5.0.1: Define the map $x \xrightarrow{f} \frac{x}{\|x\|+1}$ for $x \in \mathbb{R}^n$, and $g\colon (B^n)^\circ \to \mathbb{R}^n$ the inclusion. Then $H\colon \mathbb{R}^n \times I \to \mathbb{R}^n$ by

$$H\left(x,t\right) = tf(x) + (1-t)x$$

gives the homotopy $g \circ f \simeq \mathbb{1}_{\mathbb{R}^n}$ since $H(x,0) = \mathbb{1}_{\mathbb{R}^n}$ while $H(x,1) = g \circ f$, and H is continuous as $\mathbb{R}^n$ is convex.

Similarly, let $G\colon B^{n\circ} \times I \to B^{n\circ}$ be given by

$$G(x,t) = tf(x) + (1-t)x.$$

This is also a homotopy since $G(x,0) = \mathbb{1}_{B^{n\circ}}$ and $G(x,1) = f \circ g$, and it is continuous since the set $B^{n\circ}$ is convex and at all times

$$\|G(x,t)\| < 1$$

and is the linear homotopy connecting x and $f(x)$.

5.0.2: We have that $\mathbb{R}P^n$ is the quotient of S^n by the antipodal map. This is equivalent to taking a hemisphere D^n and identifying the antipodal points of ∂D^n. But since ∂D^n with antipodal points identified is $\mathbb{R}P^{n-1}$, we have that $\mathbb{R}P^n$ is just $\mathbb{R}P^{n-1}$ with an n-cell attached. By induction, we find that $\mathbb{R}P^n$ has a cell complex structure $e^0 \cup e^1 \cup \ldots \cup e^n$.