Norwegian University of Science
and Technology
Department of Marine Technology

TMR4247 - Marine Technology 3
Spring 2020

Exercise set 3
Due February 2, 2020

1 Consider regular waves propagating along the positive x-axis with period 12 s and wave height 8 m. The water depth is 50 m.

a) Find the velocity components and acceleration components of the water particles, and the dynamic pressure at the depth of $z = -10$ m.

Choose a time instant and sketch these variables as functions of x. Also sketch the wave profile.

b) Make vector plots of the velocity and acceleration fields and a contour plot of the pressure field under the wave. Change the different wave parameters and notice how this affects the results. NB: Remember that velocities and accelerations are functions of both vertical position z and horizontal position x, in addition to time. Choose a time instant, e.g. t = 0, for plotting.

Hint: Use a suitable computer program to assist you in solving this exercise, for example Matlab or Python. We have created a demonstration on how to create similar plots in a tool called *IPython Notebook*, which is located at `http://nbviewer.ipython.org/url/folk.ntnu.no/hakostra/TMR4247/Waves.ipynb` (be patient when loading that page - it's not so fast). You should be able to capture the general concepts from there, and transfer them into a script of your own. Examples of how to create vector field plots and contour plots of field variables can be found together with this exercise on Blackboard.

2 A linear, regular wave in deep water can be described by the velocity potential

$$\phi = \frac{g\zeta_a}{\omega} e^{kz} \cos(\omega t - kx) \tag{1}$$

a) Which assumptions about the fluid do we have to make in order to describe the fluid motion by a velocity potential?

b) Which boundary conditions do we have to satisfy at the free surface?

c) Show that the wave's surface profile, with the given velocity potential, becomes

$$\zeta = \zeta_a \sin(\omega t - kx) \tag{2}$$

d) Show that this wave propagates in the positive x-direction.

e) What is the phase velocity (wave velocity) of the wave if the period is 10 s? What is the maximum water particle velocity in x-direction in the wave when the wave amplitude is 1 m?

f) Sketch the pressure contours (dynamic pressure) below the wave profile (over one wave length) for different depths, and show on the figure how the amplitudes vary with depth.

3 Given the velocity potential

$$\phi = \frac{g\zeta_a}{\omega}\frac{\cosh[k(z+h)]}{\cosh kh}\cos(\omega t - kx\cos\beta - ky\sin\beta) \tag{3}$$

In which direction do the waves propagate? Make a sketch.

Simple numerical solutions

1. (a) $\dot{u} \approx 0.92\sin(0.0307x - 0.5236t)\ \mathrm{m\,s^{-1}}$
   $\dot{w} \approx -0.77\cos(0.0307x - 0.5236t)\ \mathrm{m\,s^{-2}}$
   $p_d \approx 3.1 \cdot 10^4\cos(0.0307x - 0.5236t)\ \mathrm{N\,m^{-2}}$
2. (e) $c = 15.6\,\mathrm{m\,s^{-1}}$, $u_{\max} = 0.63\,\mathrm{m\,s^{-1}}$