I didn't finish the problem as I started quite late, and I'm not sure about the solutions. I'll work on these when I get the time, but comments would be helpful. Thank you.

Problem 0.1. (1) Let $p\colon X \to Y$ and $f\colon Z \to Y$ be maps. Let
$$E = \{(z,x) \in Z \times X \mid f(z) = p(x)\}$$
equipped with the subspace topology from $Z \times X$, and define $p'\colon E \to Z$ by $p'(z,x) = z$. Prove that if p is a covering map, then p' is also a covering map.

(2) Give a covering p, a map f such that p' is trivial and p is not trivial.

(3) If p is a trivial covering, can p' be non-trivial.

Solution. (i) Let $z \in Z$. Since p is a covering map, we can find some evenly covered neighborhood U of $f(z)$. Since $p|_{p^{-1}(U)}\colon p^{-1}(U) \to U$ is a trivial covering, the fiber of $f(z)$ over p splits into some discrete set of elements all mapped to $f(z)$ under p. Suppose $x \in p^{-1}(f(z))$. Then $(z,x) \in E$, so in particular, $V := E \cap \left(f^{-1}(U) \times p^{-1}(U)\right)$ contains $p^{-1}(z) \times \{z\}$ and is an open set in E. The set $W := f^{-1}(U)$ is open in Z.

We claim that W is an evenly covered neighborhood of z under p'. We first show that $p'^{-1}(W) = V$. If $(z,x) \in p'^{-1}(W)$ then $z \in W = f^{-1}(U)$, so $p(x) = f(z) \in U$, so $(z,x) \in E \cap \left(f^{-1}(U) \times p^{-1}(U)\right) = V$. Conversely, $V \subset W$ is clear. Letting $\varphi\colon p^{-1}(U) \to U \times F$ be the trivial covering for p, we define $\psi\colon p'^{-1}(W) \to W \times F$ by $\psi(z,x) = (z, \pi_F(\varphi(x)))$. Since φ is bijective, clearly, ψ is too. Continuity of ψ follows from continuity of each of the coordinate functions. Now, suppose $(z,x) \in p'^{-1}(W)$ and A is a neighborhood of $\varphi(z,x) = (z, \pi_F(\varphi(x)))$ in $W \times F$. Let B be a basis open set in W and hence in Z (since W is open in Z) containing z such that we have $B \times \{\pi_F(\varphi(x))\}$ open and contained in A. If necessary, we can shrink B such that $B \times \{\pi_F(\varphi(x))\}$ is evenly covered under φ. In particular, φ is a homeomorphism of $B' = \varphi^{-1}(B \times \{\pi_F(\varphi(x))\})$ onto $B \times \{\pi_F(\varphi(x))\}$, so B' is thus open in X. Now $E \cap (B \times B')$ is an open set in E containing (z,x) and $\varphi(z,x) \in \psi(E \cap (B \times B')) = B \times \{\pi_F(\varphi(x))\} \subset A$. So ψ^{-1} is also continuous, and hence ψ is a homeomorphism.

(ii) Let $p\colon \mathbb{R} \to S^1$ be the usual covering map of S^1 by $x \mapsto e^{2\pi i x}$. This is nontrivial. Let $f\colon V := (0, \frac{1}{2}) \to S^1$ be the restriction of p to V. Then $E = \{(z,x) \in V \times \mathbb{R} \mid f(z) = p(x)\} = \{(z,x) \in V \times \mathbb{R} \mid x \in z + \mathbb{Z}\}$. Then $p'\colon E \to V$ becomes the projection $E \to V$ onto the first coordinate. The map $\varphi\colon E \to V \times \mathbb{Z}$ by $(z,x) \mapsto (z, z-x)$ is a bijective map. For any basis open set $U \subset V$, we have that $\varphi^{-1}(U \times \{n\}) = T_{-n}(U)$, where $T_n\colon \mathbb{R} \to \mathbb{R}$ is the map $T_n(x) = x + n$, which is open in E since it is open in $V \times \mathbb{R}$. Thus φ is continuous.
Let $(z,x) \in V \times \mathbb{Z}$ and let B be an open neighborhood of $\varphi^{-1}(z,x) = (z, z+x)$. We can find an open neighborhood W in V of z such that $(z, z+x) \in E \cap (W \times T_x(W))$. But $(z,x) \in W \times \{x\}$ and $\varphi^{-1}(W \times \{x\}) \subset E \cap (W \times T_x(W)) \subset B$, so φ^{-1} is also continuous. Hence φ is a homeomorphism.

(iii) I didn't get to this one in time.