

# Kernel Density Estimates and Mean Shift Clustering

Jonas Spinner | February 4, 2019



#### **Outline**



- 1 Introduction
- 2 Kernel Density Estimates
- Mean Shift Clustering
- 4 Application

Jonas Spinner - KDE and MSC

#### **History**



- Fukunaga & Hostetler (1975)
- Comaniciu & Meer (2002)
- Comaniciu et al. (2003)

$$\hat{f}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

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4/11

#### **Locally Weighted Mean**



Weighted mean

$$\mu^* = \frac{\sum_{i=1}^n \mathbf{x}_i w_i}{\sum_{i=1}^n w_i}$$

With weights as a function around a point

$$w_i = w_{\mathbf{x}}(\mathbf{x}_i) = g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)$$
$$\mu_h^*(\mathbf{x}) = \frac{\sum_{i=1}^n \mathbf{x}_i g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^n g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)}$$

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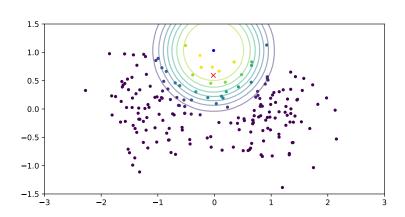
 $\mathbf{x}^{(t+1)} = \frac{\sum_{i=1}^{n} \mathbf{x}_{i} g\left(\left\|(\mathbf{x}^{(t)} - \mathbf{x}_{i})/h\right\|^{2}\right)}{\sum_{i=1}^{n} g\left(\left\|(\mathbf{x}^{(t)} - \mathbf{x}_{i})/h\right\|^{2}\right)}$ 

 $= \mathbf{x}^{(t)} + \mathbf{m} \left( \mathbf{x}^{(t)} \right)$ 

6/11

for i = 1, ...

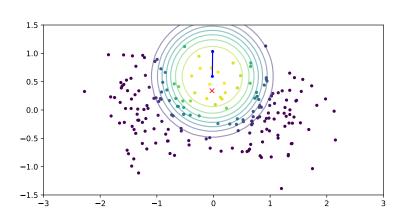
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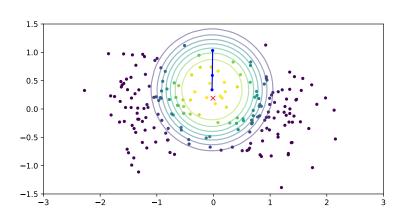


Mean Shift Clustering ○●

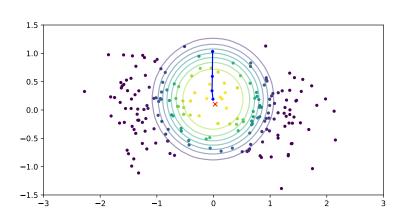
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February 4, 2019 7/11



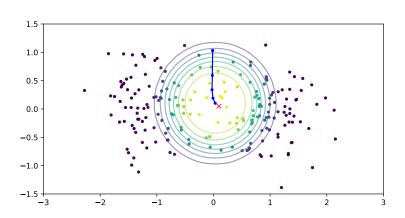














Introduction Kernel Density Estimates

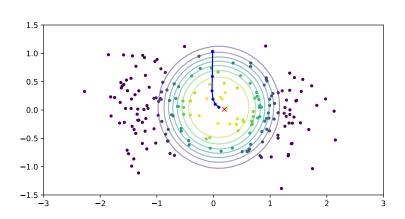
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Mean Shift Clustering ○●

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Application



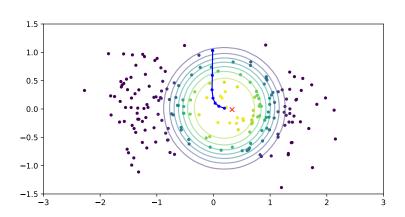


Introduction Kernel Density Estimates

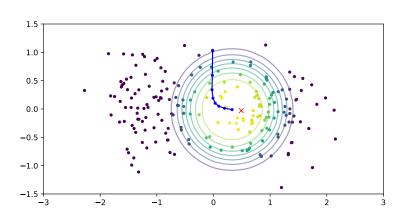
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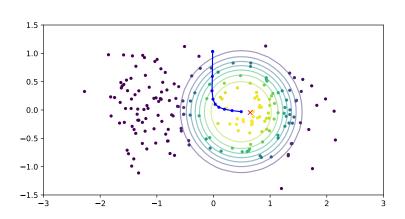
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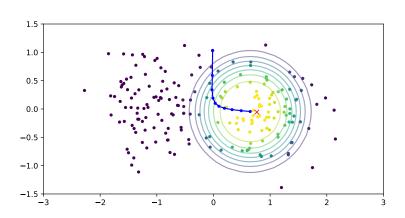




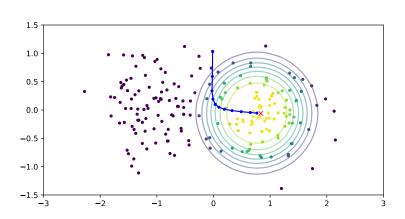




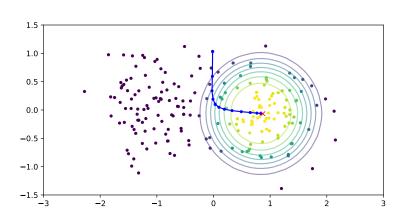
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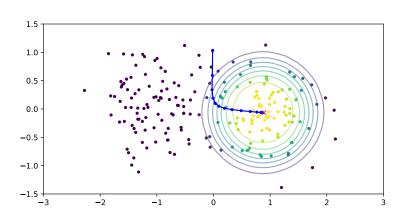




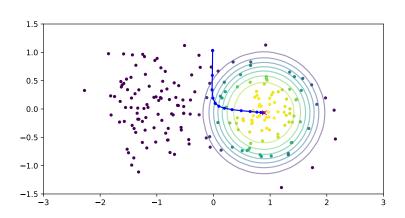






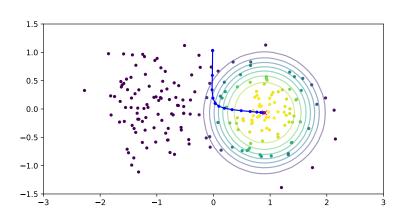








Application





#### Image segmentation – Gaussian kernel









(c) h = 0.3

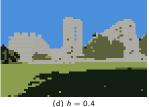


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## Image segmentation – Gaussian kernel



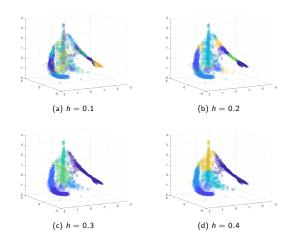


Abbildung: caption

Kernel Density Estimates



Introduction

## Image segmentation – Uniform kernel







(a) h = 0.1





(c) h = 0.3

Kernel Density Estimates

(a) " = 0.4
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Introduction

Mean Shift Clustering

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#### Image segmentation – Uniform kernel



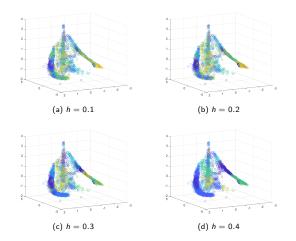


Abbildung: caption



#### References I



- Comaniciu, D. & Meer, P. (2002), 'Mean shift: a robust approach toward feature space analysis', *IEEE Transactions on Pattern Analysis and Machine Intelligence* **24**(5), 603–619.
- Comaniciu, D., Ramesh, V. & Meer, P. (2003), 'Kernel-based object tracking', *IEEE Transactions on Pattern Analysis and Machine Intelligence* **25**(5), 564–577.
- Fukunaga, K. & Hostetler, L. (1975), 'The estimation of the gradient of a density function, with applications in pattern recognition', *IEEE Transactions on Information Theory* **21**(1), 32–40.

