

#### 1.2.4:

(a) We show it by induction.

For  $[a_0] = [b_0]$ , we have  $a_0 = b_0$ .

For  $[a_0; a_1] = [b_0; b_1]$ , we have  $\frac{h_1}{k_1} = \frac{a_0 a_1 + 1}{a_1} = [a_0; a_1] = [b_0; b_1] = \frac{b_0 b_1 + 1}{b_1} = \frac{h'_1}{k'_1}$ , and by corollary 1.36,  $\frac{a_0 a_1 + 1}{a_1}$  and  $\frac{b_0 b_1 + 1}{b_1}$  are in reduced form with  $a_1, b_1 \geq 1$ , so since they are equal, we have  $a_1 \mid (a_0 a_1 + 1) b_1 \implies a_1 \mid b_1$  and similarly,  $b_1 \mid a_1$  so  $b_1 = a_1$ . And then  $a_0 = b_0$  too.

Now assume  $[a_0, \dots, a_{n-1}] = [b_0, \dots, b_{n-1}]$ . Then we have

$$a_0 + \underbrace{\frac{1}{[a_1; \dots, a_n]}}_{\in (0,1]} = [a_0; a_1, \dots, a_n] = [b_0; b_1, \dots, b_n] = b_0 + \underbrace{\frac{1}{[b_1; b_2, \dots, b_n]}}_{\in (0,1]}$$

where the element in  $(0, 1]$  follows since  $[a_1; \dots, a_n]$  and  $[b_1; \dots, b_n]$  are positive since each  $a_i, b_j$  is positive. Since  $a_0$  and  $b_0$  are integers, we therefore have

$$|a_0 - b_0| = \left| \frac{1}{[a_1; \dots, a_n]} - \frac{1}{[b_1; \dots, b_n]} \right| \in [0, 1) \implies a_0 = b_0$$

since  $a_0 - b_0 \in \mathbb{Z}$ .

We then get

$$[a_1; \dots, a_n] = [b_1; \dots, b_n]$$

and by the inductive hypothesis, we then have  $a_i = b_i$  for all  $i = 0, \dots, n$ .

#### 1.2.5 - (5 points):

A = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1]

**def** conv(S, n):

    H = [0, 1]

    K = [1, 0]

**for** i **in** range(1, n+1):

        H.append(S[i-1] \* H[i] + H[i-1])

        K.append(S[i-1] \* K[i] + K[i-1])

**return** [H[-1], K[-1]]

**for** i **in** range(1, len(A)+1):

**print**(f"{i}: {conv(A, i)}")

The output was:

1: [2, 1]

2: [3, 1]

3: [8, 3]

4: [11, 4]

5: [19, 7]

6: [87, 32]

7: [106, 39]

8: [193, 71]

9: [1264, 465]

10: [1457, 536]

Where nth:  $[h_n, k_n]$  gives the  $n$ th convergent as  $\frac{h_n}{k_n}$ .