Definition 0.1. The semi-simplex category consists of intervals $[0, \dots, p]$ and order preserving injections.

Remark. The standard simplicies, the boundary maps and the lemma for composition of face maps gives a functor $\Delta_{\text{semi}} \to \text{Top}$.

Lemma 0.2. *If G is a continuous left acting group on a space X on which it acts properly discontinuously and $H \leq G$ then the quotient map $X/H \to X/G$ is a covering space.*

Proof. We know that p_H and p_G are local homeomorphisms. Suppose $V \subset X$ is open such that $gV \cap V = \varnothing$ for all $g \neq e$. Then $p_H|_{gV} \colon gV \to p_H(gV)$ is a homeomorphism (indeed it is a continuous injective open map, hence an embedding), so if $U = p_H(gV)$, then $p'|_U = p_G|_{gV} p_H|_U^{-1} \colon U \to p_G(gV)$ is a homeomorphism. Hence p' is a local homeomorphism. But also, $p'^{-1}(p_G(V)) = \bigcup_{gH \text{ cosets}} p_H(gV)$, so in particular, this is a disjoint union of sets of the form above, so p' restricts to a homeomorphism on each of them. $\square$