Definition 0.1. For a map $f\colon M \to N$ between manifolds,

- f is called an immersion if locally at each point of M, it is iso to $\mathbb{R}^m \to \mathbb{R}^n$ sending $x \mapsto (x, 0)$.
- f is an embedding if it is an immersion, injective and induces iso with its image.
- f is a submersion if it is locally iso to $(x, y) \mapsto x$.

Definition 0.2. f is a bundle if it is locally on N of the form $X \times V \xrightarrow{\pi_2} V$.

Lemma 0.3. *The bundle $\Lambda^k E$ of k-fold exterior powers is a vector bundle when (E, π, X) is an n-dimensional vector bundle with bundle atlas $\mathcal{U}$. We construct this by forming a pre-vector bundle. Define $\Lambda^k E := \bigsqcup_{x\in X} \Lambda^k E_x$. Each $\Lambda^k E_x$ has dimension $\binom{n}{k}$, hence $\varphi_x\colon \Lambda^k E_x \cong \mathbb{R}^{\frac{n!}{k!(n-k)!}}$. Let the projection be the canonical one. Let the atlas be given by $\{(f_\alpha, U_\alpha)\}$ where $f_\alpha\colon \pi^{-1}(U_\alpha) \to U_\alpha \times \mathbb{R}^{\frac{n!}{k!(n-k)!}}$ is given by $\pi(-) \times \varphi_-$ and $(f, U_\alpha) \in \mathcal{U}$.*

Lemma 0.4 (Orientation cover)**.** *Using the exerior power bundle, we can construct the 1-dimensional bundle $\Lambda^n E$ for (E, π, X) an n-dimensional vector bundle. Define the equivalence relation in $\Lambda^n E - \{$zero section $\}$ by $x \sim y \iff y = \lambda x$ for some $\lambda > 0$. Give the equivalence classes $\tilde{X}(E)$ the quotient topology. Then we obtain a two sheeted cover of X by the canonical projection*

$$\tilde{X}(E) \xrightarrow{\tilde{\pi}} X$$

which is called the orientation cover of E.