# Reinforcement learning and robot navigation using MDPs

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# Introduction

## The problem

- Framework : the Disopt robot which can follow lines
- The problem : the robot should adapt its speed with respect to traffic lights
- How : using Markov Decision Process (MDP) and Reinforcement Learning (RL)

# MDPs

## Definition

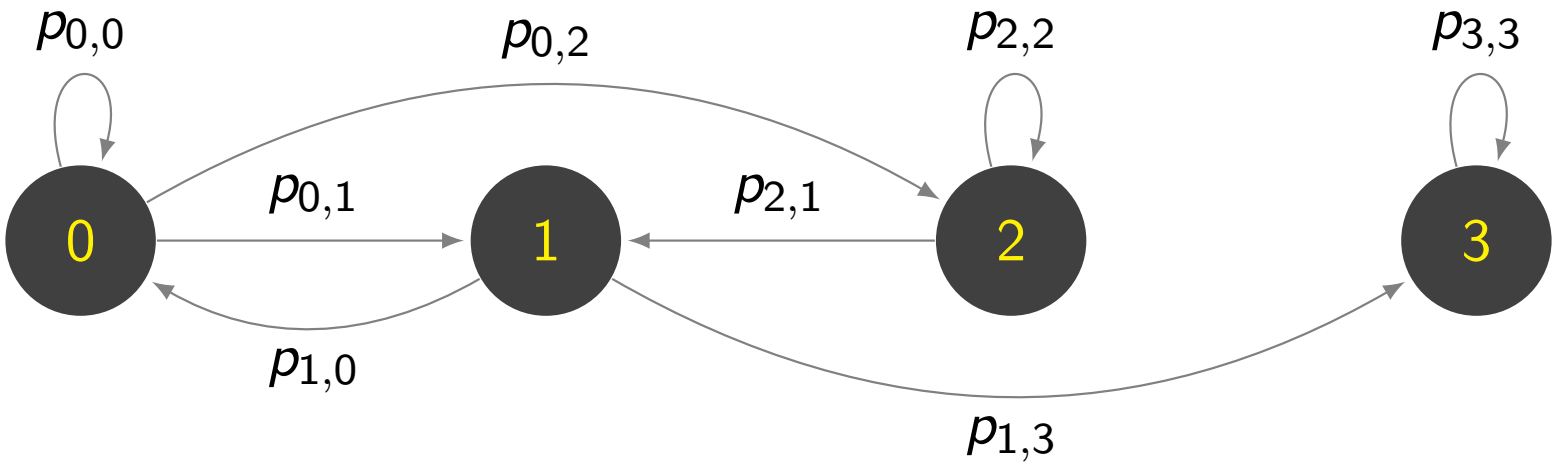
- A set of states $\mathcal{S} = \{s_0, s_1, s_2, \ldots, s_n\}$
- A set of actions $\mathcal{A} = \{a_1, a_2, a_3, \ldots, a_k\}$
- A transition function $T(a, s, s', r) = \mathbb{P}[s', r \mid a, s]$
- A reward function $R : \mathcal{S} \mapsto \mathbb{R}$
- A discount factor $0 \leq \gamma < 1$

## Markov Property

The transitions only depends on the current state and the current action.

# MDP example

MDPs can easily be represented by graphs :




The constraints are $\sum_j p_{i,j} = 1 \quad \forall i \in \mathcal{S}$

# how to pick actions

**Definition**

*A policy $\pi$ is a probabilistic mapping from the set of states to the set of actions :*

$$\pi : \mathcal{S} \mapsto \mathcal{A}$$

# Issue

**How to ?**

How to asses the goodness of policies so we can find the best one ?
What is the best policy ?

# how to asses the goodness of policies

## Discounted return

$$G_t = \sum_{k=0}^{\infty} \gamma^k * R_{t+k+1}$$

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## state value under policy $\pi$

$$\begin{aligned} v_\pi(s) &= \mathbb{E}[G_t \mid S_t = s] \\ &= \sum_a \pi(a \mid s) \sum_{r,s'} p(s', r \mid s, a)[r + \gamma v_\pi(s')] \end{aligned} \tag{2}$$

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## how to compare two policies

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## Optimal policy

$$\pi_* \quad s.t. \quad \forall \pi : \pi_* \geq \pi$$

# Bellman optimality equations

The optimal policy $\pi_*$ has value functions : $v_*$ and $q_*$

$$v_*(s) = \max_a \sum_{s',r} p(s', r \mid s, a)[r + \gamma v_*(s')] \tag{3}$$

$$q_*(s, a) = \sum_{s',r} p(s', r \mid s, a)[r + \gamma \max_{a'} q_*(s', a')] \tag{4}$$

# Another issue

**computational issue**

If we wanted to solve these equations directly, it would cost a lot of computational power to know exactly the value functions first and then to solve. So how do we do it ?

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Approximation of value function

# solving MDPs using dynamic programming

## policy iteration

update rule :

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s', r} p(s', r \mid s, a)(r + \gamma v_k(s'))$$

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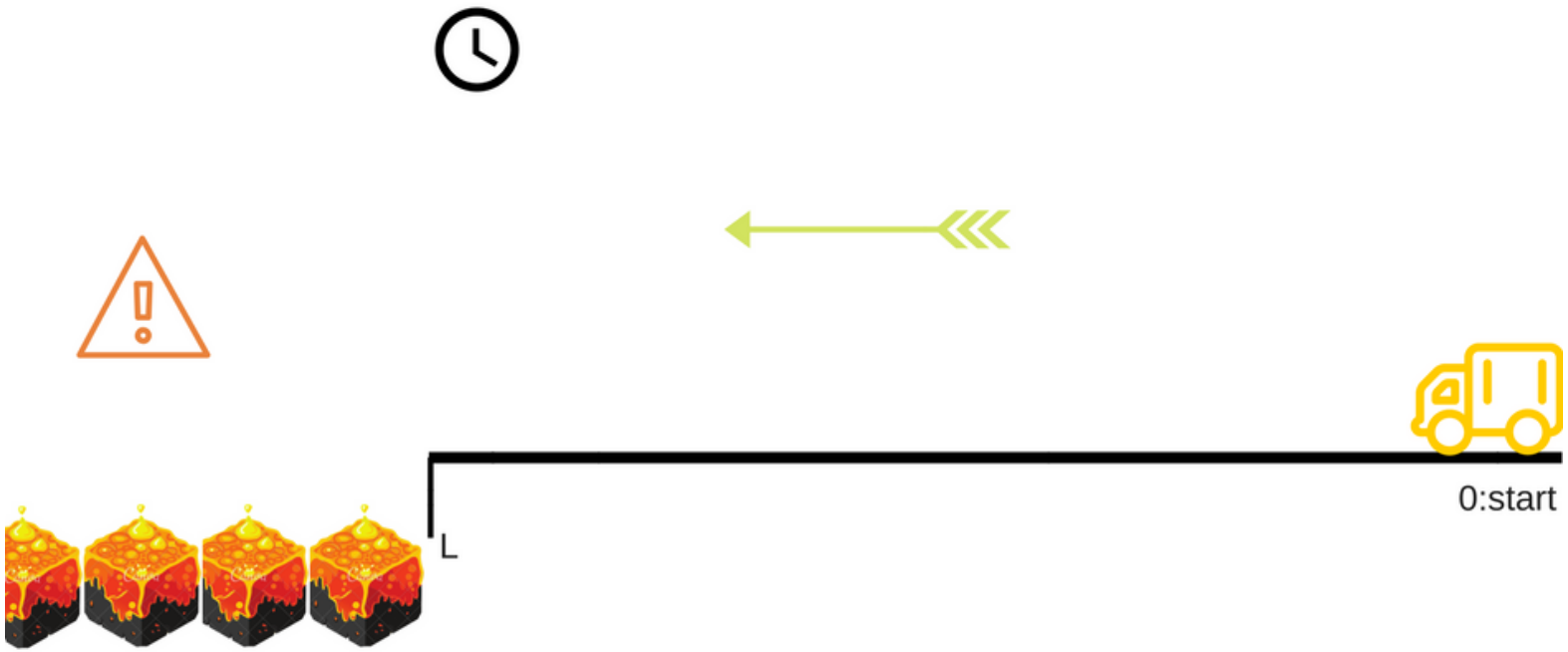
## Policy Improvement

$\pi / \pi'$ : old/new policy.

$$\pi'(s) = argmax_{a \in \mathcal{A}} q_\pi(s, a)$$

# what have we done so far



## States

# modelization

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- position $\{0,1,2,\ldots,L,$ Lava $\}$

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- maintaining speed

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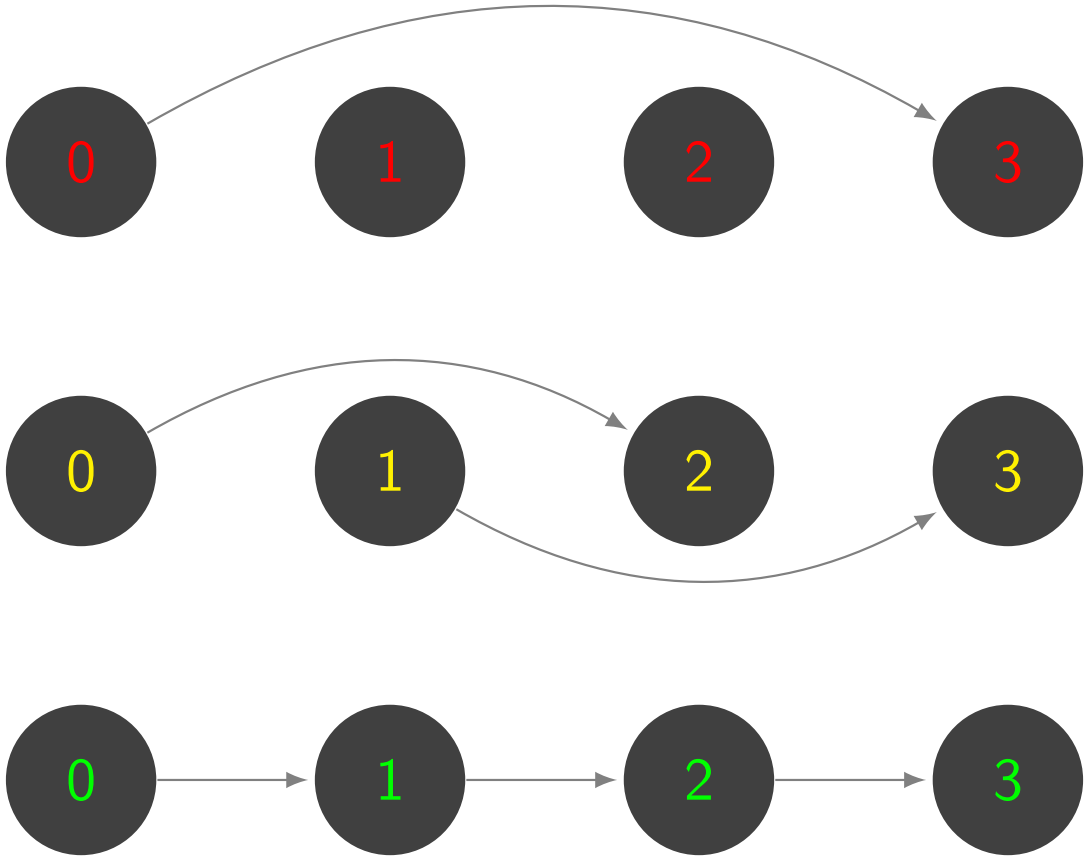
## Actions

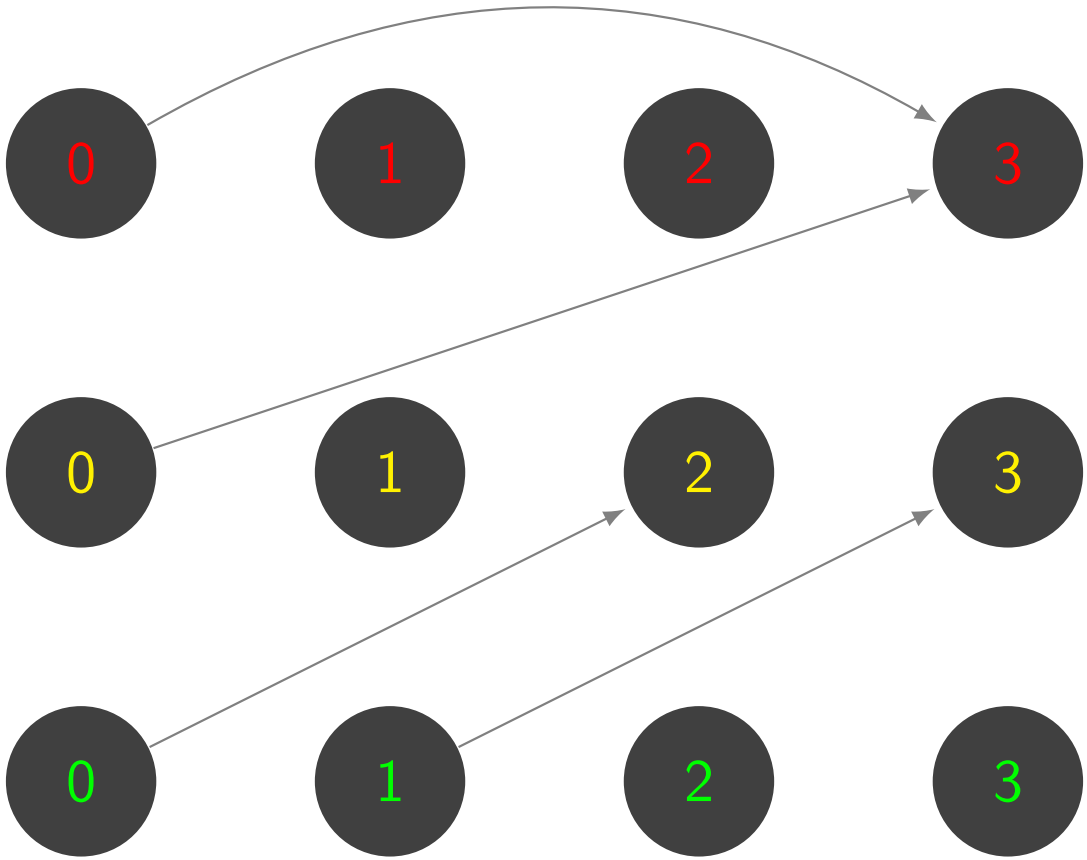
- decelerating
- maintaining speed
- accelerating

# accelerating graph

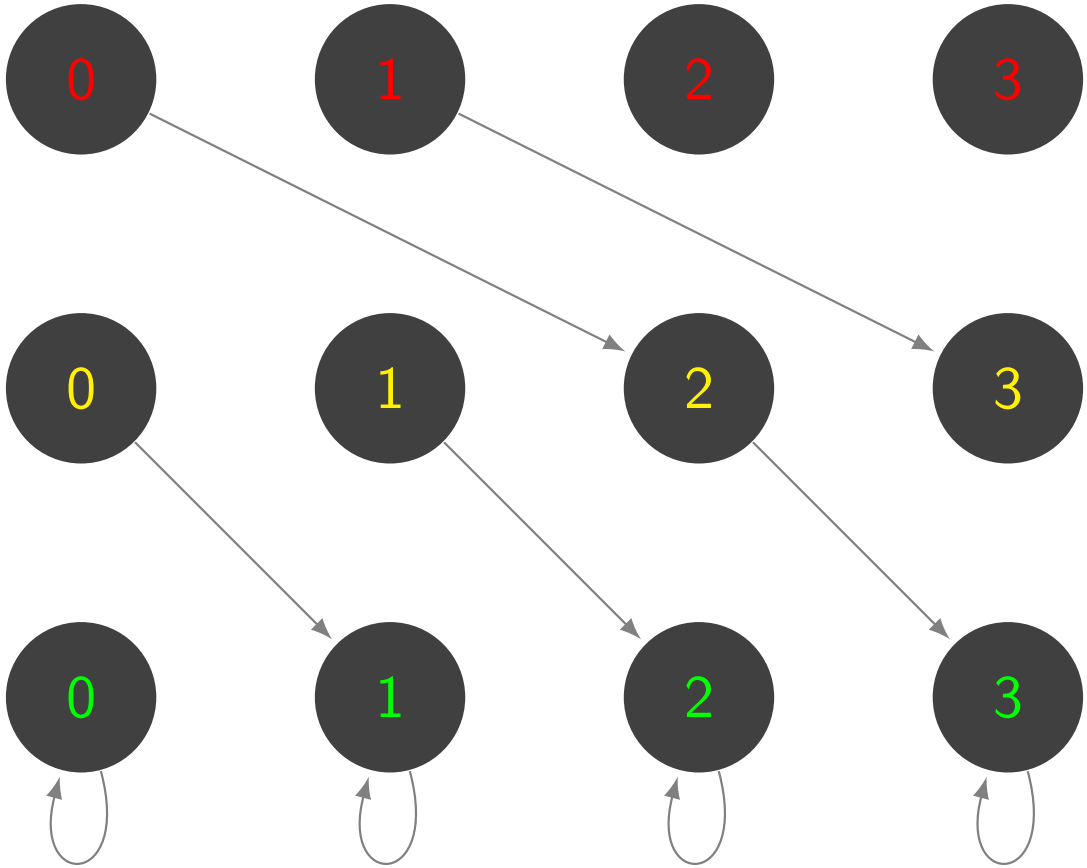



red : high speed
yellow : medium speed
green : low speed

# keeping the same speed graph

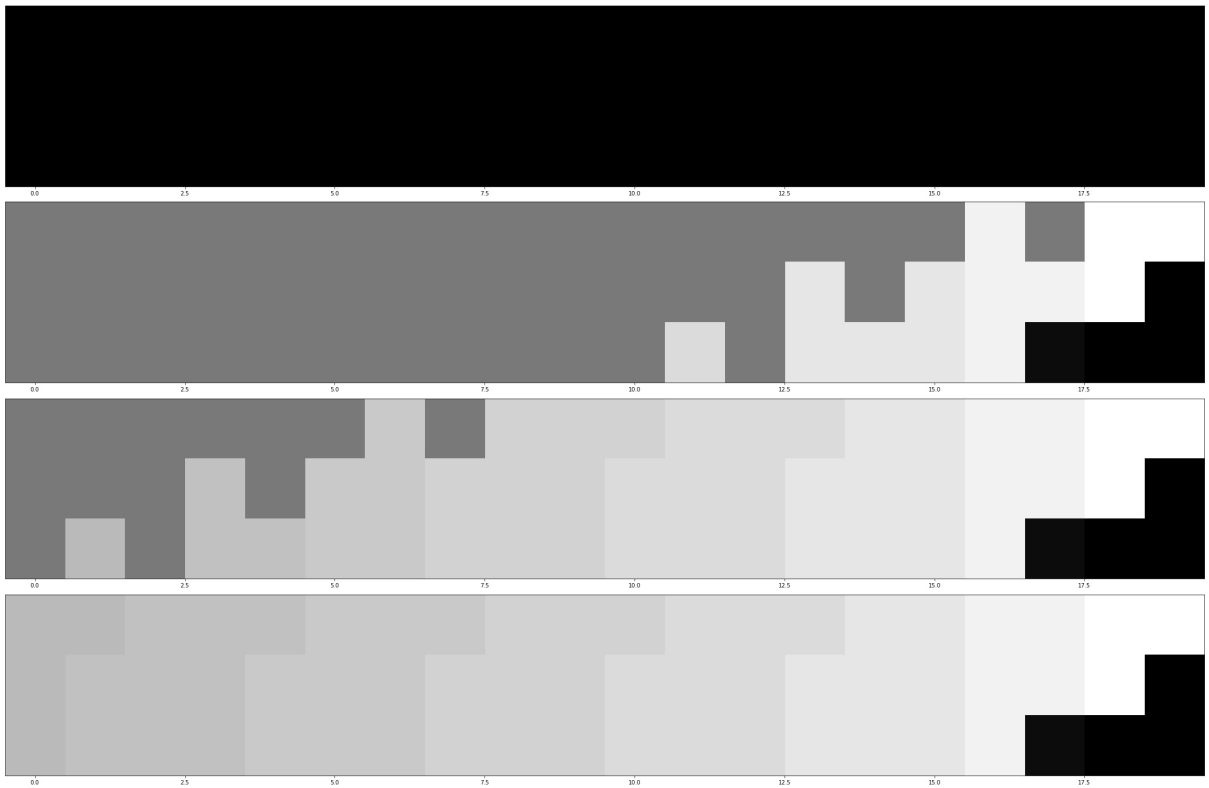


# decelerating

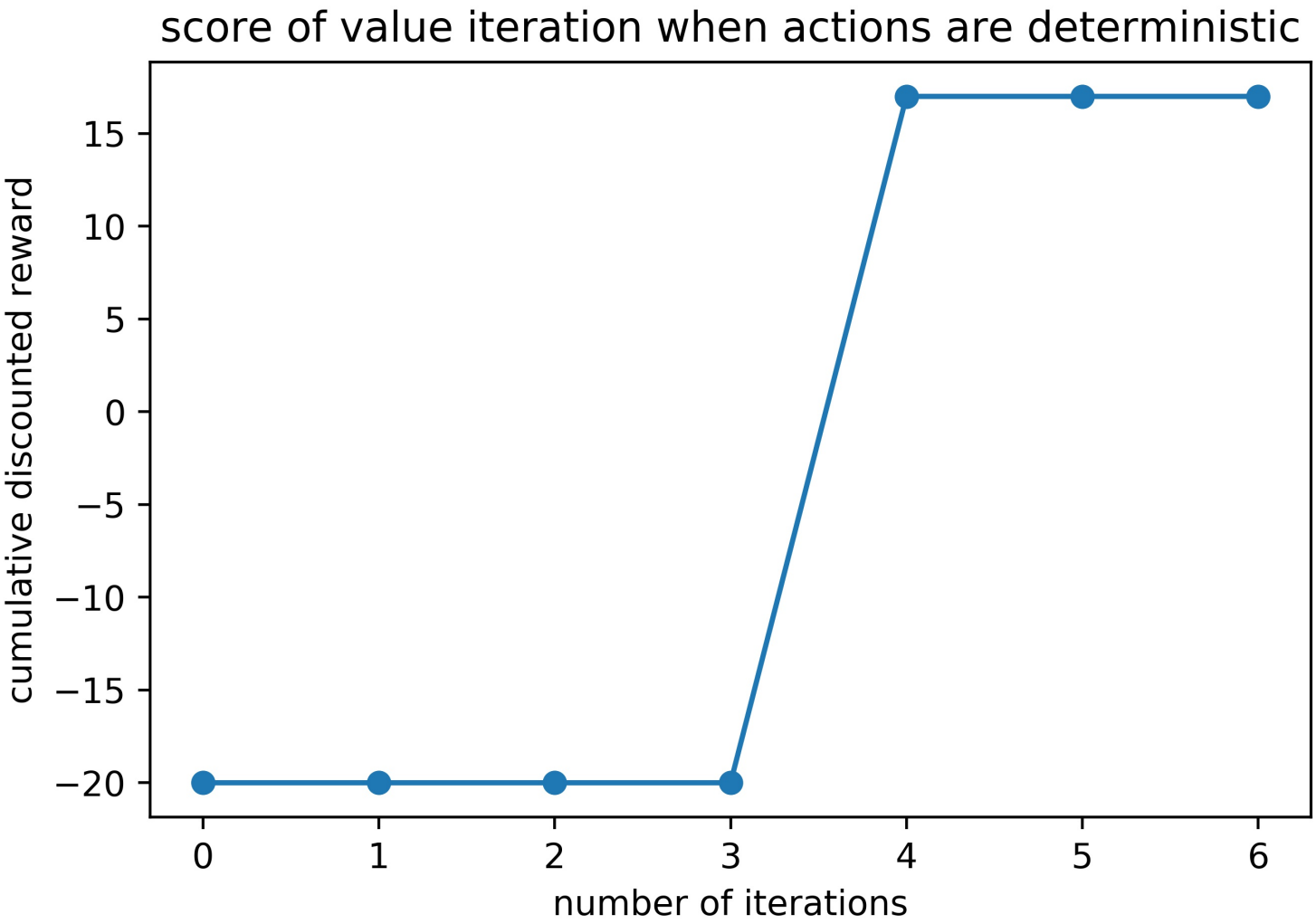


# Results
## States value evolution

States values at iterations $0, 2, 4$ and $6$ (where stable policy is attained)



# Results
## Discounted reward



# What's next ?

## already working on

- Add the traffic light into this setting

## In a not so distant future

- Finding the distance from the robot's camera to the traffic light
- implement on the robot...

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## In a not so distant future

- Finding the distance from the robot's camera to the traffic light
- implement on the robot... and pray that everything works well on the first try

## Ideas

- explore other algorithm and compare them : Monte-Carlo methods, time difference methods, Q learning, ...
- Neuro-dynamic programming ?