University of Texas at Dallas

Assignment #3

Last Name:	**First Name and Initial:** ....
Course Name: Real Analysis 1	**Number:** MATH 6301
Instructor: Wieslaw Krawcewicz	**Due Date:** September 29, 2022
E-mail Address:	**Student's Signature:** ....

Instructions:

1. Print this booklet
2. Use the space provided to write your solutions in this booklet
3. Hand in your assignment to your instructor on the due date during the class time.

Question	**Weight**	**Your Score**	**Comments**
1.	10		
2.	10		
3.	10		
4.	10		
5.	10		
Total:	50		

Problem 1. Consider the Banach space $\mathscr{E} :=]C([a,b];\mathbb{R})$ consisting of all continuous functions from the interval $[a,b]$ to $\mathbb{R}$, equipped with the norm

$$\|\varphi\|_\infty := \sup_{t\in[a,b]} |\varphi(t)|, \quad \varphi \in \mathscr{E}.$$

Take c, $d > 0$ and define the set $A \in \mathscr{E}$ by

$$A := \left\{ \psi \in \mathscr{E} : \begin{cases} \text{(i)} \;\; \psi \text{ is of class } C^1, \\ \text{(ii)} \;\; \sup_{t\in[a,b]} |\psi(t)| < c, \\ \text{(iii)} \;\; \sup_{t\in[a,b]} |\psi'(t)| < d. \end{cases} \right\}$$

Show that the set $\overline{A}$ is compact in $\mathscr{E}$.

SOLUTION:

Problem 2: Consider $X := \mathbb{R}$ and the family $\mathscr{A} \subset \mathscr{P}(X)$ given by $\mathscr{A} := \big\{\{n\} : n \in \mathbb{N}\big\}$. Describe the σ-algebra $\mathscr{S}(\mathscr{A})$ generated by $\mathscr{A}$.

SOLUTION:

Problem 3: Let X be a given space.

(a) Assume that $A \in \mathscr{P}(X)$ is a set such that $A \neq \emptyset$, X. Find $\mathscr{S}(\{A\})$.

(b) For given sets A_1, A_2, ..., $A_n \in \mathscr{P}(X)$, find $\mathscr{S}(\{A_1, A_2, \ldots, A_n\})$.

SOLUTION:

Problem 4: For two given spaces X and Y and two sets $A \in \mathscr{P}(X)$ and $B \in \mathscr{P}(Y)$, describe the σ-algebra

$$\mathscr{S}_1 \times \mathscr{S}_2, \qquad \text{where } \mathscr{S}_1 := \mathscr{S}(\{A\}) \in \mathscr{P}(X), \mathscr{S}_2 := \mathscr{S}(\{B\}) \in \mathscr{P}(Y).$$

SOLUTION:

Problem 5: For two given spaces X and Y, $f : X \to Y$ a map and $\mathscr{S} \subset \mathscr{P}(Y)$ a σ-algebra in Y, define

$$f^{-1}(\mathscr{S}) := \left\{ A \subset X : \exists_{B \in \mathscr{S}} \ \ A := f^{-1}(B) = A \right\}.$$

Is $f^{-1}(\mathscr{S})$ a σ-algebra in X? Justify your answer.

SOLUTION: