University of Texas at Dallas

Assignment #1

Last Name:	First Name and Initial:
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Course Name: Real Analysis 1	**Number:** MATH 6301
Instructor: Wieslaw Krawcewicz	**Due Date:** September 6, 2022
E-mail Address:	**Student's Signature:** ....

Instructions:

1. Print this booklet
2. Use the space provided to write your solutions in this booklet
3. Hand in your assignment to your instructor on the due date during the class time.

Question	Weight	Your Score	Comments
1.	10		
2.	10		
3.	10		
4.	10		
5.	10		
Total:	50		

Problem 1. A set A in a metric space (X, d) is called *bounded* if

$$\exists_{R>0}\ \exists_{x_o \in X}\ \ A \subset B_R(x_o).$$

Use the mathematical induction to show that if a set $A \subset X$ is *unbounded* (i.e. it is not bounded), then there exists a sequence $\{x_n\} \subset A$ such that $d(x_n, x_m) \geq 1$ for all $m \neq n$.

SOLUTION:

Problem 2: Let $(V, \|\cdot\|)$ be a normed vector space. Show that an open unit ball

$$B_1(0) := \{v \in V : \|v\| < 1\}$$

is a convex set, i.e.

$$\forall_{u,v \in B_1(0)}\ \forall_{t \in [0,1]}\ \ tu + (1-t)v \in B_1(0).$$

SOLUTION:

Problem 3: Suppose that (X, d) is a metric space and A, $B \subset X$ are such that $A \subset B$. Show that $\mathrm{int}(A) \subset \mathrm{int}(B)$ and $\overline{A} \subset \overline{B}$.

SOLUTION:

Problem 4: Let (X_1, d_1) and (X_2, d_2) be two metric spaces. Show that each of the following functions d is a metric on $X = X_1 \times X_2$ (Hint: Knowing that d_1 and d_2 satisfies the three conditions of a metric, show that d also satisfies these conditions)

(a) $d((x_1, x_2), (y_1, y_2)) = \max\{d_1(x_1, y_1), d_2(x_2, y_2)\}$;

(b) $d((x_1, x_2), (y_1, y_2)) = \sqrt{(d_1(x_1, y_1))^2 + (d_2(x_2, y_2))^2}$;

(c) $d((x_1, x_2), (y_1, y_2)) = d_1(x_1, y_1) + d_2(x_2, y_2)$;

SOLUTION:

Problem 5: Let (X, d) be a metric space and $A \subset X$ a closed non-empty set. For a given point $x \in X$ we define the *distance from x to A* by the formula

$$d(x, A) := \inf\{d(x, a) : a \in A\}. \tag{1}$$

(a) Show that $x \in A \iff d(x, A) = 0$;

(b) Show that the function $\chi_A : X \to \mathbb{R}$ defined by $\chi_A(x) = d(x, A)$ is continuous on X.

SOLUTION: