MATH5301 Elementary Analysis. Homework 2. Due: 09/10/2021, 11:59 pm

First Name: **Last Name:**

2.1

For a function $f : A \to B$ Show that for any $X \subset A$, $Y, Z \subset B$

(a) $X \subset f^{-1}(f(X))$

(b) $f(f^{-1}(Y)) \subset Y$

(c) $f^{-1}(Y \cup Z) = f^{-1}(Y) \cup f^{-1}(Z)$

(d) $f^{-1}(Y \cap Z) = f^{-1}(Y) \cap f^{-1}(Z)$

2.2

Show that

(a) $A \cap \bigcup_{\lambda \in \Lambda} A_\lambda = \bigcup_{\lambda \in \Lambda} (A_\lambda \cap A)$

(b) $\left(\bigcap_{\lambda \in \Lambda} A_\lambda \right) \cup \left(\bigcap_{\lambda \in \Lambda} B_\lambda \right) = \bigcap_{\lambda \in \Lambda} (A_\lambda \cup B_\lambda)$

2.3

Which of those are equivalence relations?

(a) for $a, b \in \mathbb{R}$, let $a\mathcal{R}b$ if $a - b \in \mathbb{Q}$.

(b) for $a, b \in \mathbb{R}$, let $a\mathcal{R}b$ if $a - b \notin \mathbb{Q}$.

(c) for $a, b \in \mathbb{R}$, let $a\mathcal{R}b$ if $a - b$ is a square root of rational number.

(d) Let $X = \mathbb{Z} \times \mathbb{N}$, let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $\mathcal{R}$ if $x_1 y_2 = x_2 y_1$.

2.4

For the relation $(x, y) \succeq (a, b)$ if $(x \geq a) \wedge (y \geq b)$ on the set of ordered pairs of $\{1, 2, 3\} \times \{1, 2, 3\}$

(a) Show that the above relation is an order relation.

(b) Can you make it the total order?

(c) How many different total ordering can be constructed?

2.5

Provide an example of $f : \mathbb{Z} \to \mathbb{N}$ such that

(a) f is surjective, but not injective,

(b) f is injective, but not surjective,

(c) f is surjective, and injective,

(d) f is niether surjective nor injective.

2.6 Is the following statement correct?

Theorem 1. *If the relation $\mathcal{R}$ on A is symmetric and transitive, then it is reflexive.*

Proof. For any $a \in A$ let $b \in A$ is such that $a\mathcal{R}b$. Then, by symmetry $b\mathcal{R}a$. Then by symmetry $a\mathcal{R}a$. $\square$