**Theorem 0.1** (Linear function $\mathbb{R}^m \to \mathbb{R}$)**.** *A linear function $f\colon \mathbb{R}^m \to \mathbb{R}$ is uniformly continuous.*

*Proof.* Let $e_1, \ldots, e_m$ be the usual basis for $\mathbb{R}^m$. By linearity,

$$\begin{aligned} |f(x)|^2 &= |x_1 f(e_1) + \ldots + x_m f(e_m)|^2 \\ &\leq \left(x_1^2 + \ldots + x_m^2\right) \left(f\left(e_1\right)^2 + \ldots + f\left(e_m\right)^2\right) && \text{(Cauchy-Schwarz)} \end{aligned}$$

□

So for all $x \in \mathbb{R}$, we have

$$|f(x)| \leq M\|x\|, \quad M = \left(f(e_1)^2 + \ldots + f(e_m)^2\right)^{\frac{1}{2}}.$$

It follows that

$$|f(x) - f(x_0)| = |f\left(x - x_0\right)| \leq M\|x - x_0\|.$$

*Note.* This is only a weak version. In fact, we have the following theorem:

*Theorem* 0.2. [Lee, p 638] *A linear map between finite-dimensional normed linear spaces is continuous.*

**Corollary 0.3.** *Linear isomorphisms between finite-dimensional normed vector spaces are homeomorphisms.*

## References

[Lee] John Lee. Introduction to Smooth Manifolds, 2nd edition