Problem 1: Show homotopy equivalence is an equivalence relation.

Solution: Let $A \simeq B \simeq C$. Let $f\colon A \to B, g\colon B \to A$ and $\overline{f}\colon B \to C, \overline{g}\colon C \to B$ with $fg \simeq \mathbb{1}$ and $\overline{f}\overline{g} \simeq \mathbb{1}$ by $\gamma_1 = \mathbb{1}, \gamma_0 = fg$ and $\alpha_1 = \mathbb{1}, \alpha_0 = \overline{f}\overline{g}$. Then

$$F_t = \begin{cases} g\alpha_{2t}f, & t \in \left[0, \frac{1}{2}\right] \\ \gamma_{2t-1}, & t \in \left[\frac{1}{2}, 1\right] \end{cases}$$

is a homotopy equivalence between A and C hence showing transitivity.

1.1.7: The homotopy

$$F\colon \left(S^1 \times I\right) \times I \to S^1 \times I$$

by

$$F\left(\theta, s, t\right) = \left(\theta + 2\pi st, s\right)$$

Then $F\left(\theta, s, 0\right) = \mathbb{1}$ while $F\left(\theta, s, 1\right) = f\left(\theta, s\right)$ and F is continuous hence a homotopy - also, it fixed the circle for $s = 0$.

Rest done on paper.

1.1.8: No, the proof of borsuk-ulam fails when we conclude η is nullhomotopic on $S^1 \times S^1$ which it is not.
More specifically, the natural projection of the torus onto the 2d-plane works.
1.1.18: Assume $\alpha \in \pi_1\left(X\right)$. Let $\varphi\colon S^1 \to X$ be the attaching map. We can let $p\colon I \to S^1$ by $t \to (\cos 2\pi t, \sin 2\pi t)$. Then $\varphi \circ p$ is the attaching loop.
Let $x_0 = \left(\varphi \circ p\right)(0) \in A$. Now, since A is path-connected, there exists a path $\gamma\colon I \to A$ from $\alpha(0)$ to x_0. Then $\gamma\alpha\overline{\gamma}$ is a loop at x_0 in A.

Now choose a neighborhood around x_0 and let A_1 be the union of this neighborhood and the attached n-cell. Let A_2 be an ε-neighborhood of A in X that deformation retracts onto A - this becomes A union an annulus of the attached n-cell. And the annulus deformation retracts onto the attaching circle of A.

The intersection of A_1 and A_2 is path-connected - the union of the annulus with the neighborhood around x_0 -, so by lemma 1.15, the loop can be decomposed into running in A_1 and A_2. Any loop in A_1 however is nullhomotopic in X, so it can be decomposed into loops running in A_2 only. Now deformation retract A_2 to A and we find the loop α can be decomposed into loops running in A only.

1.1.19: Let $\gamma\colon I \to X$ be a loop in X. The CW complex is composed of a 0-skeleton with edges which are path connected.