Given a Δ-complex $(X, \Sigma = (\Sigma_p)_{p\in\mathbb{N}})$, we can consider the evaluation map $ev\colon \Sigma_n \times \Delta^n \to X^n$ by $(\sigma, t) \mapsto \sigma(t)$ where X^n is the n-skeleton

$$X^n = \cup_{p\leq n} \cup_{\sigma\in\Sigma_p} \sigma(\Delta^p)$$

.
We give each Σ_p the discrete topolgoy so that the evaluation map becomes continuous. We then are supposed to get a homeomorphism of quotients

$$Y := \frac{\Sigma_n \times \Delta^n}{\Sigma_n \times \partial\Delta^n} \xrightarrow{ev} \frac{X^n}{X^{n-1}}$$

.
I wanted to do this by showing that ev is closed, but I'm not sure if the details work out. Here it is:
Let $A \subset Y$ be closed and let $Z = \Sigma_n \times \partial\Delta^n$ and $\overline{Z} = \pi(Z)$ in the quotient. If $\overline{Z} \in A$ then $\pi^{-1}(Y - A) \subset \Sigma_n \times (\Delta^n - \partial\Delta^n)$ is open and ev is injective on here. I claim ev is a local homeomorphism on $\Sigma_n \times (\Delta^n - \partial\Delta^n)$: let $(\sigma, t) \in \Sigma_n \times (\Delta^n - \partial\Delta^n)$. Then we can find some open neighborhood U of t contained in $\Delta^n - \partial\Delta^n$ and since σ is injective on U and the subsets $\tau(\Delta^n - \partial\Delta^n)$ are disjoint for τ ranging over Σ_n, we have that $U = \sigma^{-1}(\sigma(U))$, hence $\sigma(U)$ is open in $X^n - X^{n-1}$, thus also saturated with respect to the quotient, hence also open in the quotient space. Bijective local homeomorphisms are homeomorphisms, so ev restricted to $\Sigma_n \times (\Delta^n - \partial\Delta^n)$ is an embedding, so $\frac{X^n}{X^{n-1}} - ev(A) = ev(\pi^{-1}(Y - A))$ is open, hence A is closed.
In the case where $\overline{Z} \not\subset A$, we have $\pi^{-1}(A) \subset \Sigma_n \times (\Delta^n - \partial\Delta^n)$ is closed, so by the above, its image under ev is closed.