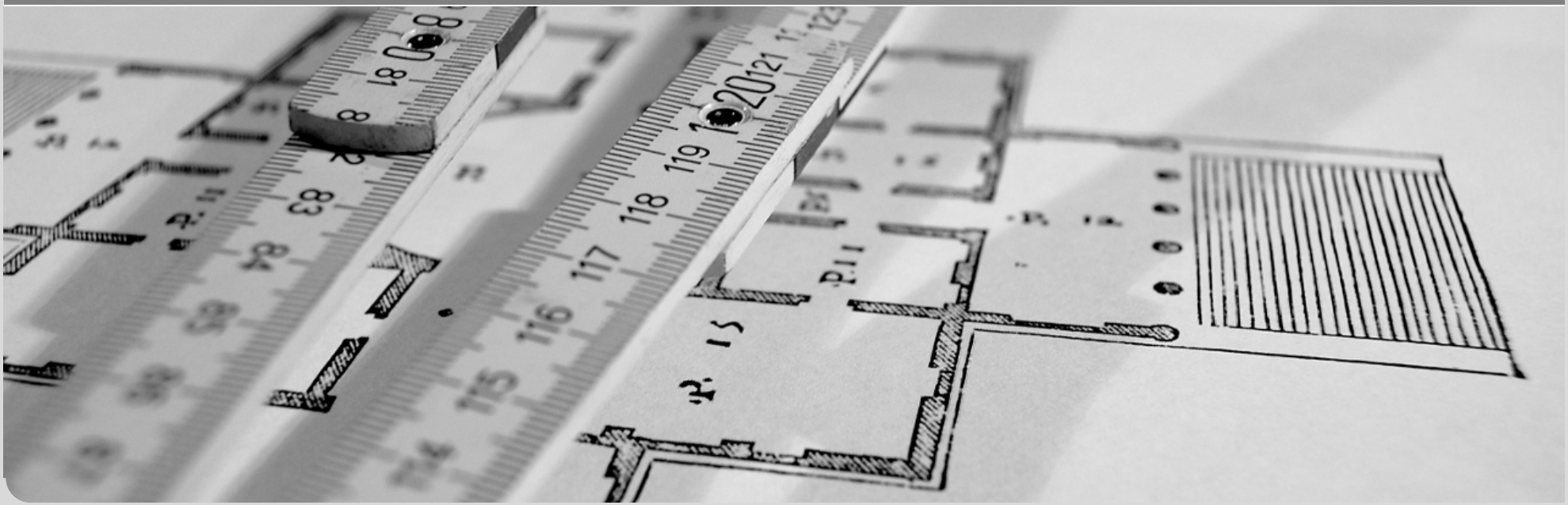
Karlsruher Institut für Technologie

Kernel Density Estimates and Mean Shift Clustering

Jonas Spinner | February 4, 2019

ANALYTICS AND STATISTICS AT THE INSTITUTE OF OPERATIONS RESEARCH



Outline

History

- Fukunaga & Hostetler (1975)
- Comaniciu & Meer (2002)
- Comaniciu et al. (2003)

$$\hat{f}(\boldsymbol{x}) = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\boldsymbol{x} - \boldsymbol{x}_i}{h}\right)$$

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Locally Weighted Mean

Weighted mean

$$\boldsymbol{\mu}^* = \frac{\sum_{i=1}^{n} \boldsymbol{x}_i w_i}{\sum_{i=1}^{n} w_i}$$

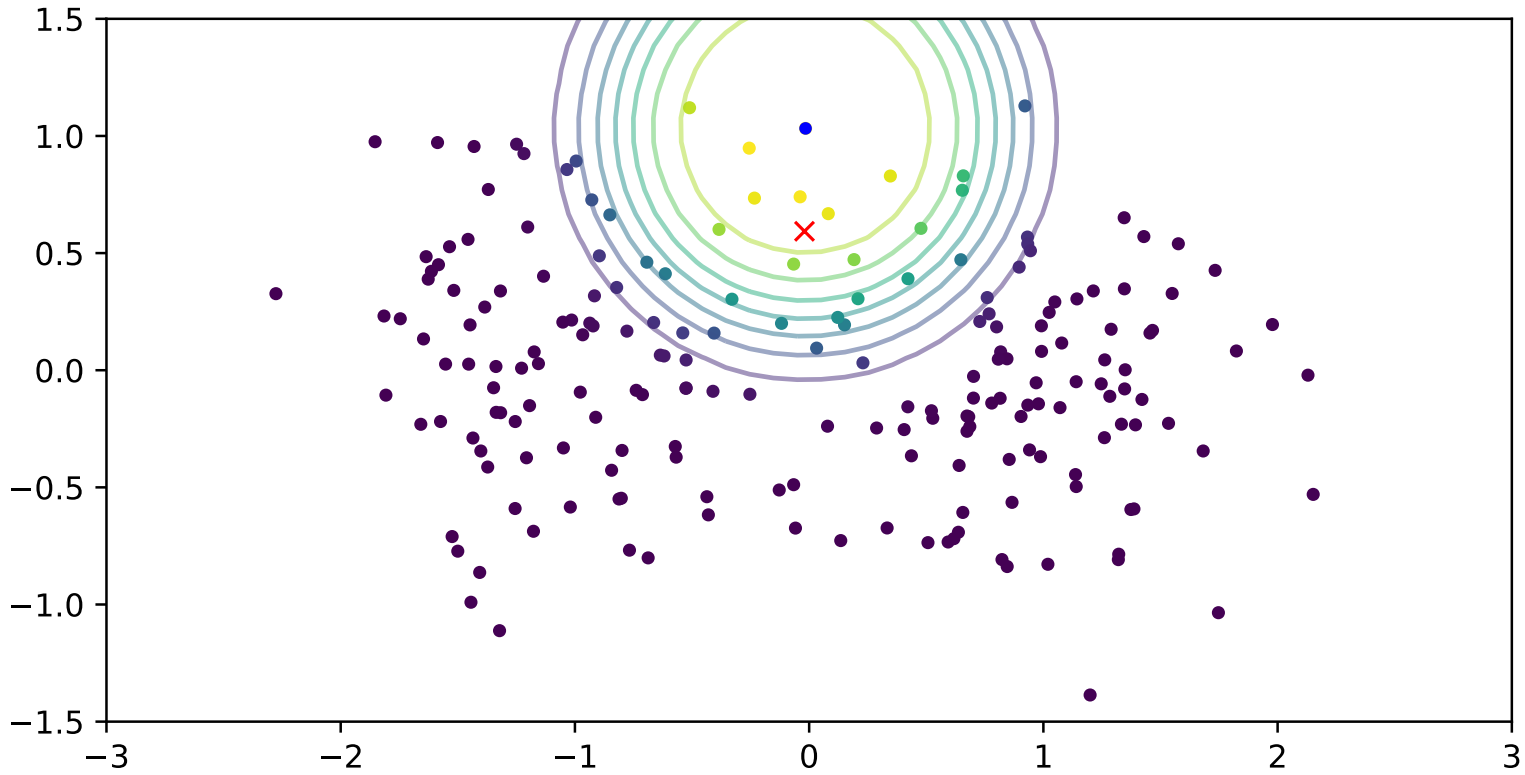
With weights as a function around a point

$$w_i = w_{\boldsymbol{x}}(\boldsymbol{x}_i) = g\left(\left\|\frac{\boldsymbol{x} - \boldsymbol{x}_i}{h}\right\|^2\right)$$

$$\boldsymbol{\mu}_h^*(\boldsymbol{x}) = \frac{\sum_{i=1}^{n} \boldsymbol{x}_i g\left(\left\|\frac{\boldsymbol{x}-\boldsymbol{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\boldsymbol{x}-\boldsymbol{x}_i}{h}\right\|^2\right)}$$

$$\boldsymbol{x}^{(t+1)} = \frac{\sum_{i=1}^{n} \boldsymbol{x}_i \; g\left(\left\|(\boldsymbol{x}^{(t)} - \boldsymbol{x}_i)/h\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|(\boldsymbol{x}^{(t)} - \boldsymbol{x}_i)/h\right\|^2\right)} \qquad \text{for } i = 1, \ldots$$

$$= \boldsymbol{x}^{(t)} + \boldsymbol{m}\left(\boldsymbol{x}^{(t)}\right)$$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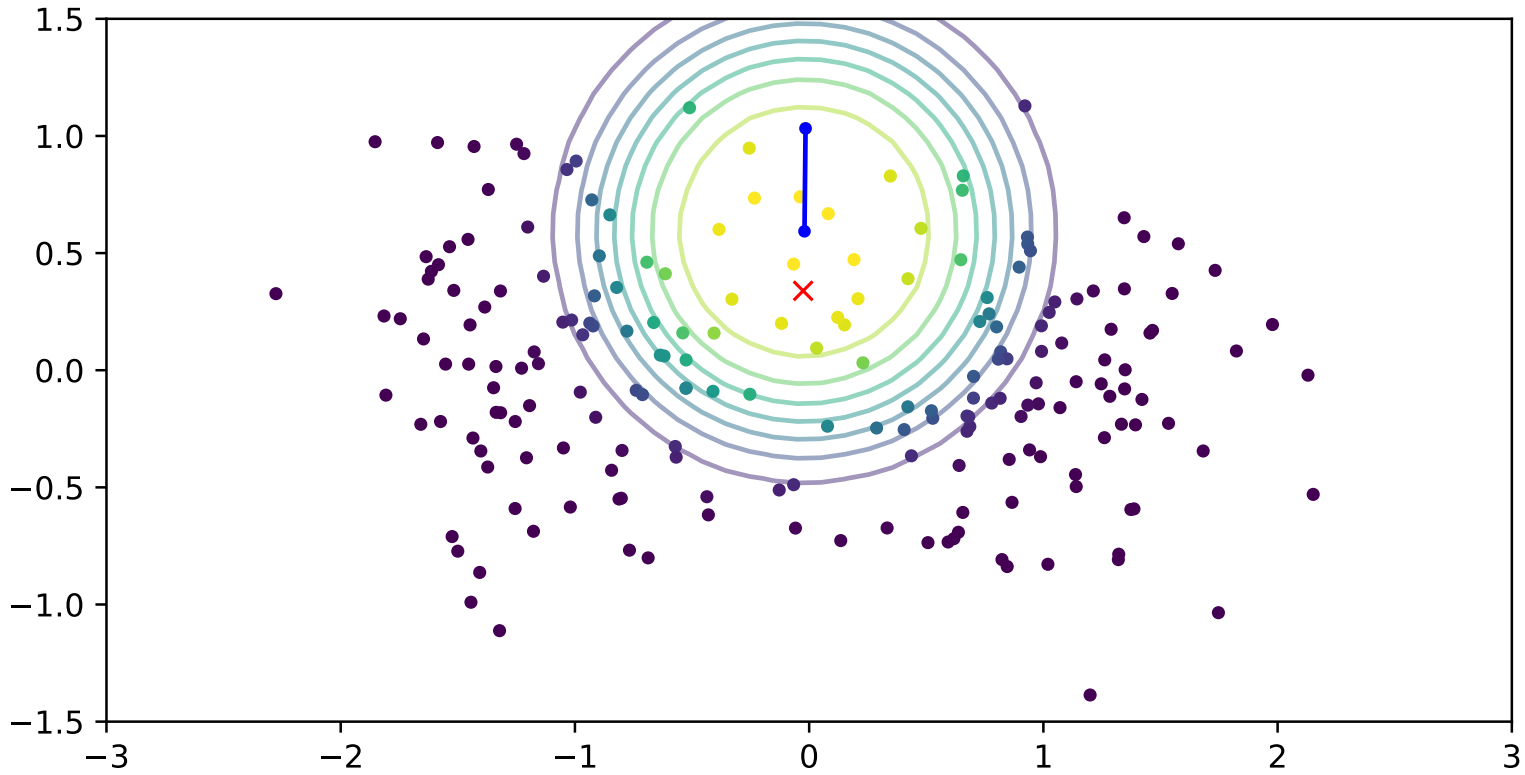
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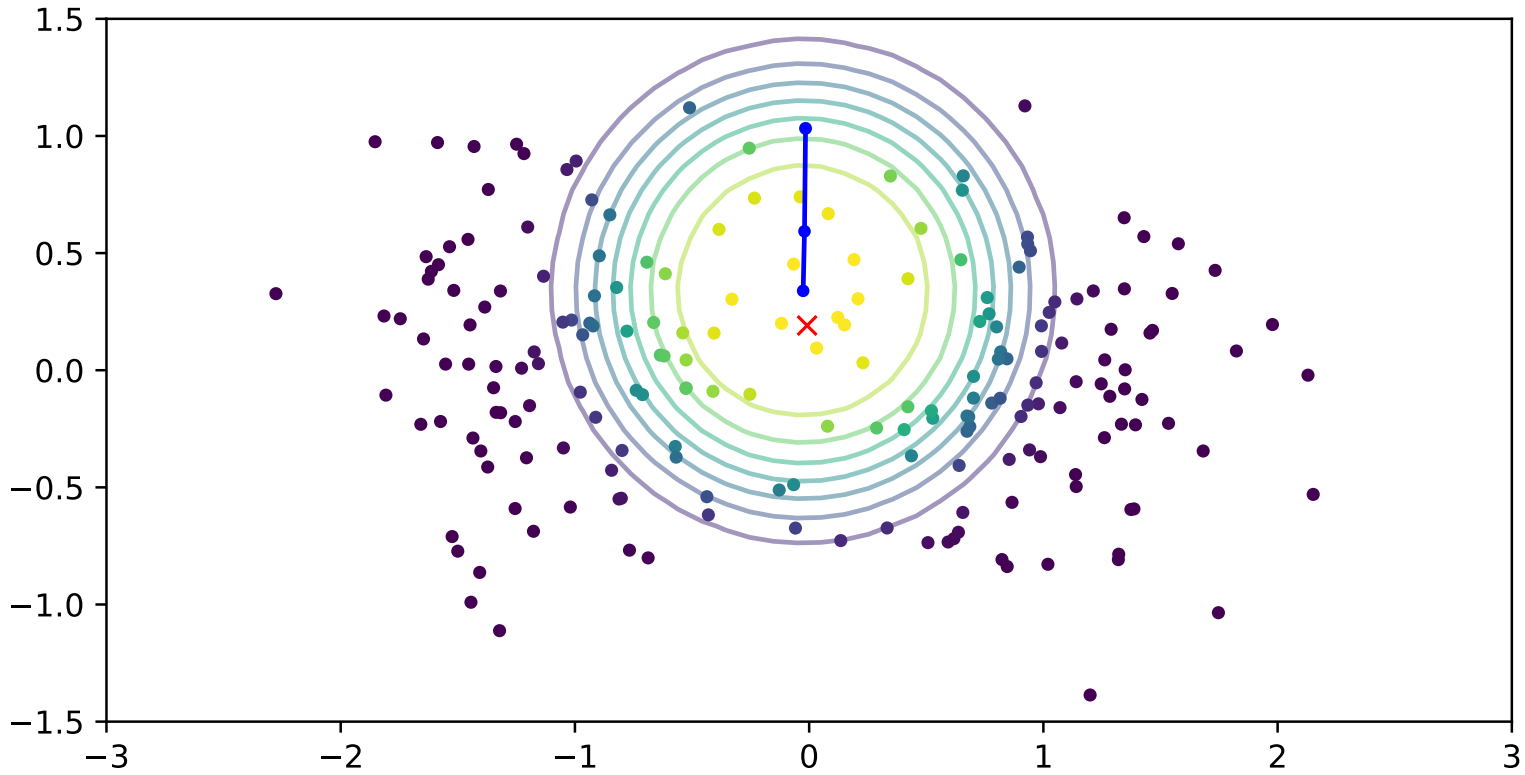
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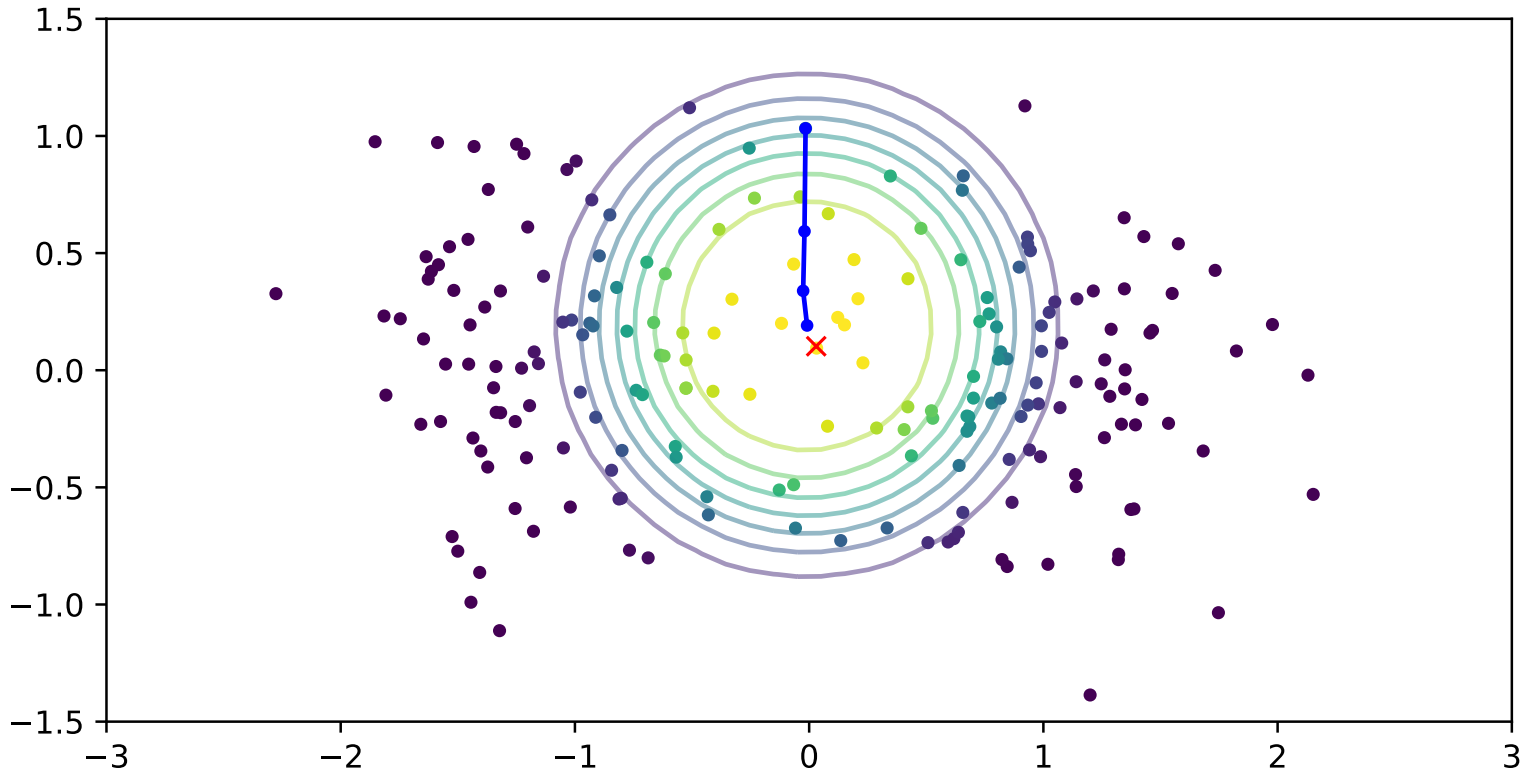
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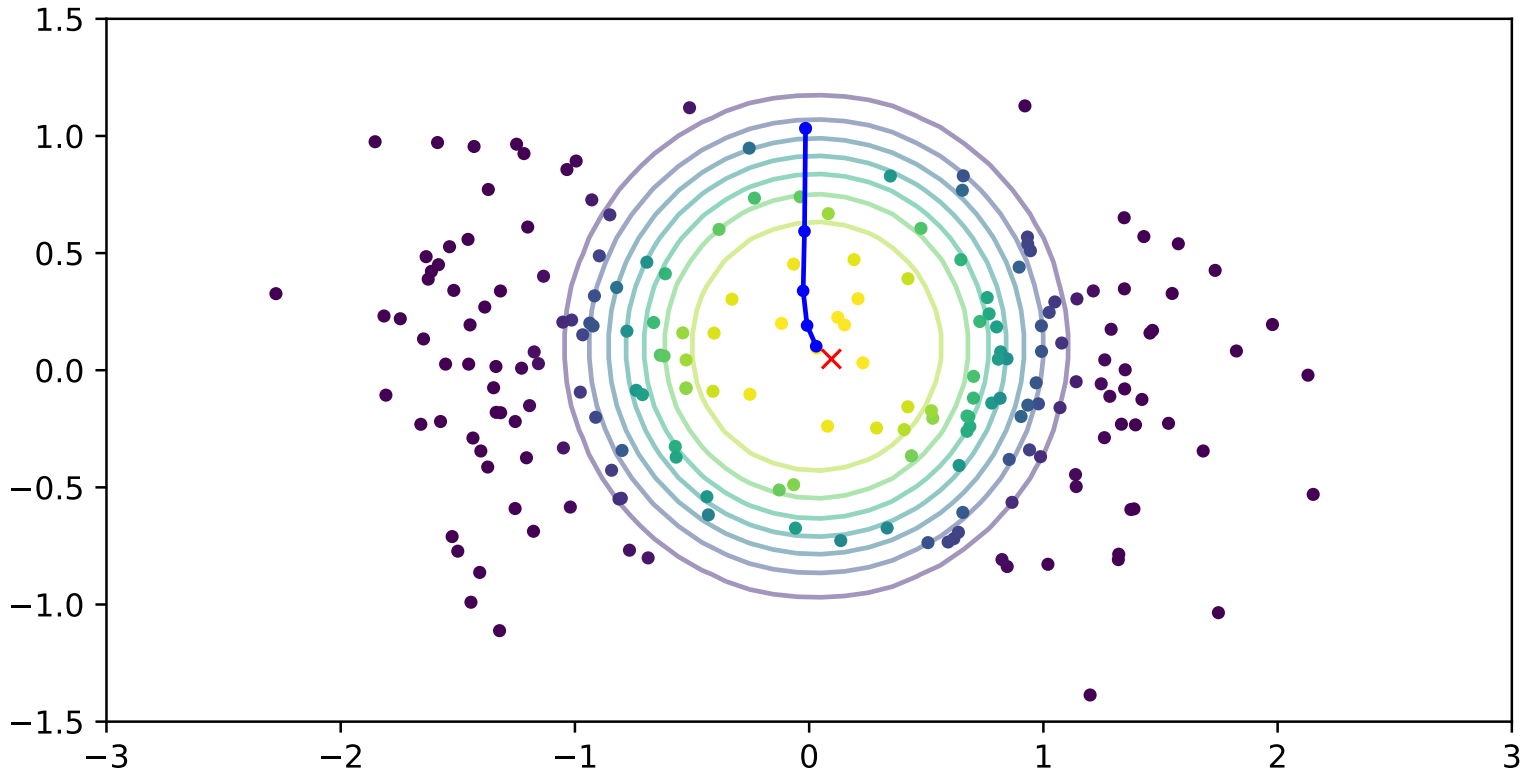
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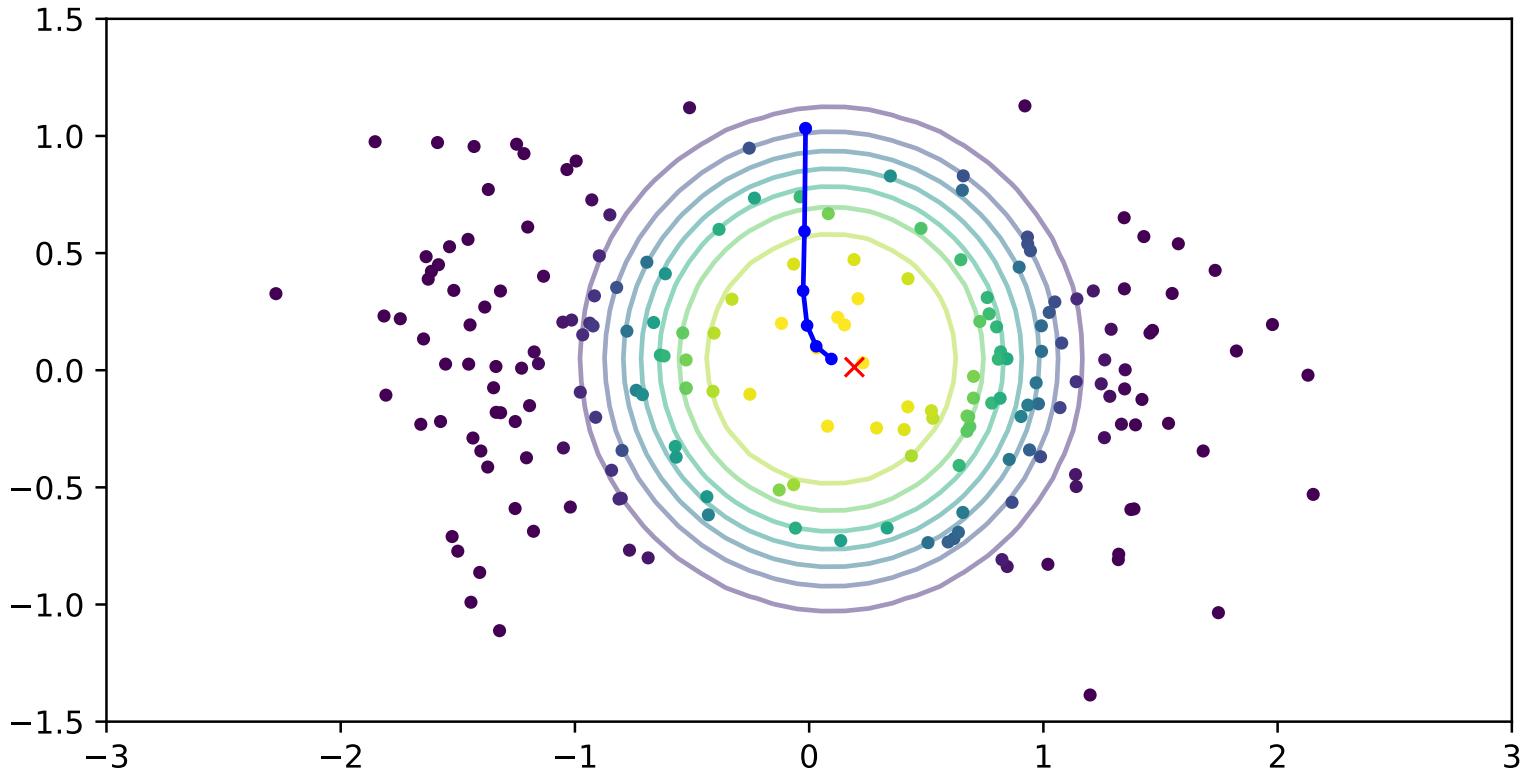
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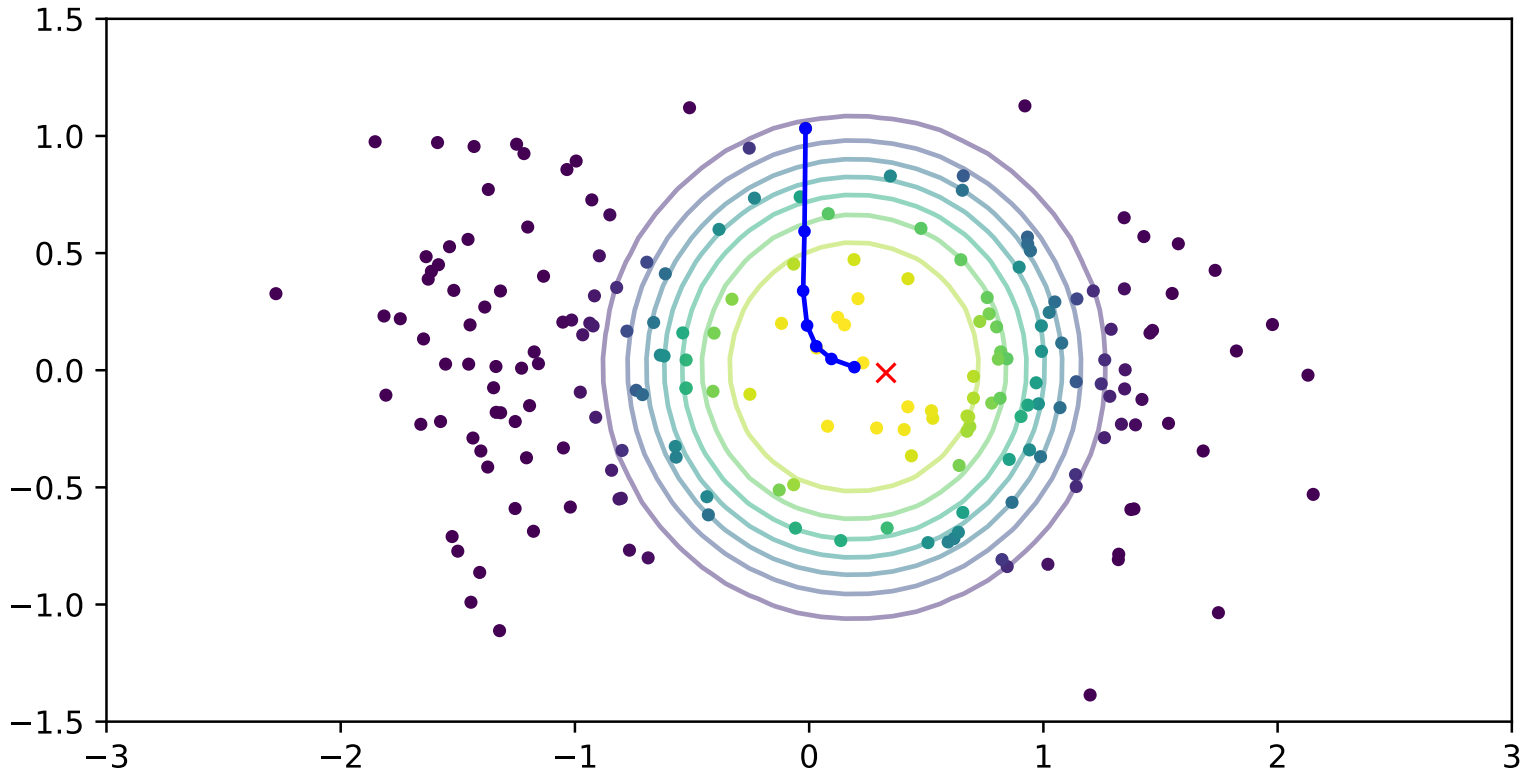
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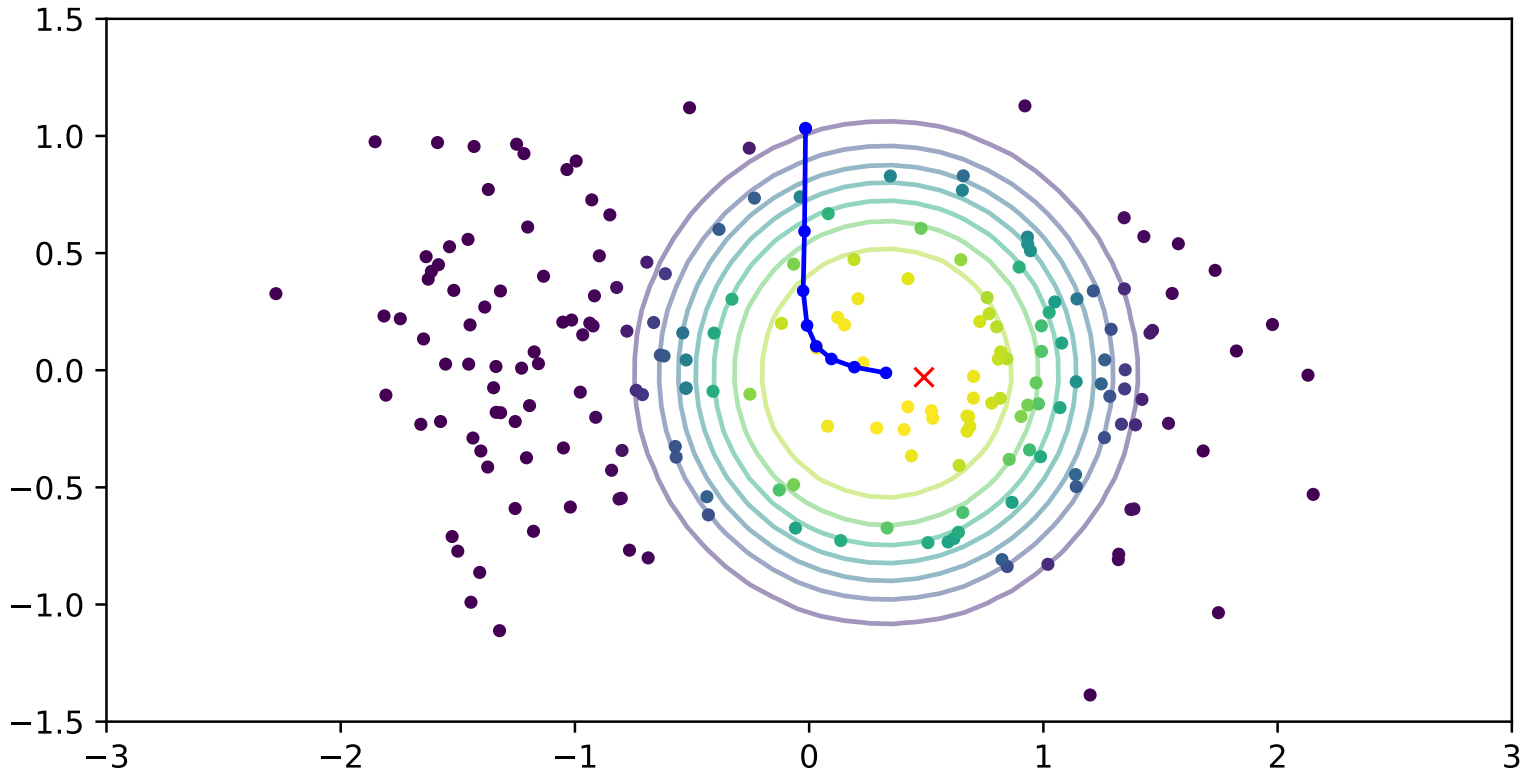
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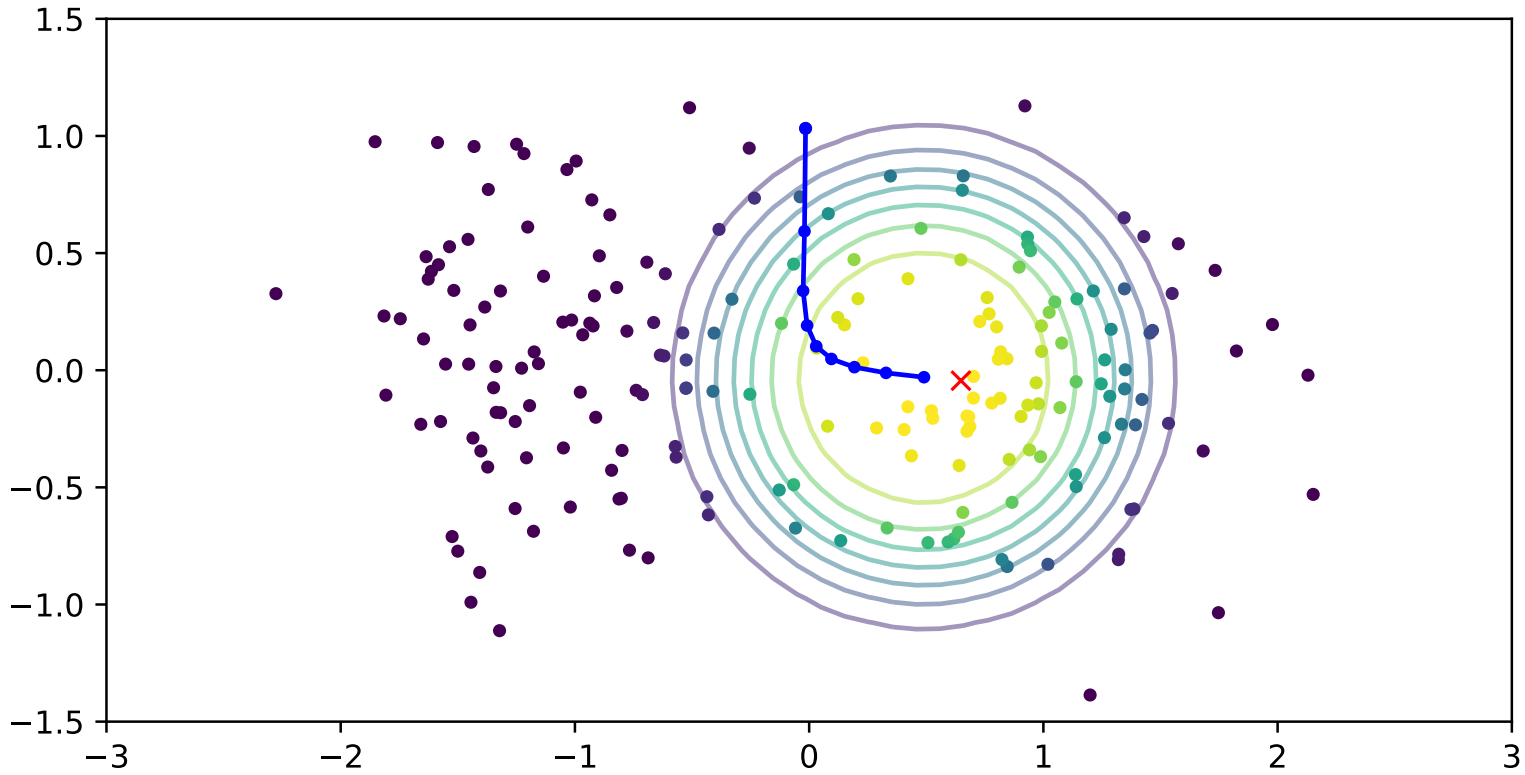
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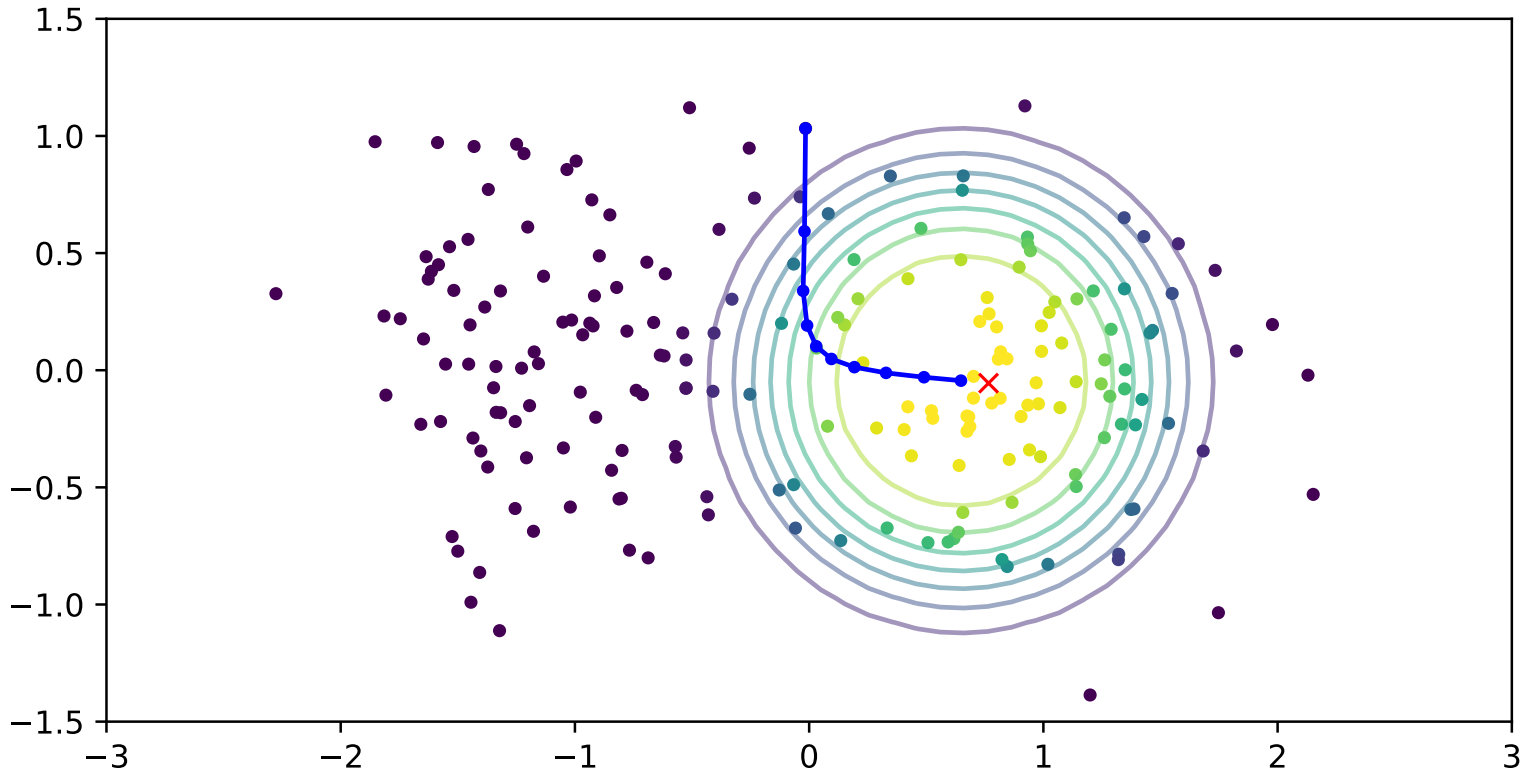
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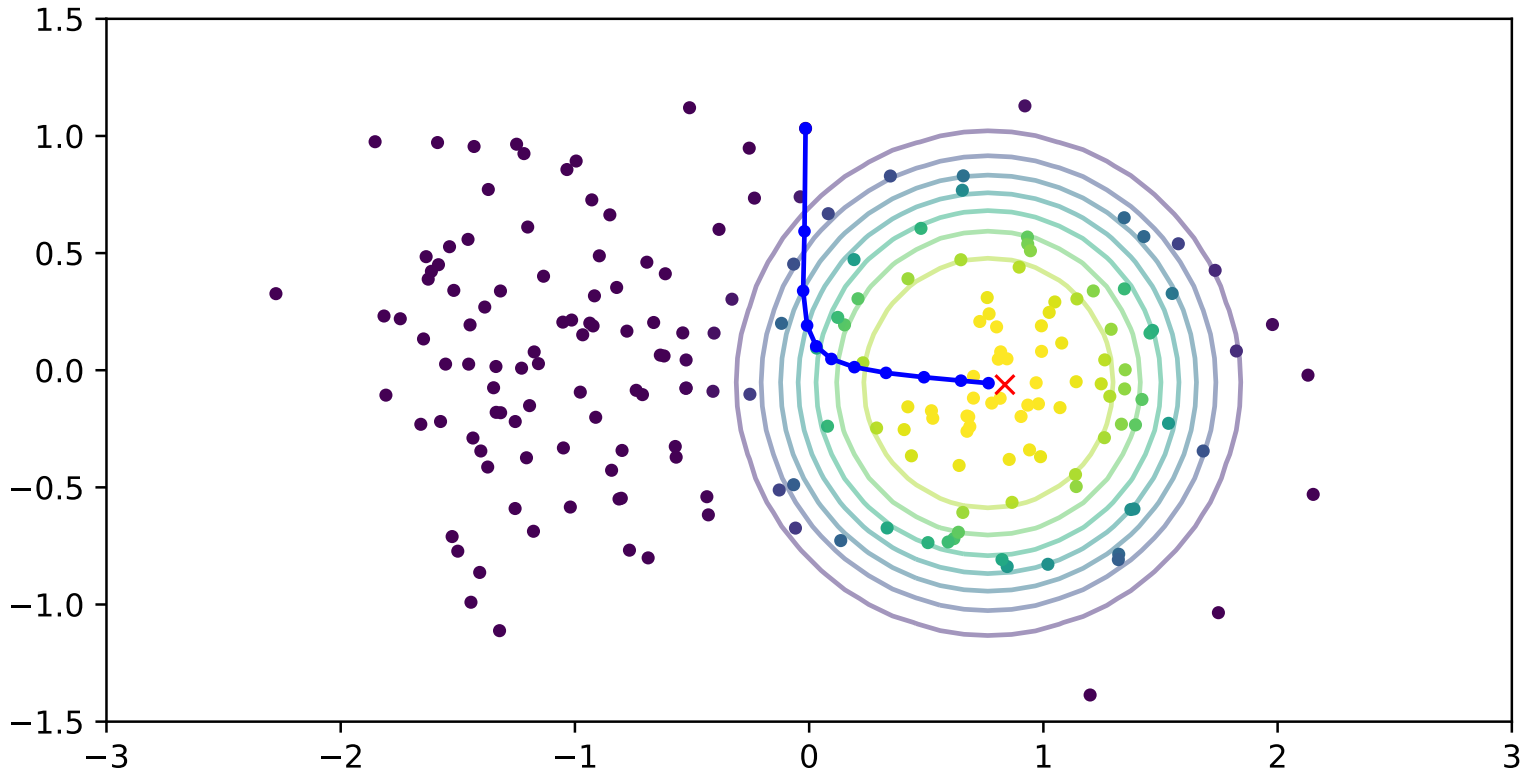
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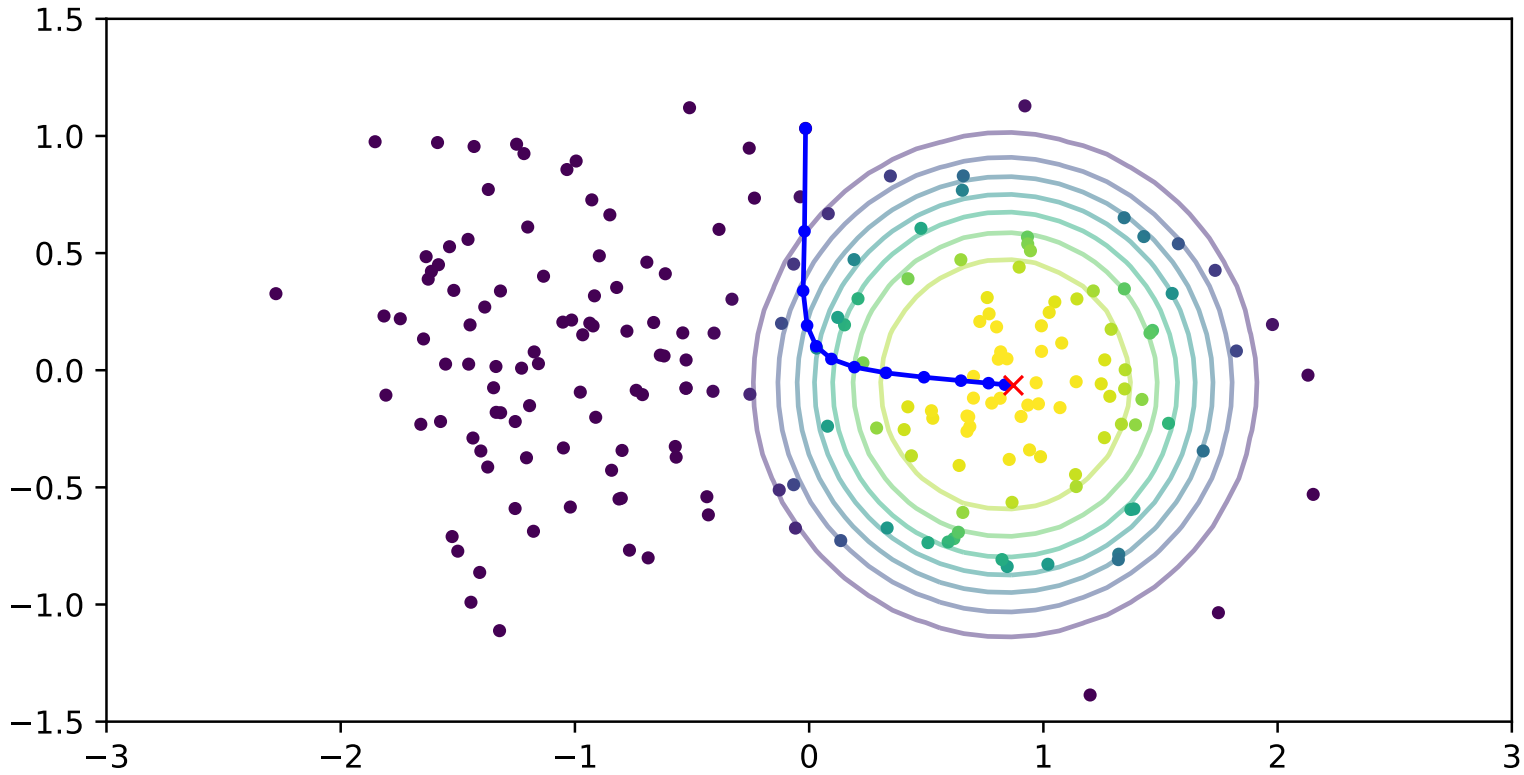
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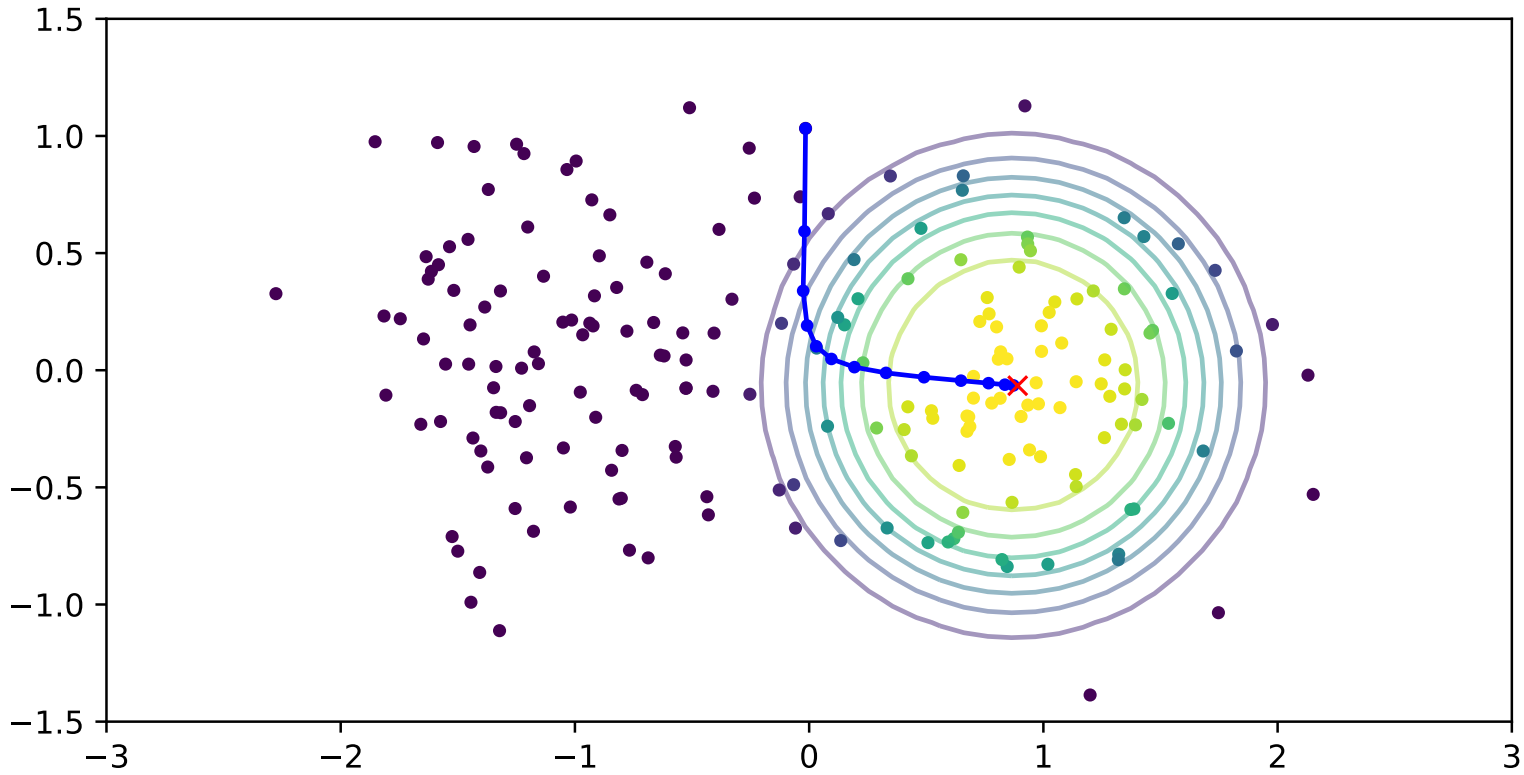
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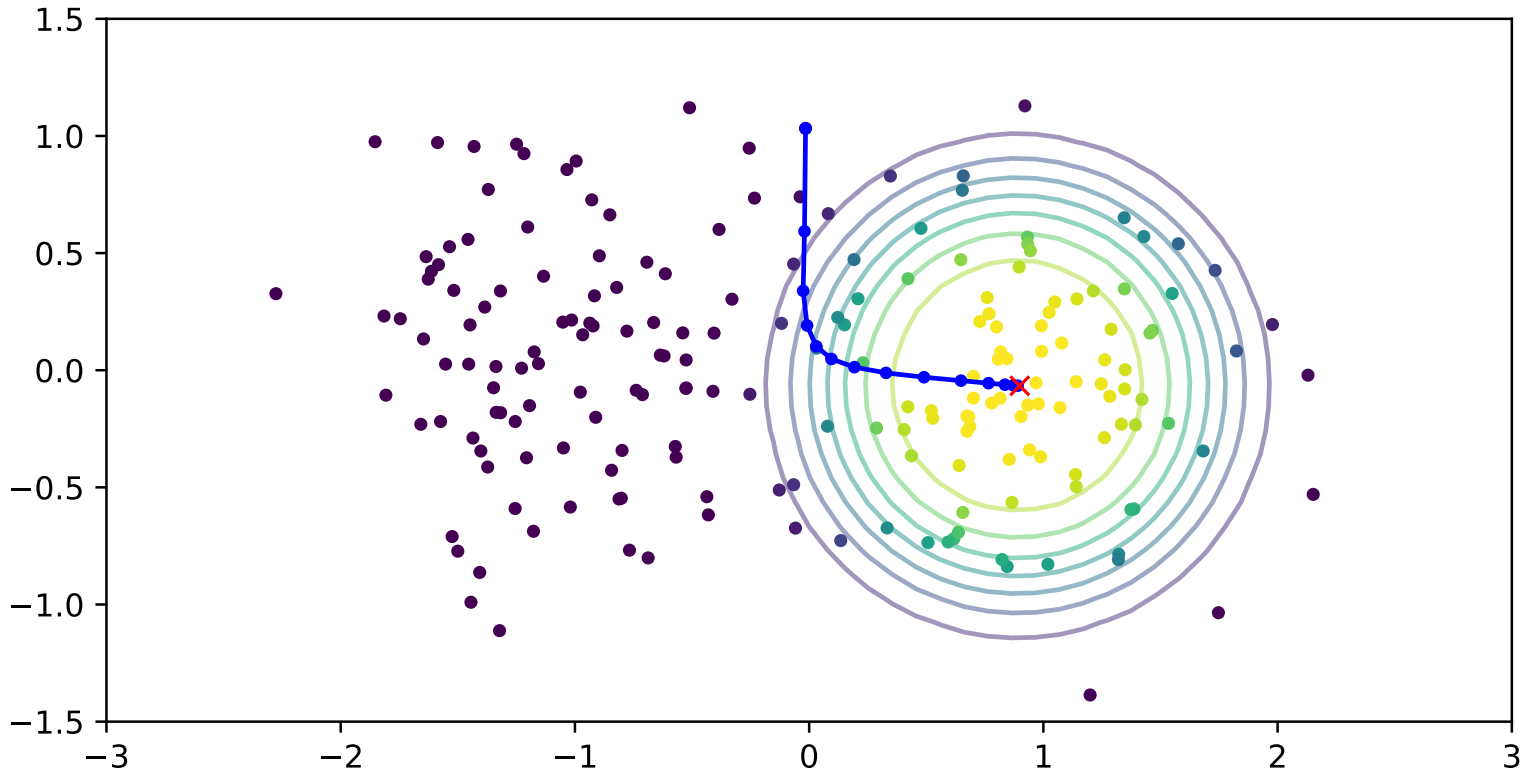
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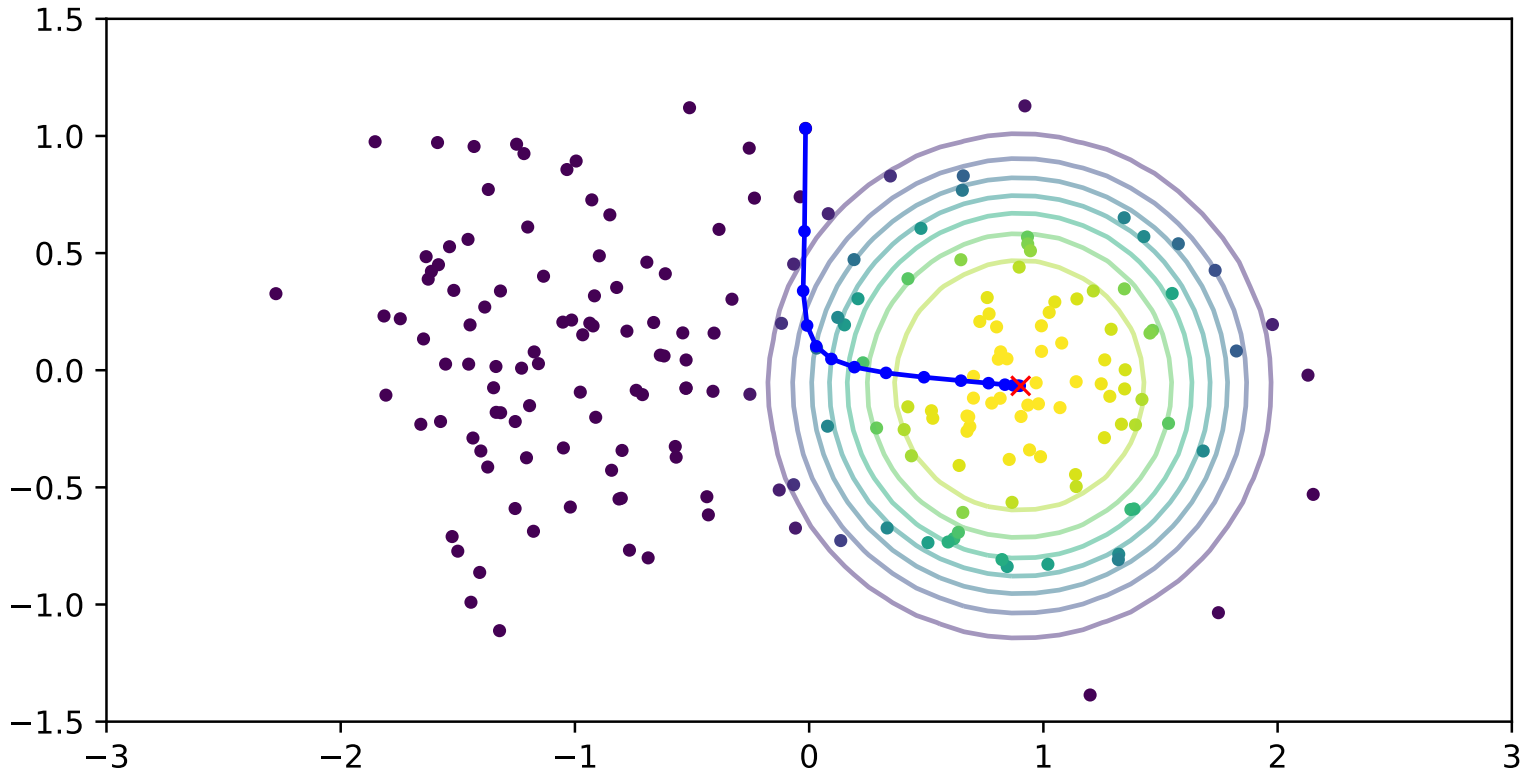
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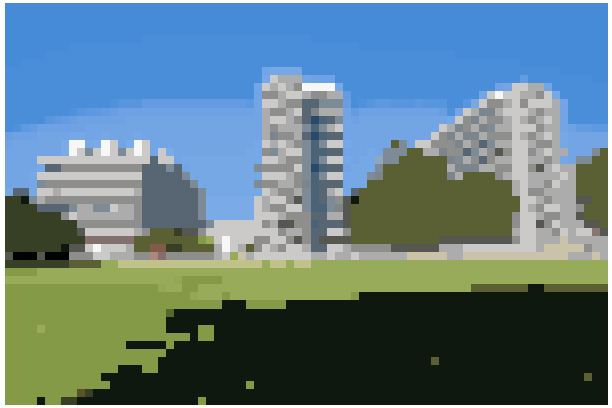
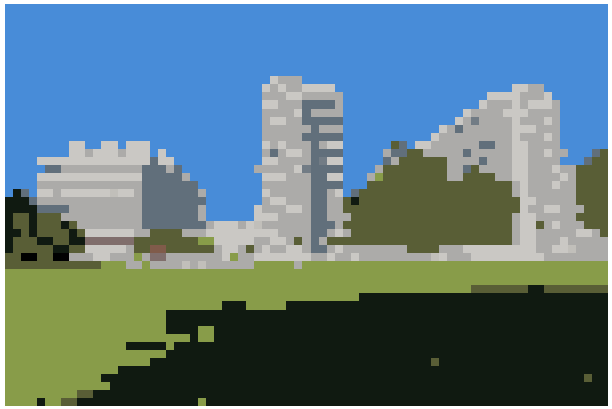
Image segmentation – Gaussian kernel

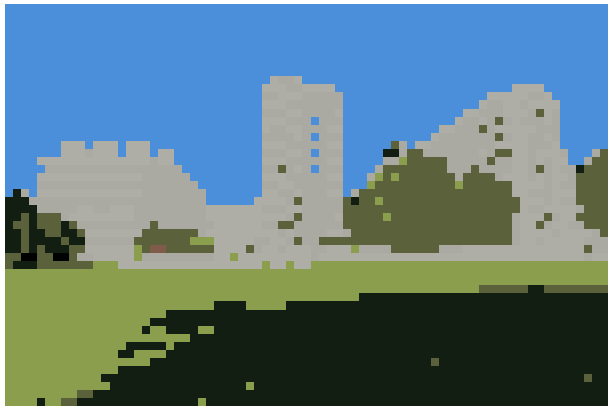
(a) $h = 0.1$



(b) $h = 0.2$



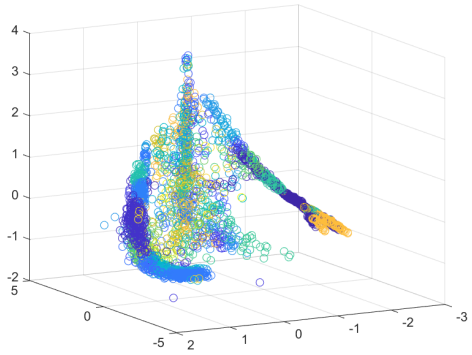
(c) $h = 0.3$



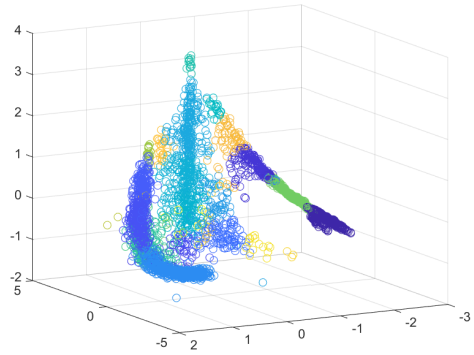
(d) $h = 0.4$

Abbildung: caption

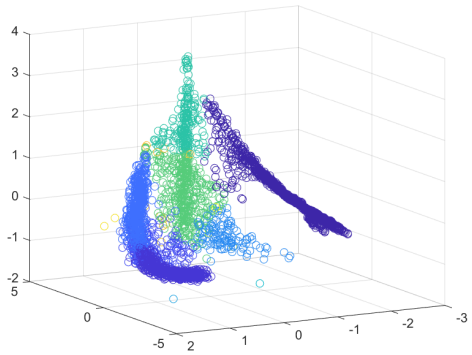
Image segmentation – Gaussian kernel



(a) $h = 0.1$



(b) $h = 0.2$



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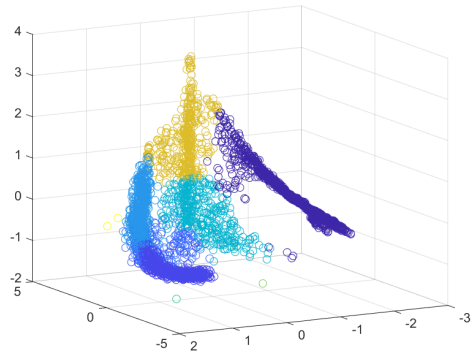


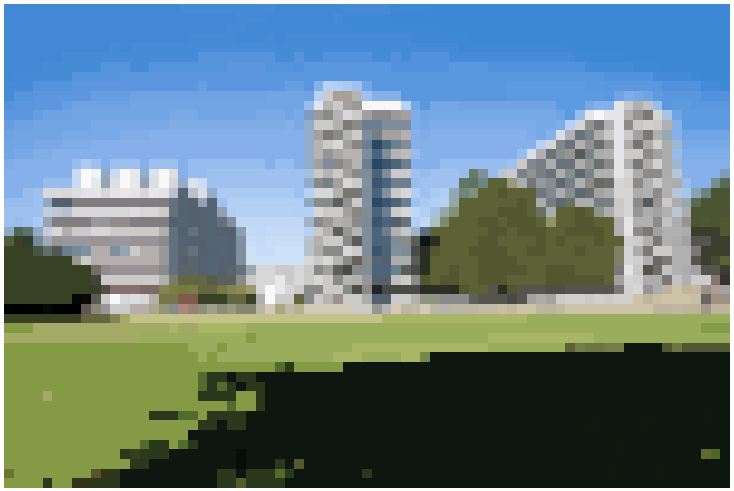
(d) $h = 0.4$

Abbildung: caption

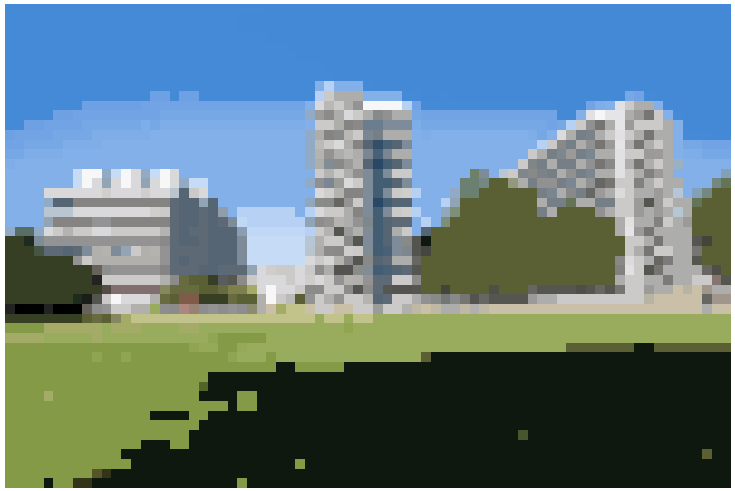
Image segmentation – Uniform kernel

(a) $h = 0.1$

(b) $h = 0.2$

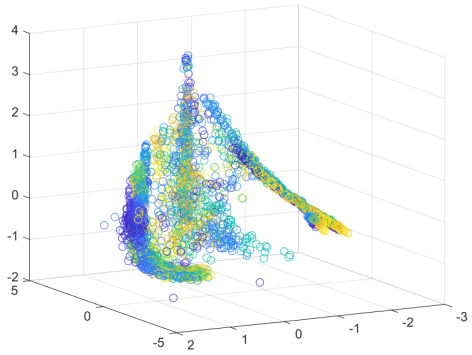


(c) $h = 0.3$



(d) $h = 0.4$

Image segmentation – Uniform kernel



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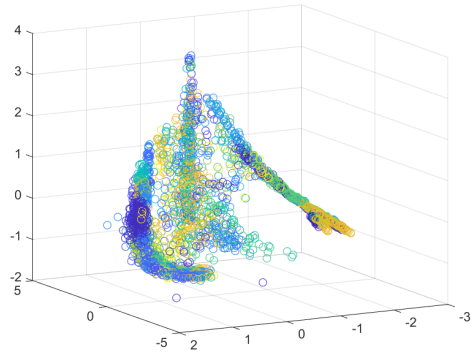

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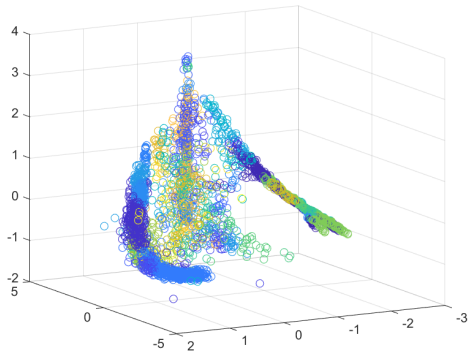

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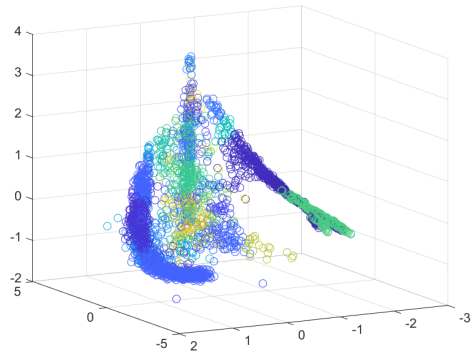

(d) $h = 0.4$

Abbildung: caption

References I

Comaniciu, D. & Meer, P. (2002), 'Mean shift: a robust approach toward feature space analysis', *IEEE Transactions on Pattern Analysis and Machine Intelligence* **24**(5), 603–619.

Comaniciu, D., Ramesh, V. & Meer, P. (2003), 'Kernel-based object tracking', *IEEE Transactions on Pattern Analysis and Machine Intelligence* **25**(5), 564–577.

Fukunaga, K. & Hostetler, L. (1975), 'The estimation of the gradient of a density function, with applications in pattern recognition', *IEEE Transactions on Information Theory* **21**(1), 32–40.