Faster Computations with Generative Expressions

Jonas Östlund

February 23, 2019

This talk

- Background and motivation
- Do more during code generation, less at runtime
- How it works and how to use it: Numerical optimization example
- Results

About Jonas

- Software engineer at MindMaze in Lausanne
- Co-founder of Anemomind
- Research in computer vision
- This project in my spare time
- Github: `https://github.com/jonasseglare`
- Blog: `http://ostlund.rocks/`

Squaring a number in Geex

Example: $5^2 = 5 \cdot 5 = 25$
Using Generative Expressions (Geex)

```
(require '[geex.java :as java]
         '[geex.common :as c])

(java/typed-defn geex-square [Double/TYPE x]
   (c/* x x))
```

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```
(geex-square 5)
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```

```
(geex-square 5)
```

```
25.0
```

Println debugging

```
(java/typed-defn geex-square [Double/TYPE x]
  (println "Input:" x)
  (let [output (c/* x x)]
    (println "Output:" output)
    output))
```

Println debugging

```
(java/typed-defn geex-square [Double/TYPE x]
  (println "Input:" x)
  (let [output (c/* x x)]
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    output))
```

```
Input: #object[geex.DynamicSeed 0x22351330 ISeed(type=double, id=5 ...
Output: #object[geex.DynamicSeed 0x2158d1d1 ISeed(type=double, id= ...
```

Println debugging

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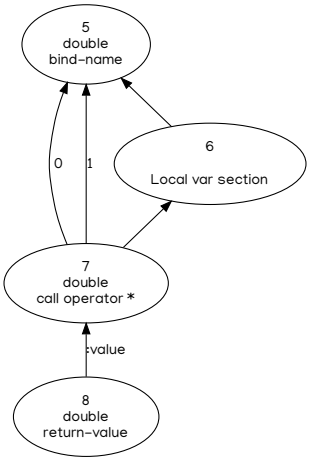
    output))
```

```
Input: #object[geex.DynamicSeed 0x22351330 ISeed(type=double, id=5 ...
Output: #object[geex.DynamicSeed 0x2158d1d1 ISeed(type=double, id= ...
```

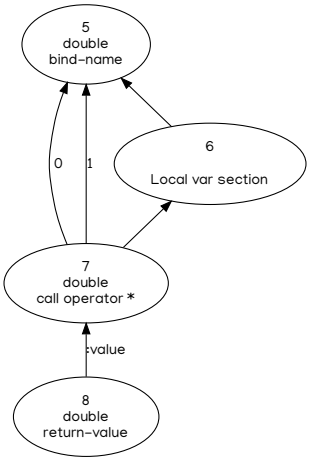
```
#'cljd.square/geex-square
```

Function body executed when loading the code

Computational graph



Computational graph



Seeds

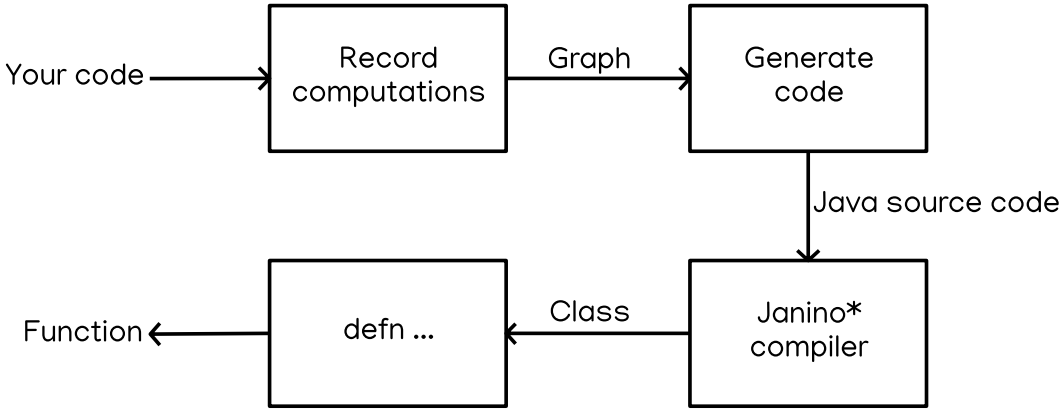
- Vertices in the graph
- Replace unknown runtime values
- Generate code
- Contain type information

Generated code

```
package cljd_psquare;

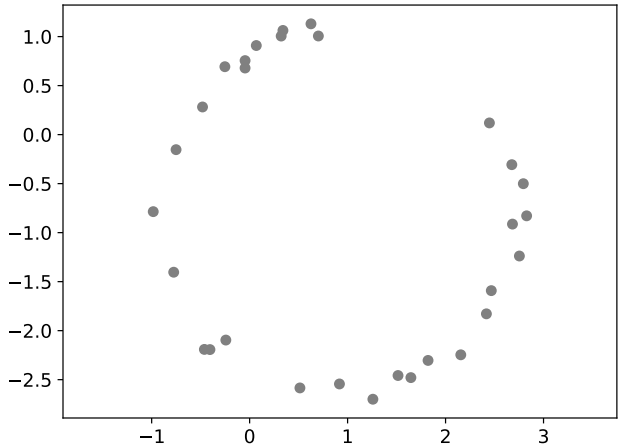
public class TypedDefn__geex_dsquare {
  /* Various definitions */
  public double apply(final double arg00) {
    return (arg00 * arg00);
  }
}
```

Code generation pipeline

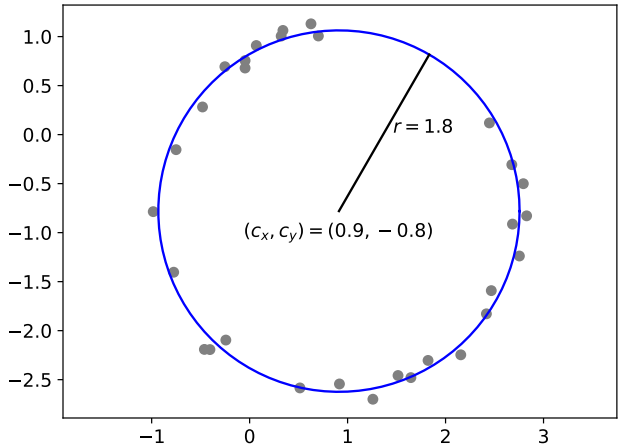


* https://janino-compiler.github.io/janino/

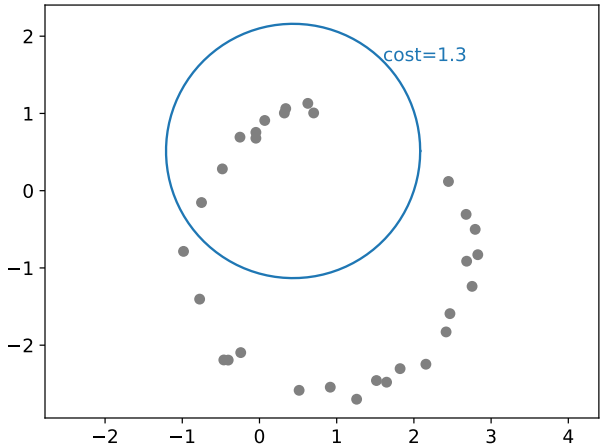
Circle fitting



Circle fitting



Optimization problem



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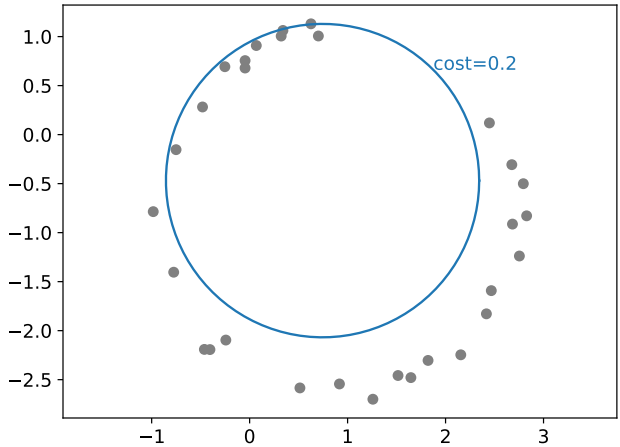

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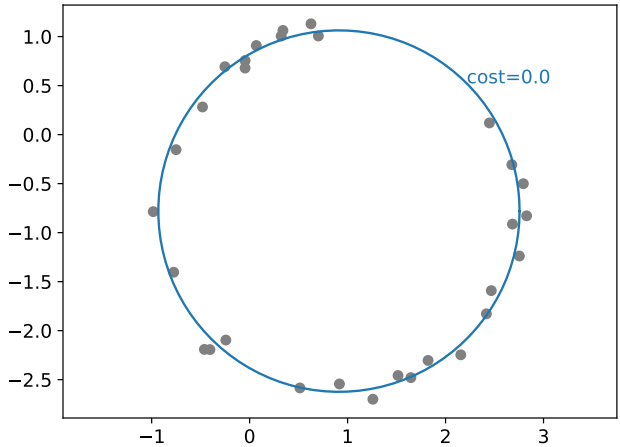
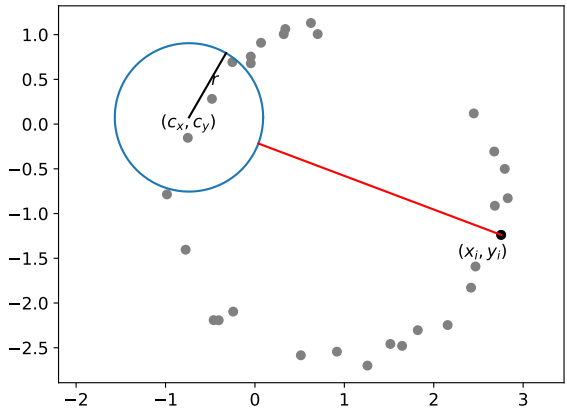


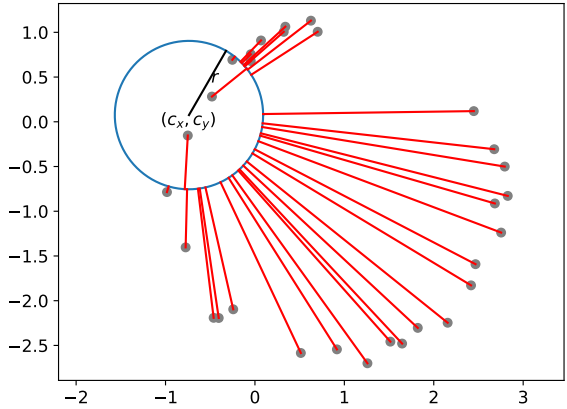
Cost of a single point

$$f_i(c_x, c_y, r) = \left(\underbrace{\sqrt{(c_x - x_i)^2 + (c_y - y_i)^2} - r}_{\text{distance from point to circle}} \right)^2$$



Cost of all points

$$f(c_x, c_y, r) = \frac{1}{N} \sum_{i=1}^{N} \left(\sqrt{(c_x - x_i)^2 + (c_y - y_i)^2} - r \right)^2$$



Cost of a single point

$$\underbrace{f_i(c_x, c_y, r)}_{\texttt{evaluate-point}} = \left(\sqrt{(c_x - x_i)^2 + (c_y - y_i)^2} - r\right)^2$$

```
(require '[geex.common :as c])

(defn sqr [x] (c/* x x))

(defn evaluate-point [{:keys [cx cy r]} ;; ← Circle parameters
                      [x y]] ;; ← The point
  (let [dist-to-centre (c/sqrt (c/+ (sqr (c/- x cx))
                                    (sqr (c/- y cy))))
        dist-to-circle (c/- dist-to-centre r)]
    (sqr dist-to-circle)))
```

Objective function

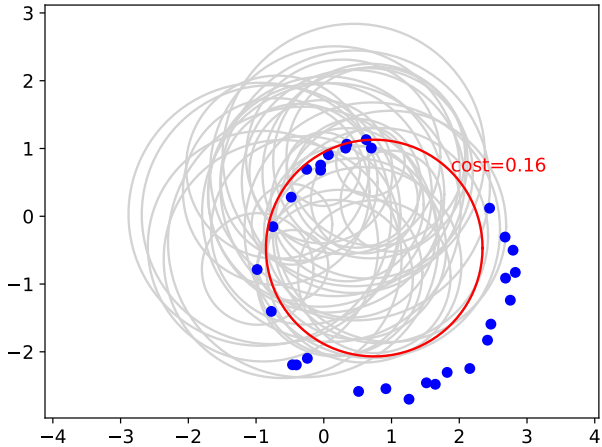
$$f(c_x, c_y, r) = \frac{1}{N} \sum_{i=1}^{N} \left(\sqrt{(c_x - x_i)^2 + (c_y - y_i)^2} - r \right)^2$$

```
(defn circle-fitness-cost [circle-params point-array init-cost]
  (let [n (c/quot (c/cast Long/TYPE (c/count point-array)) 2)]
    (c/* (c// 1.0 n)
         (c/transduce
          (c/map (comp (partial evaluate-point circle-params)
                       (partial get-point-2d point-array)))
          c/+
          init-cost ;; ← Typically 0
          (c/range n)))))
```

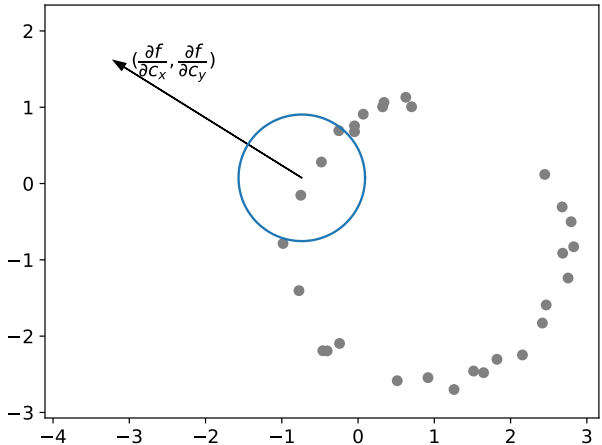
Generated objective function

```
// ... 26 lines of code not shown
    while (true) {
      final double s0041 = lvar0;
      final long s0042 = lvar1;
      final long s0043 = lvar2;
      final long s0044 = lvar3;
      if ((s0043 <= 0)) {
        lvar4 = s0041;
      } else {
        final long s0055 = (2 * s0042);
        final double s0059 = (arg01[((int) (s0055 + 0))]);
        final double s0063 = (arg01[((int) (s0055 + 1))]);
        final double s0064 = (s0059 - s0010);
        final double s0066 = (s0063 - s0015);
        final double s0070 = ((java.lang.Math.sqrt(((s0064 * s0064) + (s0066 * s0066)))) - s0020);
        lvar0 = (s0041 + (s0070 * s0070));
        lvar1 = (s0042 + s0044);
        lvar2 = (s0043 - 1);
        lvar3 = s0044;
        continue;
      }
      final double s0085 = lvar4;
      lvar5 = s0085;
      break;
    }
    final double s0091 = lvar5;
    return ((1.0 / s0026) * s0091);
// ... 2 lines of code not shown
```

Minimizing the cost (naïve approach)



Gradient descent



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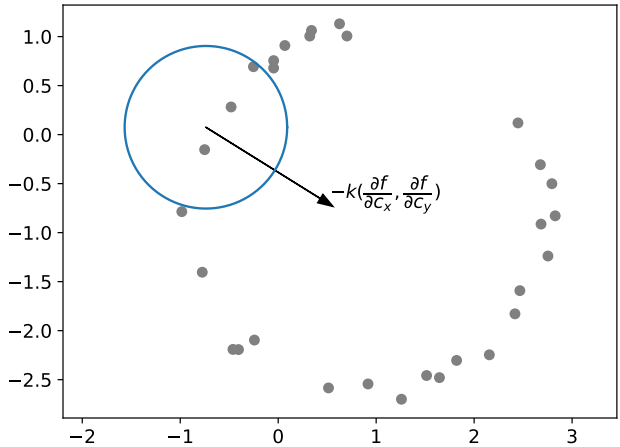
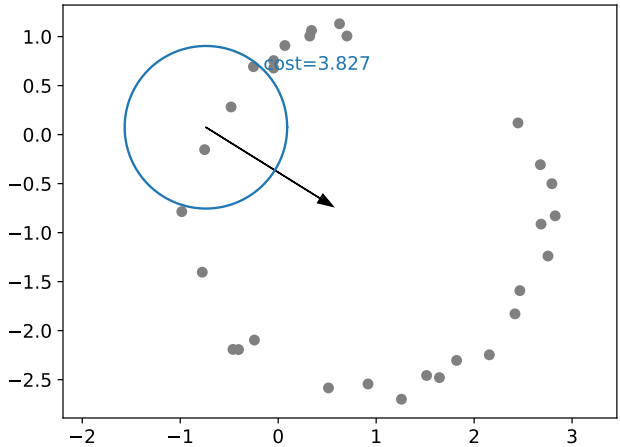


Gradient descent



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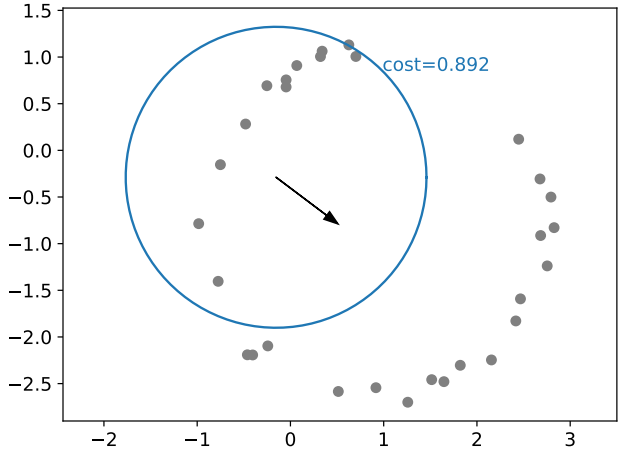
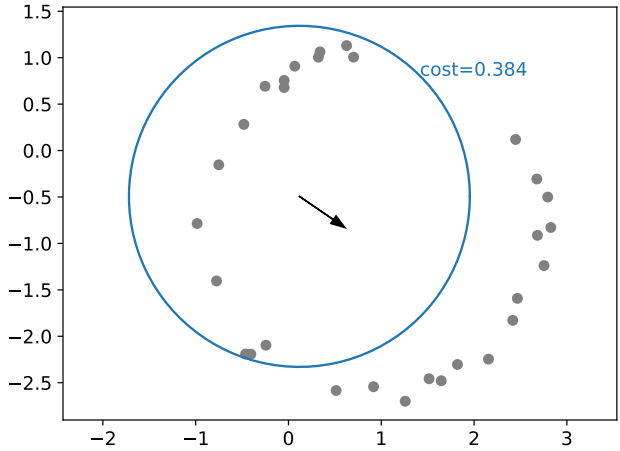


Gradient descent



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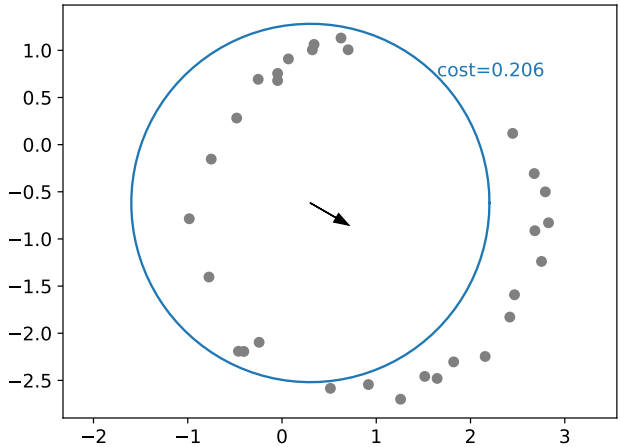

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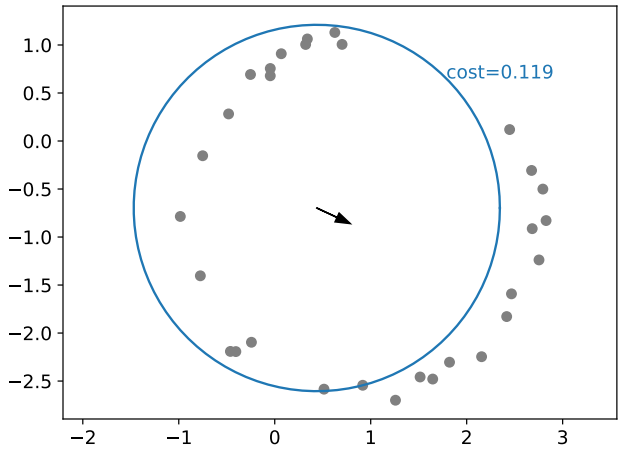

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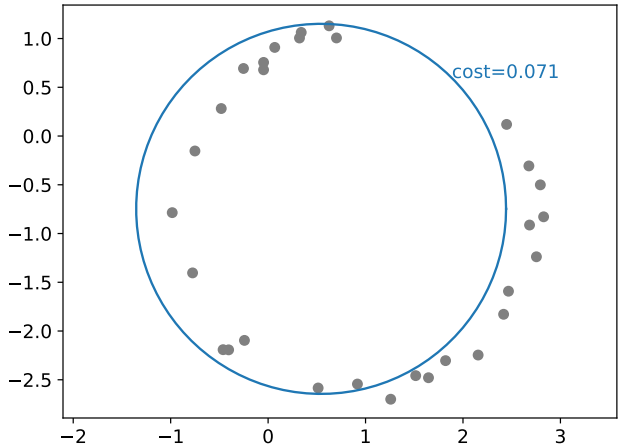

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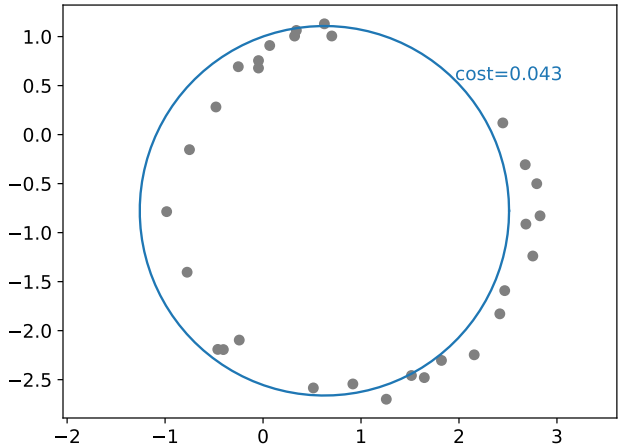
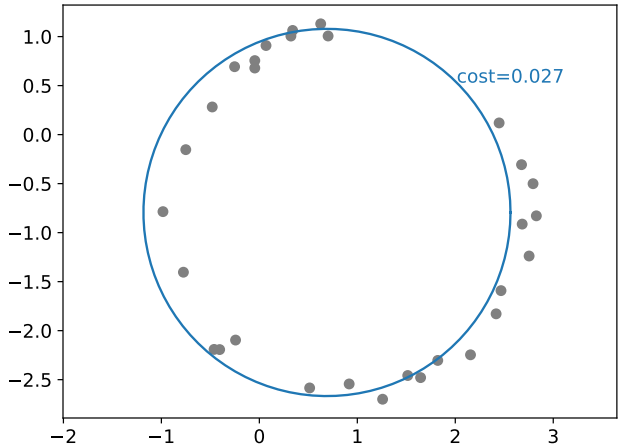
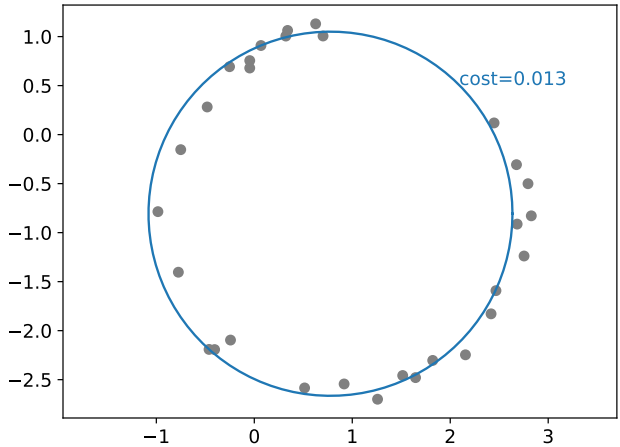


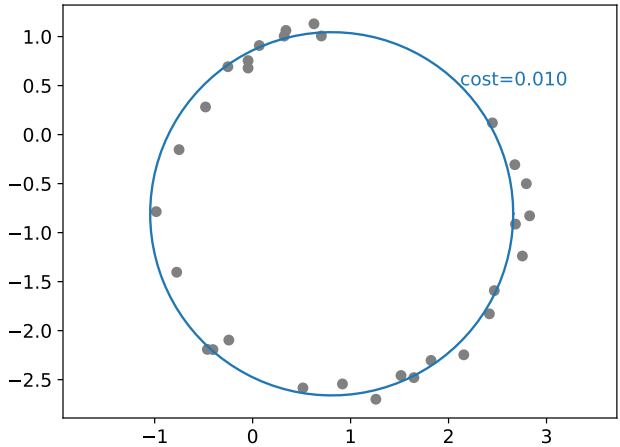
Gradient descent



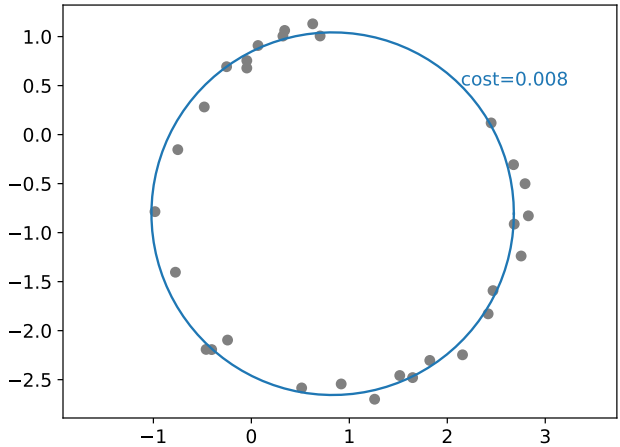
Gradient descent



Gradient descent



Gradient descent



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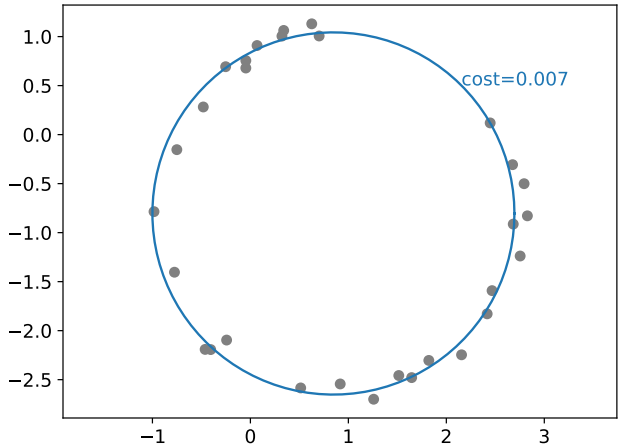

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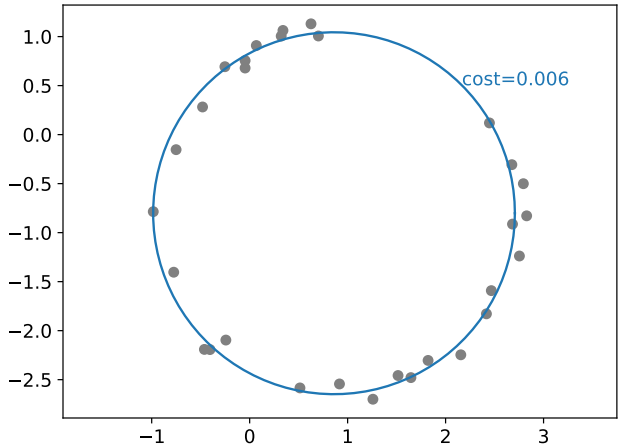

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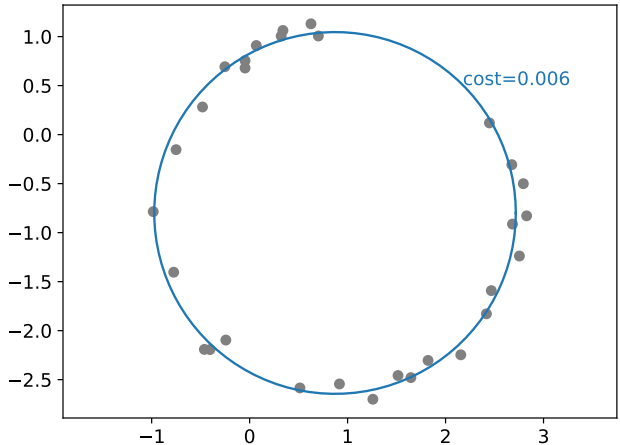
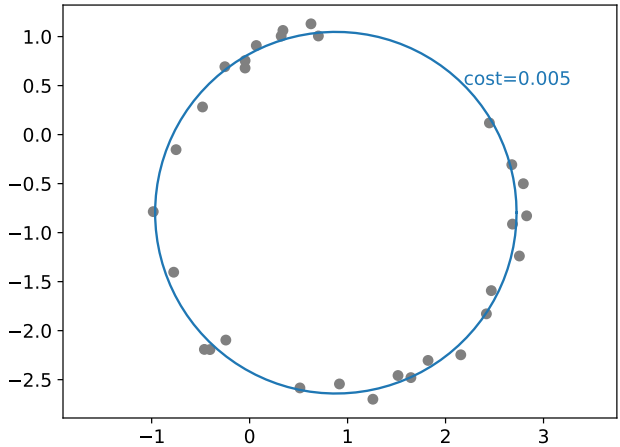
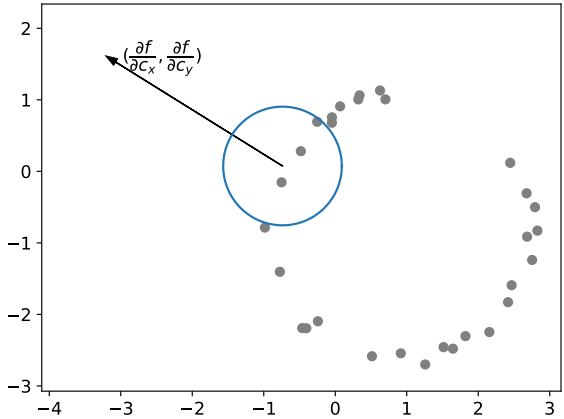


Gradient descent (good enough)



Computing the gradient

Differentiate the cost with respect to circle centre and radius: $\frac{\partial f}{\partial c_x}$, $\frac{\partial f}{\partial c_y}$ and $\frac{\partial f}{\partial r}$



A new numeric “type”: AD numbers

```clojure
(defn variable [x]
  {:value x
   :deriv 1.0}) ;; dx/dx = 1
```

```clojure
(defn constant [c]
  {:value c
   :deriv 0.0}) ;; dc/dx = 0, c being a constant
```

AD stands for a “automatic differentiation”.

Example-based multiple dispatch

Register our type with name `::ad`

```
(require '[bluebell.utils.ebmd :as ebmd])

(defn ad-number? [x]
  (and (map? x) (contains? x :value) (contains? x :deriv)))

(ebmd/def-arg-spec
  ::ad ;; ← Name of the spec
  {:pred ad-number? ;; ← Predicate function

   ;; Examples disambiguate overlapping predicates:
   :pos [(variable 3.0) (constant 5.0)] ;; ← Matching examples
   :neg [2.0 :kwd {:kattskit 119}]}) ;; ← Non-matching examples
```

Extending multiplication for automatic differentiation

```
(ebmd/def-poly c/binary-mul [::ad a
                             ::ad b]
  {:value (c/* (:value a) (:value b))

   ;; Recall that (a · b)' = a' · b + a · b'
   :deriv (c/+ (c/* (:value a)
                    (:deriv b))
               (c/* (:deriv a)
                    (:value b)))})
```

The product of two AD numbers is a new AD number.

Derivative of a cube

$g(x) = x^3$ e.g. $g(9) = 729$

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```
(let [x (variable 9.0)]
  (c/* x x x))
```

Derivative of a cube

$g(x) = x^3$ e.g. $g(9) = 729$

$g'(x) = 3x^2$ e.g. $g'(9) = 243$

```
(let [x (variable 9.0)]
  (c/* x x x))
```

```
{:value 729.0, :deriv 243.0}
```

Gradient code generation

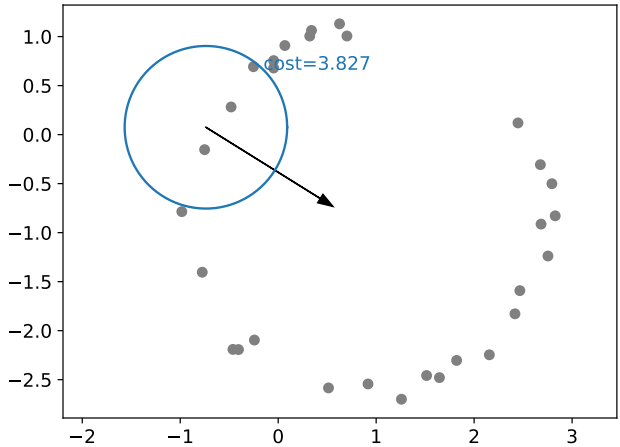
```
(defn derivative-for-key [circle-params points k]
  (:deriv (circle-fitness-cost ;; ← Derivative of objective function
           (update circle-params k variable) ;; ← 'k' is a variable
           points
           (constant 0.0))))

(java/typed-defn gradient
   [{:cx Double/TYPE :cy Double/TYPE :r Double/TYPE} circle-params
    (c/array-class Double/TYPE) points]
   {:cx (derivative-for-key circle-params points :cx) ;; ← df/dc_x
    :cy (derivative-for-key circle-params points :cy) ;; ← df/dc_y
    :r (derivative-for-key circle-params points :r)}) ;; ← df/dr
```

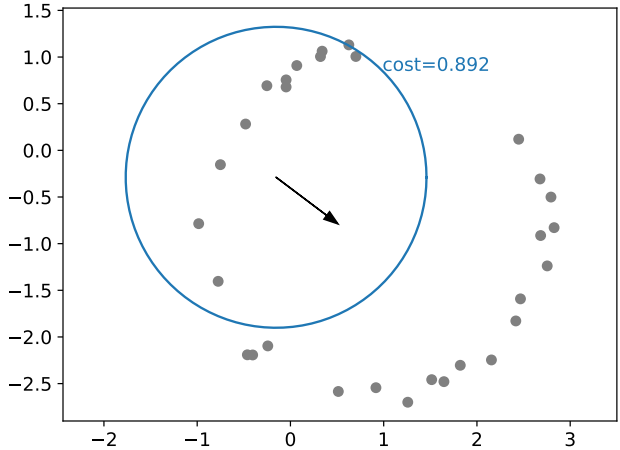
Generated gradient code

```
// ... 50 lines of code not shown
    while (true) {
      final double s0043 = lvar0;
      final double s0044 = lvar1;
      final long s0045 = lvar2;
      final long s0046 = lvar3;
      final long s0047 = lvar4;
      if ((s0046 <= 0)) {
        lvar5 = s0043;
        lvar6 = s0044;
      } else {
        final long s0059 = (2 * s0045);
        final double s0063 = (arg01[((int) (s0059 + 0))]);
        final double s0067 = (arg01[((int) (s0059 + 1))]);
        final double s0068 = (s0063 - s0010);
        final double s0075 = (s0067 - s0015);
        final double s0080 = (java.lang.Math.sqrt(((s0068 * s0068) + (s0075 * s0075))));
        final double s0084 = (s0080 - s0020);
        final double s0086 = ((((0.5 / s0080) * (((s0068 * -1.0) + (-1.0 * s0068)) + 0.0)) - 0.0);
        lvar0 = (s0043 + ((s0084 * s0086) + (s0086 * s0084)));
        lvar1 = (s0044 + (s0084 * s0084));
        lvar2 = (s0045 + s0047);
        lvar3 = (s0046 - 1);
        lvar4 = s0047;
        continue;
      }
      final double s0106 = lvar5;
      final double s0107 = lvar6;
      lvar7 = s0106;
      lvar8 = s0107;
      break;
    }
// ... 89 lines of code not shown
```

Gradient descent



Gradient descent



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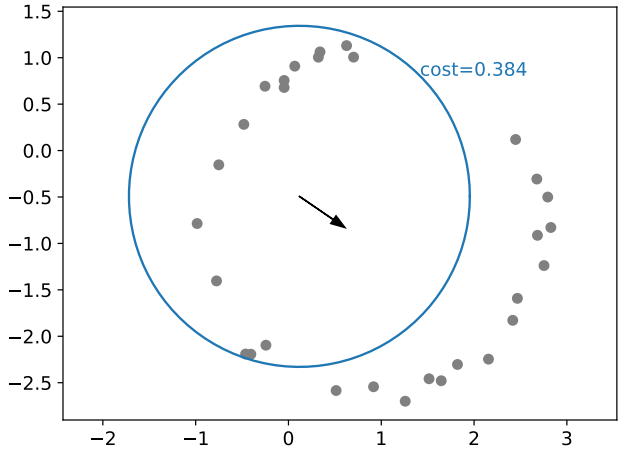

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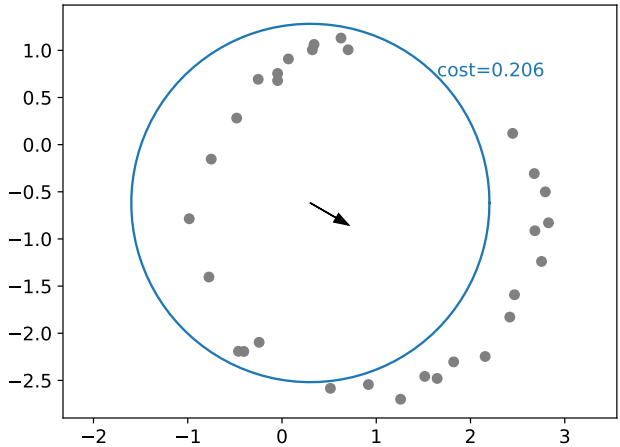
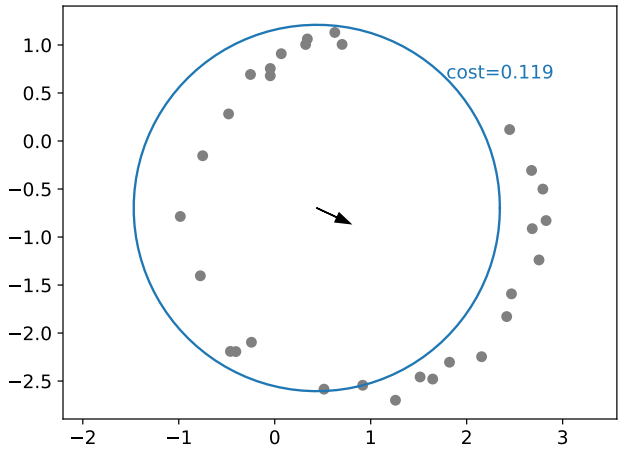


Gradient descent



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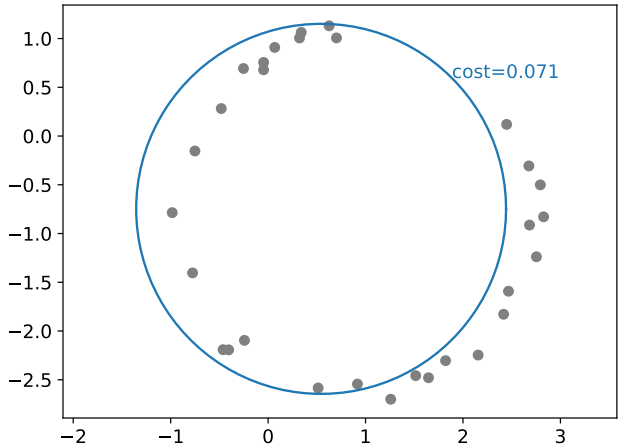

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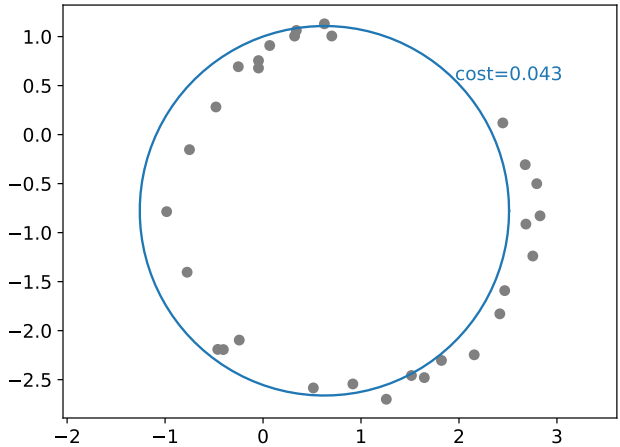
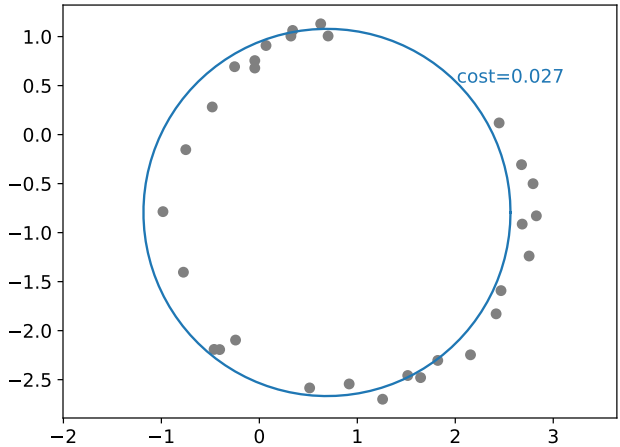
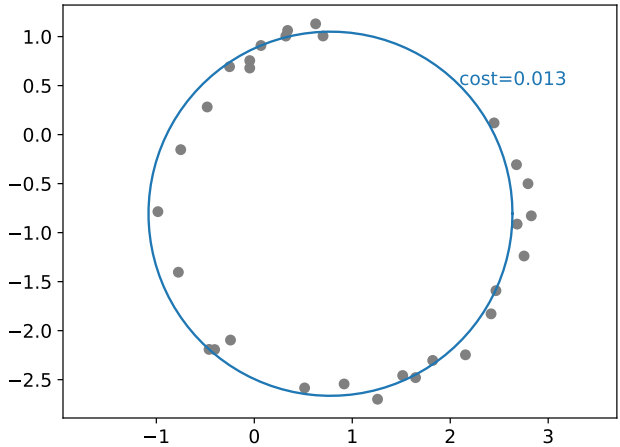


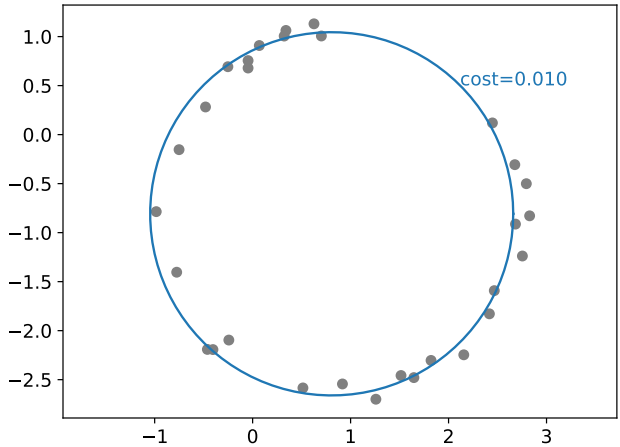
Gradient descent



Gradient descent



Gradient descent



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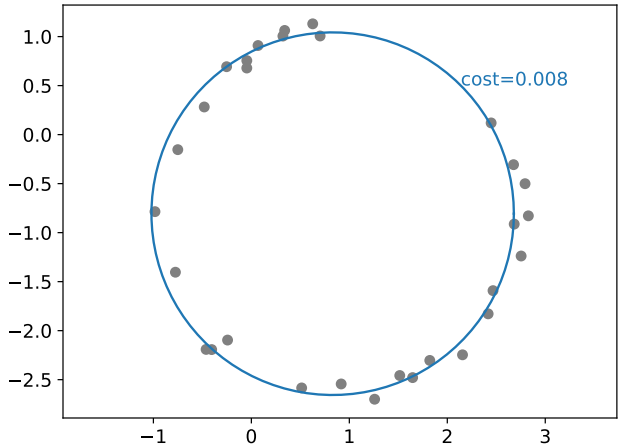
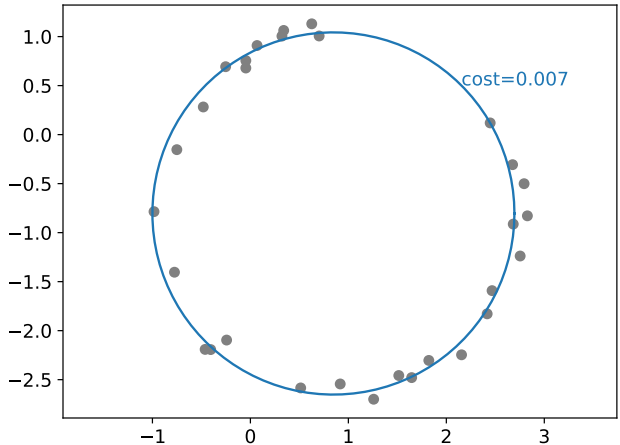
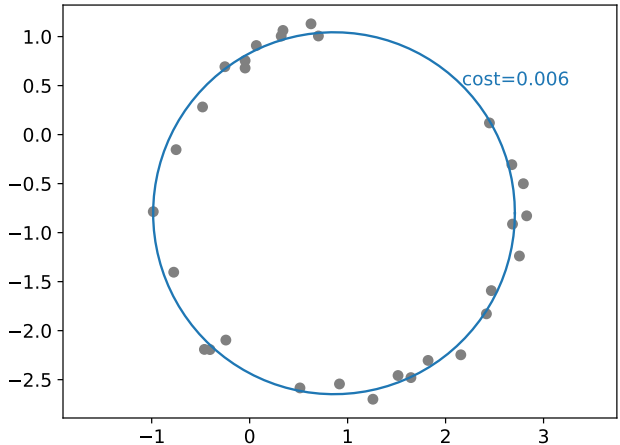


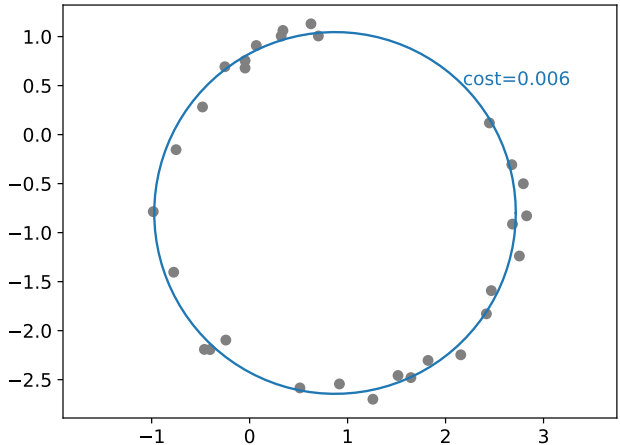
Gradient descent



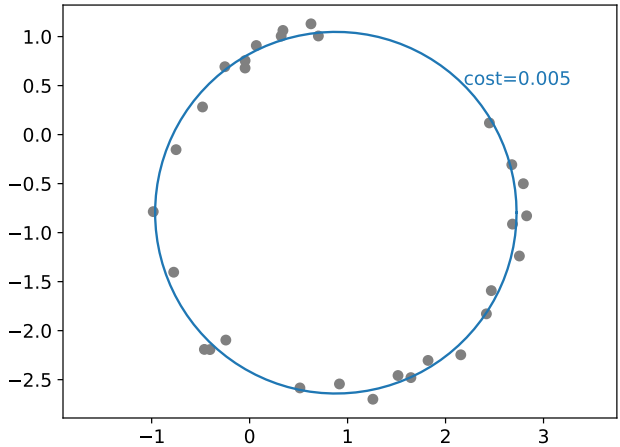
Gradient descent



Gradient descent



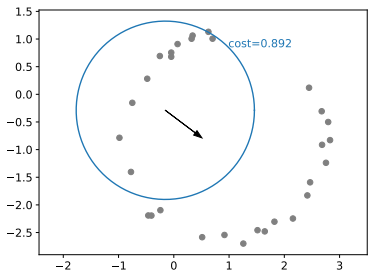
Gradient descent (good enough)



Measuring the time

Repeatedly optimize the circle parameters using gradient descent and automatic differentiation.

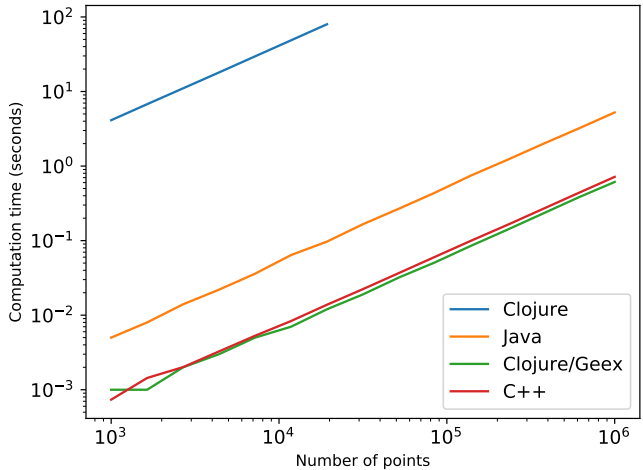
- Vary the number of points that we fit to
- Measure the computation time
- High-level implementations in
  - Clojure
  - Java
  - Clojure with Geex (what we just implemented)
  - C++



Example: Java implementation

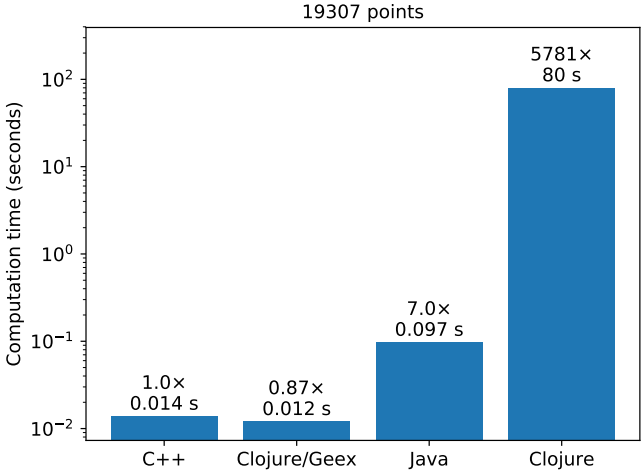
```
// ... 24 lines of code not shown
    public ADNumber evaluate(double[] data, ADCircleParameters params) {
        int N = data.length/2;
        ADNumber sum = ADNumber.constant(0.0);

        for (int i = 0; i < N; i++) {
            int at = 2*i;
            ADNumber x = ADNumber.constant(data[at + 0]);
            ADNumber y = ADNumber.constant(data[at + 1]);

            ADNumber distToCentre = x.sub(params.cx).square()
                .add(y.sub(params.cy).square())
                .sqrt();
            ADNumber distToCircle = distToCentre.sub(params.r);
            sum = sum.add(distToCircle.square());
        }

        return sum.mul(ADNumber.constant(1.0/N));
// ... 30 lines of code not shown
```

Circle fitness optimization problem



Comparison of high-level implementations that have not been optimized.

Circle fitness optimization problem



Comparison of high-level implementations that have not been optimized.

Conclusion

- Tool for code generation
- Write high-level code
- Write fast code
- General purpose
- Complement to Clojure
- Just another library
- To be published in the next few days:
  - Geex source code: `https://github.com/jonasseglare/geex`
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Thank you for listening!