations.

4. Data transfer modelling

In CUDA applications, the execution of a kernel can generally be split into three steps:

1. Reading data from global memory.
2. Processing data using registers (and possibly shared memory).
3. Writing processed data to global memory.

Code 1 follows this scheme in the case where the amounts of data read and written are equal. Function `launch_kernel` calls function `kernel` with an execution grid containing L^3 threads. One may notice some syntactic specificities of the CUDA programming language: the use of the triple angle brackets for kernel invocation, the `__global__` keyword for

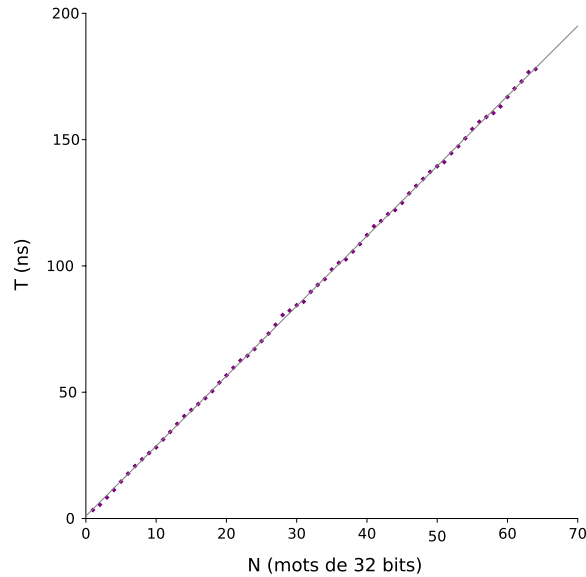


Fig. 4. Average transfer time.

kernel definition, the `__device__` keyword for auxiliary functions. The kernel performs the reading and writing of N 32-bit words. The second step is simplistic, though not suppressed in order to ensure actual data transfer to the GPU. Global memory accesses are optimal, provided L is set to an appropriate value. In the present study, $L = 128$ was chosen.

```
#define id(j, k) k + SIZE*(j)

__device__ int index(void)
{
    int x = threadIdx.x;
    int y = blockIdx.x;
    int z = blockIdx.y;
    return x + y*L + z*L2;
}

__global__ void kernel(int N, float* t)
{
    int k = index();

    for (int j = 0; j <= N; j++)
    {
        t[id(j+1, k)] = t[id(j, k)]*0.5;
    }
}

extern "C" void launch_kernel(int N, float* t)
{
    dim3 grid(L, L);
    dim3 block(L);

    kernel<<<grid, block>>>(N, t);
    cudaThreadSynchronize();
}
```

Code 1. Data transfer benchmark kernel.

Measuring the execution time of the `launch_kernel` function enables us to estimate the average time T for data transfer between the GPU and global memory relatively to the amount N of data exchanged. Fig. 4 shows the results for one warp with T in nanoseconds and N in 32-bit words, obtained using a GeForce GTX 295 graphics board.

The quasi-linear aspect of these measurements reveals that, in ideal cases, the hardware scheduler is able to hide the latency of global memory. Numerically, we obtain

$$T \approx 2.78 \times N + 0.99 \quad (1)$$

with again T in nanoseconds and N in 32-bit words.

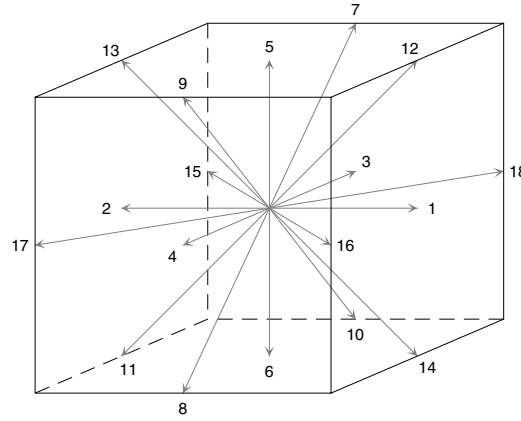


Fig. 5. The D3Q19 stencil.

The average throughput is almost constant relative to N and is about 90.7 GB/s for the GeForce GTX 295. Reckoning the characteristics of the benchmark program, we consider the value obtained as the effective maximal throughput for data transfer between the GPU and global memory. This upper bound is useful in evaluating the performances of a CUDA application leaving aside the hardware in use. The value obtained is about 81% of the theoretical maximal throughput, which is comparable to the result found in [5].

For the same hardware, the `bandwidthTest` program from the CUDA SDK gives 91.3 GB/s, which is rather close to the value that we obtained. However, this program uses only memory copy functions instead of a kernel, hence yielding less relevant results from a practical standpoint.

5. The lattice Boltzmann method

The lattice Boltzmann method is based on a threefold discretisation of the Boltzmann equation: time, space and velocity (see [6]). Velocity space reduces to a finite set of well chosen velocities $\{\mathbf{e}_i \mid i = 0, \dots, N\}$ where $\mathbf{e}_0 = \mathbf{0}$. Fig. 5 illustrates the D3Q19 stencil that we used.

Instead of reviewing the well-known Lattice Bhatnagar–Gross–Krook (LBGK) model (see [7]), we will outline the multiple-relaxation-time model presented in [8]. The analogue of the one-particle distribution function f is a set of $N + 1$ mass fractions f_i . We define

$$|f(\mathbf{x}, t)\rangle = (f_0(\mathbf{x}, t), \dots, f_N(\mathbf{x}, t))^T$$

for given lattice node \mathbf{x} and time t , T being the transpose operator. The mass fractions can be mapped to a set of moments $\{m_i \mid i = 0, \dots, N\}$ by an invertible matrix M as follows:

$$|f(\mathbf{x}, t)\rangle = M^{-1} |m(\mathbf{x}, t)\rangle \quad (2)$$

where $|m(\mathbf{x}, t)\rangle$ is the moment vector. With the D3Q19 stencil, the density is $\rho = m_0$, the momentum is $\mathbf{j} = (m_3, m_5, m_7)$. Higher order moments as well as the matrix M are given in detail in [8, App. A]. Using this notation, the lattice Boltzmann equation can be written as

$$|f(\mathbf{x} + \delta t \mathbf{e}_i, t + \delta t)\rangle - |f(\mathbf{x}, t)\rangle = M^{-1} S (|m(\mathbf{x}, t)\rangle - |m^{(eq)}(\mathbf{x}, t)\rangle) \quad (3)$$

where $|m^{(eq)}(\mathbf{x}, t)\rangle$ is the equilibrium-moment vector and

$$S = \text{diag}(s_0, \dots, s_N)$$

is the relaxation rates matrix. The LBGK model is a special case of MRT where all relaxation rates $s_i = 1/\tau$. From a numerical point of view, MRT should be preferred to LBGK, being more stable and accurate.

6. Previous work

Data organisation scheme for LBM are mainly of two kinds. The first is the Array of Structures (AoS) type, which for D3Q19 is equivalent to $L^3 \times 19$ arrays. The second is the Structure of Arrays (SoA) type, which for D3Q19 is equivalent to $19 \times L^3$ arrays. For CPU implementations of the LBM, the AoS is relevant insofar as it improves the locality of mass fractions associated with the same node (see [9]). Up to now, for all GPU implementations of LBM, a thread is allocated to each lattice node, which is probably the simplest way to take advantage of the massively parallel structure of the GPU. With this approach, ensuring coalescence of global memory accesses requires one to use a SoA kind of organisation.

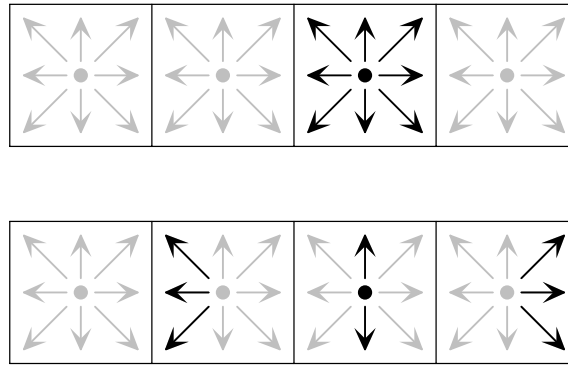


Fig. 6. Propagation using shared memory.

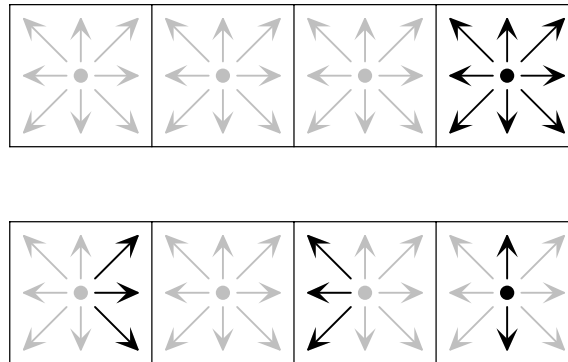


Fig. 7. Storage of outgoing mass fractions.

With values of L divisible by 16, every mass fraction associated with a half-warp lay in the same segment of global memory. Yet, this is not sufficient to ensure optimal memory transaction. Indeed, for the minor spatial dimension, propagation corresponds to one-unit shifts of memory addresses. In other words, for most mass fractions, the propagation phase leads to misalignments. Getting round this problem has up to now been the main issue as regards GPU implementations of the LBM.

The first attempt at implementing a D3Q19 model using CUDA was due to Ryoo et al. (see [10]). It consists mainly in a port of the 470 . 1bm code from the SPEC CPU2006 benchmark (see [11]). In terms of optimisation, switching from AoS to SoA is the only important modification undertaken. To the best of our knowledge, misalignment problems caused by propagation are not taken into consideration. The announced speed-up factor of 12.3 is rather low compared to subsequent results.

The two-dimensional D2Q9 implementation submitted by Tölke in [5] solves the misalignment problems using one-dimensional blocks and shared memory. More precisely, propagation within one block is split into two steps: a longitudinal shift in shared memory followed by a lateral shift in global memory. This approach is outlined in Fig. 6.

Since blocks follow the minor dimension, no more misalignment arises. Nevertheless, because of the limited scope of shared memory, mass fractions leaving or entering a block require specific handling. The retained solution is to store outgoing mass fractions in places temporarily left vacant by incoming mass fractions (see Fig. 7).

A drawback of the shared memory approach in two dimensions is consequently the need for a second kernel exchanging data in order to place properly mass fractions located at the blocks' boundaries. Obviously, this further processing has a non-negligible cost. In three dimensions, the narrowest dimension is likely to fit into one single block. Hence, the use of a second kernel can be avoided in most cases.

Following the same method as Tölke, Habich in [12] describes an implementation of a D3Q19 model. The transition from D2Q9 to D3Q19 leads to lower performances, achieving only 51% of the effective maximal throughput on GeForce 8800 GTX. Habich assumes that this decrease is due to the low occupancy rate. As a matter of fact, the number of threads run in parallel on a SP is bounded by the amount of available registers.

A way to obtain better performances for three-dimensional LBM consists in using stencils containing fewer mass fractions, like D3Q13. This approach was studied by Tölke and Krafczyk in [13], obtaining 61% of the effective maximal throughput on GeForce 8800 Ultra. The D3Q13 stencil, which corresponds to the points of contact in a close packing of spheres, is the simplest three-dimensional structure sufficiently isotropic for LBM. Yet, due to the lesser amount of information processed, D3Q13 is less accurate than D3Q19. Furthermore, node addressing becomes quite complex.

Bailey et al. in [14] announce a 20% improvement of maximal performances for their implementation of D3Q19 compared to those published in [12]. The description of the tested optimisations is not very explicit, but it seems that the main intention

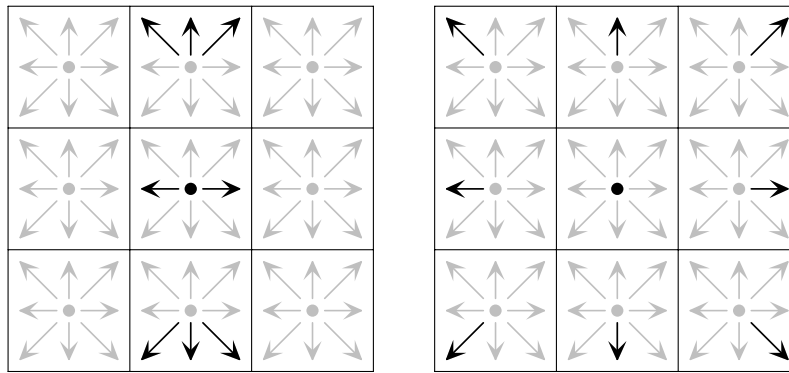


Fig. 8. Split propagation scheme.

was to increase occupancy. One of the proposed techniques consists in imposing at compile time an upper bound to the number of registers used by the computation kernel. Of course this directive causes the compiler to fall back on register spilling. Taking the cost of global memory accesses into account, we consider this approach as not relevant.

All but one of the CUDA implementations of LBM mentioned in the present section use shared memory for propagation. Although they are rather efficient, these implementations are very demanding as regards hardware, not leaving much room for possible extensions. It would be difficult, using the same approach, to implement models more complex than the usual isothermal D3Q19, for instance double-population or hybrid models (see [15,16]). This led us to strive for an alternative implementation of LBM on GPUs that would combine efficiency and lower pressure as regards hardware.

7. Proposed implementations

Though rather basic, the CUDA profiler allows one to gather some information during kernel run time (see [17]). Using this tool on GT200 hardware, we investigated the two-dimensional LBM code published by Tölke in [5], as well as modified versions. This led us to make two assumptions:

1. The additional cost caused by misalignment has the same order of magnitude as the one arising from the exchange kernel.
2. The cost of a misaligned read is less than the cost of a misaligned write.

Hence we adopted the following approach for our implementations of D3Q19:

- SoA type of data organisation.
- One-dimensional blocks following the minor dimension.
- Propagation performed by global memory transactions.
- Deferment of misalignment on reading.

To avoid overwriting problems, separate source and destination lattices reside in global memory. They are alternated at each time step. We experimented with two propagation schemes: a split scheme and a reversed scheme. The split scheme was tested with a LBGK model and on-grid boundary conditions. The reversed scheme was tested with a MRT model and mid-grid boundary conditions. To ease cross platform development, we employed the CMake build system (see [18]). Moreover, we used the XML based VTK format for output (see [19]).

7.1. The split scheme

With the split scheme, propagation is parted into two components: shifts that induce misalignment are performed at reading; the others are performed at writing, as outlined in Fig. 8. For the sake of simplicity, the diagram shows the two-dimensional case.

Boundary conditions are implemented using on-grid bounce back: nodes at the cavity's borders, except the lid, are considered as solid and simply return the incoming mass fractions in the opposite direction.

To summarise, the corresponding kernel breaks up into:

1. Reading along with propagation in minor dimensions.
2. On-grid bounce back boundary conditions.
3. Computations using the LBGK model.
4. Writing along with propagation in major dimensions.

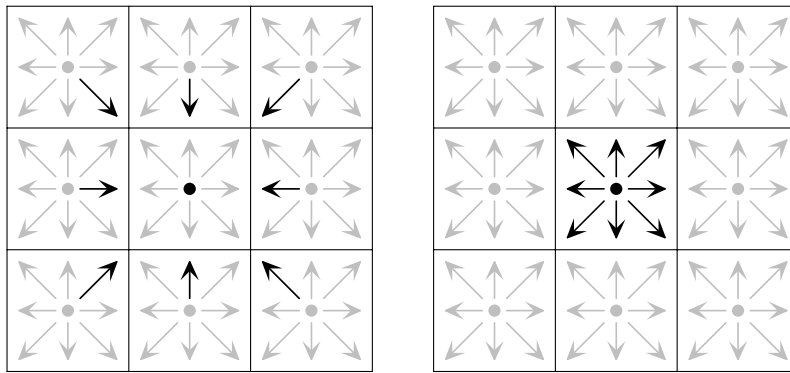


Fig. 9. Reversed propagation scheme.

Table 3

Algorithmic complexity of LBGK and MRT kernels.

	add	sub	mul	div	rcp	mad	cycles
LBGK	63	30	48	0	1	34	716
MRT	76	80	51	0	0	18	900

7.2. The reversed scheme

With the reversed scheme, propagation is entirely performed at reading, as outlined in Fig. 9. Again, the diagram shows the two-dimensional case only.

Boundary conditions are implemented using mid-grid bounce back: nodes at the cavity's borders, except the lid, are considered as fluid with null velocity. Unknown mass fractions are determined using

$$f_i - f_i^{\text{eq}} = f_j - f_j^{\text{eq}} \quad (4)$$

with i and j such that $\mathbf{e}_i = -\mathbf{e}_j$ (see [20]). For null velocity, $f_i^{\text{eq}} = f_j^{\text{eq}}$. Hence the former equation yields $f_i = f_j$.

To summarise, the corresponding kernel breaks up into:

1. Reading carrying out propagation.
2. Mid-grid bounce back boundary conditions.
3. Computations using the MRT model.
4. Writing without propagation.

8. Results

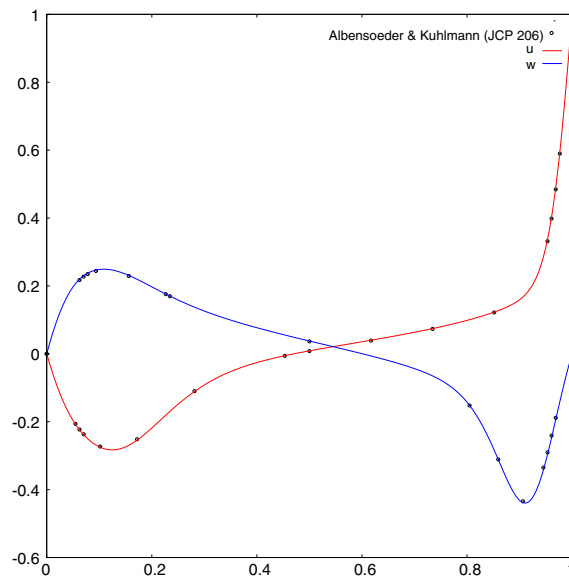
8.1. Validation

Numerical validation is an important issue in GPU computing, since most calculations are performed using single precision. This topic having been thoroughly studied for GPU implementations of LBM in [21], we will instead focus on physical validation. We used the well-known lid-driven cavity test case, comparing velocity coordinates with results published by Albensoeder and Kuhlmann in [22]. More precisely, for velocity $\mathbf{u}(u, v, w)$, we gathered u on line $x = L/2$ and $y = L/2$, as well as w on line $z = L/2$ and $y = L/2$. For Reynolds number $\text{Re} = 1000$, our LBGK code outcomes are in quite good accordance with the reference values. Not surprisingly, the MRT implementation achieves almost perfect correspondence, as shown in Fig. 10.

8.2. Performances

Binaries for nVidia GPUs are generated through a two-stage process (see [23]). First, the `nvopencc` program compiles CUDA code into Parallel Thread eXecution (PTX) pseudo-assembly language. Second, the `ocg` assembler translates the PTX code into actual binary. Analysing PTX outputs allows one to enumerate the floating point operations in a kernel and hence to evaluate the actual algorithmic complexity of the computations. Table 3 assembles the results obtained for both the LBGK and the MRT kernels (`rcp` stands for reciprocal, `mad` for multiply-add).

Though being more complex than LBGK, MRT has almost the same computational cost. It is worth noting that this cost is of the same order of magnitude as one single global memory transaction, that is to say 400 to 600 cycles. Thus, taking the hardware scheduler into account, the impact of computations on global processing time is negligible. Most of the

Fig. 10. Validation for $Re = 1000$.**Table 4**

Performance for LBGK.

	64^3	96^3	128^3	160^3
Performance (MLUPS)	471	512	481	482
Ratio to max. throughput	79%	86%	81%	81%
Occupancy rate	31%	19%	25%	16%

Table 5

Performance for MRT.

	64^3	96^3	128^3	160^3
Performance (MLUPS)	484	513	516	503
Ratio to max. throughput	81%	86%	86%	84%
Occupancy rate	25%	19%	25%	16%

execution time of our kernels is consumed by data transfer, the remaining being probably induced by scheduling. In terms of optimisation, increasing the occupancy rate of the SMs is not especially crucial.

The former opinion is supported by the analysis of the performances of our implementations. A Million Lattice node Updates Per Second (MLUPS) is the usual unit for performance measurement in LBM. For both implementations, memory addressing is analogous to that used in code 1. Therefore, the size of the blocks corresponds to the size of the cavity. Tables 4 and 5 show the performances obtained using a Debian GNU/Linux 5.0 workstation fitted with a GeForce GTX 295 graphics card.

One may notice that the data transfer rate is rather close to maximum. Global memory throughput is at present the limiting factor for LBM computations on GPU. Moreover, it is worth mentioning that these satisfactory performances are achieved with quite low SM occupancy.

Comparisons of the performances obtained with published results should be considered with care, since these studies were carried out using previous generation hardware. We matched a D2Q9 version of our code to the shared memory version published in [5] for the GTX 295, obtaining a 15% improvement of the performance. This lower performance is probably due to the cost of the exchange kernel. For D3Q19 implementations, since the use of the second kernel may be avoided in most cases, shared memory versions are likely to have slightly better performance than our codes with GT200 hardware.

9. Summary

The present study proposes a model for data transfer on the latest generation of nVidia GPUs. Optimisation principles, leading to efficient implementations of 3D LBM on GPUs, are derived as well. We state the impact of global memory transfer as the main limiting factor at present. Our implementations achieved up to 86% of the effective maximal throughput of global memory. We show that, compared to LBGK, the more stable and accurate MRT, despite its higher computational cost,

yields equivalent performances on GPUs. Our approach, being simpler than the previous ones, exerts less pressure as regards hardware. Hence, our method will allow us to implement more complex models in the near future.

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