1.3.iii: Let $\mathcal{C}$ be a category consisting of objects $1, 2, 3, 4$ and morphisms $1 \to 2$ and $3 \to 4$ with the identities on each object as well. This is clearly a category.
Now define a category $\mathcal{D}$ consisting of a, b, c with morphisms $a \to b, b \to c, a \to c$ and all identities. This is also a category.
Define the funcotr $F\colon \mathcal{C} \to \mathcal{D}$ collapsing $2, 3$ to b - i.e. $1 \to a$, $2, 3 \to b$ and $4 \to c$ with $(1 \to 2) \to (a \to b)$ and $(3 \to 4) \to (b \to c)$ and identities mapped to identiteis. This satisfies the functoriality axioms since the only composable maps in $\mathcal{C}$ were with identities, and since identities are mapped to identities. However, $a \to c$ is not in the image of F while $a \to b$ and $b \to c$ are in the image, hence the image is not a category.

1.3.iv: The functors $F = C\left(c, -\right)$ and $G = C\left(-, c\right)$ have been described in terms of objects and morphisms, so it remains to check the functoriality axioms.
Let $f = x \to y$ and $g = y \to z$ and $h = gf = x \to z$ be morphisms in C. Then $F(gf) = F\left(h\right) = h_*$ and $F(g)F(f) = g_* f_*$. Now, for any $\alpha \in C\left(c, x\right)$, we have

$$h_*\left(\alpha\right) = h\alpha = gf\alpha = g\left(f\alpha\right) = g_*\left(f\alpha\right) = g_*\left(f_*\left(\alpha\right)\right) = g_* f_*\left(\alpha\right)$$

so $F(gf) = F(g)F(f)$ for any composable f, g in C.
For each object $a \in C$, we have $F(a) = C(c, a)$ and $F(1_a) = 1_{a*}$ given by: for any $\gamma \in C(c, a)$, we have $1_{a*}(\gamma) = 1_a \gamma = \gamma$ by composition in C, so $F(1_a) = 1_{a*} = 1_{C(c,a)} = 1_{F(a)}$.

Taking the dual, we get that the opposite functor C^{op} is also a functor.

1.4.i: By the natural transformation $F \implies G$, we have that since $F(f)\alpha_c = F(f)\alpha_{c'}$ for an arbitrary morphism $f\colon c \to c'$, we have also $F(f)\alpha_c^{-1} = \alpha_{c'}^{-1}\alpha_{c'}F(f)\alpha_c^{-1} = \alpha_{c'}^{-1}G(f)\alpha_c\alpha_c^{-1} = \alpha_{c'}^{-1}G(f)$, so the following diagram commutes, and hece $\alpha^{-1}\colon G \implies F$ is a natural transformation.

$$\begin{tikzcd}
G(c) \arrow[r, "\alpha_c^{-1}"] \arrow[d, "G(f)"] & F(c) \arrow[d, "F(f)"] \\
G(c') \arrow[r, "\alpha_{c'}^{-1}"] & F(c')
\end{tikzcd}$$