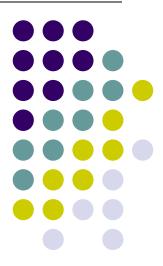
ME964 High Performance Computing for Engineering Applications

CUDA Optimization

Tiling as a Design Pattern in CUDA Vector Reduction Example

March 13, 2012



Before We Get Started...



- Last time
 - Shared memory & bank conflicts
 - Synchronization issues
 - Atomic operations

Today

- Tiling a programming design pattern in CUDA [quick discussion]
- CUDA Optimization/Best Practices issues
 - Example: Vector Reduction
 - Execution Configuration Heuristics

Other issues

- HW7 posted online, due on Th at 11:59 PM
- The Midterm Project race is underway
 - Default project: solving dense banded linear system
 - For default project: I will provide guidelines in terms of profiling, comparison to existing solutions, etc.
- The syllabus has been updated online, check it out...

Putting Things in Perspective...



- Here's what we've covered so far:
 - CUDA execution configuration (grids, blocks, threads)
 - CUDA scheduling issues (warps, thread divergence, synchronization, etc.)
 - CUDA Memory ecosystem (registers, shared mem, device mem, L1/L2 cache, etc.)
 - Practical things: building, debugging, profiling CUDA code

- Next: CUDA GPU Programming Examples & Code Optimization Issues
 - Tiling: a CUDA programming pattern
 - Example: CUDA optimization exercise in relation to a vector reduction operation
 - CUDA Execution Configuration Optimization Heuristics: Occupancy issues
 - CUDA Optimization Rules of Thumb

Tiling [Blocking]:

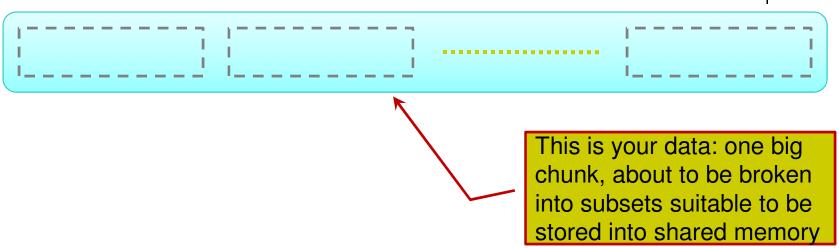
A Fundamental CUDA Programming Pattern



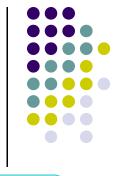
- Partition data to operate in well-sized blocks
 - Small enough to be staged in shared memory
 - Assign each data partition to a block of threads
 - No different from cache blocking!
 - Except you now have full control over it

- Provides several significant performance benefits
 - Working in shared memory reduces memory latency dramatically
 - More likely to have address access patterns that coalesce well on load/store to shared memory

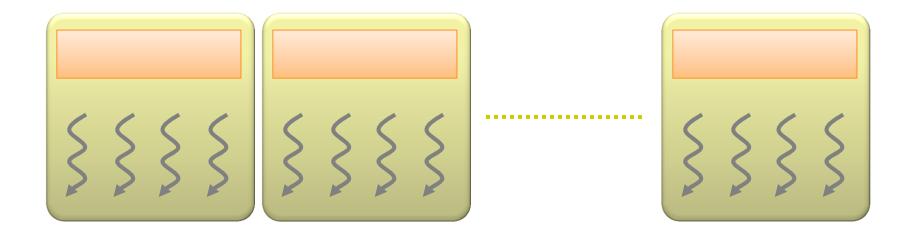




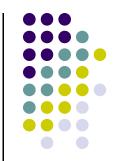
Partition data into subsets that fit into __shared__ memory

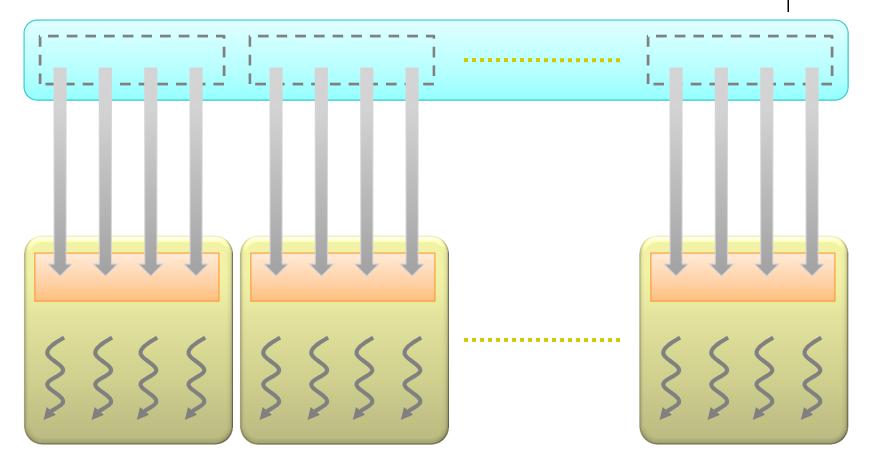




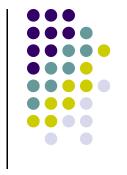


Process each data subset with one thread block

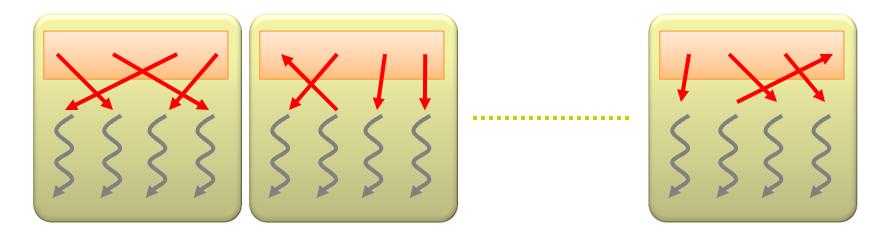




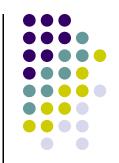
 Load the subset from global memory to shared memory, using multiple threads to exploit memory-level parallelism

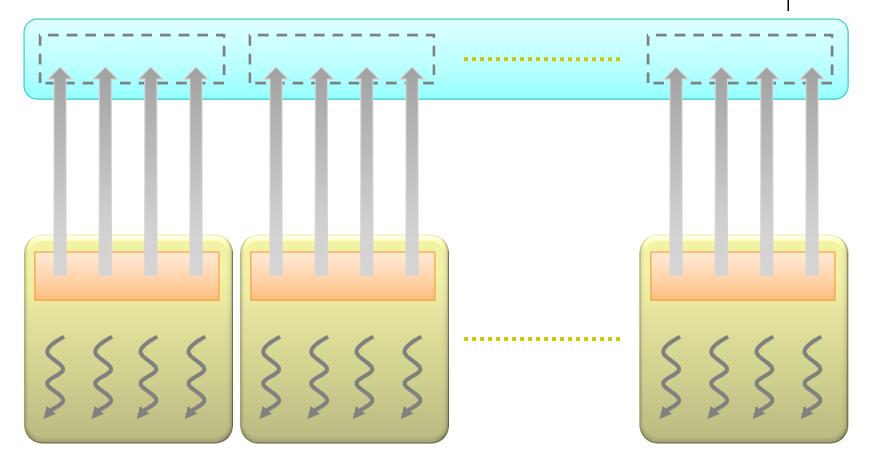






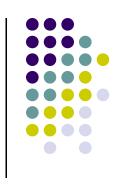
Perform the computation on the subset from shared memory





Copy the result from <u>shared</u> memory back to global memory

Tiling: Closing Remarks



- A large number of CUDA kernels are built this way
- However, tiling [blocking] may not be the only approach to solving a problem, sometimes it might not apply...
- Two questions that can guide you in deciding if tiling is it:
 - Does a thread require several loads from global memory to serve its purpose?
 - Could data used by a thread be used by some other thread in the same block?
 - If answer to both questions is "yes", consider tiling as a design pattern
- The answer to these two questions above is not always obvious
 - Sometime it's useful to craft an altogether new approach (algorithm) that is capable of using tiling: you force the answers to be "yes"



A CUDA Optimization Exercise

[A Demonstration Using the Parallel Reduction Application]

Parallel Reduction in CUDA

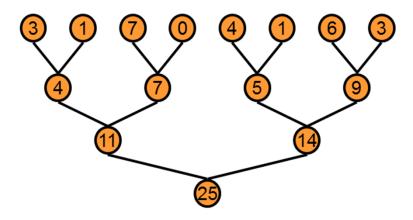


- Exercise draws on material made available by Mark Harris of NVIDIA
 [acknowledgement at bottom of slides]
- Parallel Reduction: Common and very important data parallel primitive
 - Example: Used to compute the norm of a large vector
- Easy to implement in CUDA
 - Challenging to get it right though
- Serves as a great optimization example
 - Walk step by step through several different versions
 - Demonstrates several important optimization strategies

Parallel Reduction



Basic Idea: tree-based approach used within each thread block



- Need to be able to use multiple thread blocks
 - To process very large arrays
 - To keep all multiprocessors on the GPU busy
 - Each thread block reduces a portion of the array to one single value
- Q: How do we communicate partial results between thread blocks?

Problem: Global Synchronization



- If we could synchronize across all thread blocks, could easily reduce very large arrays, right?
 - Global sync after each block produces its result
 - Once all blocks reach sync, continue recursively
- But CUDA has no global synchronization. Why?
 - Expensive to build in hardware for GPUs with high processor count
 - Would force programmer to run fewer blocks (no more than number of SMs times the number of resident blocks / SM) → this may reduce overall efficiency
- Solution: decompose into multiple kernels
 - Kernel launch serves as a global synchronization point
 - Kernel launch has negligible HW overhead, low SW overhead

Multiple Kernel Calls

[An Example, and how it all works out...]

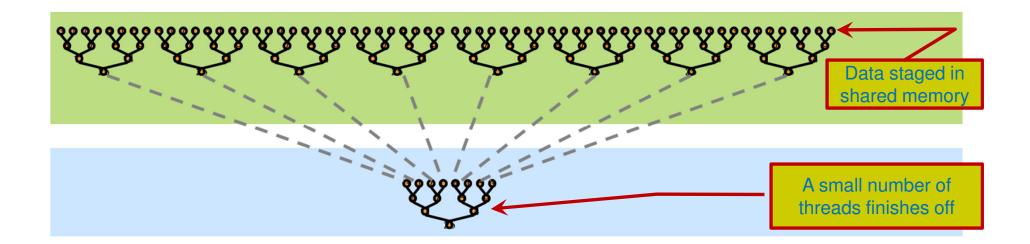


- Imagine you launch a grid in which each block has 256 threads
- Assume that the number or elements in the array is N=100,000
 - Note that 100,000= 390*256 + 160, therefore ceil[N/256.0]=391
- For the first stage, you launch 391 blocks of 256 threads
 - At the end of this stage you still have to operate on 391 elements
- For the second stage, you launch two blocks of 256 threads
 - At the end of this stage you only have to operate on two elements
- For the third and last stage, you launch one block of 256 threads
 - Almost all threads idle...
- NOTE: after the first stage, each subsequent stage operates on a number of entries equal to the number of blocks in the previous stage

Vector Reduction: 30,000 Feet Perspective



- At the block level: Bring data in shared memory, then start adding in parallel
- Fewer and fewer threads of a block participate
- The process is memory bound, low arithmetic intensity...



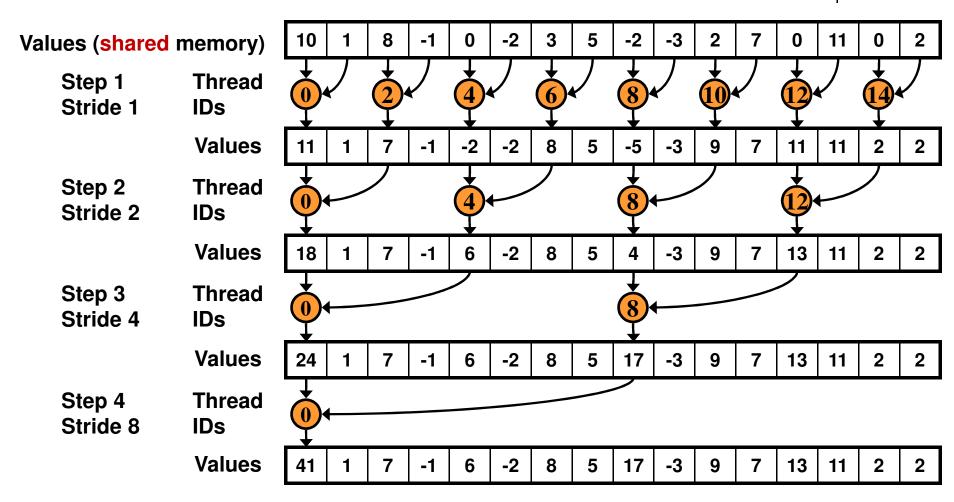
What is Our Optimization Goal?



- We should strive to reach GPU peak performance
- Choose the right metric:
 - GFLOP/s: for compute-bound kernels
 - Bandwidth: for memory-bound kernels
- Reductions have very low arithmetic intensity
 - 1 flop per element loaded (bandwidth-optimal)
- Therefore we should strive for peak bandwidth
- This example uses results generated using a G80 GPU
 - Compute capability (CC) 1.0
 - 384-bit memory interface, 900 MHz DDR
 - 384 * 900 *2 / 8 = 86.4 GB/s
 - Example carries over to other CCs, this algorithm will be memory bound

Parallel Reduction: Interleaved Addressing





Reduction #1: Interleaved Addressing



```
global void reduce1(int *g_idata, int *g_odata) {
    extern __shared__ int sdata[];
    // each thread loads one element from global to shared mem
    unsigned int tid = threadIdx.x;
    unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;
    sdata[tid] = g idata[i];
    __syncthreads();
   // do reduction in shared mem
    for(unsigned int s=1; s < blockDim.x; s *= 2) {</pre>
        if (tid % (2*s) == 0) {
            sdata[tid] += sdata[tid + s];
                                                Problem: highly divergent
                                                warps are very inefficient,
         __syncthreads();
                                                and % operator is very slow
    }
    // write result for this block to global memory
    if (tid == 0) g odata[blockIdx.x] = sdata[0];
}
```

Performance for 4M element reduction



Time (2 ²² ints)	Bandwidth
-----------------------------	------------------

Kernel 1: interleaved addressing with divergent branching

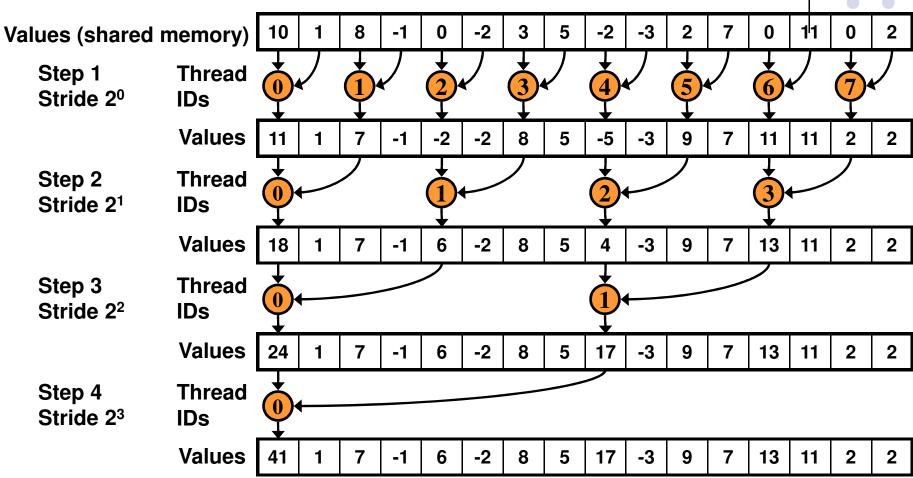
8.054 ms

2.083 GB/s

Note: Block Size = 128 threads for all tests

Parallel Reduction: Interleaved Addressing





New Problem: Shared Memory Bank Conflicts

Reduction #2: Interleaved Addressing



Just replace divergent branch in inner loop...

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {
   if (tid % (2*s) == 0) {
      sdata[tid] += sdata[tid + s];
   }
   __syncthreads();
}</pre>
```

...with strided index and non-divergent branch:

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {
   int index = 2 * s * tid;

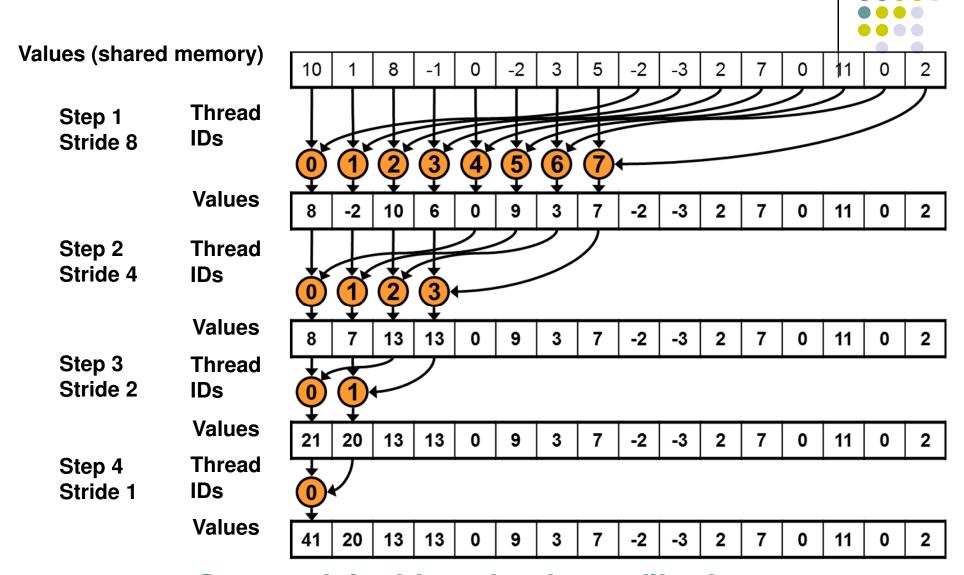
   if (index < blockDim.x) {
      sdata[index] += sdata[index + s];
   }
   __syncthreads();
}</pre>
```

Performance for 4M element reduction



	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x

Parallel Reduction: Sequential Addressing



Sequential addressing is conflict free

Reduction #3: Sequential Addressing



Just replace strided indexing in inner loop...

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {
   int index = 2 * s * tid;

   if (index < blockDim.x) {
      sdata[index] += sdata[index + s];
   }
   __syncthreads();
}</pre>
```

...with reversed loop and threadID-based indexing:

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {
   if (tid < s) {
      sdata[tid] += sdata[tid + s];
   }
   __syncthreads();
}</pre>
```

Performance for 4M element reduction



	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1:				
interleaved addressing	8.054 ms	2.083 GB/s		
with divergent branching				
Kernel 2:				
interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3:	1.722 ms	9.741 GB/s	2.01x	4.68x
sequential addressing	1./22 1115	9.741 GB/S	Z.U I X	4.00X

Idle Threads...



Current solution:

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {
   if (tid < s) {
      sdata[tid] += sdata[tid + s];
   }
   __syncthreads();
}</pre>
```

Note that half of the threads are idle on first loop iteration! This is wasteful...

Reduction #4: First Add During Load



Replace single load:

```
// each thread loads one element from global to shared mem
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;
sdata[tid] = g_idata[i];
__syncthreads();
```

...With two loads and first add of the reduction:

```
// perform first level of reduction upon reading from
// global memory and writing to shared memory
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];
__syncthreads();
```

One side effect: the number of blocks you need now is half of what it used to be...

Performance for 4M element reduction



	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first add during global load	0.965 ms	17.377 GB/s	1.78x	8.34x

Instruction Bottleneck



- At 17 GB/s, we're far from bandwidth bound
 - And we know reduction has low arithmetic intensity

- Therefore a likely bottleneck is instruction overhead
 - Ancillary instructions that are not loads, stores, or arithmetic for the core computation
 - In other words: address arithmetic and loop overhead

Strategy: unroll loops

Unrolling the Last Warp



- As reduction proceeds, the number of "active" threads decreases
 - When s <= 32, we have only one warp left
- Instructions are SIMD synchronous within a warp
 - All threads in a warp proceed in lockstep fashion
- That means when s <= 32:
 - We don't need to __syncthreads()
 - We don't need "if (tid < s)" because it doesn't save any work
- Let's unroll the last 6 iterations of the inner loop

Reduction #5: Unroll the Last Warp

```
// and use later like this...
for (unsigned int s=blockDim.x/2; s>32; s>>=1) {
   if (tid < s)
       sdata[tid] += sdata[tid + s];
       __syncthreads();
}

if (tid < 32) warpReduce(sdata, tid);</pre>
```

```
__device__ void warpReduce(volatile int* sdata, int tid) {
    sdata[tid] += sdata[tid + 32];
    sdata[tid] += sdata[tid + 16];
    sdata[tid] += sdata[tid + 8];
    sdata[tid] += sdata[tid + 4];
    sdata[tid] += sdata[tid + 2];
    sdata[tid] += sdata[tid + 1];
}
```

Note: This saves useless work in *all* warps, not just the last one!

Without unrolling, all warps execute every iteration of the for loop and if statement

32

Performance for 4M element reduction



	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first add during global load	0.965 ms	17.377 GB/s	1.78x	8.34x
Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x

Complete Unrolling



- If we knew the number of iterations (or equivalently, of threads in a block) at compile time, we could completely unroll the reduction
 - Luckily, the block size on G80 is limited by the GPU to 512 threads
 - 1024 on newer Fermi GPUs
 - Also, we are sticking to power-of-2 block sizes
- So we can easily unroll for a fixed block size
 - But we need to be generic how can we unroll for block sizes that we don't know at compile time?
- Use of templates can solve this issue...
 - CUDA supports C++ template parameters on device and host functions

Unrolling with Templates



- Specify block size as a function template parameter
- The kernel is parameterized:

```
template <unsigned int blockSize>
__global__ void reduce6(int *g_idata, int *g_odata)
```

Reduction #6: Completely Unrolled



```
if (blockSize >= 512) {
    if(tid < 256){ sdata[tid] += sdata[tid + 256]; } __syncthreads();}
if (blockSize >= 256) {
    if(tid < 128){ sdata[tid] += sdata[tid + 128]; } __syncthreads();}
if (blockSize >= 128) {
    if(tid < 64){ sdata[tid] += sdata[tid + 64]; } __syncthreads();}
if (tid < 32) warpReduce<blockSize>(sdata, tid);
```

```
template <unsigned int blockSize>
_device__ void warpReduce(volatile int* sdata, int tid) {
   if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
   if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
   if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
   if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
   if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
   if (blockSize >= 2) sdata[tid] += sdata[tid + 1];
}
```

- All code in RED will be evaluated at compile time. Results in a very efficient inner loop.
- For Fermi, you'd have one more if statement that covers the case when blockSize>=1024

Invoking Template Kernels



```
switch (threads) {
case 512:
   reduce6<512><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 256:
   reduce6<256><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 128:
   reduce6<128><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 64:
   reduce6< 64><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 32:
   reduce6< 32><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
case 16:
   reduce6< 16><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 8:
   reduce6< 8><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 4:
   reduce6< 4><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 2:
   reduce6< 2><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
case 1:
   reduce6< 1><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;
}
```

Performance for 4M element reduction



	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
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Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: completely unrolled	0.381 ms	43.996 GB/s	1.41x	21.16x

Parallel Reduction Complexity



- Assume that the number of elements in array is of the form $N=2^{D}$
- Log(N) parallel stages, each stage S requires $N/2^S$ independent ops
 - Stage Complexity is O(log N)
- For $N=2^D$, approach requires a total of $\sum_{S \in [1...D]} 2^{D-S} = N-1$ operations
 - Work Complexity is O(N) It is work-efficient
 - That is, it does not perform more operations than a sequential algorithm
- Time complexity, for P threads physically in parallel (P processors): $O(N/P + \log N)$
 - Compare to O(N) for sequential reduction
 - In a thread block, N=P, so O(log N)

What About Cost?



- Cost of a parallel algorithm is processors × time complexity
 - Allocate threads instead of processors: O(N) threads
 - Time complexity is O(log N), so cost is O(N log N): not cost efficient!
- Brent's theorem suggests O(N/log N) threads
 - Each thread does O(log N) sequential work
 - Then all O(N/log N) threads cooperate for O(log N) stages
 - Cost = O(($N/\log N$) * log N) = O(N) \rightarrow cost efficient
- Sometimes called algorithm cascading
 - Can lead to significant speedups in practice

Algorithm Cascading



- Combine sequential and parallel reduction
 - Each thread loads and sums multiple elements into shared memory
 - Tree-based reduction in shared memory
- Brent's theorem says each thread should sum O(log n) elements
 - i.e. 1024 or 2048 elements per block vs. 256
- Probably beneficial to push it even further
 - Possibly better latency hiding with more work per thread
 - More threads per block reduces levels in tree of recursive kernel invocations
 - High kernel launch overhead in last levels with few blocks
- On G80, best performance with 64-256 blocks of 128 threads
 - 1024-4096 elements per thread

Kernel 7, Comments



- For the first six kernels a large number of blocks was used to "tile" the array
- Kernel 7: reduce the number of blocks and have a thread do more work than just fetch something to shared memory
- Example [cooked up, not related to actual CUDA warp size, typical CUDA block dim, etc.]:
 - Say you have 1024 elements stored in an array; you need to reduce that array
 - You start with 32 blocks, each with 4 threads
 - Then, 128 threads total. It means that a thread, say in block 11, would have to add two numbers, then two numbers, then two numbers, then two more numbers.
 - At this point, everything is in the union of the shared memory associated with the 32 blocks. At this point proceed like before with kernel 6.

Reduction #7: Multiple Adds / Thread

Replace load and add of two elements:

```
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];
__syncthreads();
```

With a while loop to add as many as necessary:

```
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*(blockSize*2) + threadIdx.x;
unsigned int gridSize = blockSize*2*gridDim.x;
sdata[tid] = 0;

while (i < n) {
    sdata[tid] += g_idata[i] + g_idata[i+blockSize];
    i += gridSize;
}
__syncthreads();

Note: gridSize loop stride to maintain coalescing!</pre>
```

Performance for 4M element reduction

Kernel 7 on 32M elements: 73 GB/s!

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
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Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: completely unrolled	0.381 ms	43.996 GB/s	1.41x	21.16x
Kernel 7: multiple elements per thread	0.268 ms	62.671 GB/s	1.42x	30.04x

Final Kernel...

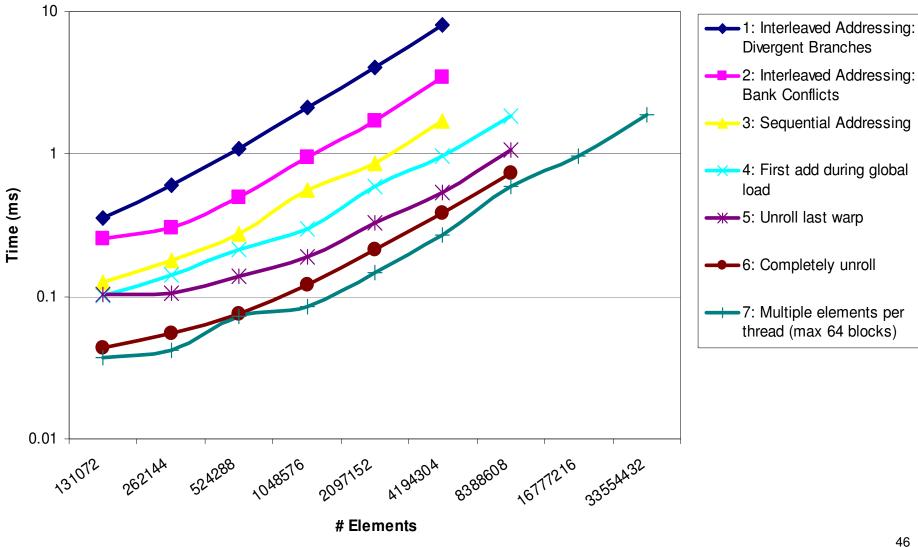


```
template <unsigned int blockSize>
device void warpReduce(volatile int *sdata, unsigned int tid) {
   if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
   if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
   if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
   if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
   if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
   if (blockSize >= 2) sdata[tid] += sdata[tid + 1];
}
template <unsigned int blockSize>
global void reduce7(int *g idata, int *g odata, unsigned int n) {
   extern shared int sdata[];
   unsigned int tid = threadIdx.x;
   unsigned int i = blockIdx.x*(blockSize*2) + tid;
   unsigned int gridSize = blockSize*2*gridDim.x;
   sdata[tid] = 0;
   while (i < n) { sdata[tid] += g_idata[i] + g_idata[i+blockSize]; i += gridSize; }</pre>
   syncthreads();
   if (blockSize >= 512) { if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }</pre>
   if (blockSize >= 256) { if (tid < 128) { sdata[tid] += sdata[tid + 128]; } syncthreads(); }</pre>
   if (blockSize >= 128) { if (tid < 64) { sdata[tid] += sdata[tid + 64]; } syncthreads(); }</pre>
   if (tid < 32) warpReduce(sdata, tid);</pre>
   if (tid == 0) g odata[blockIdx.x] = sdata[0];
}
```

NVIDIA [M. Harris]→

Performance Comparison





Sources of Efficiency Improvement



- Algorithmic optimizations
 - Changes to addressing, algorithm cascading
 - 11.84x speedup, combined!

- Code optimizations
 - Loop unrolling
 - 2.54x speedup, combined

Lessons Learned, Vector Reduction



- Understand CUDA performance characteristics
 - Memory coalescing
 - Warp divergence
 - Bank conflicts
 - Latency hiding
- Use peak performance metrics to guide optimization
- Know how to identify type of bottleneck
 - E.g. memory, core computation, or instruction overhead
- Optimize your algorithm, then unroll loops
- Use template parameters to generate optimal code
- Understand parallel algorithm complexity theory