**Problem 0.1.** Let F be the homotopy fibre of the map  $S^n \to S^n$  of degree k, for  $n \ge 2$ .

- (1) Show that  $H^i(F) = 0$  for 0 < i < n.
- (2) Using the Serre spectral sequence, compute that

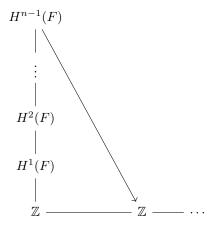
$$H^{i}(F) = \begin{cases} \mathbb{Z}, & i = 0\\ \mathbb{Z}/k, & i = 1 + m(n-1), m > 0\\ 0, & \text{otherwise} \end{cases}.$$

(3) Show that for  $x, y \in H^*(F)$ , if  $\deg(x), \deg(y) > 0$ , then  $x \smile y = 0$ .

*Proof.* (1) Since  $\pi_1 S^n = 0$ , the Serre spectral sequence to the homotopy fiber sequence

$$F \to S^n \to S^n$$

gives the following double complex:



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We apply the LSSS for cohomology and find that  $H^i(S^n) = F_0^n$ , and since  $H^i(F)$  is the only nontrivial entry on the antidiagonal in degree i, and since there are no maps to kill off  $H^i(F)$  for 0 < i < n-1, we obtain that  $H^i(F) = H^i(S^n) = 0$  for 0 < i < n-1.