1.5.iv: Show that a full and faithful functor $F: C \to D$ both reflects and creates isomorphisms. That is, show:
(i) If f is a morphism in C so that Ff is an isomorphism in D, then f is an isomorphism.

(ii) If x and y are objects in C so that Fx and Fy are isomorphic in D, then x and y are isomorphic in C.

Solution:
(i) Assume $f: c \to c'$, so $Ff: F(c) \to F(c')$.
Since Ff is an isomorphism in D there exists a morphism $g: F(c') \to F(c)$ in D such that $Ff \circ g = \mathbb{1}_{F(c')}$ and $g \circ Ff = \mathbb{1}_{F(c)}$.
Now, F is full, so since $g \in \mathrm{Hom}(F(c'), F(c))$ implies that there exists a morphism $g' \in \mathrm{Hom}(c', c)$ such that $F(g') = g$.
Now by functoriality, $F(f \circ g') = Ff \circ Fg' = Ff \circ g = \mathbb{1}_{F(c')}$ and $F(g' \circ f) = Fg' \circ Ff = g \circ Ff = \mathbb{1}_{F(c)}$.
So by faithfulness, $f \circ g' = \mathbb{1}_{c'}$ and $g' \circ f = \mathbb{1}_c$ since by functoriality, F takes $\mathbb{1}_{c'}$ and $\mathbb{1}_c$ to $\mathbb{1}_{F(c')}$ and $\mathbb{1}_{F(c)}$, respectively. Thus f is by definition an isomorphism with inverse g'.

(ii) Assume Fx and Fy are isomorphic in D.
Let $f: Fx \to Fy$ and $g: Fy \to Fx$ such that $fg = \mathbb{1}_{Fy}$ and $gf = \mathbb{1}_{Fx}$.
Again by fullness, there exist $f': x \to y$ and $g': y \to x$ such that $Ff' = f$ and $Fg' = g$.
Now $F(f'g') = Ff'Fg' = fg = \mathbb{1}_{Fy}$ and $F(g'f') = Fg'Ff' = gf = \mathbb{1}_{Fx}$.
Again by faithfulness, $f'g' = \mathbb{1}_y$ and $g'f' = \mathbb{1}_x$ since by functoriality, $F\mathbb{1}_x = \mathbb{1}_{Fx}$ and $F\mathbb{1}_y = \mathbb{1}_{Fy}$.
Hence x and y are isomorphic.

1.6.i: Show that any map from a terminal object in a category to an initial one is an isomorphism. An object that is both initial and terminal is called a **zero object**.

Solution: Let C be an arbitrary category and let $c, d \in C$ with c initial and d terminal.
Assume that $f: d \to c$ is a morphism.
Since c is initial, for every $a \in C$ there exists a unique morphism $c \to a$, so in particular, for $a = c$, there exists only the unique morphism $\mathbb{1}_c: c \to c$ in $\mathrm{Hom}(c, c)$. Similarly, since d is terminal, for every $a \in C$ there exists a unique morphism $a \to d$. Hence for $a = d$, there exists only the unique morphism $\mathbb{1}_d: d \to d$ in $\mathrm{Hom}(d, d)$. I.e. $|\mathrm{Hom}(c, c)| = 1 = |\mathrm{Hom}(d, d)|$.
Now, in particular, choosing $a = d$ in the initial condition for c, we have that there exists a unique map $g: c \to d$. Since C is a category, there exists a composition $gf: c \to c \in \mathrm{Hom}(c, c) = \{\mathbb{1}_c\}$, so $gf = \mathbb{1}_c$, and there exists a composition $fg: d \to d \in \mathrm{Hom}(d, d) = \{\mathbb{1}_d\}$, so $fg = \mathbb{1}_d$. Thus f is an isomorphism with inverse g.

1.6.ii: Show that any two terminal objects in a category are connected by a unique isomorphism.

Solution: Let C be an arbitrary category, and let $d, d' \in C$ be two terminal object.
By definition, since d and d' are terminal, for every $c \in C$ there exists a unique morphism $c \to d$ and a unique morphism $c \to d'$. I.e. $|\mathrm{Hom}(c, d)| = 1 = |\mathrm{Hom}(c, d')|$ for all $c \in C$. Choosing $c = d$, we thus find that there exists a unique morphism $f: d \to d'$ and the unique identity morphism $\mathbb{1}_d: d \to d$, and choosing $c = d'$, we find that there exists a unique morphism $g: d' \to d$ and the unique morphism $\mathbb{1}_{d'}: d' \to d'$.
Explicitly,

$$\begin{aligned}
\mathrm{Hom}(d, d') &= \{f\} \\
\mathrm{Hom}(d', d) &= \{g\} \\
\mathrm{Hom}(d, d) &= \{\mathbb{1}_d\} \\
\mathrm{Hom}(d', d') &= \{\mathbb{1}_{d'}\}\,.
\end{aligned}$$

Now, by the axioms of C being a category, there exists a composition map $gf: d \to d \in \mathrm{Hom}(d, d) = \{\mathbb{1}_d\}$, so $gf = \mathbb{1}_d$, and there exists a composition map $fg: d' \to d' \in \mathrm{Hom}(d', d') = \{\mathbb{1}_{d'}\}$, so $fg = \mathbb{1}_{d'}$.
Therefore f is an isomorphism $d \to d'$ and g is an isomorphism $d' \to d$, and since these are the unique maps in the hom-sets $\mathrm{Hom}(d, d')$ and $\mathrm{Hom}(d', d)$, respectively, these isomorphisms connecting the objects are unique.