Norwegian University of Science and Technology
Department of Marine Technology

Exercise set 4
Due February 9, 2020

1 **a)** A two-dimensional wave in deep water has a wave height of 5 m, wave length of 100 m and shape $\zeta = \zeta_a \cos(kx - \omega t)$. Show that the total wave energy is given as

$$E = \frac{1}{2}\rho g \zeta_a^2 \lambda \tag{1}$$

for one wave length and for each meter in the y-direction.
Given:

$$\int \cos^2(kx)\mathrm{d}x = \frac{\sin(2kx)}{4k} + \frac{x}{2} \tag{2}$$

b) Give a physical explanation: why do we have potential energy in a wave (over one wave length with crest and trough)?

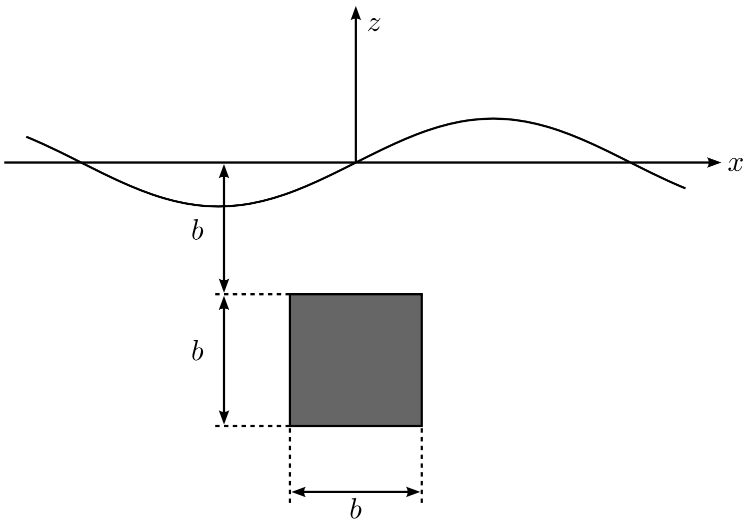
2 A towing tank is 150 m long and 5 m wide. In one end we find a wave generator capable of making regular waves. We make waves with amplitude $\zeta_a = 0.2$ m and period $T = 2.0$ s. We assume the velocity potential for the waves (deep water) can be described by

$$\phi = \frac{\zeta_a g}{\omega} \mathrm{e}^{kz} \sin(kx - \omega t) \tag{3}$$

a) Sketch the fluid particle orbits at the surface. Give quantities in your sketch. How do the fluid particle orbits look like at a depth of 0.5 m? Where in the tank is the motion of the fluid particles equal to 1% of the maximum value?

b) How large should the water depth be before you can feel safe that the "deep water" assumption is valid? How will the water particle orbits be on the surface and at the sea bottom if the "deep water" assumption is not valid? No calculations needed, only sketches.

3 We are to calculate excitation forces (per meter in y-direction) on a tank with a quadratic cross section situated with its top surface 10 m below the still water level. Waves of length 100 m and amplitude 2 m propagate in the positive x-direction. The side surfaces of the tank are $b = 10$ m. The density of sea water is 1025 kg m^{-3}.



Calculate the Froude-Kriloff forces on the cross section in the horizontal and vertical direction. Assume the velocity potential for waves in deep water is given as

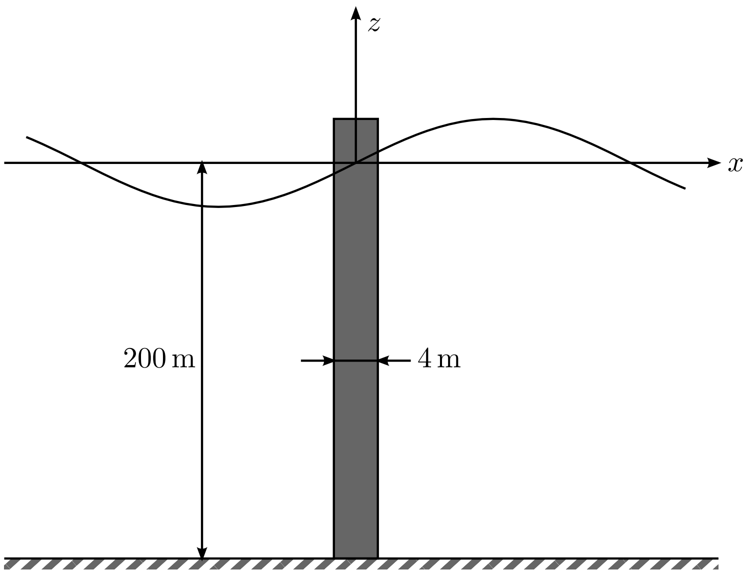
$$\phi = \frac{\zeta_a g}{\omega} \mathrm{e}^{kz} \sin(kx - \omega t) \tag{4}$$

It might be useful to know of the following trigonometric identities, which can be found for instance on page 87 of "Matematisk Formelsamling" by Karl Rottmann:

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \tag{5}$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) \tag{6}$$

4 A rigid, inelastic circular cylinder with a diameter of 4 m is fixed to the seabed and exposed to waves with wave length 100 m and wave height 4 m. The water depth is 200 m.



a) What is the Reynolds number? Choose $C_M = 2.0$ and $C_D = 0.7$ and find the mass and drag forces on the pole by using Morison's equation.

b) How large is the phase angle between the mass and drag forces?

c) How large is the maximum resulting force? Note: This task is difficult to solve analytically, and we recommend you to use a numerical approach, e.g. using Matlab or Python.

Simple numerical solutions

2. (a) $z_{0.01} = -4.6$ m

3. $F_1 = -49.2 \times 10^3 \sin(\omega t)$ N
   $F_3 = -49.2 \times 10^3 \cos(\omega t)$ N

4. (a) $F_{M\max} \approx 505 \times 10^3$ N
   $F_{D\max} \approx 28.2 \times 10^3$ N
   (b) $\pi/2$ radians $(90°)$
   (c) $F_{\max} \approx 505 \times 10^3$ N