Exercise 2

- Assume $e = 0$
- calculate optimal trajectory x^* and corresponding optimal input sequence u^*

1. $\dot{x} = A_c x + B_c u$

$$x = \begin{bmatrix} \lambda \\ r \\ p \\ \dot{p} \end{bmatrix} \qquad u = p_c$$

$$\ddot{p} = K_1 K_{pp} (p_c - p) - K_1 K_{pd} \dot{p}$$

$$\dot{\lambda} = r$$

$$\dot{r} = -K_2 p$$

$$\Rightarrow \begin{bmatrix} \dot{\lambda} \\ \dot{r} \\ \dot{p} \\ \ddot{p} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -K_2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K_1K_{pp} & K_1K_{pd} \end{bmatrix}}_{A_c} \begin{bmatrix} \lambda \\ r \\ p \\ \dot{p} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1K_{pp} \end{bmatrix}}_{B_c} p_c$$

→ Here we are modelling travel and pitch, in addition to their derivatives. The model also includes terms from the PD pitch controller and the input sequence $u^* = P_c$.

2. Euler method:

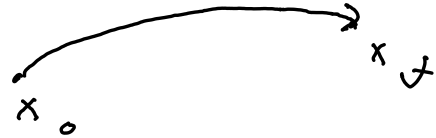
$$\dot{x}[k] \approx \frac{x[k+1] - x[k]}{T}$$

$$\Rightarrow x_{k+1} = I x_k + TAx_k + TBu_k$$

$$\Rightarrow x_{k+1} = (I + TA)x_k + TBu_k$$

$$= \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & -TK_2 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & -TK_1K_{pp} & 1 - TK_1K_{pd} \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 0 \\ TK_1K_{pp} \end{bmatrix} P_c$$

3. $\begin{bmatrix} \lambda_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \lambda_f \\ 0 \\ 0 \\ 0 \end{bmatrix}$



when $e = c$

$\lambda_o = \pi$, $\lambda_f = 0$. $|p_k| \leq \frac{30\pi}{180}$, $k \in \{1, \dots, N\}$

$\wedge \, |p_c| \leq \frac{30\pi}{180}$

cost function:

$$\phi = \sum_{i=1}^{N} (\lambda_i - \lambda_f)^2 + q \, p_{c_i}^2 \qquad q \geq 0$$

Prov

$q = 0.1, 1, 10$

Quadprog:

$$\min_x \frac{1}{2} x^T H x + f^T x \quad \text{s.t.} \begin{cases} Ax \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq bb \end{cases}$$

Constraint:

cost function:

$$\phi = \sum_{i=1}^{N} (\lambda_i - \lambda_f)^2 + q P_{c_i}^2 \quad q \geq 0$$

$$= \sum_{i=1}^{N} \lambda_i^2 - 2\lambda_i\lambda_f + \lambda_f^2 + q P_{c_i}^2, \quad q \geq 0$$

$$\lambda_f = 0$$

$$\Rightarrow \sum_{i=1}^{N} \lambda_i^2 + q P_{c_i}^2$$

Define $z = [\lambda, r, p, \dot{p}, p_c]$

$$\Rightarrow \sum_{i=1}^{N} \lambda_i^2 + q P_{c_i} = \frac{1}{2} [\lambda \; r \; p \; \dot{p} \; p_c] \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & \ddots & & & \\ 0 & & \ddots & & \\ 0 & & & \ddots & \\ 0 & & & & q \end{bmatrix} \begin{bmatrix} \lambda \\ r \\ p \\ \dot{p} \\ p_c \end{bmatrix}$$

$$x_{k+1} = A x_k + b u_k$$

$$b_{eq} = \begin{bmatrix} A x_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$