ASSIGNMENT 6

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Exercise 0.1. Assume $A \in \mathrm{End}(F^2)$ is diagonalizable with distinct non-zero eigenvalues λ and μ. Express A^{-1} as a polynomial of A.

Solution. Firstly, by corollary 6.18, since $\dim F^2 = 2$, $\sigma(A) = \{\lambda, \mu\}$. Now, since A is diagonalizable, we have $F^2 = V_\lambda \oplus V_\mu$. So choosing some non-zero $x_\lambda \in V_\lambda$ and $x_\mu \in V_\mu$, we have that with respect to the basis $\{x_\lambda, x_\mu\}$, the matrix for A is

$$[A] = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}$$

The inverse of this matrix is

$$[A]^{-1} = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & \mu^{-1} \end{pmatrix}.$$

That is $A^{-1} = \lambda^{-1}E_\lambda + \mu^{-1}E_\mu$. But $E_\lambda = p_\lambda(A) = \frac{A-\mu}{\lambda-\mu}$ and $E_\mu = p_\mu(A) = \frac{A-\lambda}{\mu-\lambda}$, so

$$\begin{aligned} A^{-1} &= \lambda^{-1}\frac{A-\mu}{\lambda-\mu} + \mu^{-1}\frac{A-\lambda}{\mu-\lambda} \\ &= \frac{\lambda^{-1}-\mu^{-1}}{\lambda-\mu}A - \frac{\lambda^{-1}\mu - \mu^{-1}\lambda}{\lambda-\mu}. \end{aligned}$$