Reinforcement learning and robot navigation

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April 13, 2018

Introduction

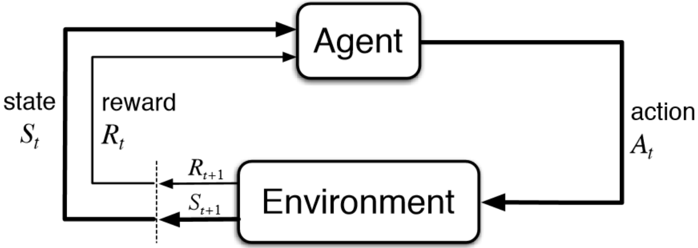
The problem

- Framework: a raspberry pie 3 robot which can follow lines
- The task: the robot should adapt its speed with respect to traffic lights
- How: using Reinforcement Learning (RL) and Markov Decision Process (MDP)

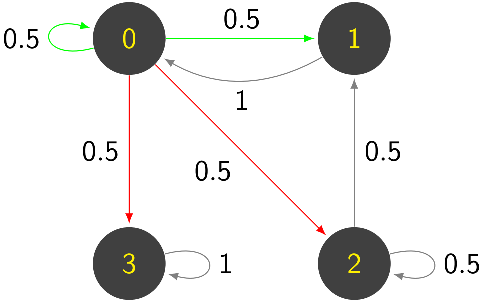
Presentation

- Part I: theoretical insight
- Part II: results from implementation

Reinforcement learning



MDP intuition



Sequence of events : $s_0, a_1, s_1, r_1, a_2, \ldots$

MDP

Definition

- A set of states $\mathcal{S} = \{s_0, s_1, s_2, \ldots, s_n\}$
- A set of actions $\mathcal{A} = \{a_1, a_2, a_3, \ldots, a_k\}$
- A transition function $T(a, s, s') = \mathbb{P}[s' \mid a, s]$
- A reward function $R : \mathcal{S} \mapsto \mathbb{R}$
- A discount factor $0 \leq \gamma < 1$

Markov property

The transitions only depends on the current state and the current action.

How to pick actions

Definition

A policy π is a probabilistic mapping from the set of states to the set of actions :

$$\pi : \mathcal{A}x\mathcal{S} \mapsto [0, 1]$$

s.t. $\sum_a \pi(a \mid s) = 1 \quad \forall s \in \mathcal{S}$

Issue

How to ?

How to asses the quality of policies so we can find the best one ?
What is the best policy ?

How to asses the quality of policies

We are interested in maximizing the discounted return

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} = R_{t+1} + \gamma G_{t+1}$$

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Action value while in a state s under π

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State value under policy π

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s]$$

Recursive definition of value functions

action value

$$
\begin{aligned}
q_\pi(s,a) &= \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\
&= \sum_{s'} \mathbb{P}(s' \mid a, s) \left[r + \gamma \mathbb{E}_\pi \left[G_{t+1} \mid S_{t+1} = s' \right] \right] \\
&= \sum_{s'} \mathbb{P}(s' \mid a, s) \left[r + \gamma v_\pi(s') \right]
\end{aligned}
$$

with $r = R(s')$

Recursive definition of value functions

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&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) \left[r + \gamma v_\pi(s') \right] \\
&= \sum_a \pi(a \mid s) q_\pi(s, a)
\end{aligned}
$$

with $r = R(s')$

How to asses the quality of policies

How to compare two policies

$$\pi \leq \pi' \iff v_\pi(s) \leq v_{\pi'}(s) \quad \forall s \in \mathcal{S}$$

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Optimal policy

$$\pi_* \quad s.t. \quad \forall \pi : \pi_* \geq \pi$$

Bellman optimality equations

The optimal policy π_* has value functions v_* and q_*

$$
\begin{aligned}
v_*(s) &= \max_a \; q_*(s, a) \\
&= \max_a \sum_{s'} \mathbb{P}(s' \mid s, a)[R(s') + \gamma v_*(s')]
\end{aligned}
$$

$$
\begin{aligned}
q_*(s, a) &= \mathbb{E}\left[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') \mid S_t = s, A_t = a\right] \\
&= \sum_{s'} \mathbb{P}(s' \mid s, a)[R(s') + \gamma \max_{a'} q_*(s', a')]
\end{aligned}
$$

Finding optimal policy from value functions

$$\pi_*(s) = \underset{a}{\operatorname{argmax}}\ q_*(s, a)$$

Another issue

computational issue

$\mid \mathcal{S} \mid$ equations to solve to evaluate policy:

$$O(\mid \mathcal{S} \mid^2)\text{sdljnsojgbsogbsj}$$

$\mid \mathcal{S} \mid$ non linear equations to solve the Bellman optimality equation:

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Approximation of value function
and
Policy iteration

Solving MDP using dynamic programming

iterative policy evaluation

update rule :

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s'} T(a, s, s') \left[R(s') + \gamma v_k(s') \right]$$

Solving MDP using dynamic programming

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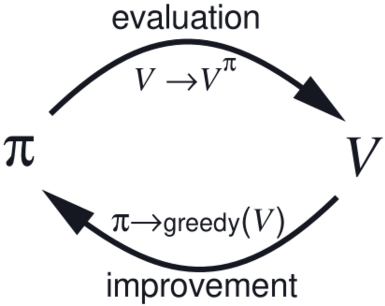
policy improvement

π / π' : old/new policy.

$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \; q_\pi(s, a)$$

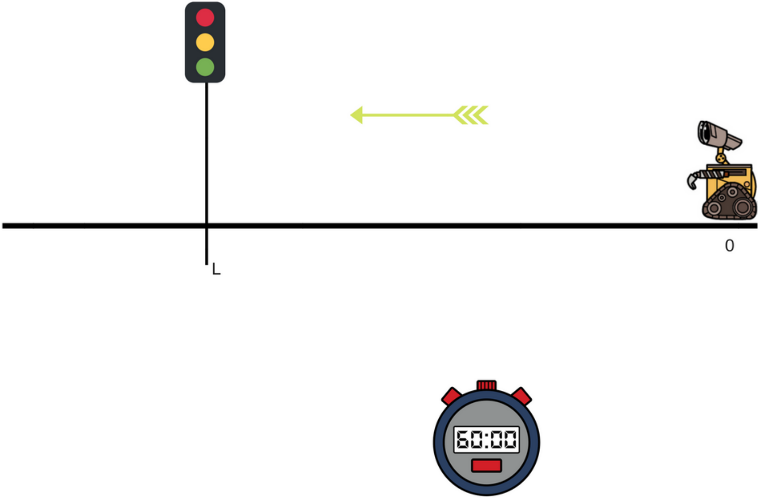
Policy iteration algorithm



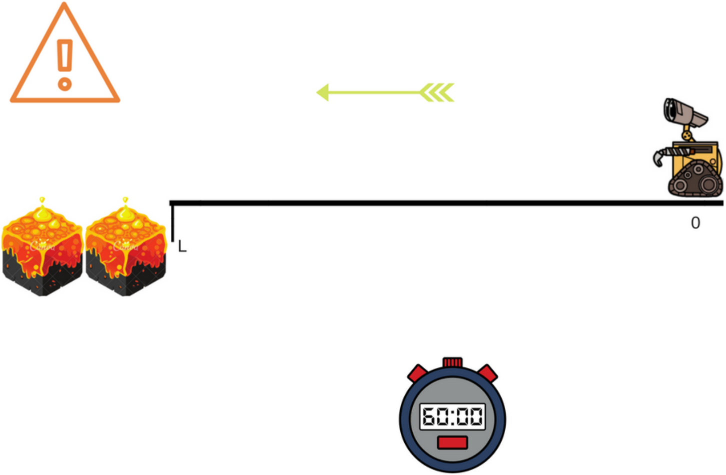
[1]

[1] From (Sutton & Barto, 1998)

Our problem



First a simpler problem



Modelization

states

- position $\{0,1,2,\ldots,L, \text{Lava}\}$
- speed $\{\text{low, medium, high}\}$

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- decelerating
- maintaining speed
- accelerating

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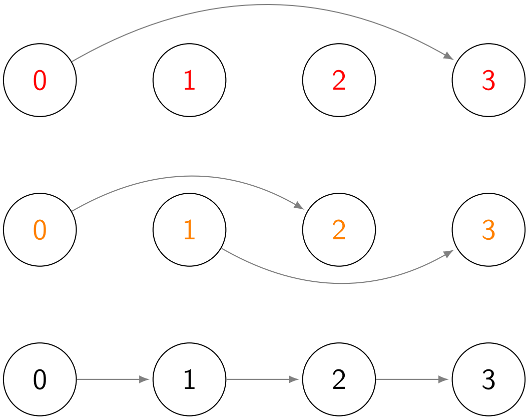
reward function

- Lava:reward of -L
- L in low speed: reward of $+$L
- any other state : reward of -1

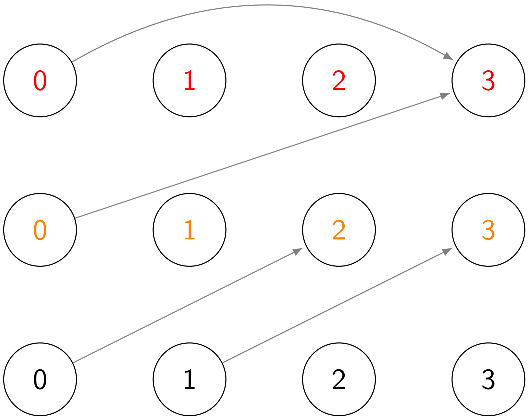
keeping the same speed graph



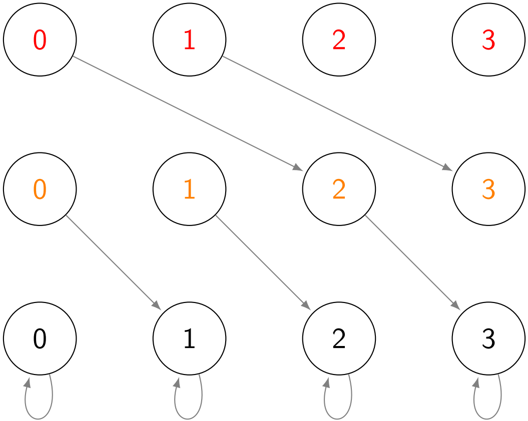
red : high speed

orange : medium speed

black : low speed


0
1
2
3
0
1
2
3
0
1
2
3

decelerating



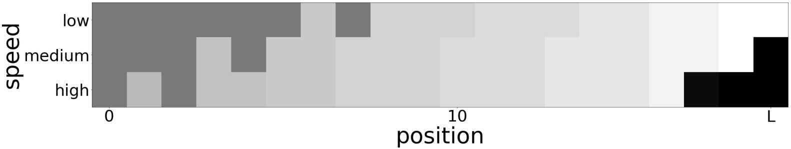
State value function at different iterations

States values at iteration 0

State value function at different iterations

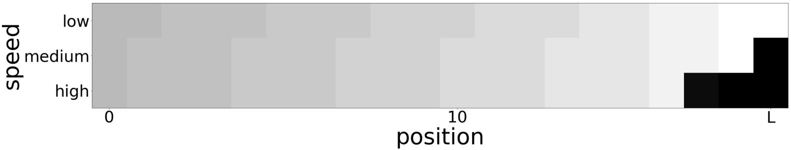
States values at iteration 2

State value function at different iterations



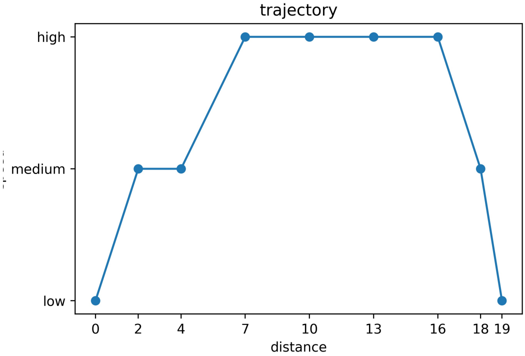
States values at iteration 4

State value function at different iterations



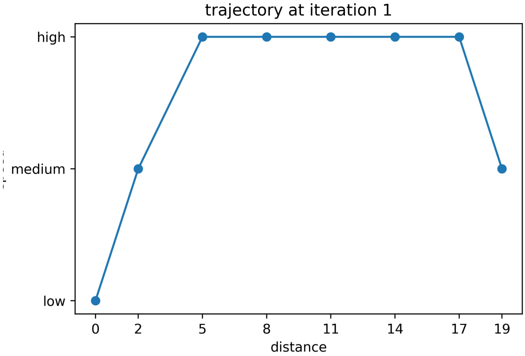
States values at iteration 6 (where stable policy is attained)

Results for deterministic actions
One of the optimal trajectories



Results for stochastic actions

decelerating results results in keeping the same speed with probability 1/4



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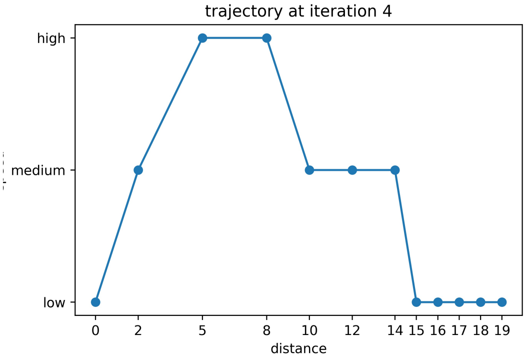
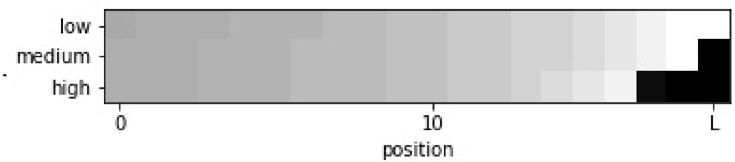


where stable policy is attained

Results for stochastic actions
state-value function at the end of the iterations



Conclusion, what have we done ?

- We introduced the theoretical framework: Reinforcement learning and Markov Decision Processes
- We solved a simple problem where we know the model

What's next ?

already working on

- Add the traffic light into this setting
- Find the distance from the robot's camera to the traffic light

In a not so distant future

- Explore other algorithm and compare them : Monte-Carlo methods, temporal difference learning, Q learning, ...
- Neuro-dynamic programming ?

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Implement everything on the robot ...

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Implement everything on the robot ... and pray that everything works well