# Reinforcement learning and robot navigation

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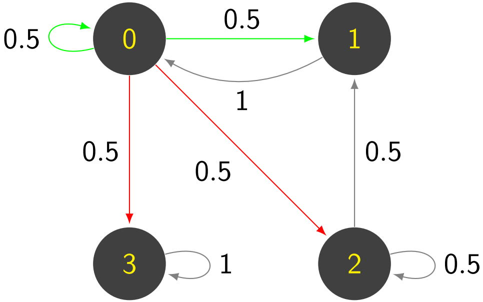
### The problem

- Framework : the Disopt robot which can follow lines
- The problem : the robot should adapt its speed with respect to traffic lights
- How : using Markov Decision Process (MDP) and Reinforcement Learning (RL)

# MDP Intuition




Sequence of events : $s_0, a_1, s_1, r_1, a_2, \ldots$

# MDPs

## Definition

- A set of states $\mathcal{S} = \{s_0, s_1, s_2, \ldots, s_n\}$
- A set of actions $\mathcal{A} = \{a_1, a_2, a_3, \ldots, a_k\}$
- A transition function $T(a, s, s') = \mathbb{P}[s' \mid a, s]$
- A reward function $R : \mathcal{S} \mapsto \mathbb{R}$
- A discount factor $0 \leq \gamma < 1$

## Markov Property

The transitions only depends on the current state and the current action.

# how to pick actions

**Definition**

*A policy* $\pi$ *is a probabilistic mapping from the set of states to the set of actions :*

$$\pi : \mathcal{A}x\mathcal{S} \mapsto [0,1]$$

$$s.t. \sum_a \pi(a \mid s) = 1 \quad \forall s \in \mathcal{S}$$

# Issue

**How to ?**

How to asses the goodness of policies so we can find the best one ?
What is the best policy ?

# how to asses the goodness of policies

**We are interested in maximizing the discounted return**

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} = R_{t+1} + \gamma G_{t+1}$$

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**state value under policy $\pi$**

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s] \tag{2}$$

# Recursive definition of function values

## state value

$$
\begin{aligned}
v_\pi(s, a) &= \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) \left[R(s') + \gamma \mathbb{E}_\pi \left[G_{t+1} \mid S_{t+1} = s'\right]\right] \\
&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) \left[R(s') + \gamma v_\pi(s')\right]
\end{aligned} \tag{3}
$$

# how to asses the goodness of policies

**how to compare two policies**

$$\pi \leq \pi' \iff v_\pi(s) \leq v_{\pi'}(s) \quad \forall s \in \mathcal{S}$$

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## Optimal policy

$$\pi_* \quad s.t. \quad \forall \pi : \pi_* \geq \pi$$

# Bellman optimality equations

The optimal policy $\pi_*$ has value functions : $v_*$ and $q_*$

$$v_{\pi_*}(s) = \max_a \sum_{s',r} \mathbb{P}(s' \mid s, a)[R(s') + \gamma v_*(s')] \tag{4}$$

$$q_{\pi_*}(s, a) = \sum_{s',r} \mathbb{P}(s' \mid s, a)[R(s') + \gamma \max_{a'} q_*(s', a')] \tag{5}$$

# Another issue

### computational issue

If we wanted to solve these equations directly, it would cost a lot of computational power to know exactly the value functions first and then to solve since they are not linear. So how do we do it ?

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Approximation of value function

# solving MDPs using dynamic programming

## iterative policy evaluation

update rule :

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s',r} T(a, s, s') \left[r + \gamma v_k(s')\right]$$

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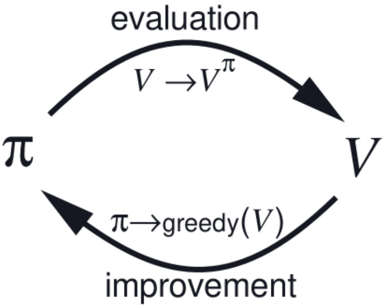
## Policy Improvement

$\pi / \pi'$ : old/new policy.

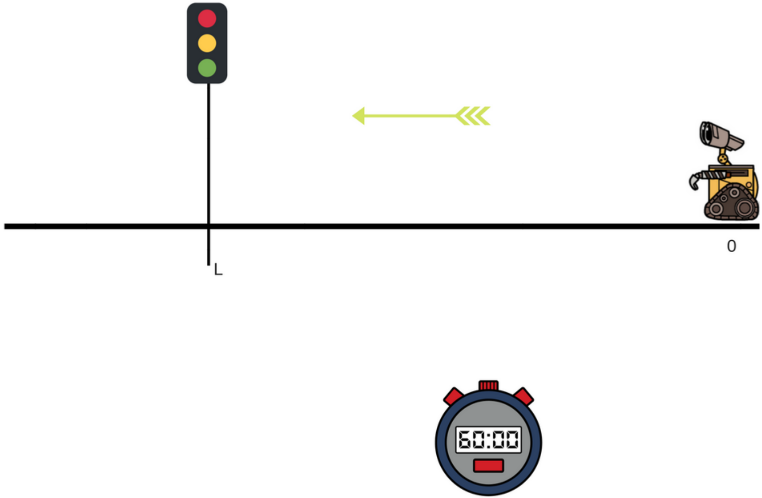
$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \ q_\pi(s, a)$$

# Policy iteration algorithm

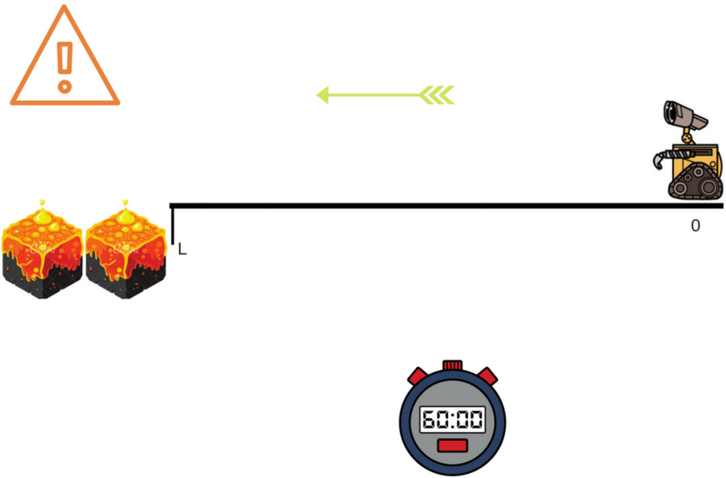



From (Sutton & Barto, 1998)

# Our problem



# First a simpler problem



## States

# modelization

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- position $\{0,1,2,\ldots,L, \text{Lava}\}$

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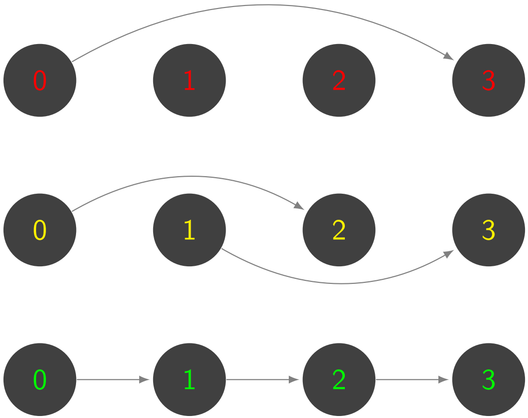
## Actions

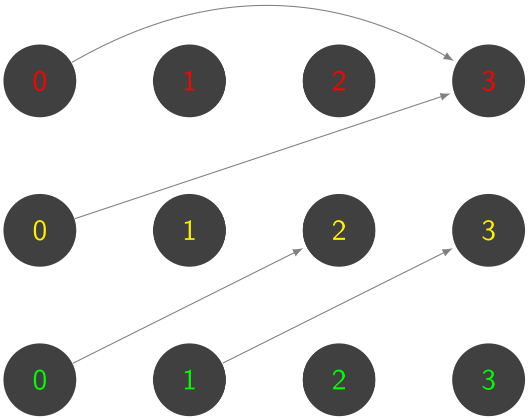
- decelerating
- maintaining speed
- accelerating

# keeping the same speed graph

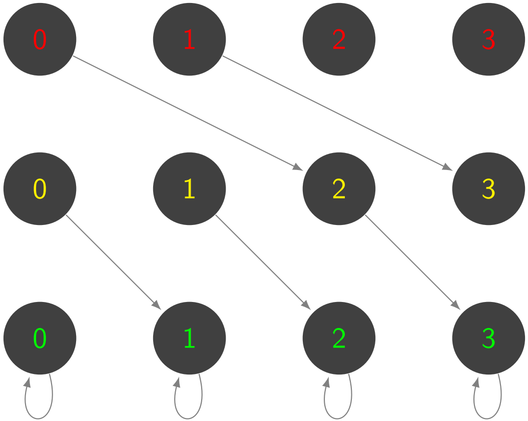



red : high speed
yellow : medium speed
green : low speed

# accelerating graph

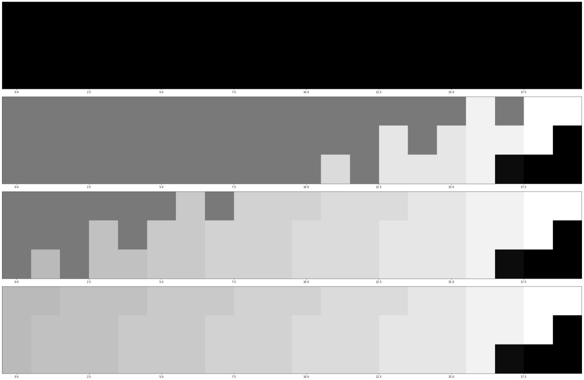


# decelerating



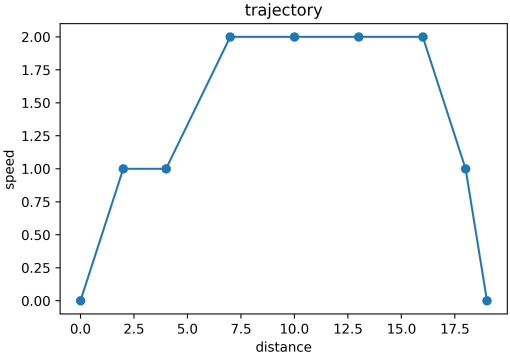
# Results
## States value evolution

States values at iterations $0, 2, 4$ and $6$ (where stable policy is attained)



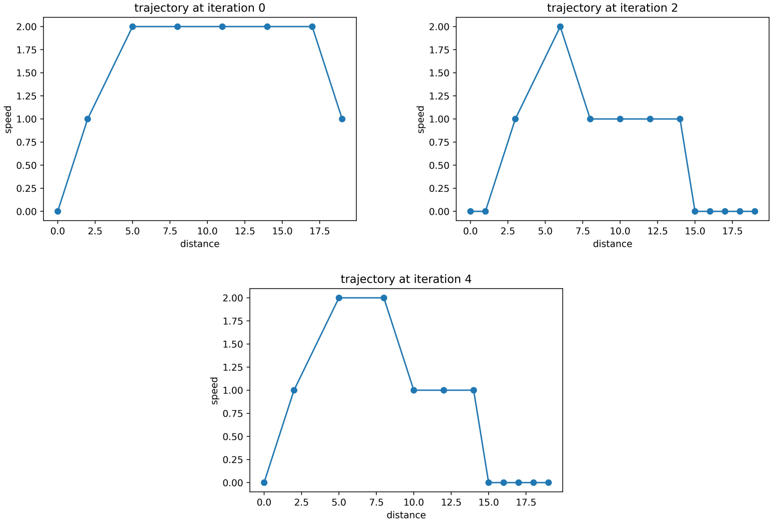
# Results for deterministic actions

## One of the optimal trajectories



# Results when uncertainty introduced into brakes

## Evolution of the trajectories during iteration 0,2, and 4 (optimal)



# What's next ?

## already working on

- Add the traffic light into this setting
- Find the distance from the robot's camera to the traffic light

## In a not so distant future

- explore other algorithm and compare them : Monte-Carlo methods, time difference methods, Q learning, ...
- Neuro-dynamic programming ?

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Implement everything on the robot ... and pray that it works !!