

MATH 6301 Real Analysis I

Homework 4

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Instructions:

1. Print this booklet
2. Use the space provided to write your solutions in this booklet
3. Hand in your assignment to your instructor on the due date during the class time.

Question	Weight	Your Score	Comments
1.	10		
2.	10		
3.	10		
4.	10		
5.	10		
Total:	50		

Problem 1

PROBLEM:

Assume that $U \subset \mathbb{R}^n$ is an open set and $f : U \rightarrow \mathbb{R}$ is a differentiable function. Show that for every $k = 1, 2, \dots, n$, the partial derivative

$$\frac{\partial f}{\partial x_k} : U \rightarrow \mathbb{R}$$

is \mathcal{B}_n -measurable (here \mathcal{B}_n stands for the σ -algebra of Borel sets in \mathbb{R}^n).

PRELIMINARIES:

Definition 1. Let $\mathcal{S} \subset P(X)$ is a σ -algebra and $E \in \mathcal{S}$. The function $f : E \rightarrow \overline{\mathbb{R}}$ is called measurable relative to \mathcal{S} (i.e. \mathcal{S} -measurable) iff

$$\forall a \in \mathbb{R} f^{-1}(a, \infty] := \{x \in E : f(x) > a\} \in \mathcal{S}$$

Remark 1. Assume that $f : E \rightarrow \overline{\mathbb{R}}$, $E \in \mathcal{S} \subset P(X)$ is \mathcal{S} -measurable. Then the following are also \mathcal{S} -measurable

1. $f^2 : E \rightarrow \overline{\mathbb{R}}$
2. $|f| : E \rightarrow \overline{\mathbb{R}}$
3. $\frac{1}{f} : E \rightarrow \overline{\mathbb{R}}$
4. $a \cdot f : E \rightarrow \overline{\mathbb{R}}$, $a \in \mathbb{R}$

SOLUTION:

Problem 2

PROBLEM:

PRELIMINARIES:

SOLUTION:

Problem 3

PROBLEM:

PRELIMINARIES:

SOLUTION:

Problem 4

PROBLEM:

PRELIMINARIES:

SOLUTION:

Problem 5

PROBLEM:

SOLUTION: