

Exercise 0.1 (4.b). Define one or two more charts to give S the structure of a Riemann surface.

Proof. Take a small ball around each vertex of P_m and intersect it with P_m . Let V_i denote the open set at vertex v_i . Define a map $\psi_i: V_i \rightarrow \mathbb{C}$ by $\psi_i(z) = f_i(z - v_i)$, where $f_j: \mathbb{C} \rightarrow \mathbb{C}$ is given by

$$f_j(re^{i\theta}) = re^{i\frac{2\pi r_j}{n}} \left[e^{-i\left(\frac{(n-2+2j)\pi}{n}\right)} e^{i\theta} \right]^{\frac{2}{n-2}}$$

where r_j is the smallest positive integer such that $r_j\left(\frac{m}{2} - 1\right) \equiv j \pmod{m}$ □