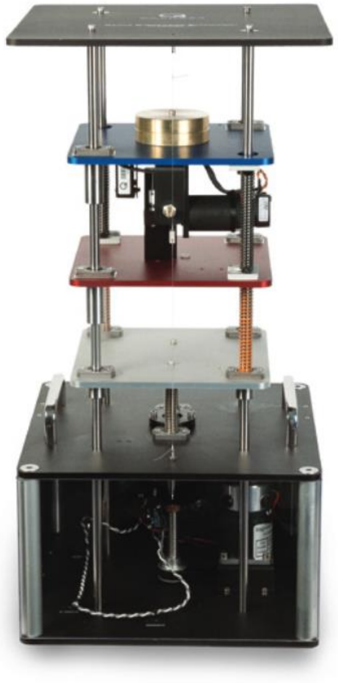
CSSC at UT Dallas

The System Identification Workshop



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Dec 4th, 2025

System Identification Workshop

- Intro to System ID
- Different Models
  - Black box, grey box, physics-based
  - Linear, nonlinear, online
- Dealing with real data
  - Noise
  - Disturbances
  - Delays
- Data collection techniques
- Brief intro to Nonlinear System ID

What are System Models?

Mathematical representations of the real world

Every model must be:

- Simple enough to be useful and computable
- Complex enough to capture the essence of the real system

Domains where models are used:

- Controller Design
- State Estimation
- Simulation
- Formal Analysis

What is System Identification?

The technique of using data to develop a model of a dynamic system

The natural generalization of curve fitting allowing the system's response to different inputs to be predicted

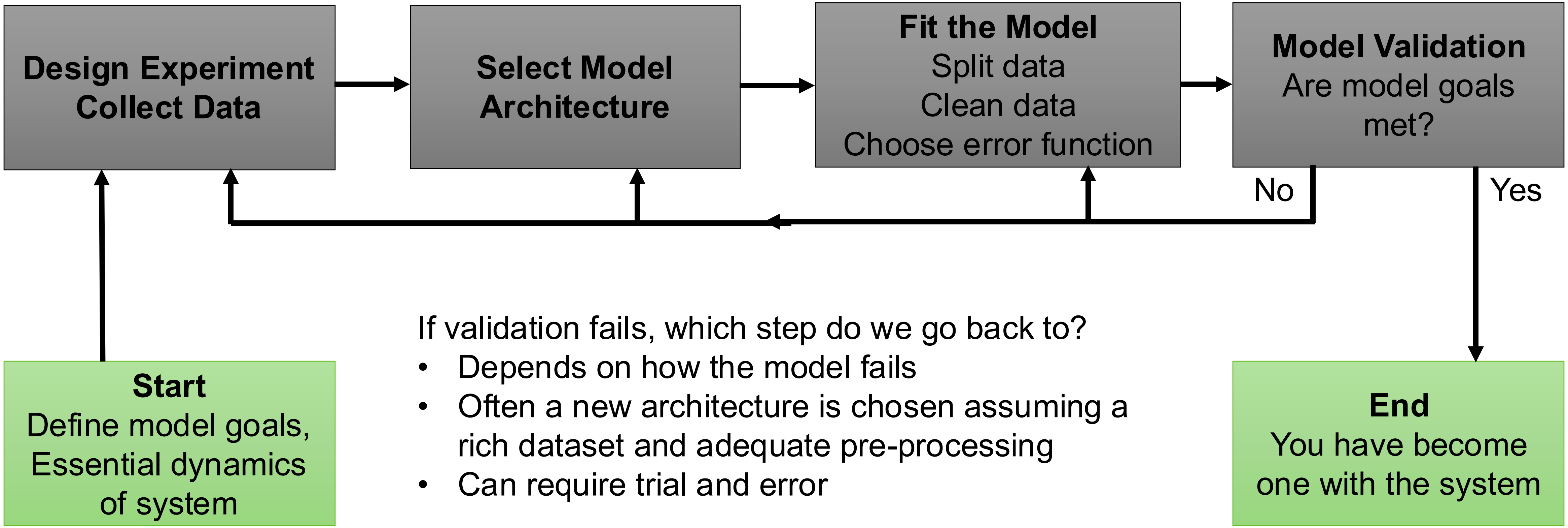
An appropriate model architecture must be chosen

- Linear Models
- Nonlinear Models
- Online and Recursive Models

High quality data must be collected to tune the model

- Does the experiment excite the modes we are interested in?
- How noisy are the sensors?
- Is there delay in the system?

What is System Identification?

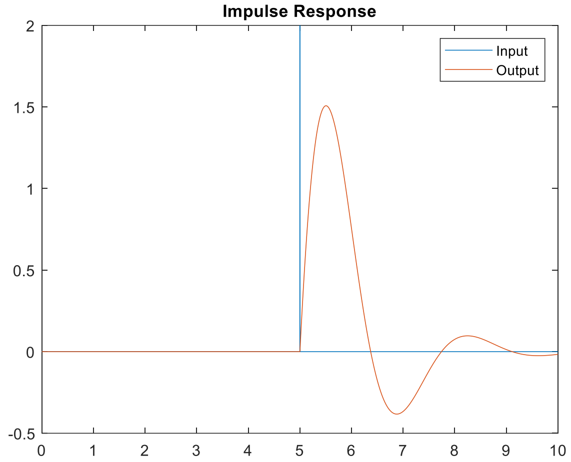


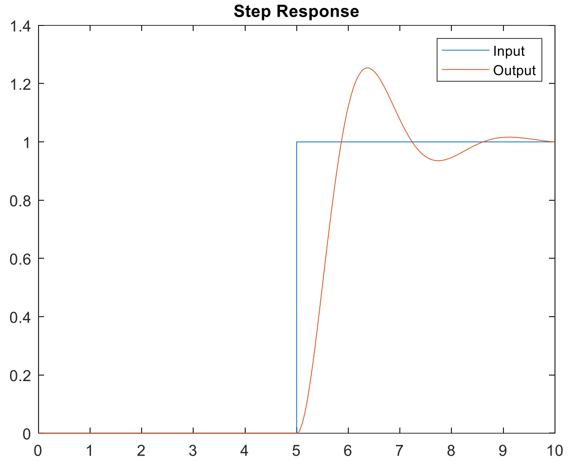
If validation fails, which step do we go back to?

- Depends on how the model fails
- Often a new architecture is chosen assuming a rich dataset and adequate pre-processing
- Can require trial and error

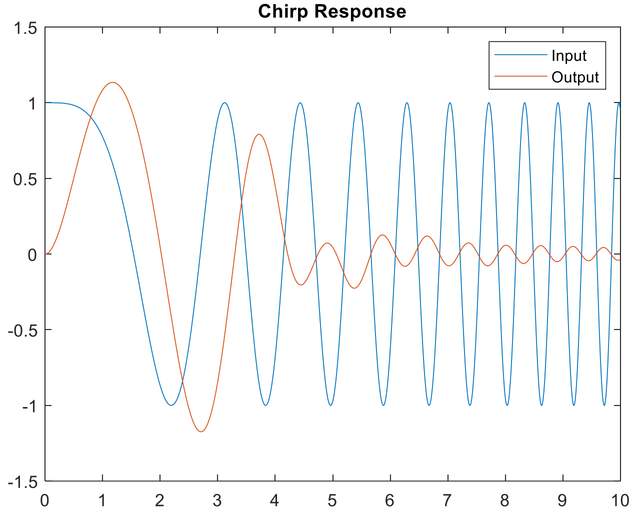
Experiment Design

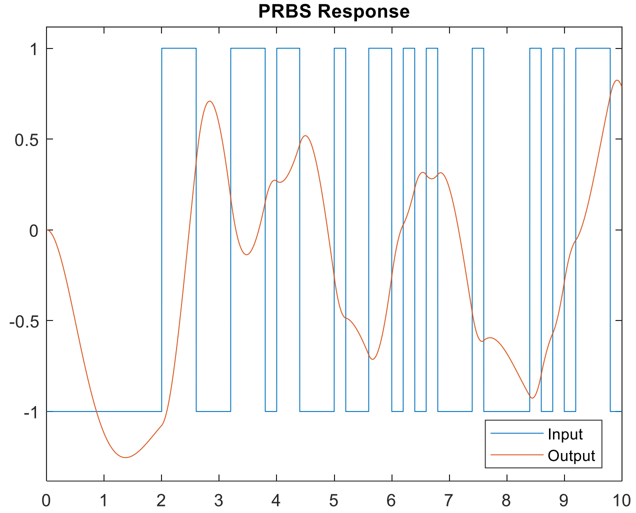
- Dataset must capture the dynamics of interest, consider:
    - Standard operating region
    - Safety and system constraints
    - What inputs and outputs can be measured?
    - What sampling rates are needed in measurement?



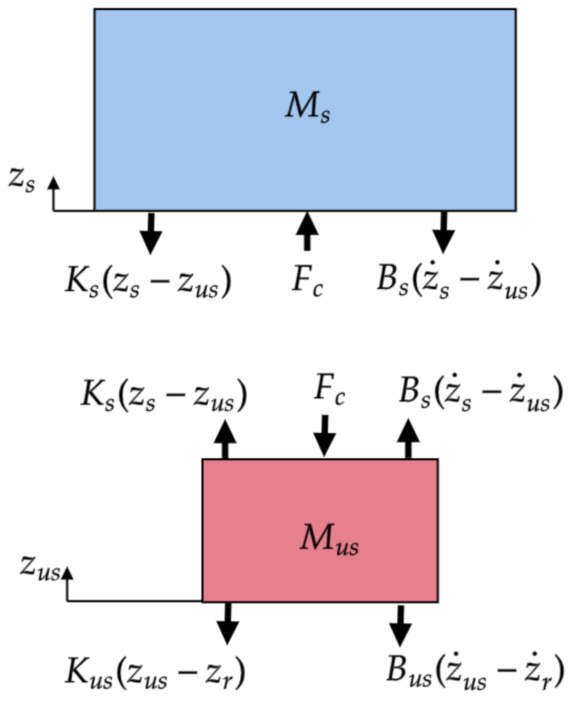


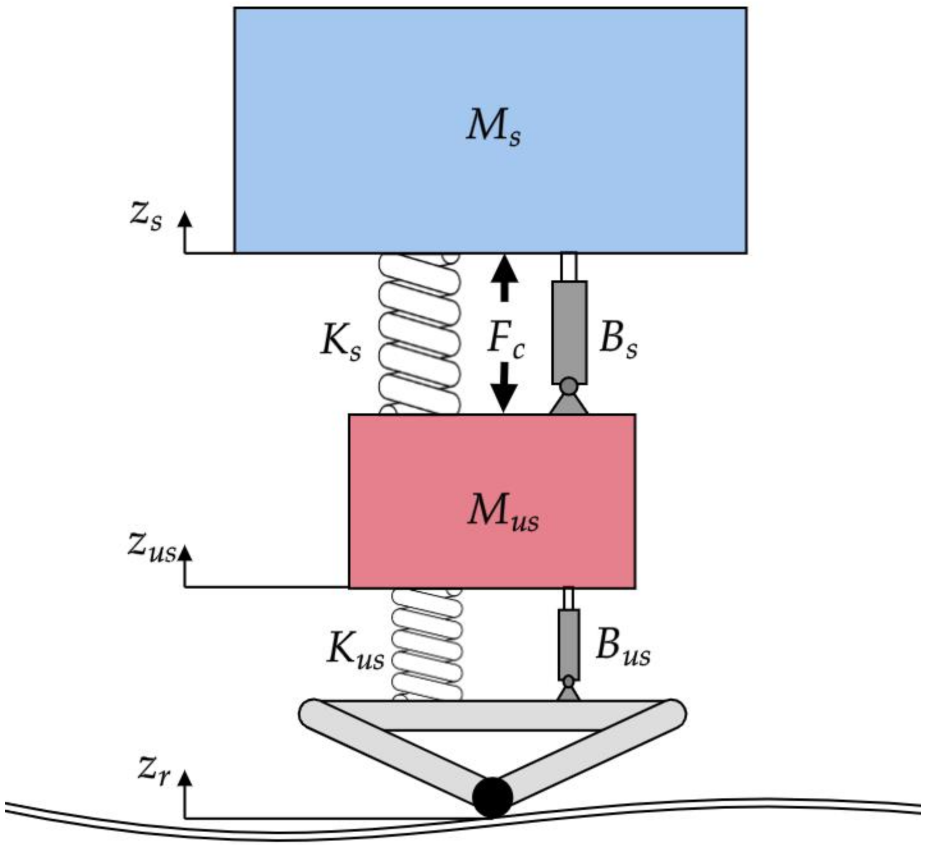
- Some common test signals
    - Step response
    - Impulse response
    - Sinusoidal and chirp responses
    - Pseudo-random binary sequences





Physics-Based Quarter Car Model



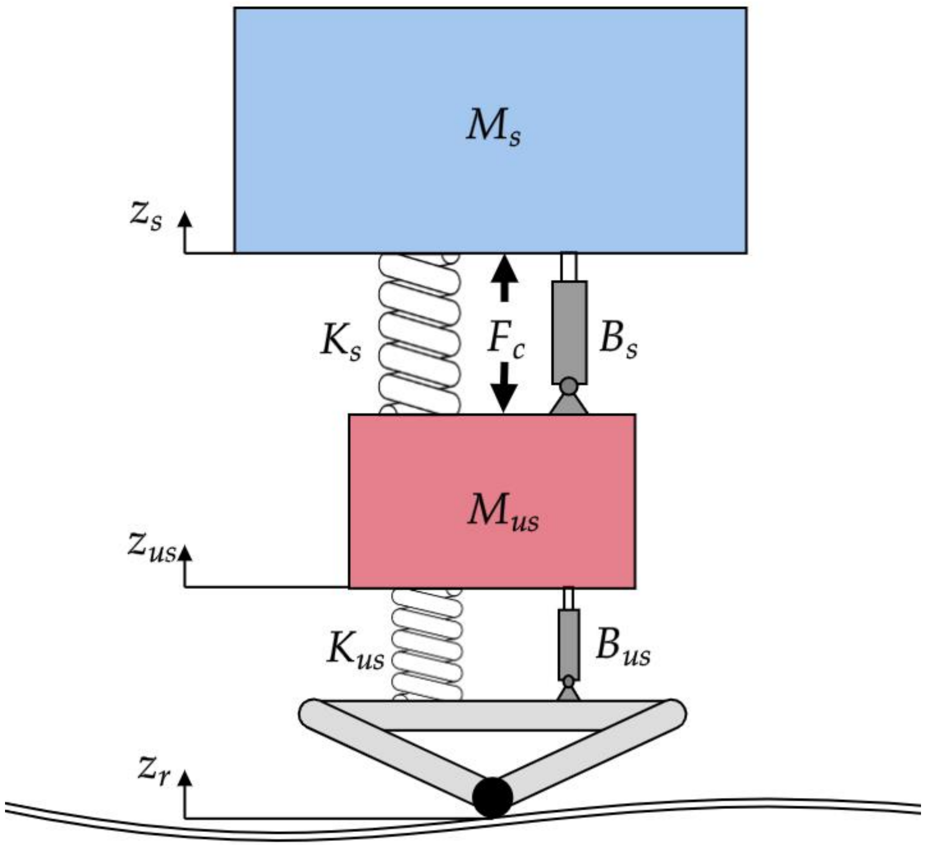


$$M_s\ddot{z}_s = F_c - K_s(z_s - z_{us}) - B_s(\dot{z}_s - \dot{z}_{us})$$
$$M_{us}\ddot{z}_{us} = -F_c + K_s(z_s - z_{us}) + B_s(\dot{z}_s - \dot{z}_{us}) - K_{us}(z_{us} - z_r) - B_{us}(\dot{z}_{us} - \dot{z}_r)$$

Physics-Based Quarter Car Model

$$
\begin{bmatrix} \dot{z}_{us} - \dot{z}_r \\ \ddot{z}_{us} \\ \dot{z}_s - \dot{z}_{us} \\ \ddot{z}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_{us}}{M_{us}} & -\frac{B_s + B_{us}}{M_{us}} & \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} z_{us} - z_r \\ \dot{z}_{us} \\ z_s - z_{us} \\ \dot{z}_s \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{M_{us}} \\ 0 \\ \frac{1}{M_s} \end{bmatrix} u + \begin{bmatrix} -1 & 0 \\ \frac{B_{us}}{M_{us}} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_r \\ z_r \end{bmatrix}
$$

$$
\begin{bmatrix} z_{us} \\ z_s \\ \ddot{z}_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} z_{us} - z_r \\ z_{us} \\ z_s - z_{us} \\ z_s \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_s} \end{bmatrix} u + \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_r \\ z_r \end{bmatrix}
$$



If all parameters are known, we already have our system model

Physics-Based Quarter Car Model

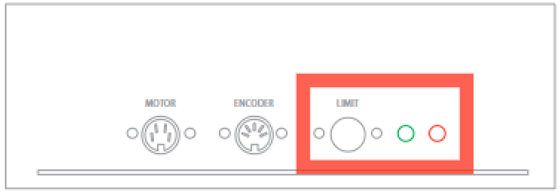
Steps to start the test bed:

1.

Amplifier (rear)

Turn ON the power switch on the AMPAQ-L2. It is located on the rear of the device.

2.



Active Suspension

Manually move the white plate until it hits the upper and lower limit switches, i.e. when the red LED is lit. Then, move the plate to the mid position and push the **Limit** button to turn on the green LED, i.e enable the motor power.

Grey box Quarter Car Model

What if we don't know all the parameters?

- Masses
- Spring constants
- Damping coefficients

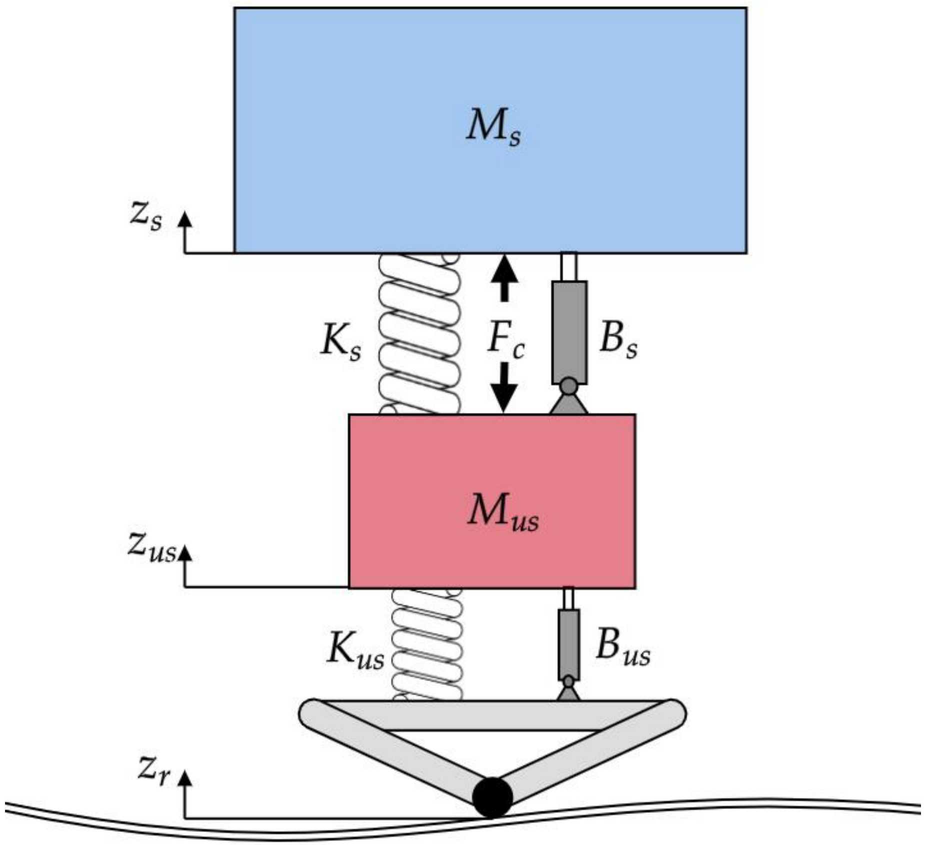
We can still use the form of the solution, but estimate the unknown parameters using the dataset

$$\begin{bmatrix} z_{us} \\ z_s \\ \ddot{z}_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} z_{us} - z_r \\ z_{us} \\ z_s - z_{us} \\ z_s \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_s} \end{bmatrix} u + \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_r \\ z_r \end{bmatrix}$$

Or

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

If we are only interested in SISO system



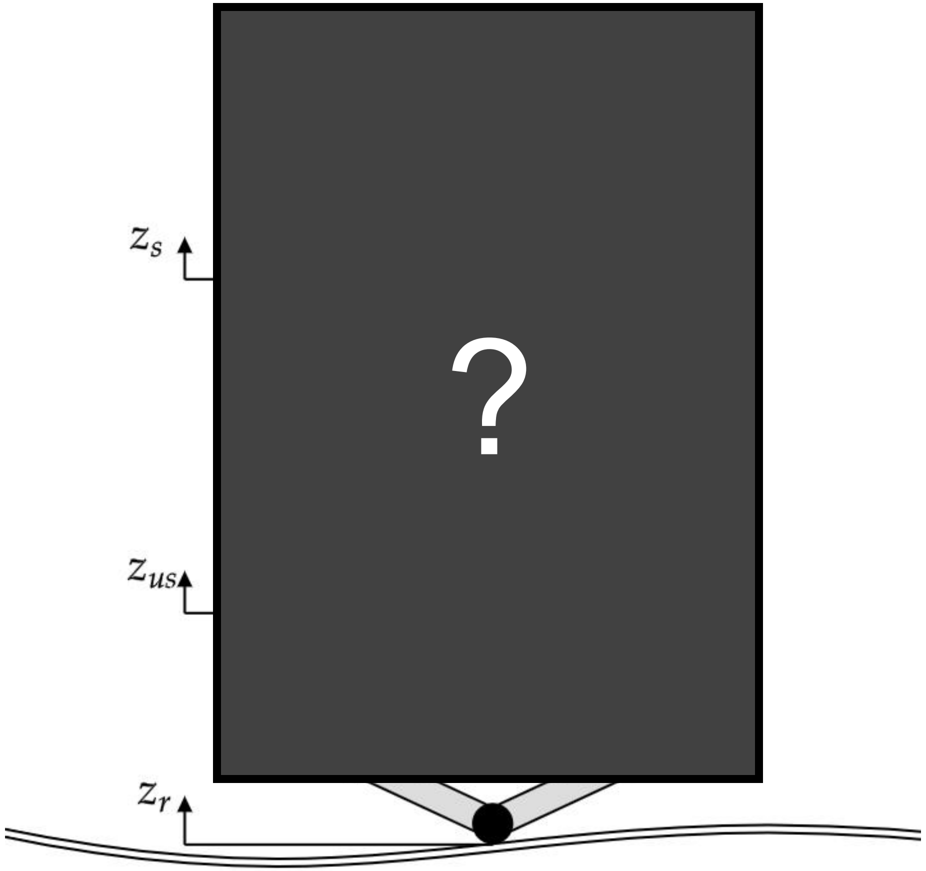
Black box Quarter Car Model

What if we don’t know the form of the solution

We can still use input-output data collected to build a model

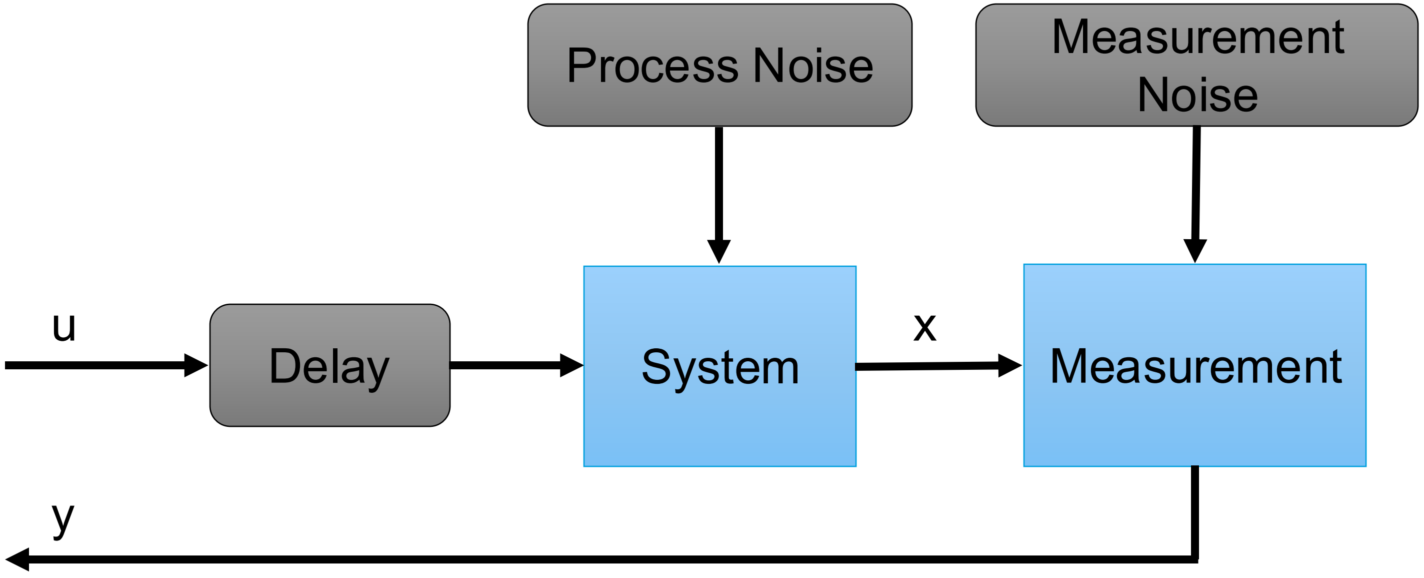
Generally multiple models are tested, refined, and compared

- Transfer functions (Continuous or Discrete), (# of poles and zeros)
- Frequency response models
- Wiener-Hammerstein
- Nonlinear ARX
- Etc.



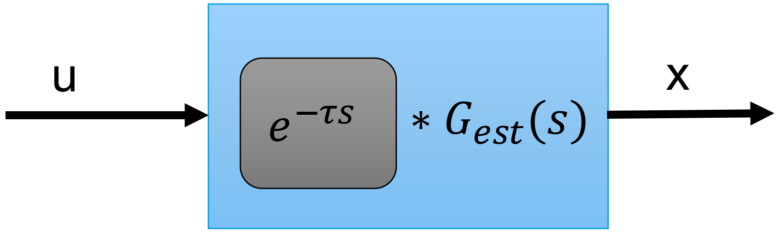
Modeling the Real World

- Not generally feasible to measure full system state
- Measurement noise
- Process noise
- Delays in the system



Two options:
- Explicitly model disturbances
  - Structured and reproducible noise
  - Consistent lag/ delay
- Filter or trim data
  - Startup transients
  - White measurement noise
  - Saturated sensor or anomalies

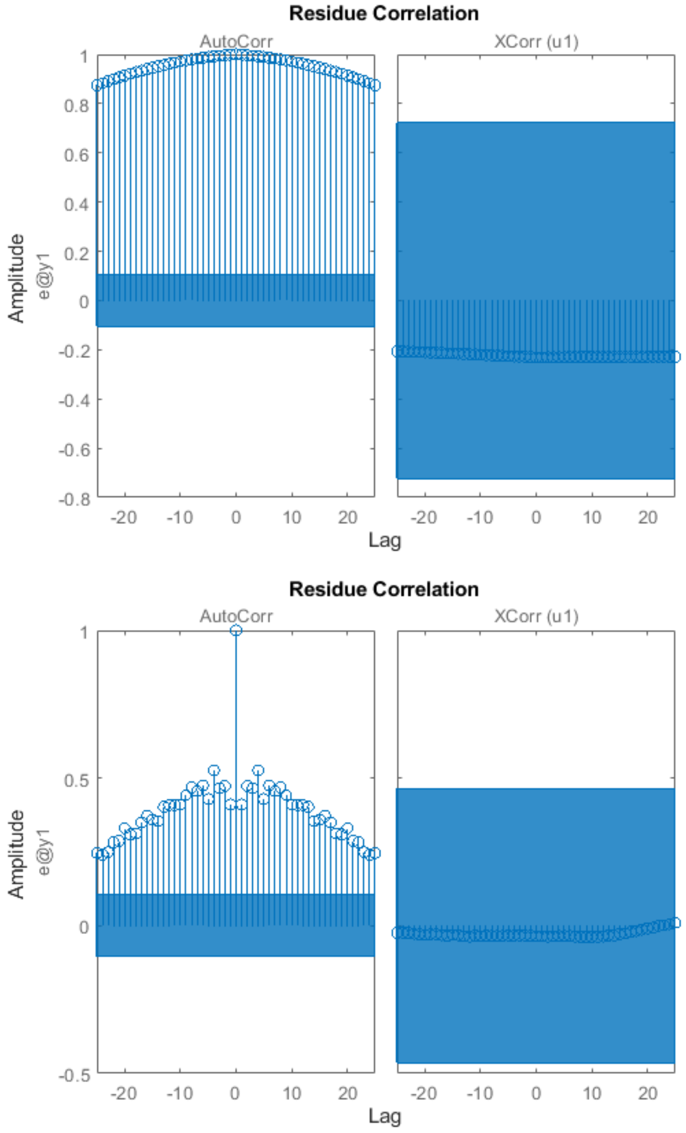
Constant delays are generally easy to estimate from the data



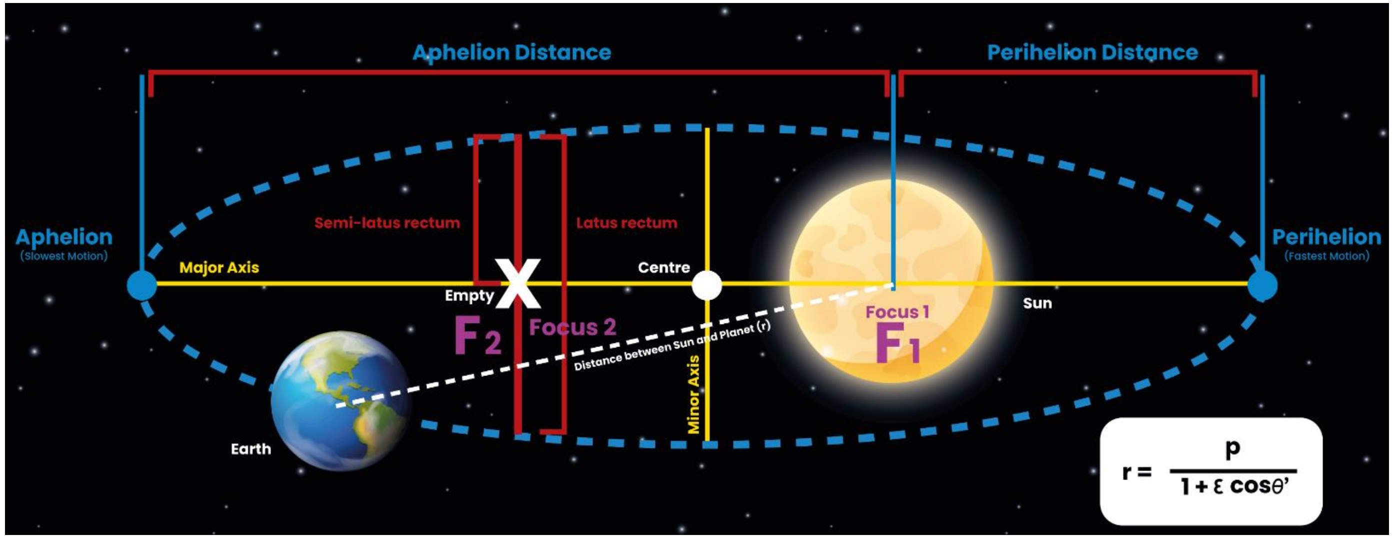
Model Validation

- Validate model with unseen data
- Define model error function
  - Mean squared error
  - Root mean squared error
  - Weighted prediction error
  - Etc.

- Residual analysis (are the prediction errors random?)
  - one step ahead prediction vs data
  - Whiteness test – autocorrelation of residuals shows unmodeled noise and system model
  - Independence test – correlation of input to prediction shows system model missing dynamics

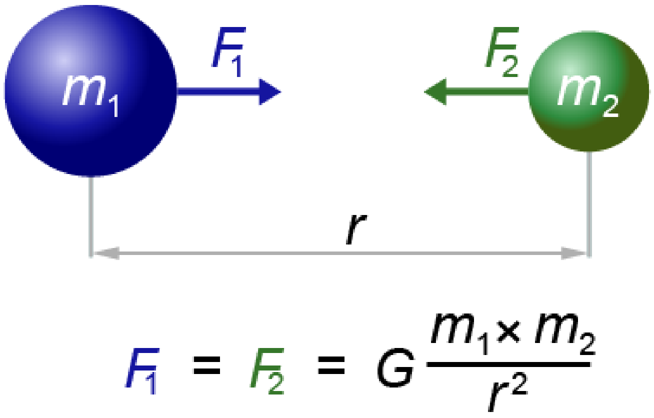


Discovering the Governing Equations



Kepler’s Orbital Model

Data-driven

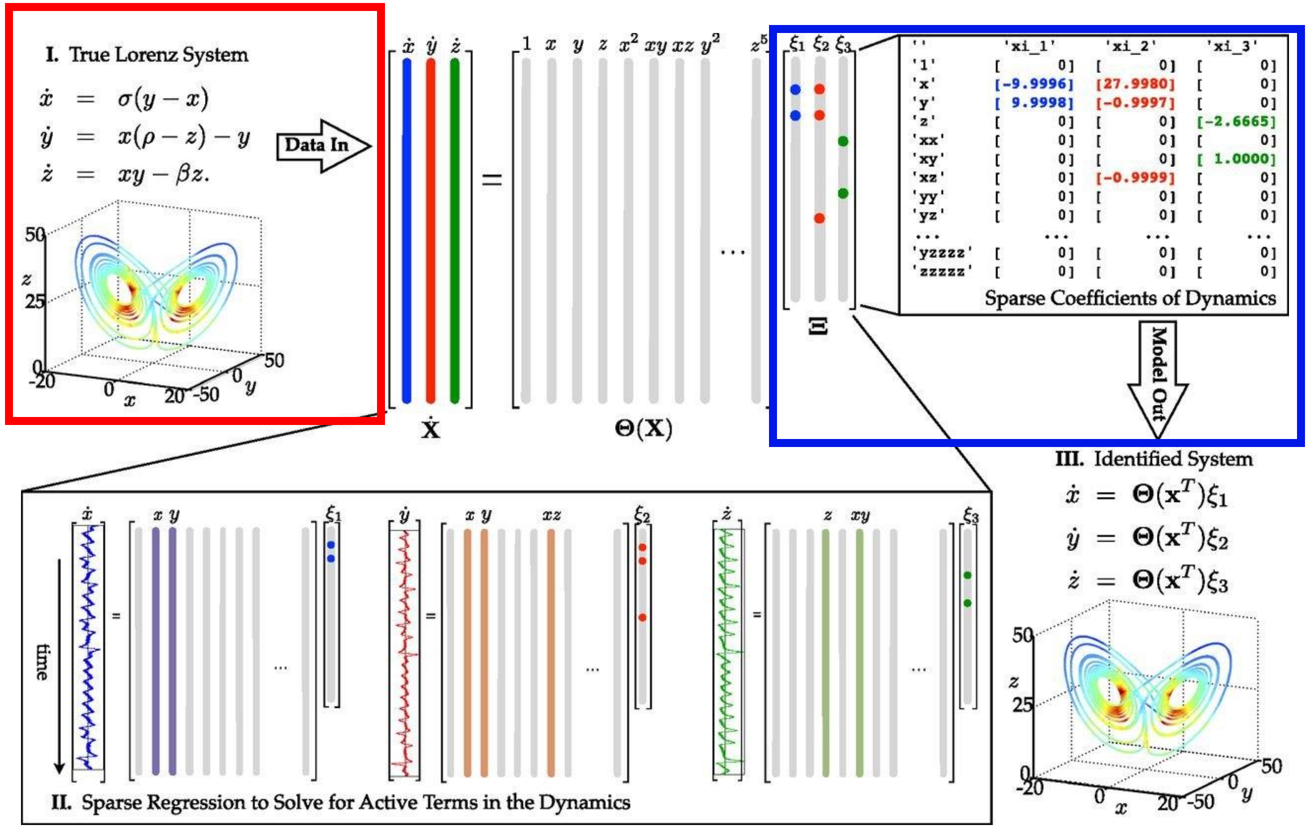


Newton's Orbital Model

Physics-based

Discovering the Governing Equations

Sparse Identification of Nonlinear Dynamics



Library of nonlinear functions

$$\Theta(\mathbf{X}) = \begin{bmatrix} | & | & | & | & | & & | \\ x(t) & \mathbf{y}(t) & \mathbf{z}(t) & \mathbf{x}(t)^2 & \mathbf{x}(t)\mathbf{y}(t) & \cdots & \mathbf{z}(t)^5 \\ | & | & | & | & | & & | \end{bmatrix}$$