ETHFE
High Frequency Electronics

AARHUS UNIVERSITY
SCHOOL OF ENGINEERING

Indholdsfortegnelse

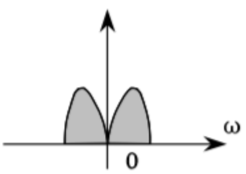
Amplitude modulation

1.1 Lektion 30-01-2018

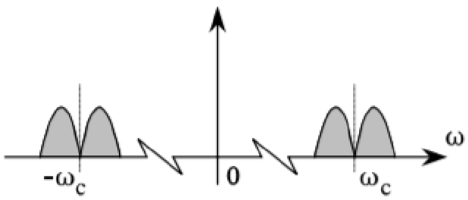
1. Intro
2. AM Modulation/demodulation

- **Pensum:** JV, Ch 1 p 1-6
- **Opgaver:** P.I-1

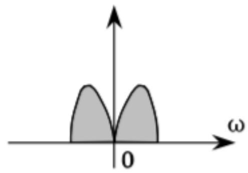
1.1.1 Basic Modulation Types and Concepts



Original baseband signal



Transmitted bandpass signal



Reconstructed baseband signal

- **Modulation:** Hvordan signaler moduleres ind på bærebølger, der efterfølgende typisk sendes ud som elektromagtetiske signaler via et transmissionsmedie.
  - Bandpass signalet er det transmitterede signal til receiveren.
  - Flere baseband signals kan blive transmitteret samtidigt gennem den samme kanal ved forskellige carrier frequencies.

- **Demodulation:** Hvordan det sendte signal demoduleres så det originale signal gendannes.
  - Receiveren gendanner det low-frequency baseband signal.
  - Scopet af demodulationen afhænger af hvilken type data der bliver sendt.
    - *In a radio telephony channel it may suffice at the receiver site to get an output with a power spectrum that contains the dominant part of the input power spectrum.*
    - *In a television video channel it is important to reconstruct in time-domain the shape of the signal being send.*
    - *In digital transmissions, the goal is to rebuild a logical bitstream representation equivalent to the input stream.*

1.1.2 Amplitude Modulations

Typer af moduleringer der er egnet for RF communication kaldes continuous wave modulations, **CW**.

- Baseband information er overlagt en sinusoidal carrier wave med amplitude A_{c0} og vinkelfrekvens ω_c.
  - *Carrier* 1.1
  - *Modulated carrier* 1.2

$$y(t) = A_{c0} \cos(\omega_c t) \tag{1.1}$$

$$y(t) = A(t) \cos(\omega_c t + \phi(t) + \phi_0) \tag{1.2}$$

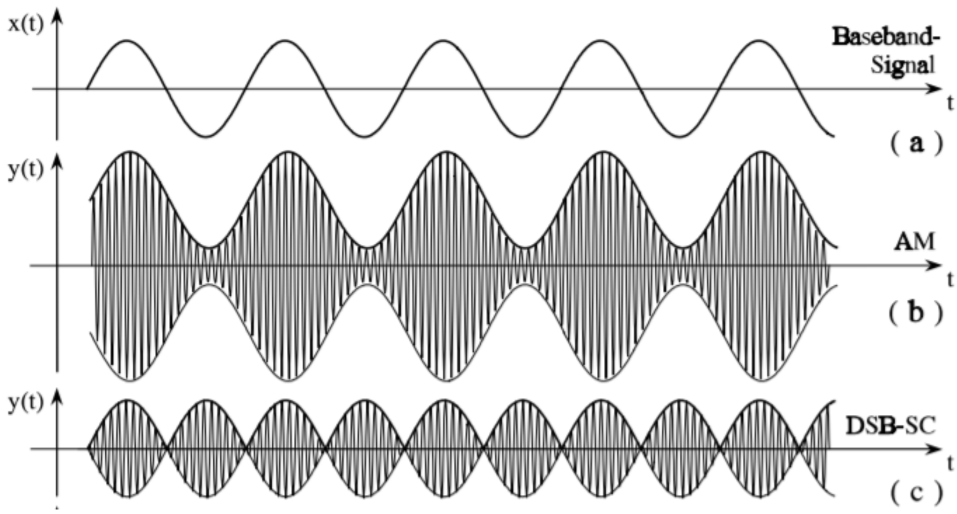
- *The time dependencies of $A(t)$ and $\phi(t)$ in 1.2 contain the baseband message and the angle ϕ_0 represents an offset phase for the carrier compared to the timing of the baseband message.*

- *If there is no synchronism between the two, the offset may be set to zero without loss of generality. Eq. 1.2 is called the envelope-phase representation of a modulated signal.*

Den største forskel mellem forskellige modulation typer er hvordan et baseband signal $x(t)$ indeholder det overlagte signal $y(t)$ som er moduleret. Amplitude modulations indebærer **AM** and **DSB-SC**.

AM

- Skalering af signalniveauerne beregnes ved modulation index m.
- Med et normaliseret baseband signal $|x(t)| \leq 1$, indebærer betingelsen $m \leq 1$ (eller 100%) et undistorted reproduction af baseband signalet.
  - m: modulation index
- Det er let at gendanne baseband signalet fra en AM modulated wave i en receiver med det simple envelope detector circuit.



Figur 1.1: Examples of modulation waveshapes (AM and DSB(-SC)) from a sinusoidal baseband signal $x(t)$.

- Amplitude modulation, $\phi(t) = 0$
  - *amplitude modulation, AM* 1.3
  - *double-sideband (supressed carrier), DSB/DSB-SC* 1.4

$$y(t) = A_{c0}(1 + mx(t))\cos(\omega_c t) \tag{1.3}$$

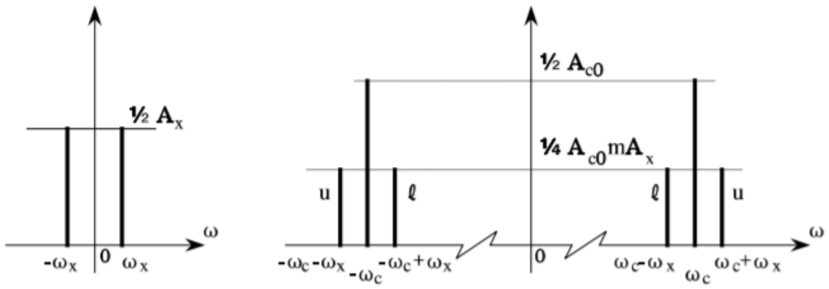
$$y(t) = A_{c0}x(t)\cos(\omega_c t) \tag{1.4}$$

- Envelope detectorens low-pass filter bandwidth skal være højere end envelope frekvensen.

- En AM modulated wave har spektrale komponenter fra baseband signalet over og under carrier signalet.

$$y(t)_{|AM} = A_{c0}(1 + mA_x \cos \omega_x t) \cos \omega_c t \Rrightarrow \tag{1.5}$$

$$A_{c0} \cos \omega_c t + \frac{A_{c0}}{2} mA_x \cos(\omega_c - \omega_x)t + \frac{A_{c0}}{2} mA_x \cos(\omega_c + \omega_x)t \tag{1.6}$$



Figur 1.2: Spectral components in a double-sided amplitude spectrum of the sinusoidal baseband signal and the AM modulated waveform from Eq. 1.6. u and l are the upper and lower sideband components.

With maximum undistorted modulation, i.e. $m = 1$ and $A_x = 1$, the power of the AM modulated wave, say it is a voltage across a $1\,\Omega$ resistor, becomes;

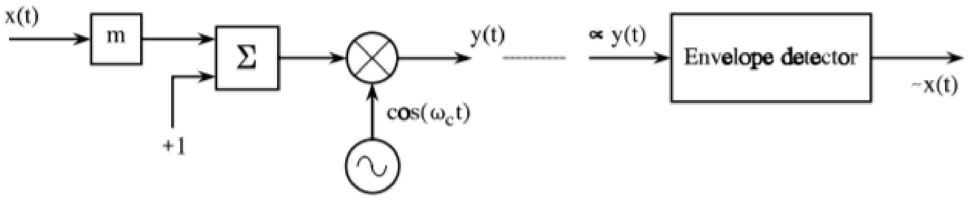
$$P_{|AM} = 2\frac{A_{c0}^2}{4} + 4\frac{A_{c0}^2 m^2 A_x^2}{16} \tag{1.7}$$

carrier *envelope*

$$P_{|AM} = \frac{1}{2}A_{c0}^2 + \frac{1}{4}A_{c0}^2 \tag{1.8}$$

so at most 33% of the transmitted power contains the message from the baseband signal.

- AM modulated signal i frekvens domænet:
  - Anvender eulers formel ($\cos 2\pi t \rightarrow e^{j2\pi\omega_c t} + e^{-j2\pi\omega_c t}$)
  - Fourier transform (gange i tidsdomænet $\cdot$ og folde i frekvenssdomænet $\circledast$)



Figur 1.3: Block schemes for simple AM modulation (left) and demodulation (right).

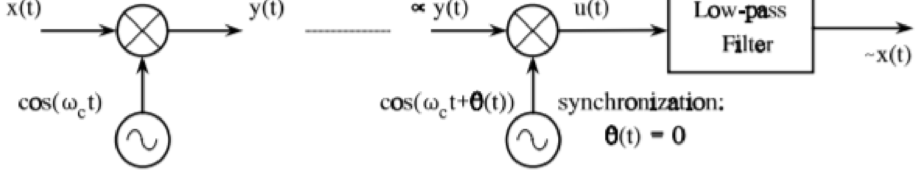
DSB-SC

DSB moduleringen fra Eq. 1.4 har ingen carrier component deraf kommer betegnelsen *suppressed carrier* eller SC.

$$y(t)_{|DSB-SC} = A_{c0}A_x \cos\omega_x t) \cos\omega_c t \tag{1.9}$$

$$y(t)_{|DSB-SC} = \frac{A_{c0}A_x}{2}\cos(\omega_c - \omega_x)t + \frac{A_{c0}A_x}{2}\cos(\omega_c + \omega_x)t \tag{1.10}$$

- Carrier componenterne indeholder ingen information, derfor er det mere kompliceret at gendanne baseband signalet i receiveren.
- For at kunne detektere baseband signalet fra et DSB moduleret signal, skal dette signal igen multipliceres med en carrier.
  - Hvis fasen $\phi(t)$ er forskellig fra nul vil cosine enten reducerer eller forvrænge signalet.
  - For at få et predictable result skal oscillatoren i demodulatoren være synkroniseret med carrier af det modtagede signal.
  - *A simple method is to let a fragment of the full carrier - a pilot carrier - follow the signal.*
    - Gøres ved at indsætte en konstant < 1 istedet for 1.



Figur 1.4: Block schemes for simple DSB-SC modulation (left) and demodulation (right).

Vinkel modulation

2.1 Lektion 06-02-2018

1. Vinkel modulation
2. Phasor repræsentation

- **Pensum:** JV, Ch 1 p 6-13, p 13-18
- **Opgaver:** P.I-2

2.1.1 Vinkel modulation

Vinkel modulation er processen når frekvensen eller phase af carrieren varrierer i forhold til baseband informationen. Her er amplituden A_{c0} konstant. En vigtig fordel ved PM og FM modulation er at de mere imun overfor channel noise, nonlinear distotion og amplitude fading i forhold til AM modulation. Vinkel modulation kræver en dobbelt så stor båndbredde som AM modulation $(2 \cdot W)$.

Vinkel modulation er delt op i frekvens (**FM**) og phase (**PM**).

- Vinkel modulation, $A(t) = A_{c0}$
  - *phase modulation, PM* 2.1
  - *frequency modulation, FM* 2.2

$$y(t) = A_{c0} \cos(\omega_c t + \beta x(t)) \tag{2.1}$$

$$y(t) = A_{c0} \cos(\omega_c t + \phi(t)) \tag{2.2}$$

Phase informationen findes ved at differentere.

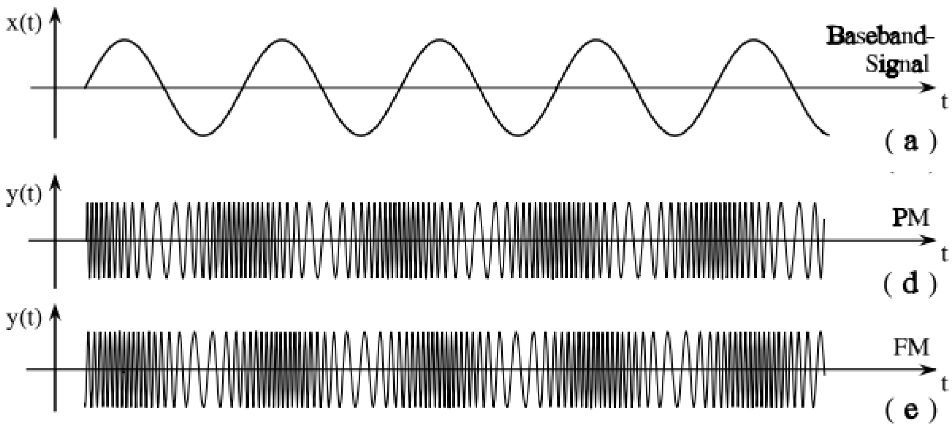
$$\frac{d\phi}{dt} = \Delta\omega(t) = \Delta\omega_{max}x(t) = 2\pi\Delta f_{max}x(t) \tag{2.3}$$

$\beta = \frac{\Delta f_{max}}{f_x}$ maximum phase deviation from the carrier phase

Δf_{max} peak frequency deviation

f_x baseband frequency component

The change in phase, changes the frequency of the modulated wave. The frequency of the wave also changes the phase of the wave.



Figur 2.1: Examples of modulation waveshapes (PM and FM) from a sinusoidal baseband signal $x(t)$.

PM

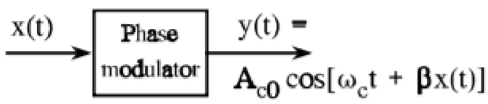
$$\theta(t) = \omega_c t + \beta x(t) + \phi_0 \tag{2.4}$$

$x(t)$ Baseband signal

$\omega_c t$ Angle of Unmodulated carrier wave

$\beta = \frac{radian}{volt}$ Phase sensitivity (const.)

$\phi_0 = 0$ Initial angle



Figur 2.2: PM angular modulation.

FM

Indirect FM

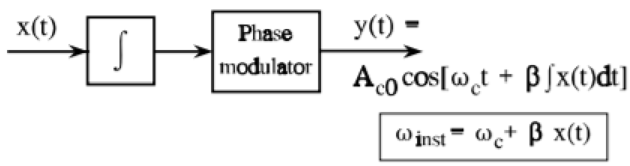
$$y(t) = A_{c0} \cos[\omega_c t + \beta \int x(t)dt] \tag{2.5}$$

$x(t)$ Baseband signal

$\omega_c t$ Angle of unmodulated carrier wave

$\beta = \frac{radian}{volt}$ Phase sensitivity (const.)

$\omega_{inst} = \omega_c + \beta x(t)$ instantaneous frequency.



Figur 2.3: FM indirect modulation.

Direct FM

$$y(t) = A_{c0} \cos[\omega_c t + 2\pi K_V \int x(t)dt] \tag{2.6}$$

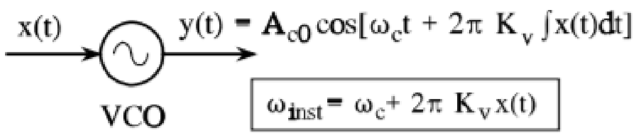
$x(t)$ Baseband signal

$\omega_c t$ Angle of unmodulated carrier wave

$\beta = \frac{radian}{volt}$ Phase sensitivity (const.)

$K_V = \frac{hertz}{volt}$ Frequency gain (const.)

$\omega_{inst} = \omega_c + 2\pi K_V x(t)$ instantaneous frequency.



Figur 2.4: FM direct modulation using VCO.

Wideband FM

Tilnærmelsen fra NBFM er ikke gældende ved wideband og derfor gælder følgende. Dette introducerer Bessel funktioner.

$$\cos(\beta_{eff} \sin \omega_x t) = J_0(\beta_{eff}) + \sum_{n-2,even}^{\infty} 2J_n(\beta_{eff}) \cos n\omega_x t \tag{2.7}$$

$$\sin(\beta_{eff} \sin \omega_x t) = \sum_{n-1,odd}^{\infty} 2J_n(\beta_{eff}) \sin n\omega_x t \tag{2.8}$$

$\beta_{eff} \ = \beta A_x$ effective modulation index

$n_{99\%} \ \approx \beta_{eff} + 1$

β_{eff}=0	1	2	3	4	5	6	7	8	9	10	n
1.0000	0.7652	0.2239	-0.2601	-0.3971	-0.1776	0.1506	0.3001	0.1717	-0.0903	-0.2459	0
	0.4401	0.5767	0.3391	-0.0660	-0.3276	-0.2767	-0.0047	0.2346	0.2453	0.0435	1
	0.1149	0.3528	0.4861	0.3641	0.0466	-0.2429	-0.3014	-0.1130	0.1448	0.2546	2
	0.0196	0.1289	0.3091	0.4302	0.3648	0.1148	-0.1676	-0.2911	-0.1809	0.0584	3
	0.0025	0.0340	0.1320	0.2811	0.3912	0.3576	0.1578	-0.1054	-0.2655	-0.2196	4
	0.0002	0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	-0.0550	-0.2341	5
		0.0012	0.0114	0.0491	0.1310	0.2458	0.3392	0.3376	0.2043	-0.0145	6
		0.0002	0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275	0.2167	7
			0.0005	0.0040	0.0184	0.0565	0.1280	0.2235	0.3051	0.3179	8
			0.0001	0.0009	0.0055	0.0212	0.0589	0.1263	0.2149	0.2919	9
				0.0002	0.0015	0.0070	0.0235	0.0608	0.1247	0.2075	10
					0.0004	0.0020	0.0083	0.0256	0.0622	0.1231	11
					0.0001	0.0005	0.0027	0.0096	0.0274	0.0634	12
						0.0001	0.0008	0.0033	0.0108	0.0290	13
							0.0002	0.0010	0.0039	0.0120	14
							0.0001	0.0003	0.0013	0.0045	15
								0.0001	0.0004	0.0016	16
									0.0001	0.0005	17
										0.0002	18

Figur 2.5: $J_n(\beta_{eff})$, expansion coefficients from Bessel functions.

Bessel funktioner indsættes.

$$y(t)|_{FMorPM} = A_{c0} J_0(\beta_{eff}) \cos(\omega_c t) \tag{2.9}$$

$$y(t)|_{FMorPM} = A_{c0} \sum_{n=-\infty}^{\infty} J_n(\beta_{eff}) \cos(\omega_c t + n\omega_x t) \tag{2.10}$$

Accumalted power

$$p_{ac,n}(\beta_{eff}) = \frac{P_{ac,n}}{\frac{A_{c0}^2}{2}} = J_0^2(\beta_{eff}) + \sum_{i=1}^{n} 2J_i^2(\beta_{eff}) \tag{2.11}$$

Bandwidth for 99% power

- *phase modulation, PM* 2.12
- *frequency modulation, FM* 2.13

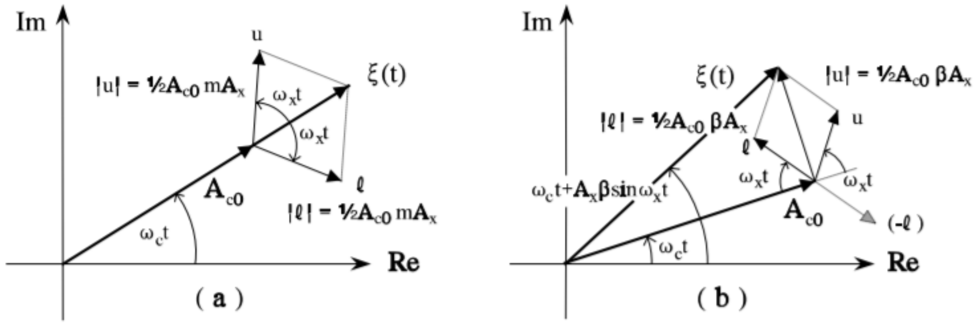
$$W_{99\%} = 2\omega_x(\beta_{eff} + 1) = 2\omega_x[\beta A_x + 1] \tag{2.12}$$

$$W_{99\%} = 2\omega_x(\beta_{eff} + 1) = 2[\Delta\omega_{max} A_x + \omega_x] \tag{2.13}$$

β_{eff}=0	1	2	3	4	5	6	7	8	9	10	n
1.0000	0.5855	0.0501	0.0676	0.1577	0.0315	0.0227	0.0901	0.0295	0.0082	0.0605	0
	0.9728	0.7154	0.2975	0.1665	0.2462	0.1758	0.0901	0.1396	0.1285	0.0643	1
	0.9992	0.9643	0.7701	0.4316	0.2505	0.2938	0.2718	0.1651	0.1705	0.1939	2
	1.0000	**0.9976**	0.9612	0.8017	0.5167	0.3201	0.3279	0.3346	0.2360	0.2008	3
		0.9999	**0.9960**	0.9598	0.8228	0.5759	0.3777	0.3568	0.3769	0.2972	4
		1.0000	0.9997	**0.9947**	0.9592	0.8381	0.6198	0.4258	0.3830	0.4068	5
			1.0000	0.9995	**0.9936**	0.9590	0.8499	0.6538	0.4665	0.4072	6
				1.0000	0.9993	**0.9926**	0.9590	0.8593	0.6809	0.5011	7
					0.9999	0.9990	**0.9918**	0.9592	0.8670	0.7032	8
					1.0000	0.9999	0.9987	**0.9911**	0.9594	0.8735	9
						1.0000	0.9998	0.9985	**0.9905**	0.9596	10
							1.0000	0.9998	0.9982	**0.9900**	11
								1.0000	0.9997	0.9980	12
									1.0000	0.9997	13
										1.0000	14

Figur 2.6: $p_{ac,n}(\beta_{eff})$, accumulated relative power of frequency components to and including order n in size. Figures in bold indicate the first 99% bound passing.

2.1.2 Phasor repræsentation



Figur 2.7: Phasor representation showing how the lower and upper side-band components, l and u, add to the carrier A_{c0} in (a) AM modulation, and (b) narrowband FM. The modulated wave becomes $y(t) = Re(\zeta)$.

Parallel og serie resonans kredsløb

3.1 Lektion 20-02-2018

1. Komponenter og systemer
2. Resonanskredsløb
3. Tabsfrie komponenter
4. Impedans transformation
5. Filter design
6. Frekvens og impedans skalering

- **Pensum:** CB, Ch 1+2+3
- **Opgaver:** Uge 3-1, Uge 3-2

3.1.1 Komponenter

Komponenters egenskaber, modstande, kondensatorer og induktorer ved radiofrekvenser og hvordan det vedrører kredsløbsdesign. De mest enkle komponenter af ethvert system undersøges ved radiofrekvenser.

- Wire (diameter, længde)
  - Skin Effect
    - As frequency is increased, an increased magnetic field at the center of the conductor presents an impedance to the charge carriers, thus decreasing the current density at the center of the conductor and increasing the current density around its perimeter.

    * It occurs in all conductors including resistor leads, capacitor leads, and inductor leads.

  - Skin Depth
    * The depth into the conductor at which the charge-carrier current density falls to $\frac{1}{e}$, or 37% of its value along the surface.
    * Is a function of the frequency and the permeability and conductivity of the medium.
    * Results in a net increase in the ac resistance of the wire.

  - Straight-Wire Inductors
    * Self-Inductance happens when current in the conductor is an alternating current. A magnetic field is alternately expanding and contracting and, thus, producing a voltage on the wire which opposes any change in current flow.

$$L = 0.002\, l \left[2.3 \log \left(\frac{4l}{d} \right) - 0.75 \right] \mu\text{H} \tag{3.1}$$

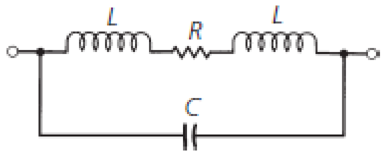
L inductance in µH

l length of wire in cm

d diameter of wire in cm

- Resistors
  - Resistor Equivalent Circuit
    * R is the resistor value itself.
    * L is the lead inductance.
    * C is a combination of parasitic capacitances which varies from resistor to resistor.
  - Resistor Types
    * *Carbon-composition resistors* consists of densely packed dielectric particulates or carbon granules. Between each pair there is a very small parasitic capacitor.
    * *Wirewound resistors* tend to exhibit widely varying impedances over various frequencies.
      · The inductor L, is much larger for a wirewound resistor than for a carbon-composition resistor.

        * *Metal-film resistors* has the best characteristics over frequency since the values of the individual parasitic elements in the equivalent circuit decrease.
            · At very high frequencies, and with low-value resistors (under $50\,\Omega$), lead inductance and skin effect may become noticeable.
        * *Thin-film chip resistors* offers very little parasitic reactance at frequencues from DC to 2 GHz.



Figur 3.1: Resistor equivalent circuit.

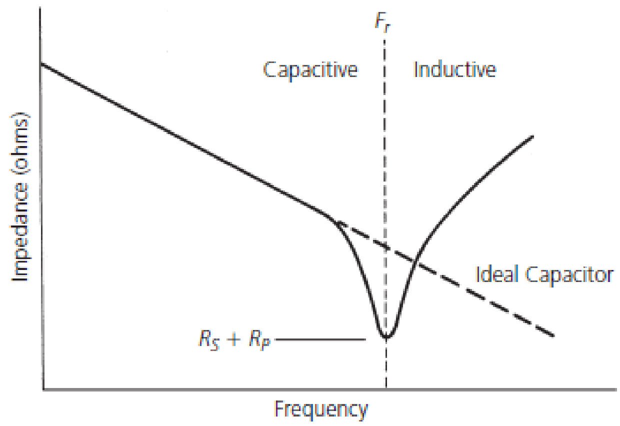
$$X_L = \omega L \tag{3.2}$$

$$X_C = \frac{1}{\omega C} \tag{3.3}$$

- Capacitors
    - Parallel-Plate Capacitor
        * Two conducting surfaces separated by an insulating material or dielectric.
        * When a potential difference exists between the conductors a capacitor storage a charge.
          $C = \dfrac{Q}{V}$
            · C is capacitance in farads
            · Q is charge in coulombs
            · V is voltage in volts
        * If the area (A) of each metal plate, the distance (d) between the plate, and the permittivity (ϵ) of the dielectric material in farads/meter are known, the capacitance of a parallel-plate capacitor can be found.

$$C = \frac{0.2249\epsilon A}{d\,\epsilon_0}\mathrm{pF} \tag{3.4}$$

- Real-World Capacitors
  * The dielectric's characteristics determine the voltage levels and the temperature extremes at which the device may be used.
  * As the frequency of operation increases, the lead inductance becomes important. Finally, at F_r, the inductance becomes series resonant with the capacitor. Then, above F_r, the capacitor acts like an inductor.



Figur 3.2: Impedance characteristic vs. frequency.

- Capacitor Equivalent Circuit
  * C equals the capacitance.
  * R_s is the heat-dissipation loss expressed either as a power factor (PF) or as a dissipation factor (DF).

    Power Factor $PF = \cos\phi$

    · In a perfect capacitor, the alternating current will lead the applied voltage by 90°. This phase angle (ϕ) will be smaller in a real capacitor due to the total series resistance ($R_s + R_p$).

    Dissipation Factor $DF = \frac{ESR}{X_c} \cdot 100\%$

    · The ratio of AC resistance to the reactance of a capacitor.

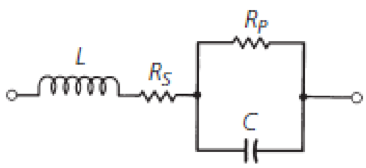
- Quality Factor Q of a capacitor is the reciprocal of DF.

  $Q = \frac{1}{DF} = \frac{X_c}{ESR}$

  Effective Series Resistance $ESR = \frac{PF}{\omega C}(1 \cdot 10^6)$

- Is the AC resistance of a capacitor, a combined equivalent of R_s and R_p.

* R_p is the insulation resistance.
  - A measure of the amount of DC current that flows through the dielectric of a capacitor with a voltage applied. No material is a perfect insulator; thus, some leakage current must flow.
  - Typically value of 100 000 MΩ or more.
* L is the inductance of the leads and plates.

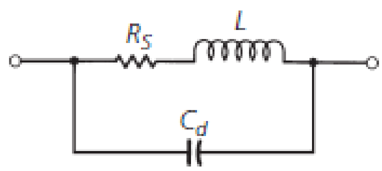


Figur 3.3: Capacitor equivalent circuit.

- Capacitor Types
  * Ceramic Capacitors
    - Vary widely in both dielectric constant k and temperature characteristics.
    - *Rule of thumb is*: "The higher the k, the worse is its temperature characteristic."
    - *Low-k ceramic capacitors* tend to have linear temperature characteristic (well suited for oscillator, resonant circuit, or filter applications).
    - *Moderately stable ceramic capacitors* typically vary ±15% of their rated capacitance over their temperature range. This variation is typically nonlinear. (used in switching circuits).

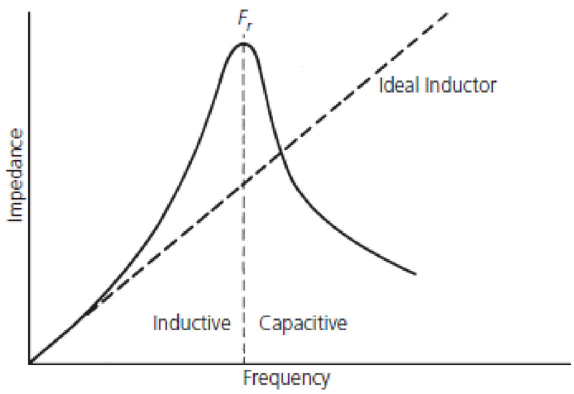
  - · *High-K ceramic capacitors* typically termed general-purpose capacitors. Temperature characteristics are very poor and their capacitance may vary as much as 80% over various temperature ranges (used only in bypass applications).
  - · *Chip capacitors* are capacitors that have no leads (specifically intended for RF applications).
- ∗ Mica Capacitors
  - · Typically have a dielectric constant k of about 6, producing an extremely good temperature characteristic (used extensively in resonant circuits and in filters where PC board area is of no concern).
  - · Are becoming increasingly less cost effective than ceramic types.
- ∗ Metalized-Film Capacitors
  - · Dialectrics of teflon, polystyrene, polycarbonate and paper (used in a number of applications, including filtering, bypassing, and coupling).
  - · Polycarbonate, polystyrene, and teflon styles are available in very tight (±2%) capacitance tolerances over their entire temperature range.
  - · Typically larger than the equivalent-value ceramic types.

- Inductors
  - – Inductor Equivalent Circuit
    - ∗ A wire wound or coiled in such a manner as to increase the magnetic flux linkage between the turns of the coil which increases the wire's inductance.
    - ∗ Distributed capacitance (C_d)
      - · Two conductors brought into close proximity but separated by a dielectric, and place a voltage differential between the two, we form a capacitor.



Figur 3.4: Inductor equivalent circuit.

- Real-World Inductors
  - Initially, at lower frequencies, the inductor's reactance parallels that of an ideal inductor. Soon, however, its reactance departs from the ideal curve and increases at a much faster rate until it reaches a peak at the inductor's parallel resonant frequency (F_r).
  - Above F_r , the inductor's reactance begins to decrease with frequency and, thus, the inductor begins to look like a capacitor.
  - Theoretically, the resonance peak would occur at infinite reactance. However, due to the series resistance of the coil, some finite impedance is seen at resonance.
  - Ratio of an inductor's reactance to its series resistance is often used as a measure of the quality of the inductor. The larger the ratio, the better is the inductor.
    - Quality Factor $Q = \frac{X}{R_S}$
  - At low frequencies, the Q of an inductor is very good (resistance in the windings is the dc resistance of the wire).
  - As frequency increases, skin effect and winding capacitance begin to degrade the quality of the inductor.



Figur 3.5: Impedance characteristic vs. frequency for a practical and an ideal inductor.

  - Methods of increasing the Q of an inductor and extending its useful frequency range:
    1. Use a larger diameter wire. This decreases the AC and DC resistance of the windings.

2. Spread the windings apart. Air has a lower dielectric constant than most insulators. Thus, an air gap between the windings decreases the interwinding capacitance.
3. Increase the permeability of the flux linkage path. This is most often done by winding the inductor around a magnetic-core material, such as iron or ferrite. A coil made in this manner will also consist of fewer turns for a given inductance.

– Single-Layer Air-Core Inductor Design
  * Formula generally used to design single-layer air-core inductors L.
    · Coil length l must be greater than $0.67r$. This formula is accurate to within one percent.

$$L = \frac{0.394 r^2 N^2}{9r + 10l} \tag{3.5}$$

r coil radius in cm

l coil length in cm

L inductance in µH

– Magnetic-Core Materials
  * Air-Core inductors cannot be used because of their size.
  * Method of decreasing the size of a coil while maintaining a given inductance:
    · Decrease the number of turns while at the same time increasing its magnetic flux density (decreasing the "reluctance"). Done by adding a magnetic-core material to the inductor.

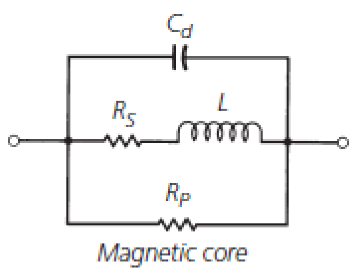
  * Advantages:
    1. Smaller size — fewer number of turns needed.
    2. Increased Q — fewer turns means less wire resistance.
    3. Variability — obtained by moving the magnetic core in and out of the windings.

- Problems:
  1. Each core tends to introduce its own losses. Thus, adding a magnetic core to an air-core inductor could possibly decrease the Q of the inductor, depending on the material used and the frequency of operation.
  2. The permeability of all magnetic cores changes with frequency and usually decreases to a very small value at the upper end of their operating range. It eventually approaches the permeability of air and becomes "invisible"to the circuit.
  3. The higher the permeability of the core, the more sensitive it is to temperature variation. Thus, over wide temperature ranges, the inductance of the coil may vary appreciably.
  4. The permeability of the magnetic core changes with applied signal level. If too large an excitation is applied, saturation of the core will result.

- Toroids
  - Core Inductors Equivalent Circuits
    - R_S = resistance of the windings
    - R_P = losses in the core itself (hysteresis)

    - To determine the new Q of the inductor:

      By what factors did the inductance and loss increase?
      - By adding a toroidal core, inductance will increase by a factor of two and its total loss will also increase by a factor of two - Q will remain unchanged.
      - The additional loss introduced by the core is not constant, but varies (usually increases) with frequency.



Figur 3.6: Equivalent circuits for magnetic-core inductor.

- Core Characteristics
  * The magnetization curve for a magnetic core indicates the magnetic-flux density (B) that occurs in the inductor with a specific magnetic-field intensity (H) applied.
    · The ratio of the magnetic-flux density to the magneticfield intensity is called the permeability (µ) of the material.
    $\mu = \frac{B}{H} \quad webers/ampere\, turn$
    · As the magnetic-field intensity is increased from zero, the magnetic-flux density that links the turns of the inductor increases quite linearly.
    · A point is reached at which the magnetic-flux intensity does not continue to increase at the same rate as the excitation. Any further increase in excitation may cause saturation to occur B_{sat}. The incremental permeability above this point is the same as air.
  * Once B_{sat} is known for the core, it is simple to determine whether or not its use in a particular circuit application will cause it to saturate. The in-circuit operational flux density (B_{op}) is given by:

$$B_{op} = \frac{E \cdot 10^8}{(4.44) f N A_e} \tag{3.6}$$

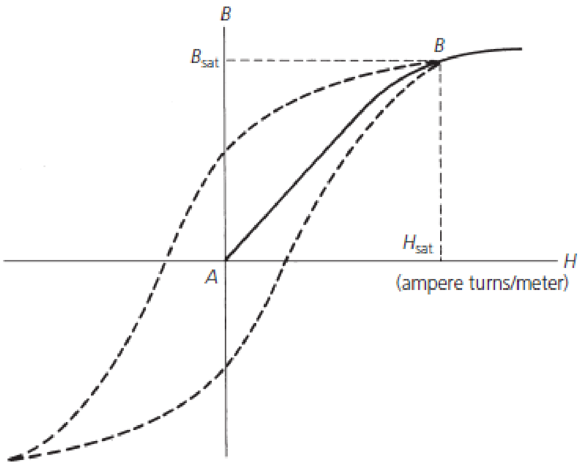
B_{op} magnetic-flux density in gauss

E maximum rms voltage across the inductor

f frequency in Hz

N number of turns

A_e effective cross-sectional area of the core in cm^2



Figur 3.7: Magnetization curve for a typical core.

- Powdered Iron vs. Ferrite
  * In general, no hard and fast rules of ferrite cores versus powdered-iron cores.
  * Special applications in which one core might outperform another:
    · Any application where high RF power levels are involved, iron cores might be the best choice.
    · Powdered-iron cores tend to yield higher-Q inductors, at higher frequencies, than an equivalent size ferrite core making them very useful in narrowband or tuned-circuit applications.
    · At very low frequencies, or in broadband circuits, ferrite seems to be the general choice since it has a much higher permeability.
    · A coil of a given inductance can usually be wound on a much smaller ferrite core and with fewer turns than a powdered-iron core and thereby save circuit board area.
- Toroidal Inductor Design
  * For a toroidal inductor operating on the linear (nonsaturating) portion of its magnetization curve, its inductance is given by:

$$L = \frac{0.4\pi N^2 \mu_1 A_c \cdot 10^{-2}}{l_e} \tag{3.7}$$

L inductance in nH

N number of turns

μ_1 initial permeability

A_e effective cross-sectional area of the core in cm^2

l_e effective length of the core in cm

- In order to make calculations easier, most manufacturers have combined μ_1, A_c, l_e, and other constants for a given core into a single quantity called the inductance index, A_L.

$$L = N^2 A_L \quad \text{nH} \tag{3.8}$$

L inductance in nH

N number of turns

A_L inductance index in $nanohenries/turn^2$

- Number of turns to be wound on a given core for a specific inductance.

$$N = \sqrt{\frac{L}{A_L}} \tag{3.9}$$

- A calculated guess of Q at low frequencies (100 kHz), the Q of the coil would be approximately:

$$Q = \frac{R_p/N^2}{X_p/N^2} = \frac{R_p}{X_p} \tag{3.10}$$