

Assignment 4 of Math 5302

Due Date: Feb. 23, 2022 at 11:59pm

1. Show that

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$$

is well-defined for $\alpha > 0$ and $\beta > 0$.

2. Show that if f is Riemann integrable on $[a, b]$, then

$$\lim_{\epsilon \rightarrow 0^+} \int_a^{b-\epsilon} f(x) dx = \int_a^b f(x) dx.$$

3. Evaluate $\int_0^1 (1 - x^{\frac{2}{3}})^{\frac{3}{2}} dx$. Hint: Express the integral in terms of the gamma function first.

4. Let

$$f(x) = \begin{cases} x \sin(\frac{1}{x}) & \text{if } 0 < x \leq 1; \\ 0 & \text{if } x = 0. \end{cases}$$

Show that f is bounded and continuous on $[0, 1]$, but not of bounded variation on $[0, 1]$.

5. Assume f is differentiable on $[a, b]$ with $|f'(x)| \leq M < \infty$ for $a \leq x \leq b$. Show that f is of bounded variation and $V_a^b(f) \leq M(b-a)$. (Hint: Use Mean Value Theorem.)