2.2.i State and prove the dual to Theorem 2.2.4, characterizing natural transformations $C(-,c) \implies F$ for a contravariant functor $F\colon C^{op} \to \text{Set}$.

Solution: Taking the dual of the theorem, we have that for any contravariant functor $F\colon C^{op} \to \text{Set}$, whose domain C^{op} is locally small and any object $c \in C^{op}$, there is a bijection

$$\text{Hom}\left(C^{op}(-,c), F\right) \cong Fc$$

that associates a natural transformation $\alpha\colon C^{op}(-,c) \implies F$ to the element $\alpha_c(1_c^{op}) \in Fc$. Moreover, this correspondence is natural in both c and F.

For the proof, we insert C^{op} in the Yoneda lemma and choosing a covariant functor $F\colon C^{op} \to \text{Set}$, so we get

$$\text{Hom}\left(C^{op}(-,c), F\right) = \text{Hom}\left(C(c,-), F\right) \cong Fc.$$

But a covariant functor $F\colon C^{op} \to \text{Set}$ is just a contravariant functor $C \to \text{Set}$ (which we usually just denote $F\colon C^{op} \to \text{Set}$ as in the statement of the dual theorem above), giving the desired bijection.
We check naturality in the functor and object.

Naturality in the functor asserts that given a natural transformation $\gamma\colon F \implies G$, the element of Gc representing the composite natural transformation $\gamma\alpha\colon C^{op}(-,c) \implies F \implies G$ is the image under $\gamma_c\colon Fc \to Gc$ of the element Fc representing $\alpha\colon C^{op}(-,c) \implies F$, i.e the diagram

$$\begin{tikzcd}
\text{Hom}\left(C^{op}(-,c), F\right) \arrow[r, "\Phi_F"] \arrow[d, "\gamma_*"] & Fc \arrow[d, "\gamma_c"] \\
\text{Hom}\left(C^{op}(-,c), G\right) \arrow[r, "\Phi_G"] & Gc
\end{tikzcd}$$

commutes in Set. By definition, $\Phi_G(\gamma\alpha) = (\gamma\alpha)_c \mathbb{1}_c$ which is $\gamma_c(\alpha_c\mathbb{1}_c)$ by the definition of vertical composition, and this is $\gamma_c\left(\Phi_F(\alpha)\right)$.

Naturality in the object asserts that given a morphism $f\colon d \to c$ in C, the element of Fd representing the composite natural transformation $\alpha f_*\colon C^{op}(-,d) \implies C^{op}(-,c) \implies F$ is the image under $Ff\colon Fc \to Fd$ of the element of Fc that represents α, i.e. the diagram

$$\begin{tikzcd}
\text{Hom}\left(C^{op}(-,d), F\right) \arrow[r, "\Phi_d"] \arrow[d, "(f_*)^*"] & Fd \arrow[d, "Ff"] \\
\text{Hom}\left(C^{op}(-,c), F\right) \arrow[r, "\Phi_c"] & Fc
\end{tikzcd}$$

commutes. Letting $\beta \in \text{Hom}\left(C^{op}(-,d), F\right)$, we get $Ff\left(\beta_d(\mathbb{1}_d)\right)$ along the top right, and $\Phi_c(\beta f_*) = (\beta f_*)_c(\mathbb{1}_c)$ along the bottom left. But $(\beta f_*)_c(\mathbb{1}_c)$ by definition of vertical composition is the function $\mathbb{1}_c \mapsto f \mapsto \beta_c(f)$ and $\beta_c(f) = Ff\left(\beta_d(\mathbb{1}_d)\right)$ by the commutative square (2.2.6) on page 58 in Riehl.
This gives naturality in the object.

2.2.ii: Explain why the Yoneda lemma does not dualize to classify natural transformations from an arbitrary set-valued functor to a represented functor.

Solution: When we formulated the dual version, we replaced C with C^{op} and considered covariant functors $C^{op} \to \text{Set}$. Taking the dual thus changed the direction of the morphisms in our category and the variance of our functor (as covariant $C^{op} \to \text{Set}$ is the same as contravariant $C \to \text{Set}$), but the direction of the functors (i.e. going $C \to \text{Set}$ or $C^{op} \to \text{Set}$) and natural transformations did not change, so the statement does not dualize to natural transformations going $F \implies C(c,-)$.