Solution to (iv): Let $f_1\colon I \times I \to \mathbb{C}$ be the "retraction" homotopy $f_1(x,t) = \gamma_1 * \gamma_2 * \overline{\gamma_1}\,(x(1-t))$. Thus $\gamma_1 * \gamma_2 * \overline{\gamma_1}$ is freely homotopic to the constant path at 0. Similarly γ_2 is also freely homotopic to the constant path at 0 by, for example, $f_2(x,t) = \gamma_2\,(tx)$. Then the map $H\colon I \times I \to \mathbb{C}$ by

$$H(x,t) = \begin{cases} f_1(x,2t), & t \in \left[0, \frac{1}{2}\right], \\ f_2(x,2t-1), & t \in \left[\frac{1}{2}, 1\right] \end{cases}$$

clearly has $H(x,0) = \gamma_1 * \gamma_2 * \overline{\gamma_1}(x)$ and $H(x,1) = \gamma_2(x)$, and as $f_1(x,1) = c_0 = f_2(x,0)$ where c_0 is the constant path at 0, we get that H is continuous by the gluing lemma since H is continuous on $I \times \left[0, \frac{1}{2}\right]$ and $I \times \left[\frac{1}{2}, 1\right]$ and agrees on the intersection.

They are not homotopic relative to the basepoint since $\pi_1\left(\mathbb{C} - \{-1,1\}\right) \approx \pi_1\left(S^1 \vee S^1\right) = \langle a, b\rangle$ where the loop γ_1 corresponds to a, say, and γ_2 corresponds to b, for example. But then $\overline{\gamma_1} = a^{-1}$, so $\gamma_1 * \gamma_2 * \overline{\gamma_1}$ corresponds to aba^{-1} which does not correspond to b which is what γ_2 corresponds to since we have no relations giving $aba^{-1} = b$.

Additional note: Since $aba^{-1}b^{-1}$ is a commutator, it is trivial in the abelianization of $\pi_1\left(\mathbb{C} - \{-1,1\}\right)$ which is the first homology group of $\mathbb{C} - \{-1,1\}$ as $\mathbb{C} - \{-1,1\}$ is path-connected, so the two loops in question are homologous.