Example 0.1. Define $f\colon \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} e^{-\frac{1}{x}}, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

By induction, one can show that f is smooth on $\mathbb{R}$ and that the derivatives $f^{(k)}(0)$ are equal to 0 for all $k \geq 0$.