Problem 0.1 (1 (Principal G-bundles as a locally trivial bundle theory)**.** Let G be a discrete group. We consider the category Sm^G, where objects are smooth manifolds equipped with a free,fixed point free action by G which is properly discontinuous: there exists a cover $\{U_\alpha\}_{\alpha \in A}$ of M so that $\{g \cdot U_\alpha\}$ are pairwise disjoint for all $\alpha \in A$ and $g \in G$. Furthermore, morphisms are smooth maps which are G-equivariant: $f \colon M \to N$ is such that $f(g \cdot x) = g \cdot f(x)$ for all $g \in G$ and $x \in M$.

(1) Show that for $M \in \mathrm{Sm}^G$, the quotient M/G admits a structure of a smooth manifold so that the map $M \to M/G$ is a local diffeomorphism.
(2) Check that the association $M \mapsto M/G$ defines a functor $\mathrm{Sm}^G \to \mathrm{Sm}$, and show that this defines a locally trivial bundle theory on smooth manifolds.