16:
(a) Assume there is a retraction $r\colon \mathbb{R}^3 \to A$ where $A \cong S^1$. Then by proposition 1.17, the homomorphism $i_*\colon \pi_1\left(\mathbb{R}^3\right) \to \pi_1(A)$ induced by the inclusion $i\colon A \to X$ is injective. However, since $A \cong S^1$, we have $\pi_1(A) \approx \pi_1\left(S^1\right) \approx \mathbb{Z}$, while $\pi\left(\mathbb{R}^3\right) = 0$ since $\mathbb{R}^3$ is convex (example 1.4) - we dropped basepoints because both spaces are path-connected.
Therefore there would be an injective map $\mathbb{Z} \to \{0\}$ by proposition 1.17 which is impossible.

(b) We have $\pi\left(S^1 \times D^2\right) \cong \pi\left(S^1\right) \times \pi\left(D^2\right) \cong \pi(S_1) \cong \mathbb{Z}$.
On the other hand, $\pi\left(S^1 \times S^1\right) \cong \pi\left(S^1\right) \times \pi\left(S^1\right) \cong \mathbb{Z} \times \mathbb{Z}$.

Where the first isomorphism in both cases follows from proposition 1.12 and the fact that S^1 and D^2 are path-connected, and the last isomorphism follows from theorem 1.7.
If there existed a retraction from $S^1 \times D^2$ to $S^1 \times S^1$, then by proposition 1.17, there would exist an injective homomorphism from $\pi_1\left(S^1 \times S^1\right) \cong \mathbb{Z} \times \mathbb{Z}$ to $\pi_1\left(S_1 \times D^2\right) \cong \mathbb{Z}$ which is impossible: assume $\varphi\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is an injective homomorphism. Let $\varphi(1,0) = a, \varphi(0,1) = b$ with $a, b \neq 0$ since φ is assumed to be injective. Then $\varphi(x,y) = ax + by$, and hence $\varphi(b,-a) = ab - ab = 0$, so $a, b = 0$, contradiction.

(c) We first have that $\pi_1\left(S^1 \times D^2\right) \cong \pi_1\left(S^1\right) \times \pi_1\left(D^2\right) \cong \pi_1(S_1) \cong \mathbb{Z}$.
Explicitly, we have: let $\varphi\colon S^1 \times D^2 \to S^1$ be the map of the filled torus to its central circle, i.e. the map which collapses each meridian circle $\{x\} \times S^1$ to a point. This is a deformation retraction, and we can thus for any loop $f\colon I \to S^1 \times D^2$ compose f with φ to get a loop on S^1.
Now take the loop in A : $\gamma\colon I \to A$ that completes exactly one cycle. Then with the inclusion $i\colon A \to S^1 \times D^2$, we have $i\gamma\colon I \to S^1 \times D^2$ is a loop in $S^1 \times D^2$. Therefore $[i\gamma] \in \pi_1\left(S^1 \times D^2\right)$. Now φ induces a homomorphism $\varphi_*\colon \pi_1\left(S^1 \times D^2\right) \to \pi_1\left(S^1\right)$ by $\varphi_*[f] = [\varphi f]$. So $\varphi_*[i\gamma] = [\varphi i\gamma]$.
Thus $\varphi i\gamma$ is generated by the generating element of $\pi_1\left(S^1 \times D^2\right)$, call it a - where we have used theorem 1.7. By the projection, we see that $\varphi i\gamma$ corresponds to aa^{-1} which is nullhomotopic to the constant loop at $\varphi i\gamma(0)$ which we choose freely as our basepoint as S^1 is path-connected. Therefore $[\varphi i\gamma] = [0]$ where 0 denotes the constant loop at the basepoint. Since φ is a deformation retraction, the induced homomorphism is an isomorphism, so $[i\gamma] = [0]$.
Now, if $S^1 \times D^2$ were retractible to A, then the induced inclusion homomorphism $i_*\colon \pi_1(A) \to \pi_1\left(S^1 \times D^2\right)$ would map $[0]$ and $[\gamma]$ to different homotopy classes, but as we have seen, $[i\gamma] = [0]$ in $\pi_1\left(S^1 \times D^2\right)$, and thus i_* is not injective, so $S^1 \times D^2$ is not retractible to A.

(d) Both $D^2 \vee D^2$ and $S^1 \vee S^1$ are path-connected, so we can consider $\pi_1\left(D^2 \vee D^2\right)$ and $\pi_1\left(S^1 \vee S^1\right)$. Since $D^2 \vee D^2$ is star shaped with respect to the connecting point, it is deformation retractible to a point and thus has trivial fundamental group. If we can show that $\pi_1\left(S^1 \vee S^1\right)$ is non-trivial, then we are done since the induced inclusion $i_*\colon \pi_1\left(S^1 \vee S^1\right) \to \{0\}$ cannot be injective, and then the result follows by proposition 1.17.

Let $r|_{S_1^1}\colon S_1^1 \to S^1 \vee S^1$ be the map sending one of the spheres of $S^1 \vee S^1$ to the connecting point of $S^1 \vee S^1$. Let $r|_{S_2^1}\colon S_2^1 \to S^1 \vee S^1$ be the identity on the other sphere. Since S^1 is closed and the intersection of the domains is the connecting point which is a closed set, we find by the pasting lemma a retraction $r\colon S^1 \vee S^1 \to S^1 \vee S^1$ where $r(S^1 \vee S^1) = S^1$ and $r|_{S_2^1} = \mathbb{1}$.
So there is a retraction onto S^1, but thus we get an injective inclusion $i_*\colon \pi_1\left(S^1\right) \to \pi_1\left(S^1 \vee S^1\right)$ from proposition 1.17, and since $\pi_1\left(S^1\right) \cong \mathbb{Z}$, $\pi_1\left(S^1 \vee S^1\right)$ cannot be trivial.

(e) Assume that X is S_1 where $(0,1)$ and $(0,-1)$ are identified. Then there is a deformation retraction $F((x,y),t) = t(\sqrt{1-y^2}, y) + (1-t)(x,y)$ sending S^1 to the right side of S^1.
Since the ends of this curve are identified, this is just a 1-cell attached to a 0-cell which is S^1.

Hence we have that if there is a retraction from the disk with two points on its boundary identified to its boundary $S^1 \vee S^1$, then by proposition 1.17, it induces an injective homomorphism $i_*\colon \pi_1\left(S^1 \vee S^1\right) \to \pi_1\left(S^1\right)$; however, by the van Kampen Theorem (Example 1.21), we have $\pi_1\left(S^1 \vee S^1\right) \cong \mathbb{Z} * \mathbb{Z}$, and a map from $\mathbb{Z} * \mathbb{Z}$ into $\mathbb{Z}$ cannot be injective since the image would have to be an abelian subgroup, e.g.

20: By lemma 1.19, we have $f_{0*} = \beta_h f_{1*}$. Let $x_0 \in X$ be any point and let $[g] \in \pi_1(X, x_0)$. Then since

f_0 and f_1 are identity maps, we have f_{0*} and f_{1*} are identity maps, so

$$[g] = f_{0*}[g] = \beta_h f_{1*}[g] = \beta_h [g] = \left[h \cdot g \cdot \overline{h}\right] = [h][g]\left[\overline{h}\right]$$

If we apply $[h]$ on the right side, we get

$$[g][f_t(x_0)] = [g][h] = [h][g]\left[\overline{h}\right][h] = [h][g] = [f_t(x_0)][g]$$

Since $[g] \in \pi_1(X, x_0)$ and $x_0 \in X$ were arbitrary, we find that $f_t(x_0)$ is in the center of $\pi_1(X, x_0)$ for any $x_0 \in X$.