Reinforcement learning and robot navigation

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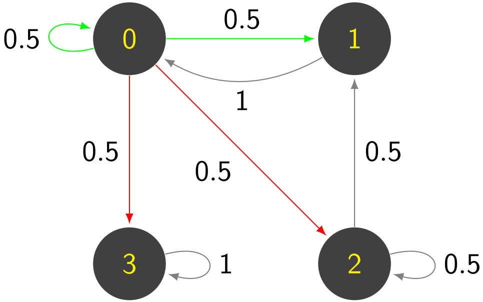
Introduction

The problem

- Framework : the Disopt robot which can follow lines
- The problem : the robot should adapt its speed with respect to traffic lights
- How : using Markov Decision Process (MDP) and Reinforcement Learning (RL)

MDP Intuition



Sequence of events : $s_0, a_1, s_1, r_1, a_2, \dots$

MDPs

Definition

- A set of states $\mathcal{S} = \{s_0, s_1, s_2, \dots, s_n\}$
- A set of actions $\mathcal{A} = \{a_1, a_2, a_3, \dots, a_k\}$
- A transition function $T(a, s, s') = \mathbb{P}[s' \mid a, s]$
- A reward function $R : \mathcal{S} \mapsto \mathbb{R}$
- A discount factor $0 \leq \gamma < 1$

Markov Property

The transitions only depends on the current state and the current action.

how to pick actions

Definition

A policy π is a probabilistic mapping from the set of states to the set of actions :

$$\pi : \mathcal{A}x\mathcal{S} \mapsto [0,1]$$

$$s.t.\ \sum_{a} \pi(a \mid s) = 1 \quad \forall s \in \mathcal{S}$$

Issue

How to ?

How to asses the goodness of policies so we can find the best one ?
What is the best policy ?

how to asses the goodness of policies

We are interested in maximizing the discounted return

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} = R_{t+1} + \gamma G_{t+1}$$

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state value under policy π

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s] \tag{2}$$

Recursive definition of function values

action value

$$
\begin{aligned}
q_\pi(s,a) &= \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\
&= \sum_{s'} \mathbb{P}(s' \mid a, s)\left[r + \gamma \mathbb{E}_\pi\left[G_{t+1} \mid S_{t+1} = s'\right]\right] \\
&= \sum_{s'} \mathbb{P}(s' \mid a, s)\left[r + \gamma v_\pi(s')\right]
\end{aligned} \tag{3}
$$

with $r = R(s')$

Recursive definition of function values

state value

$$
\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) \left[r + \gamma \mathbb{E}_\pi \left[G_{t+1} \mid S_{t+1} = s' \right] \right] \\
&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) \left[r + \gamma v_\pi(s') \right] \\
&= \sum_a \pi(a \mid s) q_\pi(s, a)
\end{aligned}
\tag{4}
$$

with $r = R(s')$

how to asses the goodness of policies

how to compare two policies

$$\pi \leq \pi' \iff v_\pi(s) \leq v_{\pi'}(s) \quad \forall s \in \mathcal{S}$$

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Optimal policy

$$\pi_* \quad s.t. \quad \forall \pi : \pi_* \geq \pi$$

Bellman optimality equations

The optimal policy π_* has value functions v_* and q_*

$$\begin{aligned} v_*(s) &= \max_a \; q_{\pi_*}(s,a) \\ &= \max_a \sum_{s'} \mathbb{P}(s' \mid s,a)[R(s') + \gamma v_*(s')] \end{aligned} \tag{5}$$

$$\begin{aligned} q_*(s,a) &= \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') \mid S_t = s, A_t = a \right] \\ &= \sum_{s'} \mathbb{P}(s' \mid s,a)[R(s') + \gamma \max_{a'} q_*(s', a')] \end{aligned} \tag{6}$$

finding optimal policy from value functions

$$\pi_*(s) = \underset{a}{\operatorname{argmax}}\ q_*(s, a)$$

Another issue

computational issue

If we wanted to solve these equations directly, it would cost a lot of computational power to know exactly the value functions first and then to solve since they are not linear. So how do we do it ?

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Approximation of value function

solving MDPs using dynamic programming

iterative policy evaluation

update rule :

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s',r} T(a, s, s') \left[r + \gamma v_k(s') \right]$$

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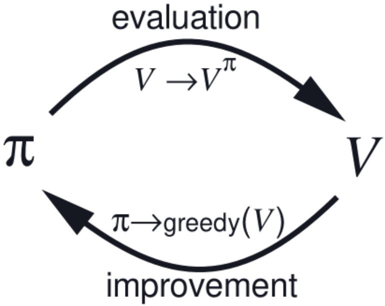
Policy Improvement

π / π' : old/new policy.

$$\pi'(s) = \operatorname*{argmax}_{a \in \mathcal{A}} q_\pi(s, a)$$

Policy iteration algorithm



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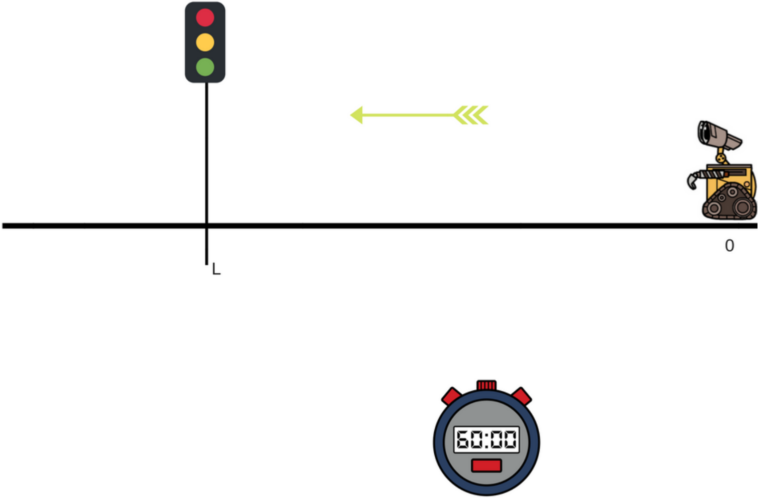
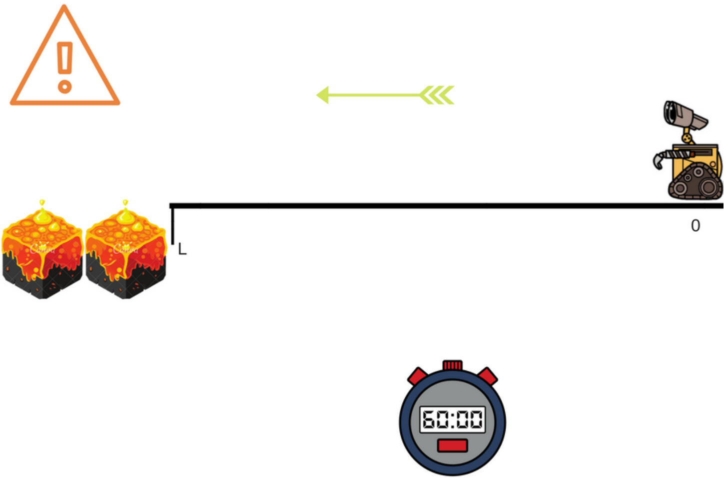
[1] From (Sutton & Barto, 1998)

Our problem



First a simpler problem



modelization

States

modelization

States

- position $\{0,1,2,\ldots,L, \text{Lava}\}$

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- position $\{0,1,2,\ldots,L, \text{Lava}\}$
- speed $\{\text{low, medium, high}\}$

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- decelerating

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Actions

- decelerating
- maintaining speed

modelization

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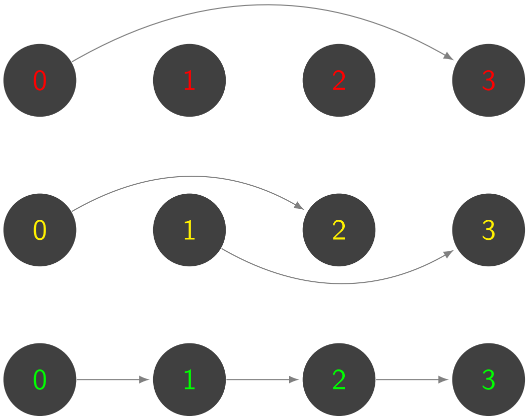
- speed $\{\text{low, medium, high} \}$

Actions

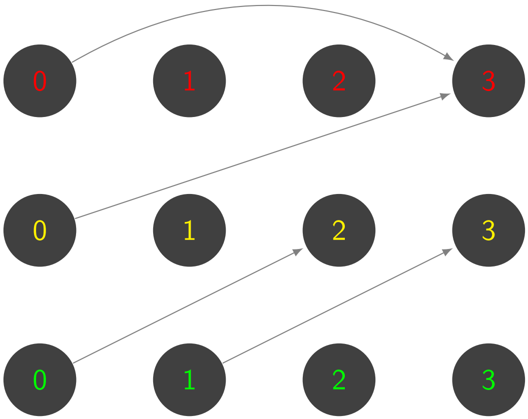
- decelerating
- maintaining speed
- accelerating

keeping the same speed graph

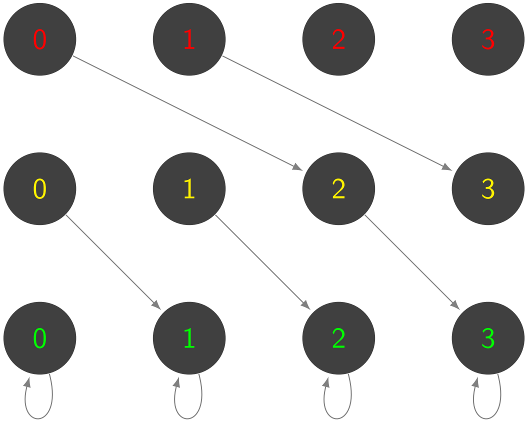


red : high speed
yellow : medium speed
green : low speed

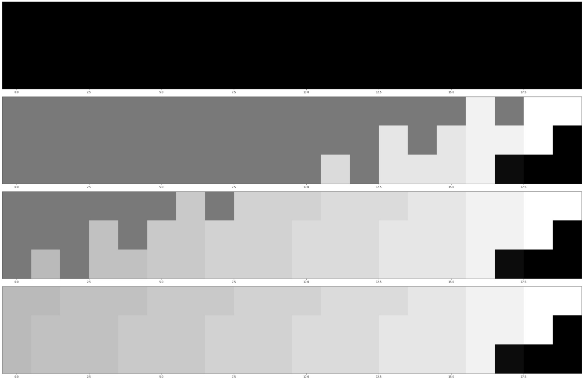
accelerating graph



decelerating

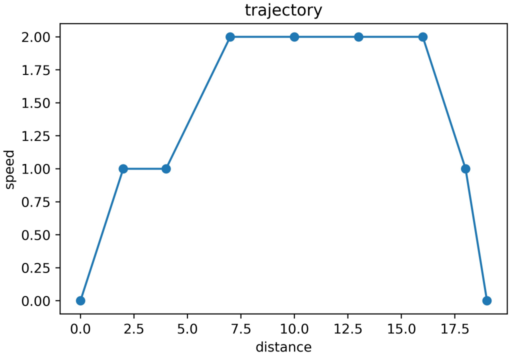


States values at iterations $0, 2, 4$ and 6 (where stable policy is attained)



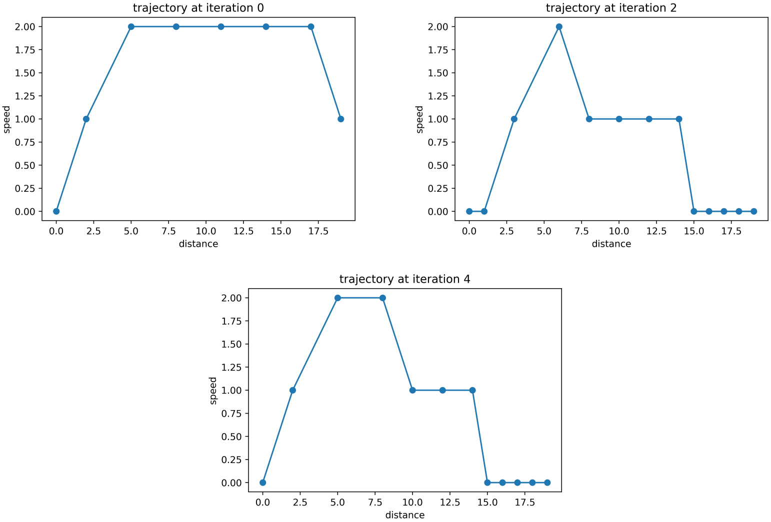
Results for deterministic actions

One of the optimal trajectories



Results when uncertainty introduced into brakes

Evolution of the trajectories during iteration 0,2, and 4 (optimal)



Conclusion, what have we done ?

- We introduced the theoretical framework,the Markov Decision Process
- We solved a simple problem where we know the model

What's next ?

already working on

- Add the traffic light into this setting
- Find the distance from the robot's camera to the traffic light

In a not so distant future

- explore other algorithm and compare them : Monte-Carlo methods, temporal difference learning, Q learning, ...
- Neuro-dynamic programming ?

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Implement everything on the robot ... and pray that everything works well