

Breaking MuSig2 with tweaking

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1 Modified Scheme

See Figure 1.

Setup (1^λ) <hr/> $(\mathbb{G}, p, g) \leftarrow \text{GrGen}(1^\lambda)$ Select three hash functions $H_{\text{agg}}, H_{\text{non}}, H_{\text{sig}} : \{0, 1\}^* \rightarrow \mathbb{Z}_p$ $par := ((\mathbb{G}, p, g), H_{\text{agg}}, H_{\text{non}}, H_{\text{sig}})$ return par	SignAgg (out_1, \dots, out_n) <hr/> for $i := 1 \dots n$ do $(R_{i,1}, \dots, R_{i,\nu}) := out_i$ for $j := 1 \dots \nu$ do $R_j := \prod_{i=1}^n R_{i,j}$ return $out := (R_1, \dots, R_\nu)$
KeyGen () <hr/> $x \leftarrow \mathbb{Z}_p$; $X := g^x$ $sk := x$; $pk := X$ return (sk, pk)	Sign' ($state_1, out, sk_1, m, (pk_2, \dots, pk_n), C$) <hr/> $\text{// Sign' must be called at most once per } state_1.$ $(r_{1,1}, \dots, r_{1,\nu}) := state_1$ $x_1 := sk_1$; $X_1' := g^{x_1}$; $t := H_{\text{contract}}(X_1', C)$; $X_1 := X_1' g^t$ $(X_2, \dots, X_n) := (pk_2, \dots, pk_n)$ $L := \{X_1, \dots, X_n\}$ $a_1 := \text{KeyAggCoef}(L, X_1)$ $\tilde{X} := \text{KeyAgg}(L)$ $(R_1, \dots, R_\nu) := out$ $b := H_{\text{non}}(\tilde{X}, (R_1, \dots, R_\nu), m)$ $R := \prod_{j=1}^\nu R_j^{b^{j-1}}$ $c := H_{\text{sig}}(\tilde{X}, R, m)$ $s_1 := ca_1(x_1 + t) + \sum_{i=1}^\nu r_{1,i} b^{i-1} \bmod p$ $state'_1 := R$; $out'_1 := s_1$ return $(state'_1, out'_1)$
KeyAggCoef (L, X_i) <hr/> return $H_{\text{agg}}(L, X_i)$	
KeyAgg (L) <hr/> $\{X_1, \dots, X_n\} := L$ for $i := 1 \dots n$ do $a_i := \text{KeyAggCoef}(L, X_i)$ return $\tilde{X} := \prod_{i=1}^n X_i^{a_i}$	
Ver (\tilde{pk}, m, σ) <hr/> $\tilde{X} := \tilde{pk}$; $(R, s) := \sigma$ $c := H_{\text{sig}}(\tilde{X}, R, m)$ return $(g^s = R\tilde{X}^c)$	SignAgg' (out'_1, \dots, out'_n) <hr/> $(s_1, \dots, s_n) := (out'_1, \dots, out'_n)$ $s := \sum_{i=1}^n s_i \bmod p$ return $out' := s$
Sign () <hr/> $\text{// Local signer has index 1.}$ for $j := 1 \dots \nu$ do $r_{1,j} \leftarrow \mathbb{Z}_p$; $R_{1,j} := g^{r_{1,j}}$ $out_1 := (R_{1,1}, \dots, R_{1,\nu})$ $state_1 := (r_{1,1}, \dots, r_{1,\nu})$ return $(out_1, state_1)$	Sign'' ($state'_1, out'$) <hr/> $R := state'_1$; $s := out'$ return $\sigma := (R, s)$

Fig. 1. The modified multi-signature scheme MuSig2Tweak[GrGen, ν]. Modifications in red.

2 Wagner

The attack relies on Wagner’s algorithm for solving the Generalized Birthday Problem, which can be defined as follows for the purpose of this paper: Given a constant value $t \in \mathbb{Z}_p$, an integer k_{\max} , and access to random oracle H mapping onto \mathbb{Z}_p , find a set $\{q_1, \dots, q_{k_{\max}}\}$ of k_{\max} queries such that $\sum_{k=1}^{k_{\max}} H(q_k) = t$. For $k_{\max} = 2^{\sqrt{\log_2(p)}-1}$ the complexity of this algorithm is $O(2^{2\sqrt{\log_2(p)}})$.

Jonas’ note: Perhaps can use BLOR? “If the attacker is able to open more sessions concurrently, the improved polynomial-time attack by Benhamouda *et al.* [add:BLOR20] assumes $k_{\max} > \log_2 p$ sessions, but then has complexity $O(k_{\max} \log_2 p)$ and a negligible running time in practice.”

3 Attack

The attack proceeds as follows. The adversary opens $k_{\max} = 2^{\sqrt{\log_2(p)}-1}$ concurrent signing sessions and receives k_{\max} nonce pairs $R_{1,1}^{(1)}, R_{1,2}^{(1)}, \dots, R_{1,1}^{(k_{\max})}, R_{1,2}^{(k_{\max})}$ from the honest signer with public key $X_1 = g^{x_1}$.

The adversary draws $\ell_{\max} \in O(2^{\sqrt{\log_2(p)}})$ values $C_1, \dots, C_{\ell_{\max}}$ at random. Let $L = \{X_1 g^{t^{(1)}}, \dots, X_1 g^{t^{(\ell_{\max})}}\}$ where $t^{(\ell)}$ are the tweaks and $\tilde{X} = \text{KeyAgg}(L)$ be the corresponding aggregate public key.

Jonas’ note: The attack probably doesn’t require such large L but can work with multiple smaller multisets of keys

Given $R_1 = \sum_{k=1}^{k_{\max}} R_{1,1}^{(k)}$, $R_2 = \sum_{k=1}^{k_{\max}} R_{1,2}^{(k)}$ and a forgery target message m^* , the adversary computes $b = H_{\text{non}}(\tilde{X}, (R_1, R_2), m)$ and $R^* = \prod_{k=1}^{k_{\max}} R_{1,1}^{(k)} (R_{1,2}^{(k)})^b$. and uses Wagner’s algorithm to find a $t^{(\ell)}, 1 \leq \ell \leq \ell_{\max}$ for each session $k, 1 \leq k \leq k_{\max}$ (in other words a function $f : [1, k_{\max}] \rightarrow [1, \ell_{\max}]$) such that

$$\sum_{k=1}^{k_{\max}} \underbrace{H_{\text{agg}}(L, X_1 g^{t^{(f(k))}})}_{=: a_1^{(k)}} \underbrace{H_{\text{sig}}(\tilde{X}, R^*, m)}_{=: c^{(k)}} = \underbrace{H_{\text{sig}}(X_1, R^*, m^*)}_{=: c^*}. \quad (1)$$

The honest signer will reply with k_{\max} partial signatures $s_1^{(k)} = r_{1,1}^{(k)} + br_{1,2}^{(k)} + c^{(k)} \cdot a_1^{(k)} (x_1 + t^{(f(k))})$. The adversary is able to obtain

$$s_1^* = \sum_{k=1}^{k_{\max}} s_1^{(k)} \quad (2)$$

$$= \sum_{k=1}^{k_{\max}} \left(r_{1,1}^{(k)} br_{1,2}^{(k)} \right) + \left(\sum_{k=1}^{k_{\max}} c^{(k)} a_1^{(k)} \right) \cdot x_1 + \sum_{k=1}^{k_{\max}} c^{(k)} a_1^{(k)} t^{(f(k))} \quad (3)$$

$$= \log_g(R^*) + c^* x_1 + \sum_{k=1}^{k_{\max}} c^{(k)} a_1^{(k)} t^{(f(k))}, \quad (4)$$

where the last equality follows from Equation (1). We want (R^*, s^*) to be a valid forgery on message m^* for public key X_1 (which is not the case here).

4 (Preliminary) Conclusion

This attack on the specific scheme doesn’t work because the adversary doesn’t know f and therefore $\sum_{k=1}^{k_{\max}} c^{(k)} a_1^{(k)} t^{(f(k))}$ when determining c^* and therefore can’t calculate a valid target public key. Even if that was possible, the adversary would only forge for some random target public key which seems fine in practical security models. However, it may be possible to mount a secondary Wagner’s attack that allows further grinding such that $t = \sum_{k=1}^{k_{\max}} t^{(f(k))}$ for some target tweak t . If the honest signer would take the tweak t directly instead of C , then this attack may be even easier.

However, Equation (2) seems to indicate that if the honest signer signs with multiple random keys instead of tweaked keys then this attack does not apply. The adversary would only be able to obtain a $s_1^* = \log_g(R^*) + \sum_{k=1}^{k_{\max}} c^{(k)} a_1^{(k)} x^{(f(k))}$.