

Problem 0.1. Let $T = S^1 \times S^1$ be the torus and $i: D^2 \hookrightarrow T$ an embedding of the unit disk that is disjoint from $S^1 \times \{s_0\}$. Define $A := (S^1 \times \{s_0\}) \cup i(S^1) \subset T$. Let $x_0 = (s_0, s_0)$ and $x_1 \in i(S^1)$.

- (1) Draw a picture of (X, A) and the two points x_0 and x_1 .
- (2) Construct an explicit bijection of sets $\pi_1(T, A, x_1) \cong \mathbb{Z}^2 \sqcup \mathbb{Z}$.
- (3) Compute the relative homotopy groups $\pi_2(T, A, x_0)$ and $\pi_2(T, A, x_1)$.

Problem 0.2. (1) Compute $\pi_1(S^1 \vee S^2)$ and describe the universal cover of $S^1 \vee S^2$.

- (2) Show that $\pi_2(S^1 \vee S^2)$ is isomorphic to $\bigoplus_{\mathbb{Z}} \mathbb{Z}$.
- (3) Explicitly describe the action of $\pi_1(S^1 \vee S^2)$ on $\bigoplus_{\mathbb{Z}} \mathbb{Z} \cong \pi_2(S^1 \vee S^2)$.

Problem 0.3. Let (X, A, x_0) be a pointed pair such that the inclusion $i: A \rightarrow X$ is based nullhomotopic (the nullhomotopy preserves the basepoint). The goal is to show that for $n \geq 2$, there is an isomorphism of groups:

$$\pi_n(X, A, x_0) \cong \pi_n(X, x_0) \times \pi_{n-1}(A, x_0).$$

- (1) Show that there is an exact sequence of groups

$$1 \rightarrow \pi_n(X, x_0) \xrightarrow{j_*} \pi_n(X, A, x_0) \xrightarrow{\partial_*} \pi_{n-1}(A, x_0) \rightarrow 1$$