

Cryptography Basics for Bitcoiners

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Kyiv Bitcoin
Meetup

Cryptography shifts
the balance of power



What is needed is an electronic payment system based on cryptographic proof instead of trust [...].

- Satoshi







- Focus: Cryptography & formal languages



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Blockstream RESEARCH

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- blog.blockstream.com
- twitter.com/blksresearch

Warm Up

bitcoin.pdf

11. Calculations

We consider the scenario of an attacker trying to generate an alternate chain faster than the honest chain. Even if this is accomplished, it does not throw the system open to arbitrary changes, such

Block



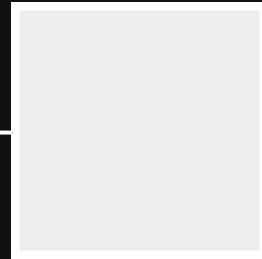
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2 Confirmations



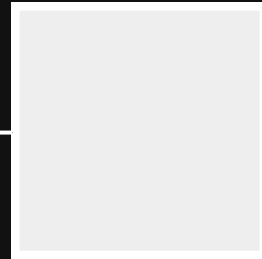
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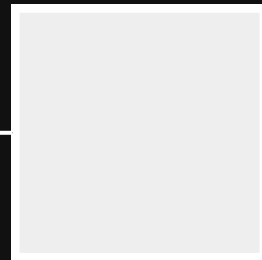
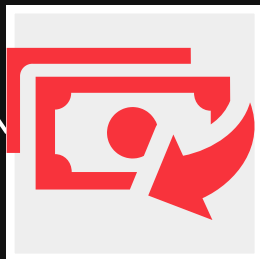
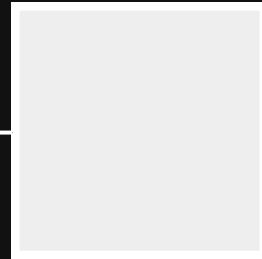
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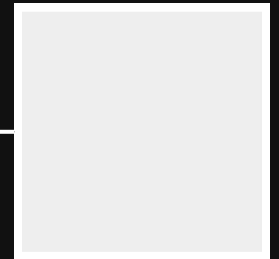
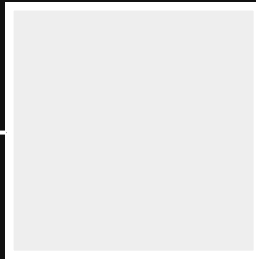
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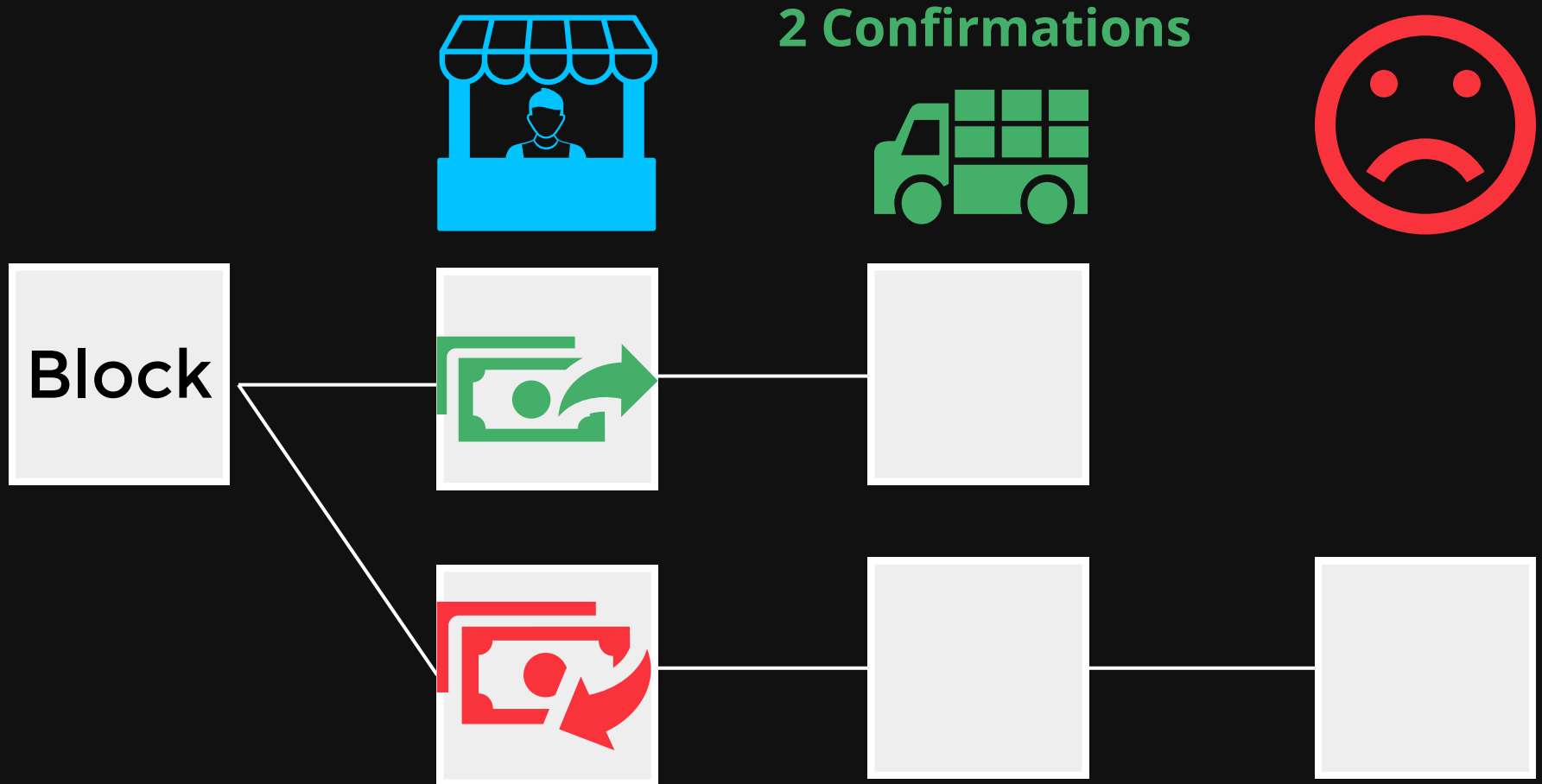
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Double Spending



How many confirmations
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The authors differ in their model of the attacker

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For example, both models ignore whether attacking is a rational strategy in the first place.

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From: Introduction to Modern Cryptography, J. Katz, Y. Lindell

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Definitions, models, assumptions and
precise security proofs play a central
role

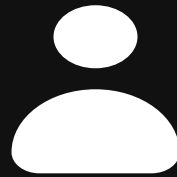
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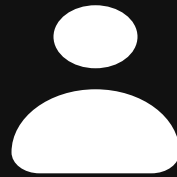


private Key

public Key

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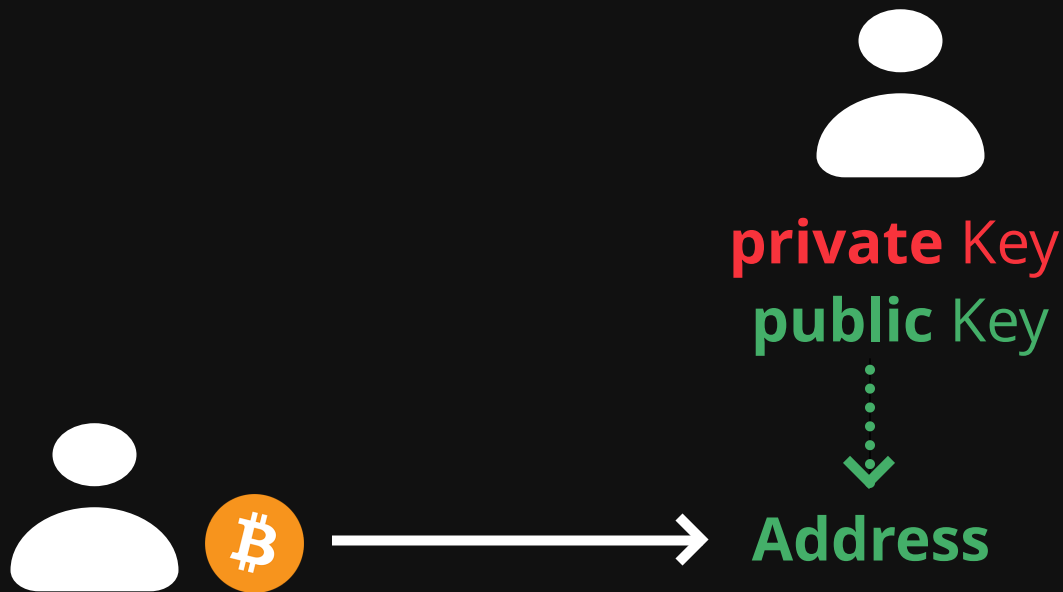
public Key



Address

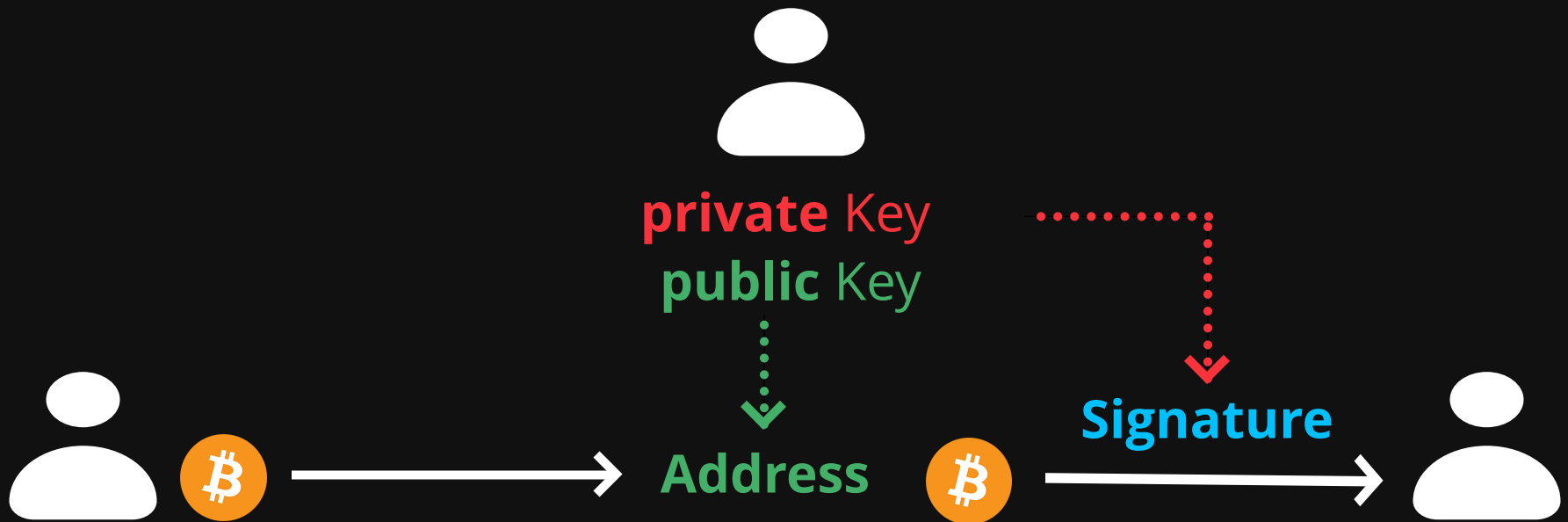
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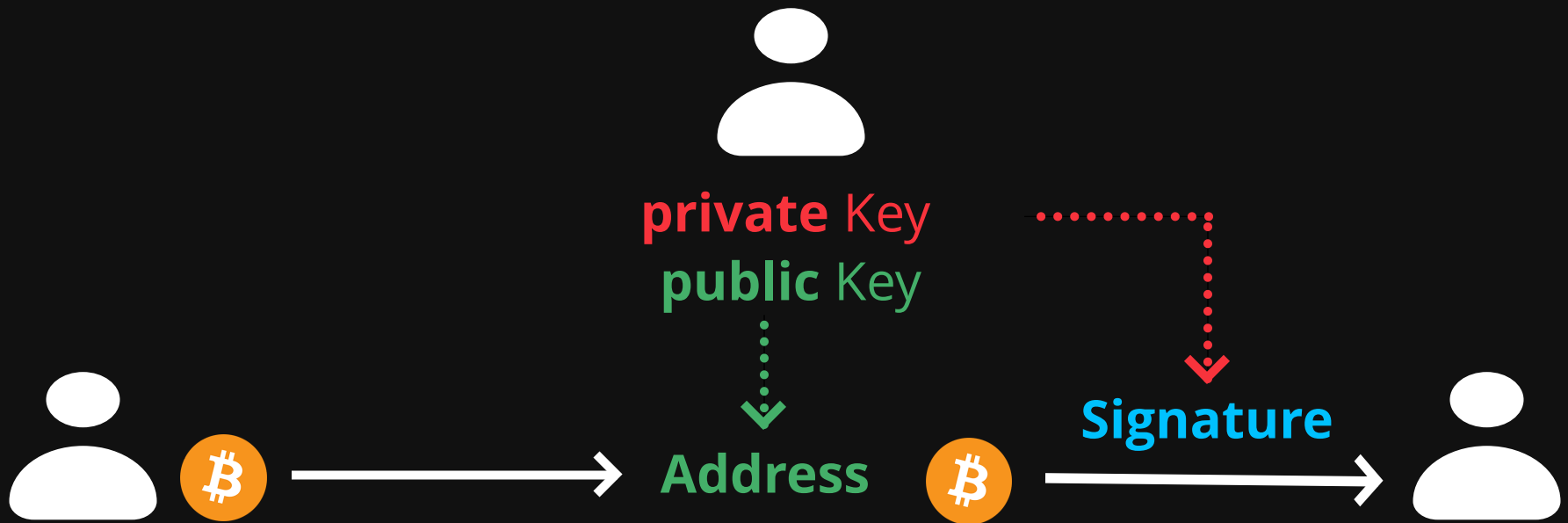
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They consist of three parts...

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Forgery: a valid signature for the public key of someone else

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 - Or the other way around: if problem X is hard, then the signature scheme is secure

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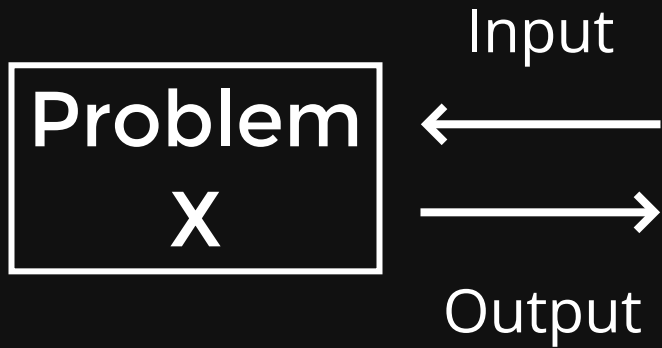
Problem
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The Security Proof

**Problem
X**

(Discrete
logarithm on
elliptic curve
secp256k1)

The Security Proof



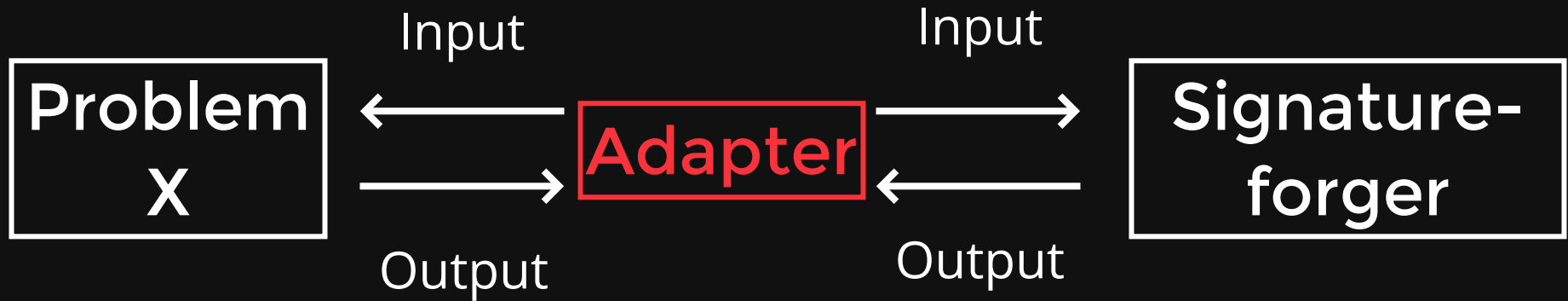
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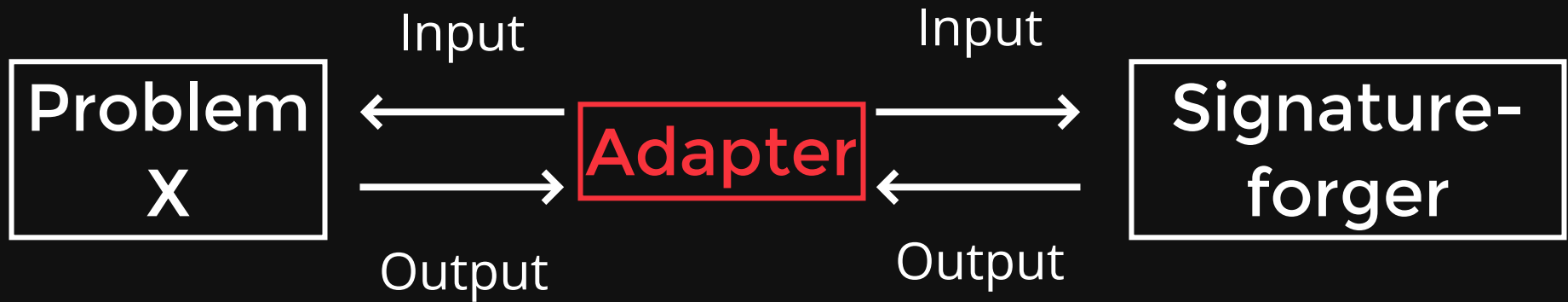
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If there is a forger, we can solve problem X

Time to catch our breath...



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 - **BIP 32** (HD-Wallets) more complicated than necessary

Back to the real world

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Paper

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Paper

Bsp.: "Let
 $P \in \mathbb{G}$ "

Back to the real world

Paper

→ Implementation

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unsigned char  
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- In the future ideally: [formal specifications](#), that allow provably correct implementations

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Hello,

We'd like to announce the release of version 0.3.2 of libsecp256k1:

<https://github.com/bitcoin-core/secp256k1/releases/tag/v0.3.2>

This is a bugfix release after 0.3.1. The impetus for this release is the discovery that GCC 13 became smart enough to optimize out a specific timing side-channel protection mechanism in the ECDH code that could leave applications vulnerable to a side-channel attack. This has been fixed in 0.3.2 [1].

What is Bitcoin Cryptography?



on-chain

(Base Layer)

off-chain

("Layer 2")

on-chain

(Base Layer)

off-chain

("Layer 2")

on-chain

(Base Layer)

validated by every Bitcoin node,
consensus

off-chain
("Layer 2")

optional, settlement on base layer

on-chain
(Base Layer)

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(Multipartly-)
Payment Channels

Sidechains

Federated
E-Cash

off-chain
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Cryptography



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Bitcoin Cryptography



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- "simple" to analyze and implement



Cryptography

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**Consensus is
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Cryptography, which

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- but would not be practical there
- allows better efficiency and privacy
- includes, for example, multisignatures, blind signatures, zero-knowledge proofs

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Post-Quantum Crypto...

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Case study: Post-Quantum Cryptography

For quantum computers, problem X is theoretically unsolvable.

CRYPTOGRAPHY

‘Post-Quantum’ Cryptography Scheme Is Cracked on a Laptop

 6 | 

Two researchers have broken an encryption protocol that many saw as a promising defense against the power of quantum computing.

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"Some people seem to be unable to rationally consider the possibility that NSA is sabotaging post-quantum cryptography."

ONE DOES NOT SIMPLY

MAKE BITCOIN QUANTUM-SECURE

Outlook

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 - Further progress on resilience (e.g., security proofs, better specifications, secure code)

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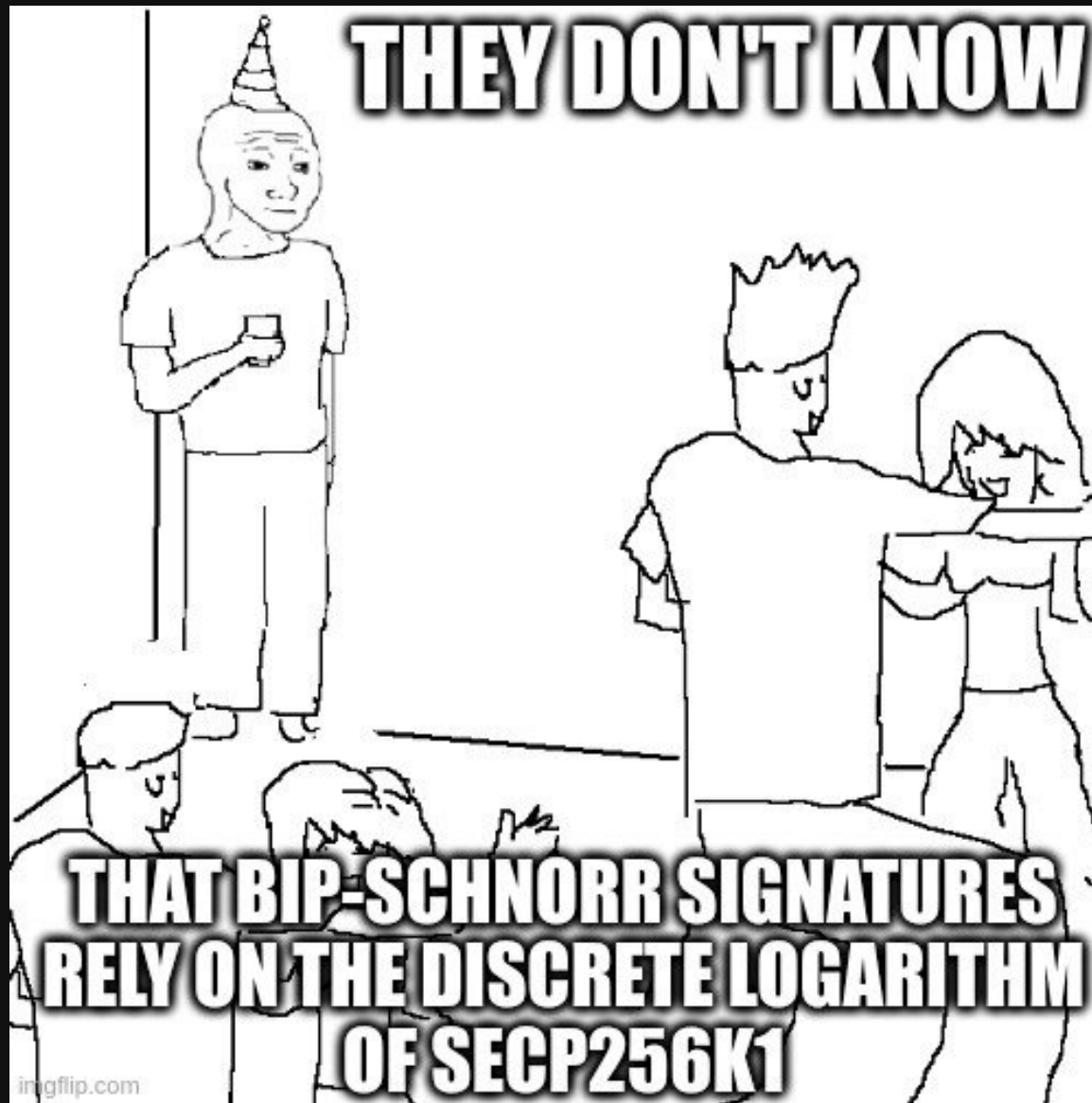
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- Layer 2 Cryptography:
 - A vast field with many open questions for theory and practice



Slides: nickler.ninja/slides