MATH5301 Elementary Analysis. Homework 1. Due: 09/03/2021, 11:59 pm

First Name: **Last Name:**

1.1 Prove the following tautologies writing the true-false table.

(a) $A \vee \sim A$

(b) $(A \vee B) \Rightarrow A$

(c) $(A \wedge B) \Rightarrow A$

(d) $(A \Rightarrow B) \Leftrightarrow (\sim B \Rightarrow \sim A)$

(e) $\sim (A \vee B) \Leftrightarrow (\sim A \wedge \sim B)$

(f) $((A \Rightarrow B) \Rightarrow A) \Rightarrow A$

(g) $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$

These tautologies are sometimes called *the axioms of logical system.* Translate each of these statements in the human language.

1.2 Prove the following identities for the set operations.

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $(A \setminus B) \cup C = ((A \cup C) \setminus B) \cup (B \cap C)$

1.3 Write the following statements using the quantifiers.

(a) Even elements of the sequence $\{a_n\}$ may be arbitrary large.

(b) The sequence $\{a_n\}$ contains arbitrary large even elements.

(c) The sequence $\{a_n\}$ contains infinitely many even elements.

1.4 Show that

(a) $\exists_x : (p(x) \vee q(x)) \Leftrightarrow (\exists_x : p(x)) \vee (\exists_x : q(x))$

(b) $(\forall_x p(x) \vee \forall_x q(x)) \Rightarrow \forall_x (p(x) \vee q(x))$

(c) Why there is no left arrow implication on the previous line?

1.5 Show that one needs only one logic operation to construct all the 16 binary operations on statements A and B.

Define $A \star B$ via the following table:

A	B	$A \star B$
0	0	1
0	1	0
1	0	0
1	1	0

.

Show that one can construct $\sim A$, $A \vee B$ and $A \wedge B$ using only operation $\star$. Then show that any other binary operation can be obtained from $\{\sim, \vee, \wedge\}$.

1.6 How many subsets does the set $A = \{a, p, p, l, e\}$ have?