1a) $u = \frac{kg\zeta_a}{\omega} \frac{\cosh(k(h+z))}{\cosh(kh)} \cos(kx - \omega t)$

$\dot{u} = kg\zeta_a$ —||— $\sin(kx-\omega t)$

$w = \frac{kg\zeta_a}{\omega} \frac{\sinh(k(h+z))}{\cosh(kh)} \sin(kx-\omega t)$

$\dot{w} = -kg\zeta_a$ —||— $\cos(kx-\omega t$

$p_d = \rho g \zeta_a \frac{\cosh(k(h+z))}{\cosh(kh)} \cos(kx-\omega t)$

At $z = -10$

$u = 1{,}76 \cos(0.0307x - 0{,}524t)$ m/s

$\dot{u} = 0{,}919 \quad \sin(\quad)$ m/s^2

$w = 1{,}477 \quad \sin(\ )$ m/s

$\dot{w} = -0{,}774 \cos(\ )$ m/s^2

$p_d = 30{,}7 \cdot 10^3 \cos(\ )$ N/m^2

2 a) $\Phi = \frac{g \zeta_A}{\omega} e^{kz} \cos(\omega t - kx)$

a) Velocity potential is only possible for incompressible, irrotational flow and inviscid Fluid.

$\nabla \times \underline{u} = 0$

$\nu = \frac{\mu}{\rho} = 0 \rightarrow \mu = 0$

No shear forces

$\nabla \cdot \underline{u} = 0$

$(= div(u) = 0)$ (also called continuity eq.)

2b At free surface kinematic and Dynamic boundary conditions must be satisfied.

Kinematic

$$\boxed{\frac{\partial \zeta}{\partial t} = \left(\frac{\partial \varphi}{\partial z}\right)_{z=0}}$$

From observations that waterparticle at surface of small wave stays at surface.

$$\boxed{\frac{\partial^2 \varphi}{\partial t^2} + g\frac{\partial \varphi}{\partial z} = 0 \quad \text{on } z=0}$$

$\leftrightarrow$ combination of Dynamic & Kinematic BC.

Dynamic BC: From Bernoulli at free surface, Pressure at surface = 1ATM.

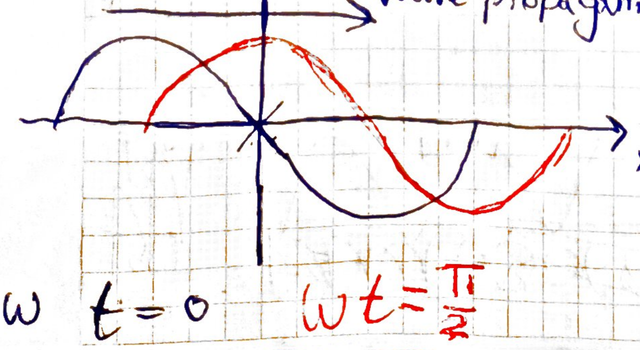
$$\boxed{g\zeta + \left(\frac{\partial \varphi}{\partial t}\right)_{z=0} = 0}$$

2C Insert φ into Dynamic BC: $g\zeta + \left(\frac{\partial \varphi}{\partial t}\right)_{z=0} = 0$

$$\frac{\partial \varphi}{\partial t} = -g\,\zeta_a e^{kz} \sin(\omega t - kx)$$

$$\zeta = -\frac{1}{g}\left(\frac{\partial \varphi}{\partial t}\right)_{z=0} = \underline{\zeta_a \sin(\omega t - kx)}$$

2d ~~Airtonud~~ ~~xxxx~~



$\boxed{2d}$ Alternative $\zeta = \zeta_a \sin(\omega t_0 - kx_0) = \zeta_a \sin(\omega t_1 - kx_1)$

$= 7.$

$\omega t_0 - kx_0 = \frac{\pi}{2}$

$\omega > 0$ and $k > 0$

$X_0 = \frac{\omega t_0}{k} - \frac{\pi}{2k}$

As t increases x must also increase.

$X_1 = \frac{\omega t_1}{k} - \frac{\pi}{2k}$

. Wave propagates in positive x-direction.

$\boxed{2e}$ $C_w = \frac{\lambda}{T} = \frac{\omega}{k}$

$\boxed{k \cdot \lambda = 2\pi}$

$T = \frac{2\pi}{\omega}$

In deep water $\omega^2 = kg$

$$\boxed{C_w = \frac{gT}{2\pi}}$$

$\frac{\omega^2}{g} \lambda = 2\pi$

$\lambda = \frac{2\pi g}{\omega^2} = \frac{2\pi g}{\left(\frac{2\pi}{T}\right)^2}$

$\lambda = \frac{gT^2}{2\pi}$

$\underline{C_w = 15{,}6 \text{ m/s}}$

$u = \frac{d\varphi}{dx} = \frac{g \zeta_a k}{\omega} \cancel{e^{zk}} \sin(\omega t - kx)$ $1,\ z=0$

$u = \omega \zeta_a \sin(\omega t - kx)$

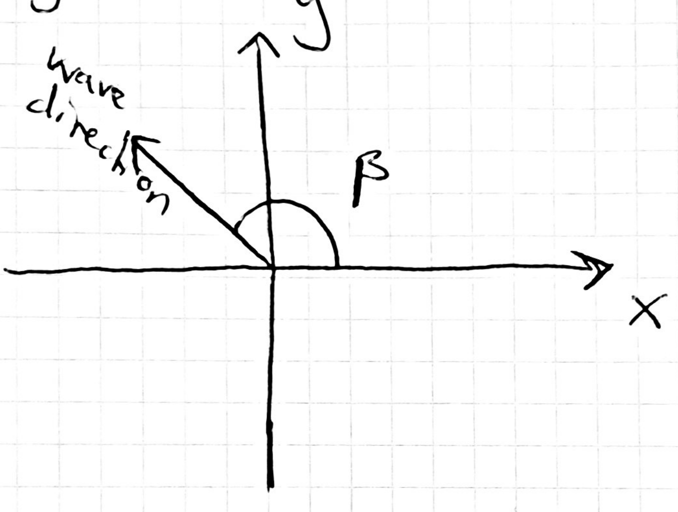
$U_{max} = \frac{2\pi}{10s} 1m = \frac{\pi}{5} = \underline{0{,}628 \text{ m/s}}$

When $\underline{\omega t - kx = \frac{\pi}{2}}$

$\boxed{3}$

$$u = \frac{d\varphi}{dx} \propto k \cos\beta \; \sin(\omega t - kx\cos\beta - ky\sin\beta)$$

$$v = \frac{d\varphi}{dy} \propto k \sin\beta \; \sin(\omega t - kx\cos\beta - ky\sin\beta)$$



<u>Max wave velocity</u>: $\frac{gk\zeta_a}{\omega} = \underline{\omega\zeta_a}$

(Horizontal)