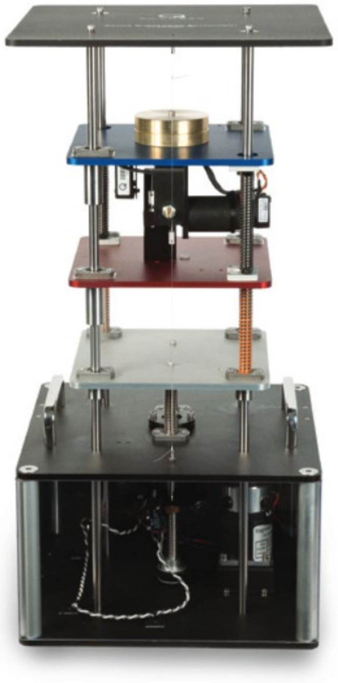
CSSC at UT Dallas

Introduction to Model Predictive Control Workshop



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May 2nd, 2025

Model Predictive Control Workshop

- Intro to MPC
- Components to MPC
- Quarter Car MPC Example
- Drawbacks of MPC

What is Model Predictive Control (MPC)?

- A receding-horizon optimal control framework that uses a dynamic model of a system to predict the future response of the system

- By solving a finite-time horizon, open-loop, optimal control problem using the current state of the system, MPC determines a sequence of control decisions which minimize the specified cost function over the prediction horizon

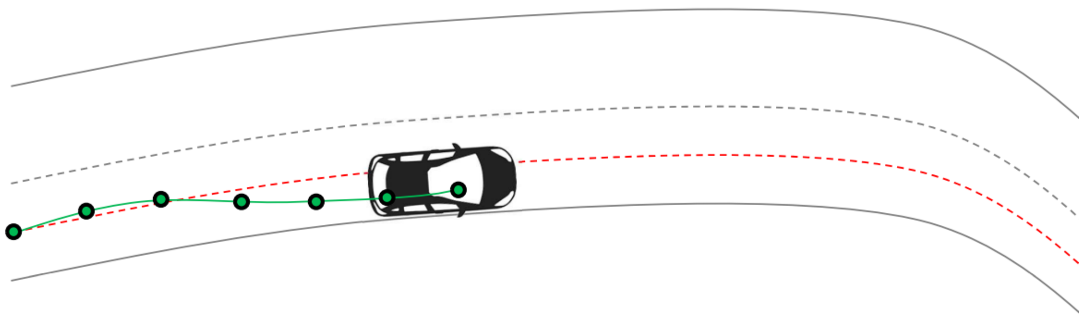
- The first element of this control sequence is applied to the system and the procedure is repeated at the next time instance.

Car Analogy PID vs. MPC

$$u = k_p e + k_i \int e\,dt + k_d \dot{e}$$

Steering is only based on past tracking error

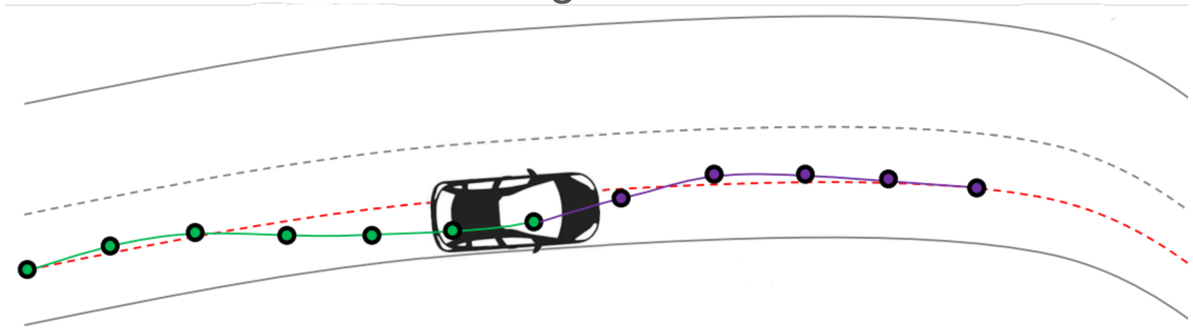
Drive using side view and rear mirrors



$$\min_{U_t} \sum_{k=t}^{t+N-1} J(x_k, u_k) + J_T(x_{t+N}, u_{t+N})$$

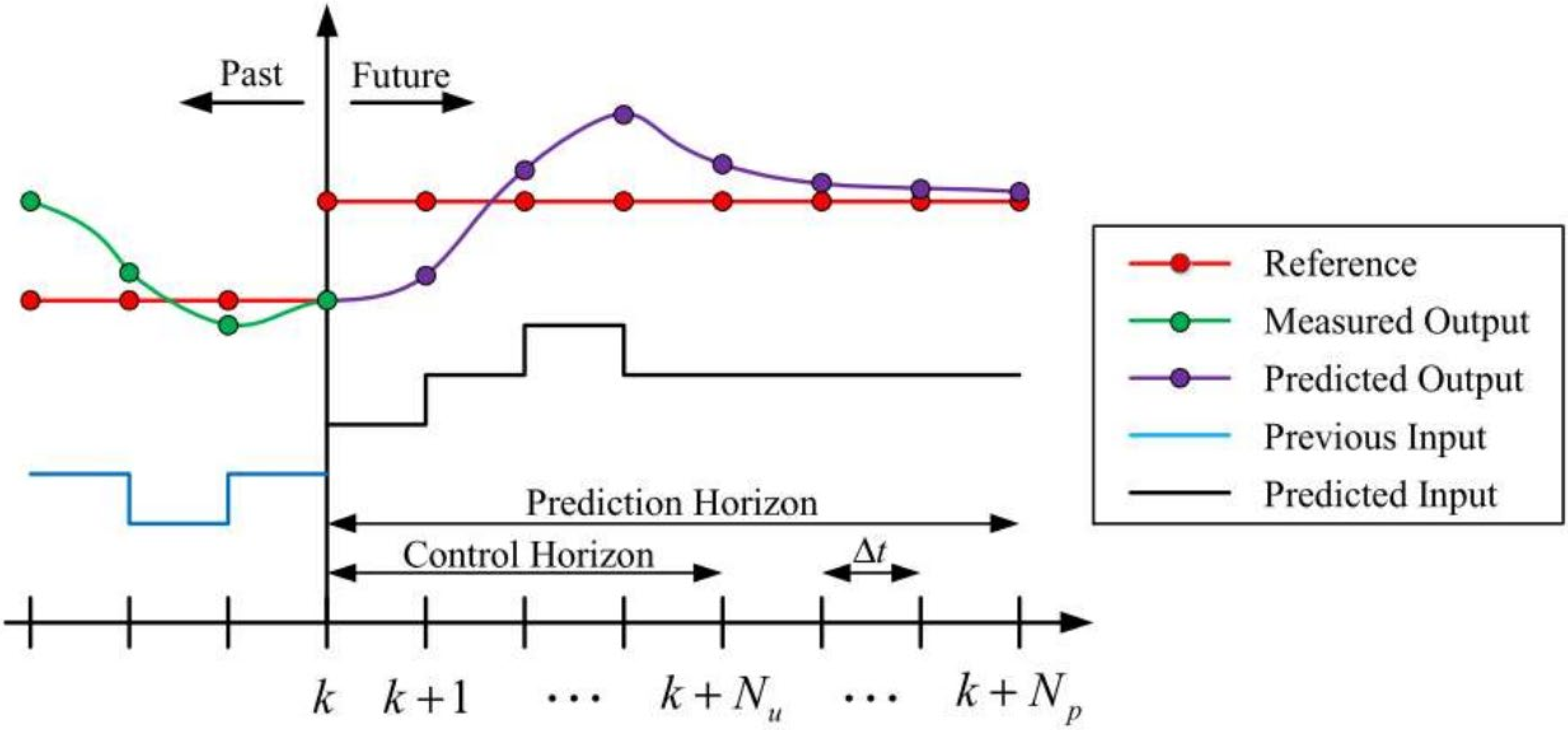
Steering is based on past tracking error and looking ahead

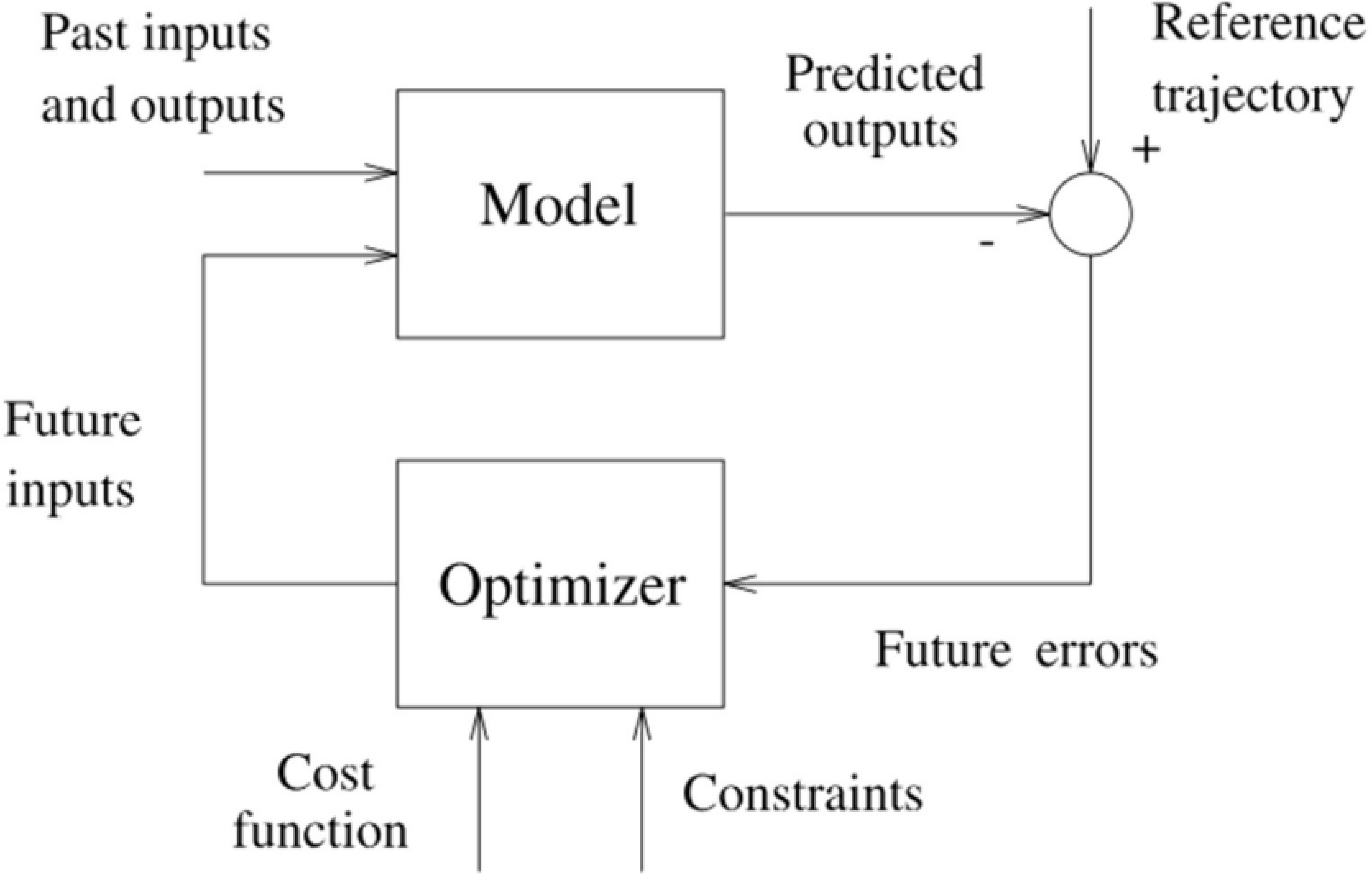
Drive using side view, rear mirrors, and looking ahead



Car Analogy PID vs. MPC

Weakest PID fan vs. Strongest MPC enjoyer


Past
Future
Reference
Measured Output
Predicted Output
Previous Input
Predicted Input
Prediction Horizon
Control Horizon
Δt
k
$k+1$
$\cdots$
$k+N_u$
$\cdots$
$k+N_p$


Past inputs
and outputs
Model
Predicted
outputs
Reference
trajectory
+
-
Future
inputs
Optimizer
Future errors
Cost
function
Constraints

Components to MPC

Model: Need a mathematical model that captures the system. A good model should enable easy predictions, result in an easy optimization problem, and faithfully react the physical system. Ideally, we want a linear model for the reasons above.

Optimization Problem: A quadratic cost function is often minimized since there is one minimum or maximum

Constraint Handling: Allows for the direct embedding of limits in the control input, states, and output.

Mathematical Formulation

$$\boxed{J(k) = x_k{}^T Q x_k + u_k{}^T R u_k}$$

Cost Function to be minimized via optimization

Subject to (s.t):

- Dynamic system constraints (i.e. state-space model)
- Input, output, state constraints
- Initial condition constraint

1. Measure current state x(t)
2. Solve for the optimal input sequence, U_t^*
3. Only apply the first input, u_t^*
4. Repeat at t+1

$$\min_{U_t} \sum_{k=t}^{t+N-1} J(x_k, u_k) + J_T(x_{t+N}, u_{t+N})$$

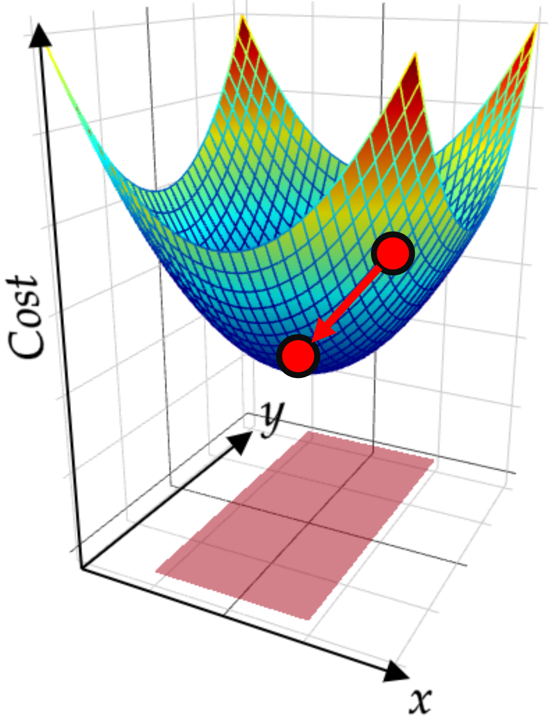
$$s.t. \;\; \forall k = t, \ldots, t+N-1$$

$$x_{k+1} = f(x_k, u_k),$$

$$u_k \in \mathcal{U}, \; x_k \in \mathcal{X}, \; y_k = g(x_k, u_k) \in \mathcal{Y}$$

$$x_t = x(t)$$

Optimization Problem



$$\boxed{J(k) = x_k{}^T Q x_k + u_k{}^T R u_k}$$

$$\min_{U_t} \sum_{k=t}^{t+N-1} J(x_k, u_k) + J_T(x_{t+N}, u_{t+N})$$

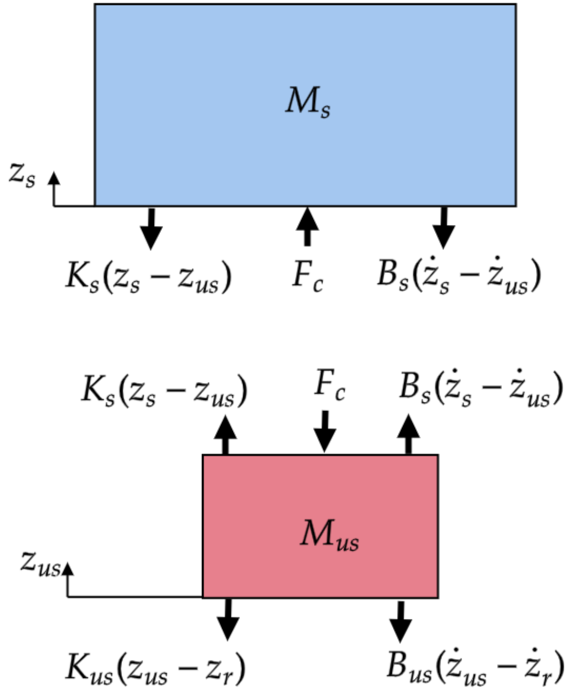
$$s.t.\ \forall k = t, \ldots, t+N-1$$

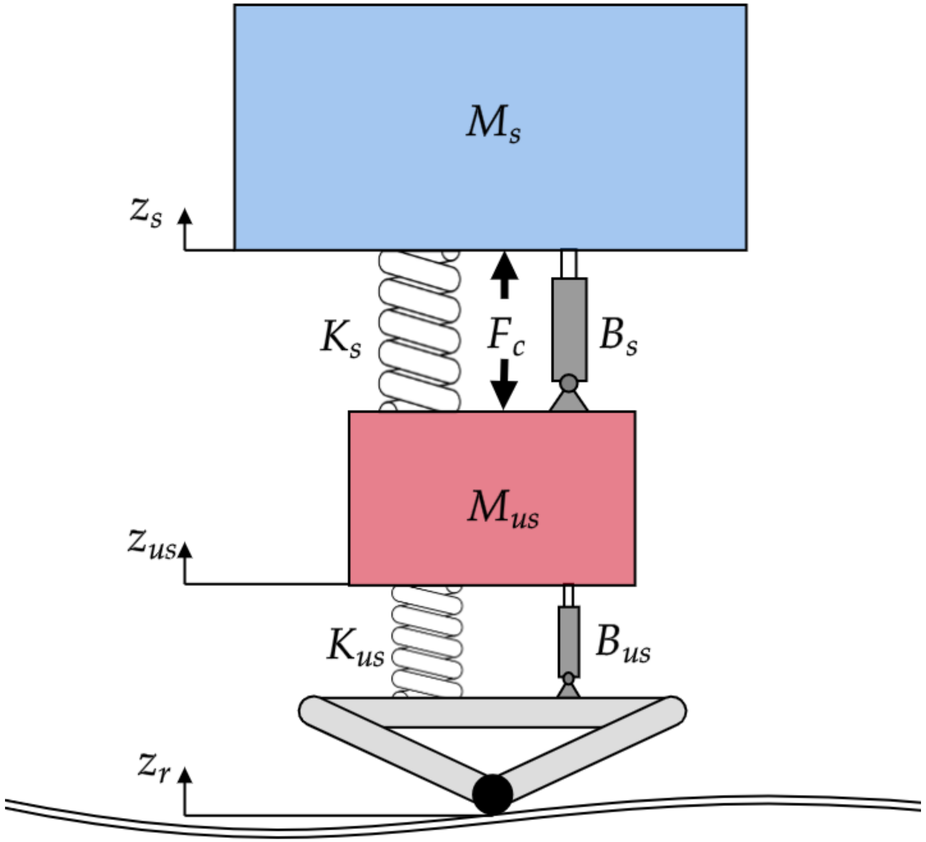
$$x_{k+1} = f(x_k, u_k),$$

$$u_k \in \mathcal{U},\ x_k \in \mathcal{X},\ y_k = g(x_k, u_k) \in \mathcal{Y}$$

$$x_t = x(t)$$

Quarter Car Model



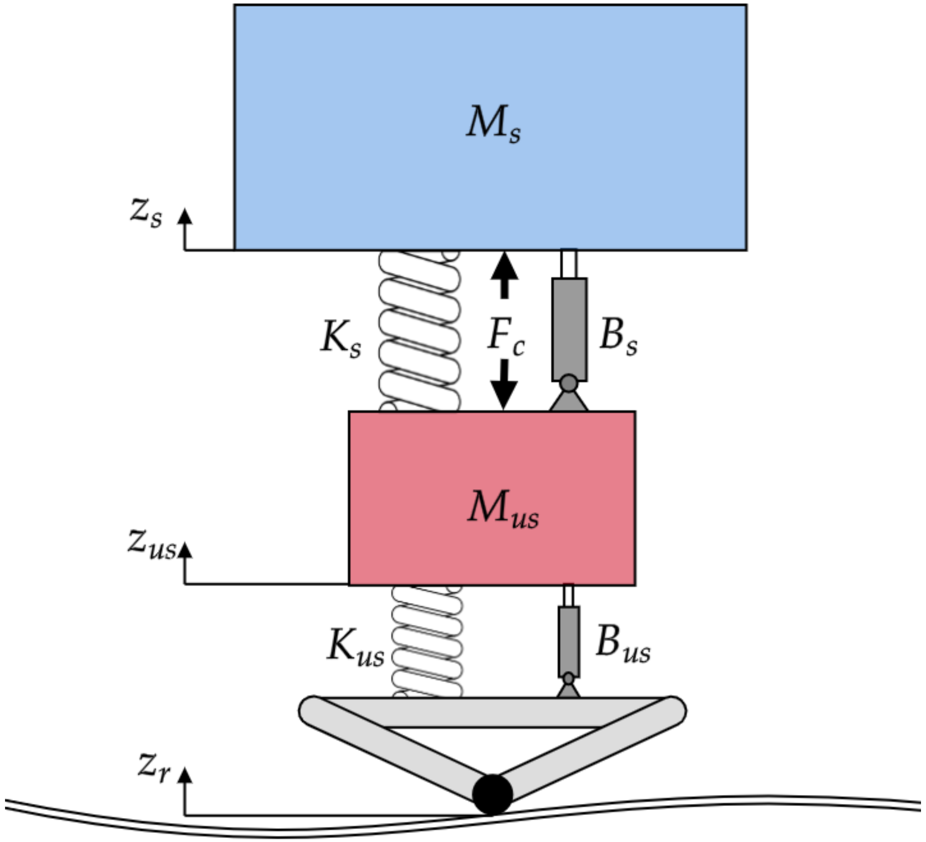


$$M_s\ddot{z}_s = F_c - K_s(z_s - z_{us}) - B_s(\dot{z}_s - \dot{z}_{us})$$
$$M_{us}\ddot{z}_{us} = -F_c + K_s(z_s - z_{us}) + B_s(\dot{z}_s - \dot{z}_{us}) - K_{us}(z_{us} - z_r) - B_{us}(\dot{z}_{us} - \dot{z}_r)$$

Quarter Car Model

$$\begin{bmatrix}\dot{z}_{us}-\dot{z}_r\\ \ddot{z}_{us}\\ \dot{z}_s-\dot{z}_{us}\\ \ddot{z}_s\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_{us}}{M_{us}} & -\frac{B_s+B_{us}}{M_{us}} & \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}}\\ 0 & -1 & 0 & 1\\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s}\end{bmatrix}\begin{bmatrix}z_{us}-z_r\\ \dot{z}_{us}\\ z_s-z_{us}\\ \dot{z}_s\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{M_{us}}\\ 0\\ \frac{1}{M_s}\end{bmatrix}u + \begin{bmatrix}-1 & 0\\ \frac{B_{us}}{M_{us}} & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{z}_r\\ z_r\end{bmatrix}$$

$$\begin{bmatrix}z_{us}\\ z_s\\ \ddot{z}_s\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s}\end{bmatrix}\begin{bmatrix}z_{us}-z_r\\ z_{us}\\ z_s-z_{us}\\ z_s\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{M_s}\end{bmatrix}u + \begin{bmatrix}0 & 1\\ 0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{z}_r\\ z_r\end{bmatrix}$$



Task 1: Define the quarter car model dynamics & perform open-loop simulation

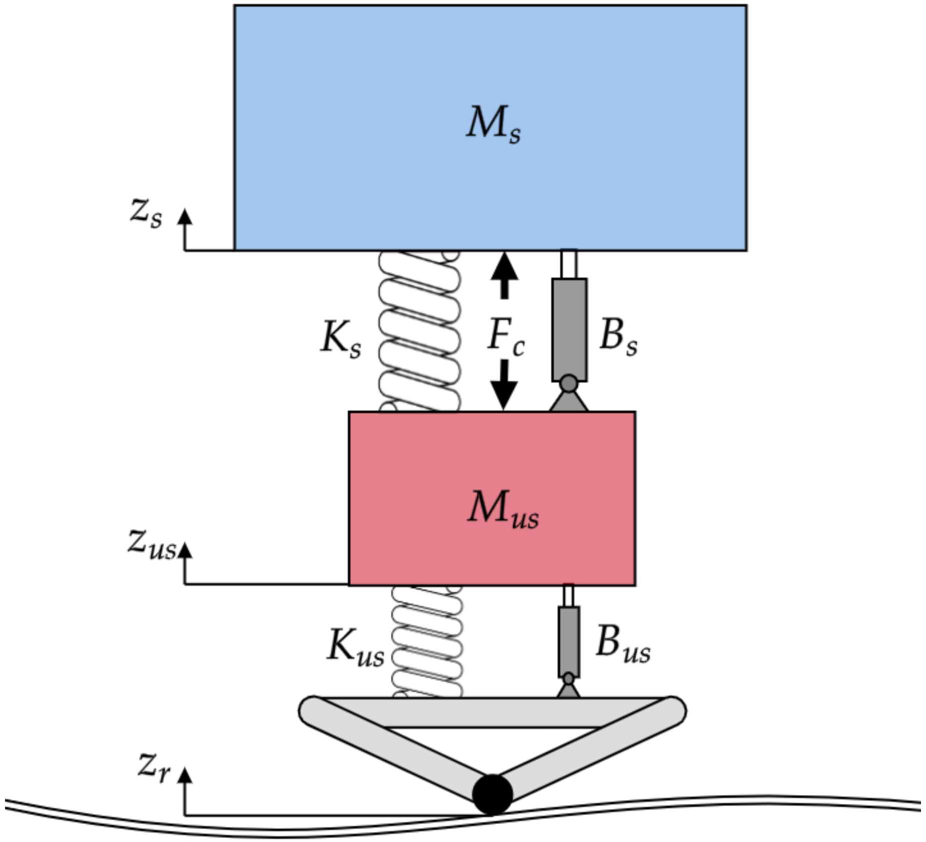
Task 2: Find the gains for PD controller that stabilizes the system

MPC Problem Formulation

$$\min_{U_t} \sum_{k=t}^{t+N-1} J(x_k, u_k) + J_T(x_{t+N}, u_{t+N})$$

$$\begin{aligned} s.t. \quad & \forall k = t, \ldots, t+N-1 \\ & x_{k+1} = Ax_k + Bu_k + Ew_k, \\ & -30 \le u_k \le 30, \\ & [\,-0.01 \;\; -1 \;\; -0.03 \;\; -1\,]^\top \le x_k \le [\,-0.01 \;\; -1 \;\; -0.03 \;\; -1\,] \end{aligned}$$

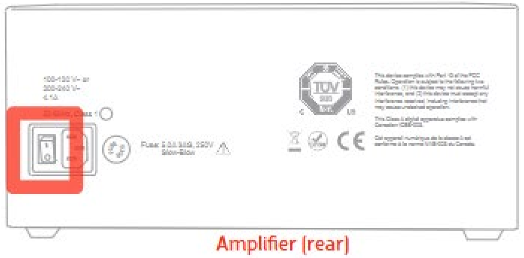
Task 3: Find the gains for the MPC cost function



MPC Experiment

Steps to start the test bed:

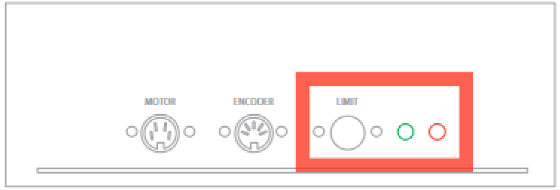
1.



Amplifier (rear)

Turn ON the power switch on the AMPAQ-L2. It is located on the rear of the device.

2.



Active Suspension

Manually move the white plate until it hits the upper and lower limit switches, i.e. when the red LED is lit. Then, move the plate to the mid position and push the **Limit** button to turn on the green LED, i.e enable the motor power.

Drawbacks of MPC

Feasibility: How can we guarantee that the optimization problem has a valid solution that satisfies all constraints?

Stability: How can we guarantee that the controller makes the system closed-loop stable?

Real-time Implementation: Solving an optimization problem can take a lot of time, can we ensure that the we solve the problem fast enough for control?

Model Sophistication: The effectiveness of the controller is sensitive on the plant model, how are we able to produce a good enough model that describes our system?

These questions are answered if you take the MPC course!