3.0.1:

(i) Let $f\colon \tilde{X} \to Y$ be a map, and $q\colon X \to \tilde{X}$ be a quotient map.
If f is continuous then $f \circ q$ is continuous as the composition of continuous maps.
If conversely $f \circ q$ is continuous, let $V \subset Y$ be open. Then $(f \circ q)^{-1}(V) = q^{-1} \circ f^{-1}(V)$ is open, and by definition $q^{-1}\left(f^{-1}(V)\right)$ is open if and only if $f^{-1}(V)$ is open in $\tilde{X}$, so $f^{-1}(V)$ is open, hence f is continuous.

(ii) Choose arbitrary $x \neq y$ of X and let f be the corresponding continuous function satisfying $f(x) \neq f(y)$.
Since f is continuous, $U = f^{-1}\left((-\infty, \frac{x+y}{2})\right)$ and $V = f^{-1}\left((\frac{x+y}{2}, \infty)\right)$ are open, disjoint and U contains $\min\{x, y\}$ while V contains $\max\{x, y\}$. Hence X is Hausdorff as x, y were arbitrary.

(iii) Define $\rho_w(P) = d(P, w) = \min\{d(w, p) \mid p \in P\}$ where d denotes standard metric inherited from $\mathbb{R}^n$.
This is continuous and gives the euclidean distance from w to P - it is in particular well defined as a minimum since there exists some closed ball D^n containing w and points from P - the intersection of D^n and P is closed hence compact, and thus by the extreme value theorem, the distance function attains a maximum on the compact subspace - which is clearly smaller than any distance outside the ball.
By (i) we have that ρ_w is continuous if and only if $\rho_w \circ q$ is continuous where $q\colon V_k^O(\mathbb{R}^n) \to Gr_k(\mathbb{R}^n)$ is the quotient map.
Now, for any interval $(a, b) \subset R$, we have $(\rho_w \circ q)^{-1}(a, b)$ is precisely all k-tuples of northonormal vectors in $\mathbb{R}^n$ such that their span contains a point whose distance to w is in (a, b).

ρ_w is naturally equal to the orthogonal projection P_U sending a vector $v = u + w$ with $u \in U$ and $w \in U^\perp$ with U as subspace of V to u, i.e. $P_U(v) = u$.
By 6.55 in LADR, we have that if we pick an orthonormal basis $(v_1, \ldots, v_k)$ representing the k-plane, we have $\rho_w(P) = P_P(w) = \langle w, v_1\rangle v_1 + \ldots + \langle w, v_k\rangle v_k$ which is continuous as it is linear (6.55.(i) and (a)).

(iv): Let $P_1, P_2 \in Gr_k(\mathbb{R}^n)$ be distinct. Choose any point $w \in P_1 - P_2$. Then $\rho_w(P_1) = 0 < \rho_w(P_2)$ where we can conclude strict inequality since we have min in our definition for ρ_w.

(i) For a 1-plane in $Gr_1(\mathbb{R}^2)$, we have $P \in U_X$ if it has trivial intersection with $X^\perp$ which is the orthogonal line of X through 0. Any line that is not $X^\perp$ will trivial intersection with $X^\perp$, so any 1-dimensional subspace of $\mathbb{R}^2$ that is not $X^\perp$ will be in U_X.

(ii) Any set in $\left\{P \in Gr_k(\mathbb{R}^n) : P \cap X^\perp = \{0\}\right\}$ is of the form $\Gamma(A) = \{Ax + x \text{ with } x \in X\}$ where $A\colon X \to X^\perp$ is a linear map. Defining $\varphi(A) = \Gamma(A)$, we find this is a bijection, so since the dimension of $\mathcal{L}(X, X^\perp)$ is $(n-k) \cdot k$ by identifying it with its matrix representation getting $M((n-k) \times k, \mathbb{R})$, we find that $U_X \cong \mathbb{R}^{(n-k)k}$.