

# Reinforcement learning and robot navigation

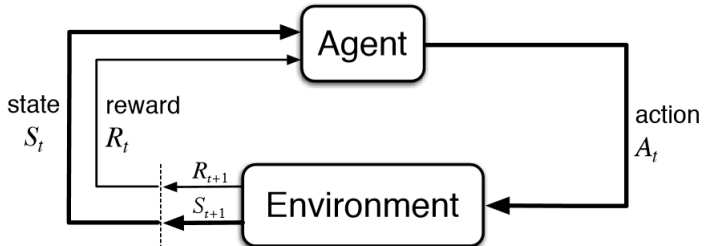
Charles Dufour

April 11, 2018

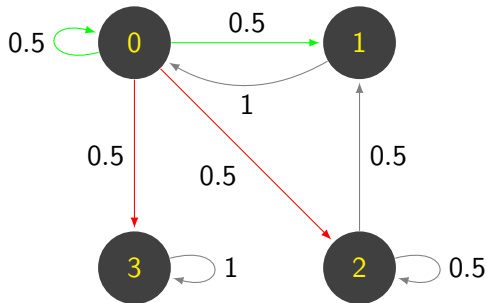
## The problem

- Framework : the Disopt robot which can follow lines
- The task : the robot should adapt its speed with respect to traffic lights
- How : using Reinforcement Learning (RL) and Markov Decision Process (MDP)

# Reinforcement learning



# MDP Intuition



Sequence of events :  $s_0, a_1, s_1, r_1, a_2, \dots$

## Definition

- A set of states  $\mathcal{S} = \{s_0, s_1, s_2, \dots, s_n\}$
- A set of actions  $\mathcal{A} = \{a_1, a_2, a_3, \dots, a_k\}$
- A transition function  $T(a, s, s') = \mathbb{P}[s' \mid a, s]$
- A reward function  $R : \mathcal{S} \mapsto \mathbb{R}$
- A discount factor  $0 \leq \gamma < 1$

## Markov Property

The transitions only depends on the current state and the current action.

## Definition

*A policy  $\pi$  is a probabilistic mapping from the set of states to the set of actions :*

$$\pi : \mathcal{A} \times \mathcal{S} \mapsto [0, 1]$$

$$\text{s.t. } \sum_a \pi(a \mid s) = 1 \quad \forall s \in \mathcal{S}$$

How to ?

How to assess the goodness of policies so we can find the best one ?  
What is the best policy ?

# how to asses the goodness of policies

We are interested in maximizing the discounted return

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} = R_{t+1} + \gamma G_{t+1}$$



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state value under policy  $\pi$

$$v_{\pi}(s) = \mathbb{E}_{\pi}[G_t \mid S_t = s]$$

## action value

$$\begin{aligned}q_{\pi}(s, a) &= \mathbb{E}_{\pi}[G_t \mid S_t = s, A_t = a] \\&= \mathbb{E}_{\pi}[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\&= \sum_{s'} \mathbb{P}(s' \mid a, s) [r + \gamma \mathbb{E}_{\pi}[G_{t+1} \mid S_{t+1} = s']] \\&= \sum_{s'} \mathbb{P}(s' \mid a, s) [r + \gamma v_{\pi}(s')]\end{aligned}$$

with  $r = R(s')$

## state value

$$\begin{aligned}v_{\pi}(s) &= \mathbb{E}_{\pi}[G_t \mid S_t = s] \\&= \mathbb{E}_{\pi}[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) [r + \gamma \mathbb{E}_{\pi}[G_{t+1} \mid S_{t+1} = s']] \\&= \sum_a \pi(a \mid s) \sum_{s'} \mathbb{P}(s' \mid a, s) [r + \gamma v_{\pi}(s')] \\&= \sum_a \pi(a \mid s) q_{\pi}(s, a)\end{aligned}$$

with  $r = R(s')$

# how to asses the goodness of policies

how to compare two policies

$$\pi \leq \pi' \iff v_{\pi}(s) \leq v_{\pi'}(s) \quad \forall s \in \mathcal{S}$$

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Optimal policy

$$\pi_* \quad s.t. \quad \forall \pi : \pi_* \geq \pi$$

# Bellman optimality equations

The optimal policy  $\pi_*$  has value functions  $v_*$  and  $q_*$

$$\begin{aligned} v_*(s) &= \max_a q_*(s, a) \\ &= \max_a \sum_{s'} \mathbb{P}(s' \mid s, a) [R(s') + \gamma v_*(s')] \end{aligned}$$

$$\begin{aligned} q_*(s, a) &= \mathbb{E} [R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \mathbb{E} \left[ R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') \mid S_t = s, A_t = a \right] \\ &= \sum_{s'} \mathbb{P}(s' \mid s, a) [R(s') + \gamma \max_{a'} q_*(s', a')] \end{aligned}$$

# finding optimal policy from value functions

$$\pi_*(s) = \operatorname{argmax}_a q_*(s, a)$$



## computational issue

If we wanted to solve these equations directly, it would cost a lot of computational power to know exactly the value functions first and then to solve since they are not linear. So how do we do it ?

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Approximation of value function

# solving MDPs using dynamic programming

iterative policy evaluation

update rule :

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a | s) \sum_{s'} T(a, s, s') [R(s') + \gamma v_k(s')]$$

# solving MDPs using dynamic programming

## iterative policy evaluation

update rule :

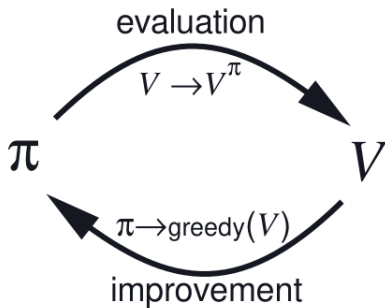
$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a | s) \sum_{s'} T(a, s, s') [R(s') + \gamma v_k(s')]$$

## Policy Improvement

$\pi/\pi'$  : old/new policy.

$$\pi'(s) = \operatorname{argmax}_{a \in \mathcal{A}} q_{\pi}(s, a)$$

# Policy iteration algorithm

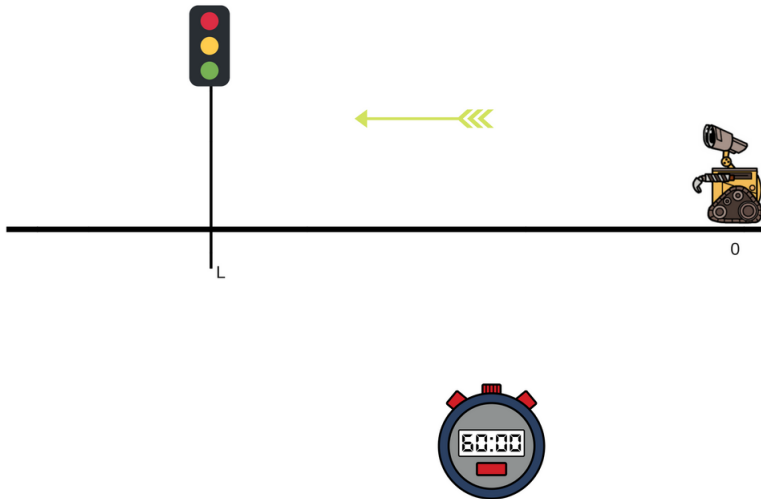


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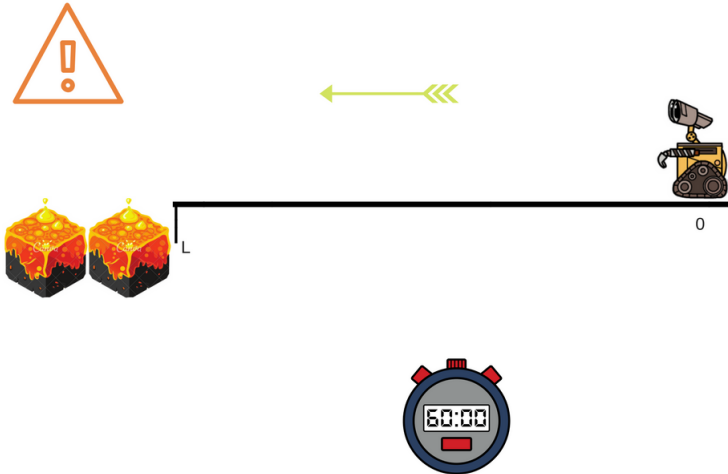
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<sup>1</sup>From (Sutton & Barto, 1998)

# Our problem



# First a simpler problem



States



## States

- position  $\{0,1,2,\dots,L, \text{Lava}\}$

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- decelerating

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- position  $\{0,1,2,\dots,L, \text{Lava}\}$
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## Actions

- decelerating
- maintaining speed

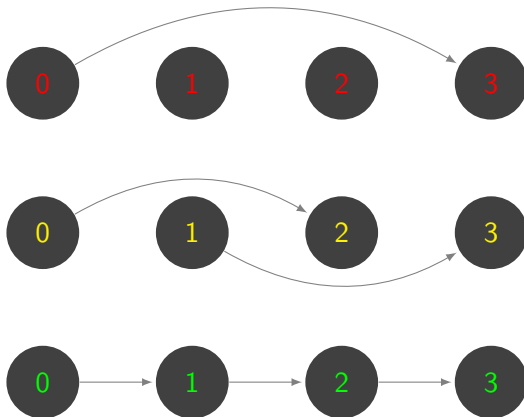
## States

- position  $\{0,1,2,\dots,L, \text{Lava}\}$
- speed  $\{\text{low, medium, high}\}$

## Actions

- decelerating
- maintaining speed
- accelerating

# keeping the same speed graph

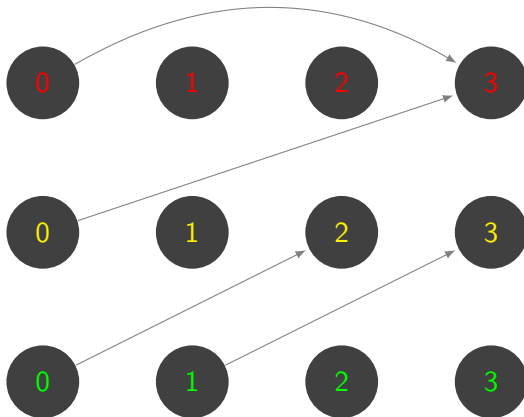


red : high speed

yellow : medium speed

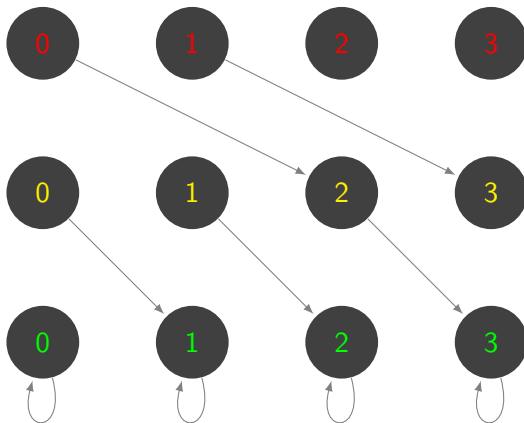
green : low speed

# accelerating graph





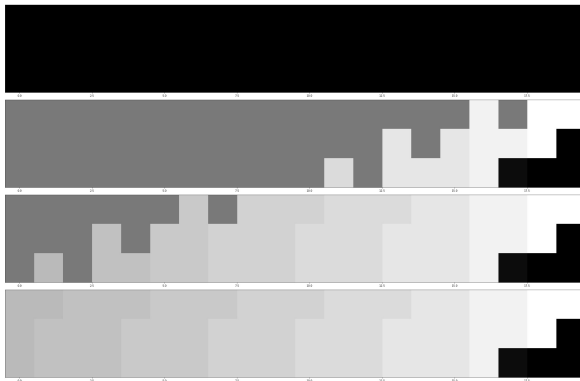
# decelerating



# Results

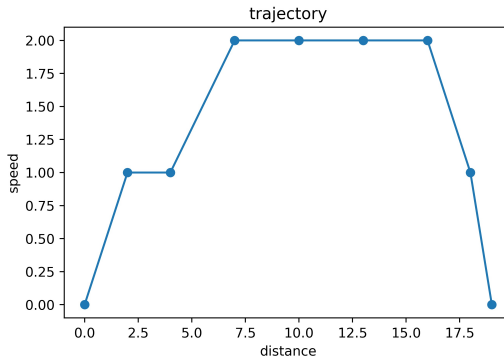
## States value evolution

States values at iterations 0, 2, 4 and 6 (where stable policy is attained)



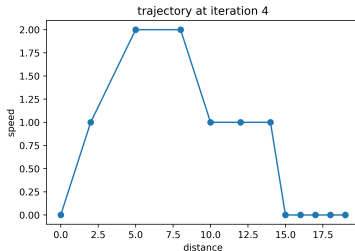
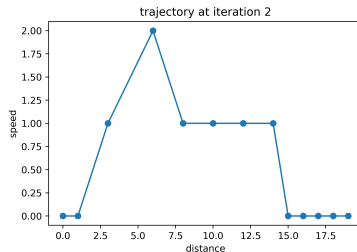
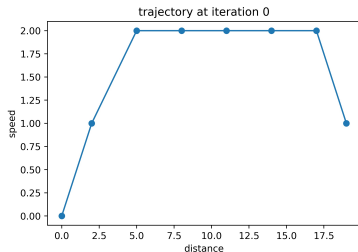
# Results for deterministic actions

One of the optimal trajectories



# Results when uncertainty introduced into brakes

Evolution of the trajectories during iteration 0,2, and 4 (optimal)



# Conclusion, what have we done ?

- We introduced the theoretical framework: Reinforcement learning and Markov Decision Processes
- We solved a simple problem where we know the model

# What's next ?

## already working on

- Add the traffic light into this setting
- Find the distance from the robot's camera to the traffic light

## In a not so distant future

- explore other algorithm and compare them : Monte-Carlo methods, temporal difference learning, Q learning, ...
- Neuro-dynamic programming ?

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Implement everything on the robot ...

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## Ultimately

Implement everything on the robot ... and pray that everything works well