1. Dummit and Foote

Exercise 1.1 (13.4.(5))**.** Let K be a finite extension of F. Prove that K is a splitting field over F if and only if every irreducible polynomial in $F[x]$ that has a root in K splits completely in $K[x]$.

Proof. Suppose $\{f_i\} \subset F[x]$ is the collection of polynomials over F that split over K. Suppose $g \in F[x]$ is irreducible and has a root $\alpha \in K$. We must show that $g \in \{f_i\}$. Since K is a finite extension, we have $K = F(\alpha_1, \ldots, \alpha_n)$ for α_i algebraic over F. Let K' be a splitting field for $\{f_i\} \cup \{g\}$ over F which we can obtain by adjoining roots of g' to K if we write $g(x) = (x - k)g'(x)$ for $k \in K$. Suppose we adjoin the roots $\beta_1, \ldots, \beta_m$. $\square$