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Smooth Sailing? A Finite Gaussian Mixture Factor Model of What Makes Safe Haven Currencies

SNB Workshop 2021, The Use of Mixture Models in Central Banking

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About us

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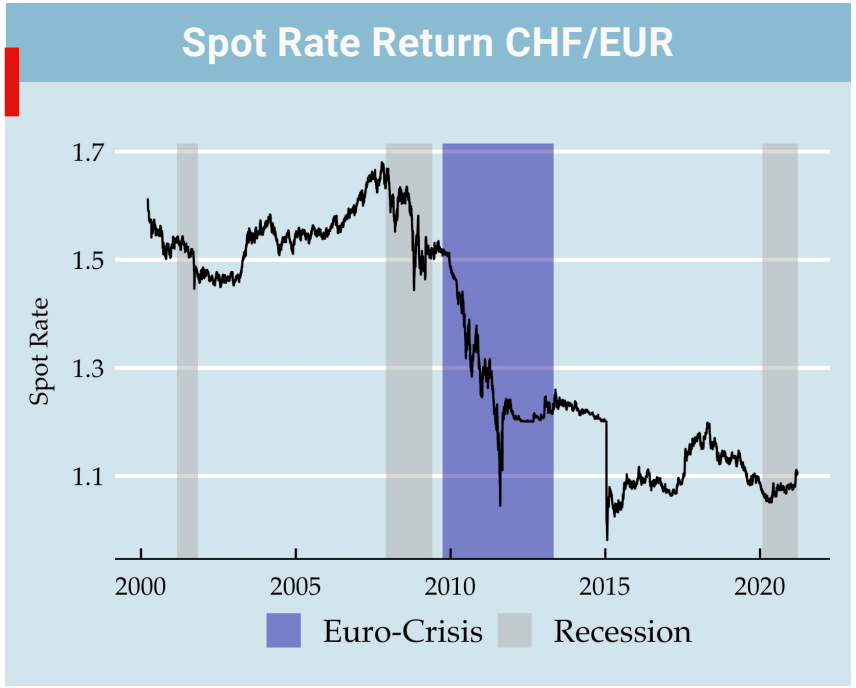
Content

- Safe Haven Currencies – A brief Overview
- How to identify FX regimes
  - Finite Gaussian Mixture Models
  - Data
- Results
  - Does the model work
  - What makes a safe haven currency
  - Are the results stable over time
  - Can we predict switches
- Policy Implications
- Future Research
- Extensions

What are Safe Haven Currencies?

- Low correlation with traditional risk factors and not too sensitive to volatility in markets or liquidity squeezes (Ranaldo & Söderlind, 2010)
  - Provide hedging benefits against a reference asset, both in times of stress and on average

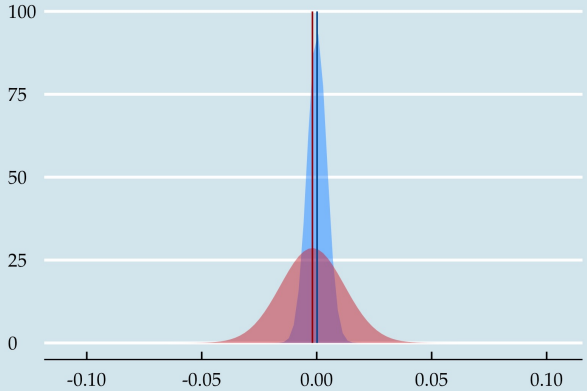
- Kugler & Weder (2002) put forward the hypothesis that Switzerland suffers from a **reverse peso problem**



Finite Gaussian Mixture Models (1)

- Idea – Separate spot rate returns into two distributions:
  - 1st component: ”Crisis"
  - 2nd component: ”Non-Crisis"
- Why? – Identify safe haven periods and detect **non-linearities** in response to risk factors
- How? – EM-Algorithm to numerically get MLE estimates of latent variables (μ, σ)

Two Regimes CHF/NOK



Univariate Mixture Model

$$p(x) = \sum_{i=1}^{K} \phi_i \mathcal{N}(x \mid \mu_i, \sigma_i)$$

$$\mathcal{N}(x \mid \mu_i, \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right)$$

$$\sum_{i=1}^{K} \phi_i = 1$$

Finite Gaussian Mixture Models (2)

- Two-step **expectation-maximizing (EM) algorithm** for a parametric mixture (Dempster, Laird, & Rubin, 1977)
- The one-dimensional model with K = 2 states is given by equations 1-3. p(x) is the a-priori **posterior probability**, the probability that an observed x is generated by component K, and ϕ are the component weights constrained to 1 for the probability distribution to not exceed 1. x are the spot returns, μ and σ the mean and standard deviation of spot returns, respectively.
- In the E step, we estimate the expected values of the **latent variables**, i.e., the probability weights by means of Bayes' theorem.
- Using γ in the M-step, E- and M-steps are then iteratively changed and repeated until the difference for parameters θ_t at iteration t | $\theta_t - \theta_{t-1}$ |≤ ε is arbitrarily small and does not change anymore.

$$p(x) = \sum_{i=1}^{K} \phi_i \mathcal{N}(x \mid \mu_i, \sigma_i)$$

$$\mathcal{N}(x \mid \mu_i, \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right)$$

$$\sum_{i=1}^{K} \phi_i = 1$$

$$\hat{\gamma}_{ik} = \frac{\hat{\phi}_k \mathcal{N}(x_i \mid \hat{\mu}_k, \hat{\sigma}_k)}{\sum_{j=1}^{K} \hat{\phi}_j \mathcal{N}(x_i \mid \hat{\mu}_j, \hat{\sigma}_j)}$$

$$\hat{\phi}_k = \sum_{i=1}^{N} \frac{\hat{\gamma}_{ik}}{N}$$

$$\hat{\mu}_k = \frac{\sum_{i=1}^{N} \hat{\gamma}_{ik} x_i}{\sum_{i=1}^{N} \hat{\gamma}_{ik}}$$

$$\hat{\sigma}_k^2 = \frac{\sum_{i=1}^{N} \hat{\gamma}_{ik} (x_i - \hat{\mu}_k)^2}{\sum_{i=1}^{N} \hat{\gamma}_{ik}}$$

Finite Gaussian Mixture Models (3)

- The Gaussian Mixture approach does have **downsides**:
  - **Ex-post** analysis in our case – we do not predict regime switches (yet)
  - **Univariate setting** – the model splits states based on the dependent variable
    - Multivariate setting possible but computationally intensive (**bootstrapping**)
  - Gaussian distribution – does not accurately capture the distribution of FX returns given the **high kurtosis**

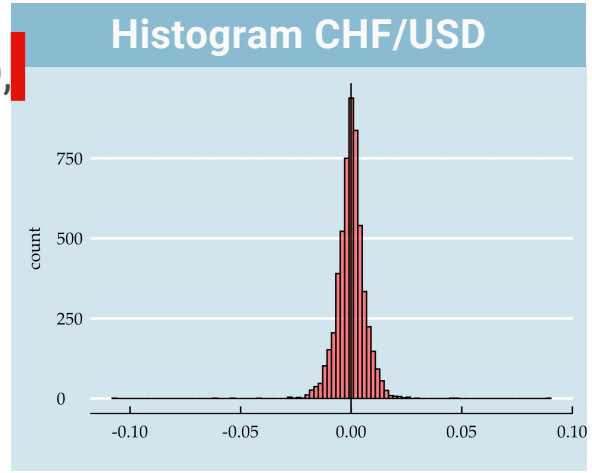
Alternatives to Gaussian Mixture models possible: Markov Switching Models, Machine Learning ….

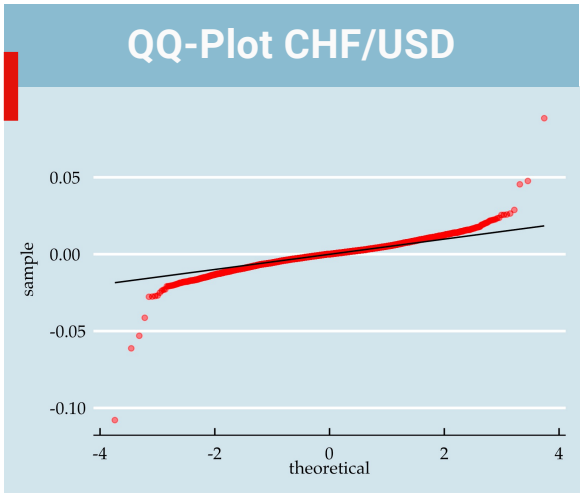
Data

- Currency Pairs: Focus on "safer" currencies such as CHF, EUR, USD, GBP & JPY

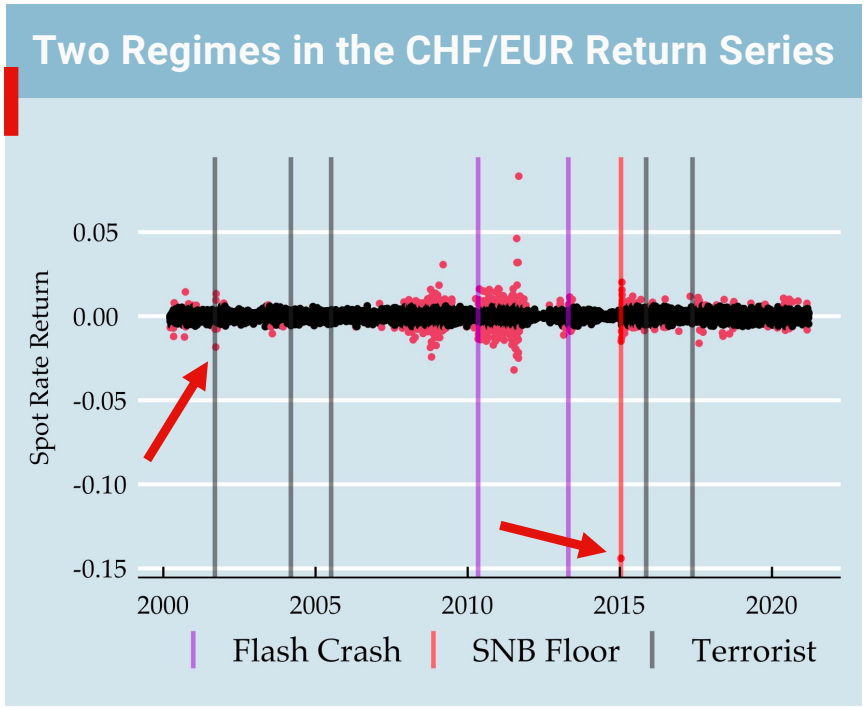
- Broad range of risk factors:
  - Equity Markets: MSCI World
  - Market Volatility: VIX & VSTOXX
  - Investor Sentiment: CBOE Put-Call Ratio
  - Credit Risk: TED-Spread
  - FX Volatility: JPM Global FX Vola Index

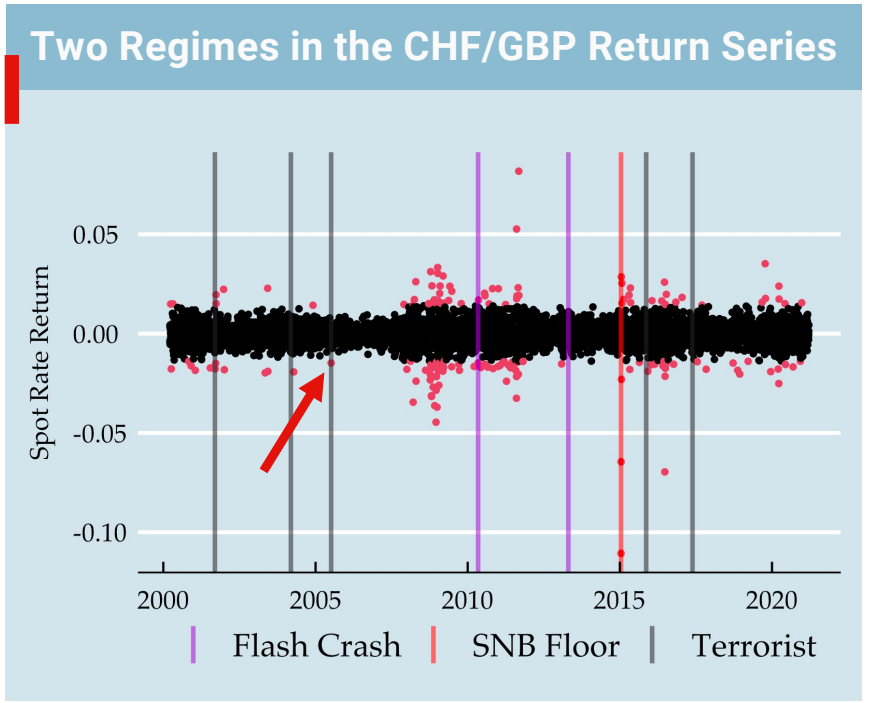
- Take simple returns of spot rates and risk factors





Does the Mixture detect FX regimes?





Results Crisis Regime

- Magnitude of coefficients larger than non-crisis
- Small sample size problematic
- ϕ_1 and σ_1 larger than non-crisis

Table 2: Results Gaussian Mixture and Regression Crisis Component

	Dependent variable:								
	CHF/EUR	CHF/GBP	CHF/USD	CHF/JPY	CHF/BRL	CHF/INR	CHF/NOK	JPY/USD	BRL/USD
ϕ_1	0.094	0.036	0.065	0.105	0.126	0.036	0.149	0.206	0.204
σ_1	0.011	0.019	0.017	0.015	0.014	0.020	0.023	0.010	0.019
MSCI	0.129*	0.183	0.066	−0.358***	0.162	0.210	0.259***	0.015	−0.348***
	(0.068)	(0.112)	(0.333)	(0.123)	(0.116)	(0.263)	(0.067)	(0.051)	(0.081)
Put-Call Ratio	−0.004	−0.012	−0.013	0.004	−0.018**	−0.005	−0.006	−0.012***	0.011**
	(0.005)	(0.009)	(0.016)	(0.009)	(0.008)	(0.021)	(0.006)	(0.003)	(0.006)
MOVE 3-months	0.018	0.064	0.057	−0.007	0.088**	−0.069	0.011	0.046***	−0.024
	(0.021)	(0.046)	(0.090)	(0.036)	(0.043)	(0.095)	(0.022)	(0.014)	(0.026)
VIX	0.013	0.040**	0.005	−0.038*	0.029	0.019	0.026**	0.009	−0.042***
	(0.011)	(0.020)	(0.047)	(0.021)	(0.021)	(0.045)	(0.012)	(0.008)	(0.014)
VSTOXX	−0.018	−0.033	−0.020	0.031	−0.094***	−0.061	−0.042***	−0.041***	0.051***
	(0.012)	(0.025)	(0.054)	(0.021)	(0.020)	(0.054)	(0.012)	(0.008)	(0.014)
TED Spread	0.019*	0.025	0.018	0.008	0.003	0.045	0.015	−0.006	−0.001
	(0.011)	(0.018)	(0.030)	(0.015)	(0.011)	(0.035)	(0.010)	(0.005)	(0.007)
Gold	−0.101**	−0.244***	−0.748***	−0.183**	−0.241***	−0.720***	0.025	−0.352***	−0.180***
	(0.051)	(0.086)	(0.170)	(0.080)	(0.087)	(0.188)	(0.056)	(0.033)	(0.055)
Global FX Vola	−0.124***	−0.199***	−0.147*	0.047	−0.192***	−0.197**	−0.047*	−0.105***	0.139***
	(0.024)	(0.046)	(0.084)	(0.037)	(0.041)	(0.089)	(0.024)	(0.017)	(0.030)
10-year Breakeven Inflation	−0.003	0.009	0.008	−0.009	0.004	−0.016	0.003	0.007	−0.007**
	(0.010)	(0.011)	(0.014)	(0.013)	(0.004)	(0.022)	(0.002)	(0.005)	(0.003)
US 10-year HY Index	0.005	−0.019	0.143	−0.013	−0.110*	0.203	−0.065*	−0.015	0.075*
	(0.034)	(0.064)	(0.160)	(0.061)	(0.057)	(0.143)	(0.033)	(0.024)	(0.041)
Bid-Ask Spread	−0.978	−2.712	−5.672*	0.901	0.673	−0.038	−0.035		
	(0.752)	(1.943)	(3.145)	(1.141)	(0.446)	(0.084)	(0.031)		
Constant	0.001	0.001	0.005	−0.002	−0.0005	0.002	−0.001	0.0003	0.001
	(0.001)	(0.002)	(0.004)	(0.002)	(0.002)	(0.006)	(0.001)	(0.001)	(0.001)
Observations	326	165	71	246	374	67	341	479	573
R^2	0.241	0.391	0.449	0.213	0.362	0.443	0.355	0.444	0.330
Adjusted R^2	0.214	0.348	0.346	0.176	0.342	0.331	0.333	0.432	0.318
Residual Std. Error	0.013	0.019	0.023	0.020	0.026	0.025	0.015	0.011	0.021
F Statistic	9.047***	8.942***	4.367***	5.751***	18.658***	3.969***	16.449***	37.352***	27.644***

Notes: This table shows the results for Gaussian Mixture model and regression for the crisis component. The data covers the period of 20-03-2000 until 18-03-2021. The figures are in percentage. Statistical significance is given by *p<0.1; **p<0.05; ***p<0.01. ϕ_1 indicates the mixture proportion for the crisis component while σ_1 indicates the daily volatility of the spot rate returns. Spot rates are reported as CHF/EUR, i.e., Swiss franc per Euro.

Results Non-Crisis Regime

- The franc appreciates with increases in various risk factors
- Results are more pronounced for the VSTOXX than the VIX
- Yen is a safer currency when risks are idiosyncratic to the FX market

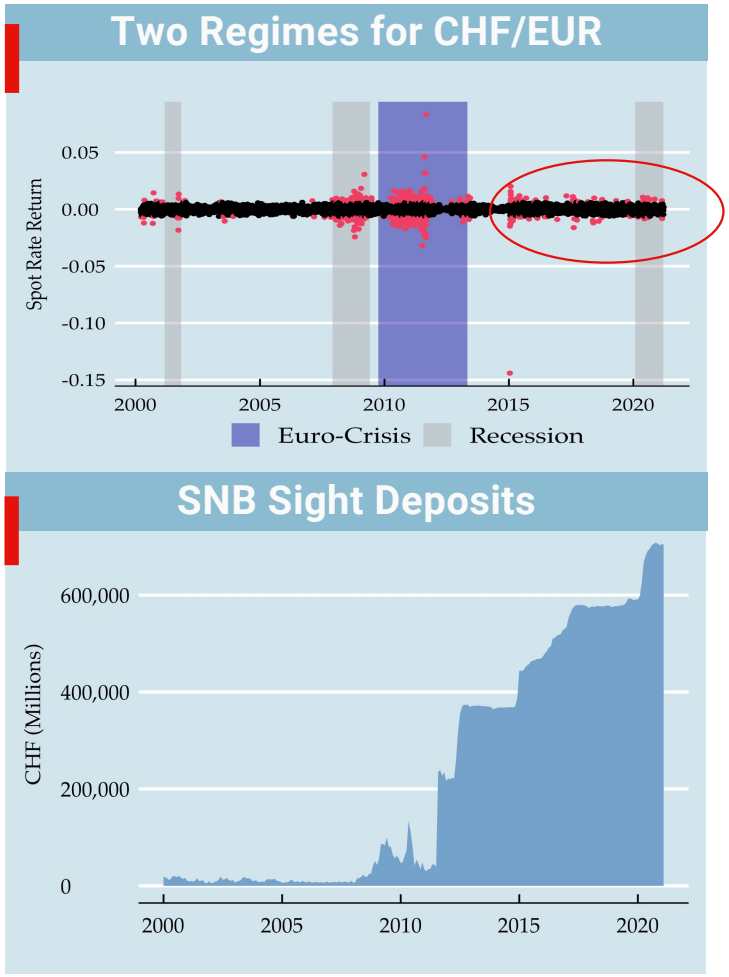
Table 3: Results Gaussian Mixture and Regression Non-Crisis Component

	Dependent variable:								
	CHF/EUR	CHF/GBP	CHF/USD	CHF/JPY	CHF/BRL	CHF/INR	CHF/NOK	JPY/USD	BRL/USD
ϕ_2	0.906	0.964	0.935	0.895	0.874	0.964	0.851	0.794	0.796
σ_2	0.002	0.006	0.005	0.005	0.004	0.006	0.008	0.004	0.006
MSCI	0.023***	0.011	−0.149***	−0.065***	0.027	−0.096***	0.069***	−0.058***	−0.180***
	(0.005)	(0.010)	(0.010)	(0.011)	(0.018)	(0.012)	(0.009)	(0.009)	(0.015)
Put-Call Ratio	0.0001	−0.001*	−0.002***	−0.001**	−0.003***	−0.001***	−0.001	−0.001***	0.0002
	(0.0002)	(0.0004)	(0.0004)	(0.0004)	(0.001)	(0.0005)	(0.0003)	(0.0004)	(0.001)
MOVE 3-months	0.001	0.003	0.002	−0.013***	0.0005	0.007**	0.001	0.012***	0.003
	(0.001)	(0.002)	(0.003)	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)	(0.003)
VIX	0.002***	0.0003	−0.008***	−0.003**	0.003	−0.004***	0.005***	−0.004***	−0.008***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)
VSTOXX	−0.003***	−0.009***	−0.014***	−0.0002	−0.026***	−0.018***	−0.009***	−0.012***	0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)
TED Spread	−0.001**	−0.001	−0.001	−0.0002	−0.003**	−0.001	−0.001	−0.001	0.0004
	(0.0003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Gold	−0.006*	−0.061***	−0.195***	−0.044***	−0.068***	−0.147***	0.003	−0.092***	−0.072***
	(0.003)	(0.006)	(0.007)	(0.007)	(0.011)	(0.008)	(0.006)	(0.006)	(0.009)
Global FX Vola	−0.012***	−0.019***	−0.001	0.039***	−0.044***	−0.023***	−0.033***	−0.023***	0.032***
	(0.002)	(0.003)	(0.004)	(0.004)	(0.006)	(0.004)	(0.003)	(0.003)	(0.005)
10-year Breakeven Inflation	−0.001***	−0.001	−0.002***	−0.001*	0.004	−0.001	0.002	−0.0003	0.001
	(0.0003)	(0.001)	(0.001)	(0.001)	(0.004)	(0.001)	(0.002)	(0.001)	(0.003)
US 10-year HY Index	−0.004**	−0.016***	−0.021***	0.017***	−0.028***	−0.040***	−0.026***	−0.031***	0.008
	(0.002)	(0.004)	(0.004)	(0.004)	(0.007)	(0.005)	(0.004)	(0.003)	(0.005)
Bid-Ask Spread	−0.027	−0.084	−0.096	0.041	−0.004	−0.00004	−0.002		
	(0.047)	(0.082)	(0.085)	(0.105)	(0.007)	(0.001)	(0.004)		
Constant	−0.00002	0.0001	0.0002**	−0.0001	−0.0001	−0.00001	0.0001*	0.0003***	0.00004
	(0.00004)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Observations	5,153	5,314	5,408	5,233	5,105	5,412	5,138	5,000	4,906
R^2	0.054	0.066	0.195	0.064	0.117	0.141	0.130	0.131	0.132
Adjusted R^2	0.052	0.064	0.193	0.062	0.115	0.139	0.128	0.129	0.130
Residual Std. Error	0.002	0.005	0.005	0.005	0.007	0.006	0.004	0.004	0.006
F Statistic	26.484***	34.141***	118.857***	32.676***	61.456***	80.251***	69.790***	74.999***	74.400***

Notes: This table shows the results for Gaussian Mixture model and regression for the non-crisis component. The data covers the period of 20-03-2000 until 18-03-2021. The figures are in percentage. Statistical significance is given by *p<0.1; **p<0.05; ***p<0.01. ϕ_2 indicates the mixture proportion for the non-crisis component while σ_2 indicates the daily volatility of the spot rate returns. Spot rates are reported as CHF/EUR, i.e., Swiss franc per Euro.

Are the Results Stable Over Time?

- Less extreme swings but consistently higher level of volatility post-SNB CHF/EUR floor removal in early 2015
- Safe haven status of CHF less clear due to additional noise in the data
  - Which appreciation as a result of safe haven status?
- Coincides with massive increase in FX interventions by the SNB post-2015



Regime Switch Detection – Thresholds

- Idea – Calculate threshold values to see if risk factors are distinct between the two regimes and which values separate the two
- Threshold values have some power as **early warning indicators**
- Proportion of deviations higher in crisis period vs non-crisis period

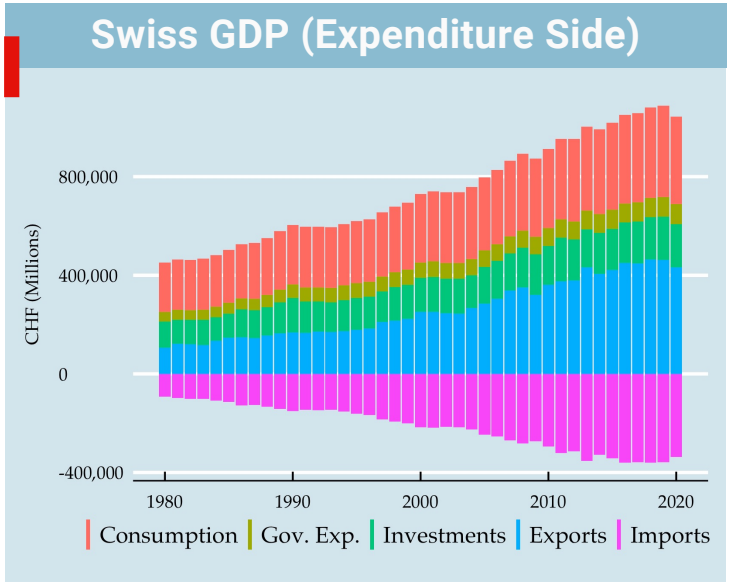
Threshold Values JPM Global FX Volatility (CHF/EUR)

Threshold Value Formula

$$x_i = \begin{cases} \varphi, & \text{if } \gamma > 0.5 \\ x_i, & \text{otherwise} \end{cases}$$

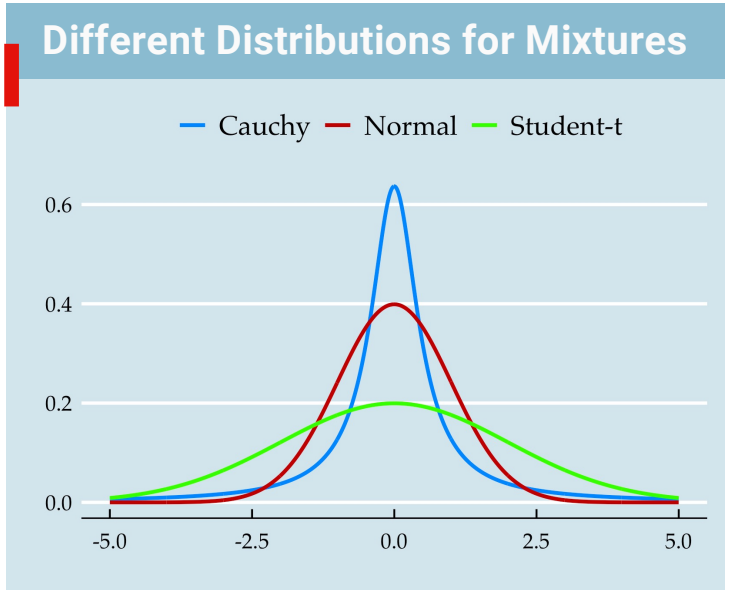
Policy Implications

- Switzerland's **high export dependency** coupled with safe haven status is problematic for monetary policy
- Monitor developments in **gold market** and European risk factors such as VSTOXX
- Safe haven status not problematic vis-à-vis all currencies (e.g. BRL)
- **Changing role** of CHF safe haven status post-2015 may alleviate reverse peso problem
- Monitor capital flows generated by switch into and out of CHF



Future Research

- Use different distributions in mixture model
- Extend results to include weekly/monthly variables with Reverse MIDAS
- Fine-tune threshold value selection with ML
- Add lagged variables in probabilistic ML model to narrow down predictability



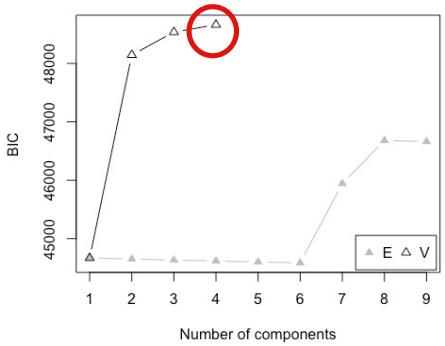
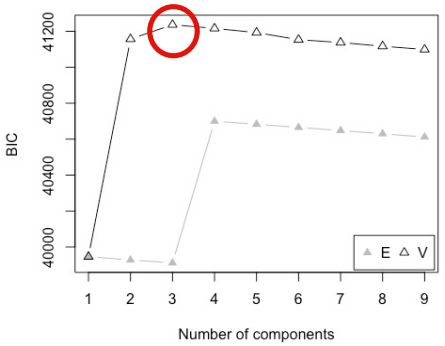
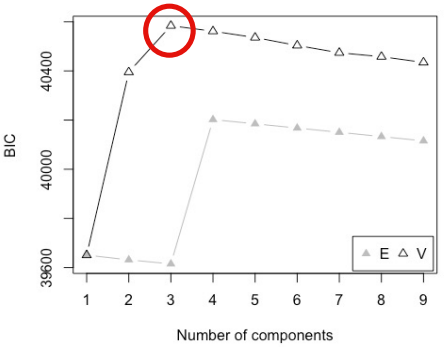
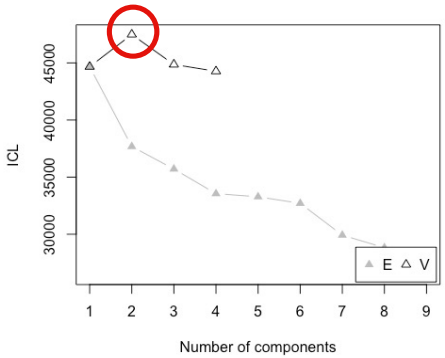
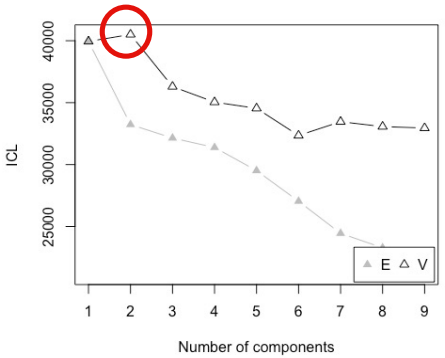
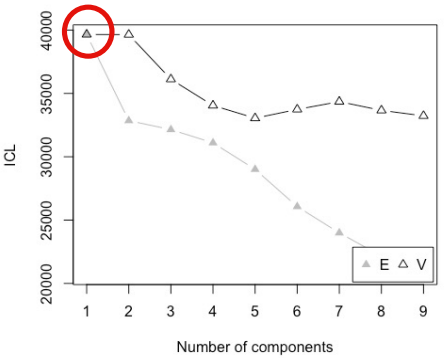
Extension – Model Selection (1)

- Select number of components in mixture model with **information criterion:**
  - Bayesian Information Criterion **(BIC)**

    $-2 * \log(L) + \mathrm{k} * \ln(n)$
  - Integrated Complete Likelihood Criterion **(ICL)**

    $BIC(K) - Mean\ Entropy$
  - Mean entropy = $\sum_{g=1}^{K}\sum_{i=1}^{n} p(z_i|x,\hat{\theta_K},K) log p(z_i|x,\hat{\theta_K},K)$
- Issues with BIC:
  - While consistent under **regularity** conditions, not focused on clustering goal
  - Tendency to over select number of components (see Biernacki et al., 2000)
- Integrated Complete Likelihood Criterion:
  - Takes clustering into account via **mean entropy**
  - Solutions with low entropy preferred (well-separated groups)

Extension – Model Selection (2)

Information Criterion	CHF/EUR	CHF/GBP	CHF/USD
BIC			
ICL			

Red circle indicates the preferred model under the respective information criterion

Extension – Predictions

- Idea based on Hauptmann et al. (2014)
  - Use probabilities derived from mixture model to compute one-step ahead predictions for safe haven regime status with **logistic regression:**

$$y_{t+1}^{j} = ln\left(\frac{p_{t+1}^{j}}{1-p_{t+1}^{j}}\right) = \beta^{j} x_{t}^{j} + \epsilon_{t}$$

- Separate model for crisis and non-crisis period
- Can capture time-dependency of error term via **ARMA(p,q) process**
- Verify economic significance with dynamic portfolio strategies
- **Alternatives** to logistic regression: support vector machines, tree-based approaches, neural networks, k-nearest neighbours

Q&A

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