MATH5301 Elementary Analysis. Homework 11. Due: 11/20/2021, 11:59 pm

First Name: **Last Name:**

11.1

Prove that the closure and the interior of a convex set $A \subset \mathbb{R}^n$ are also convex.

11.2

Prove that the intersection of an arbitrary collection of convex sets $\bigcap_{i \in I} C_i$ is also convex.

11.3

Let $\{C_i\}_{i \in \mathbb{N}}$ be a sequence of nested convex sets in $\mathbb{R}^n$, i.e. $C_i \subset C_{i+1}$. Prove that $\bigcup_{i=1}^{\infty} C_i$ is also convex.

11.4

(a) Show that the convex hull of any open set in $\mathbb{R}^n$ is open.

(b) Provide an example of a closed set $A \subset \mathbb{R}^n$, such that its convex hull is not closed.

11.5

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function and $A \subset \mathbb{R}^n$ be a bounded set. Prove that $f(A)$ is bounded in $\mathbb{R}$.

11.6

Show that the convex hull of a compact set $A \subset \mathbb{R}^n$ is compact. (*Hint:* Caratheodory theorem)