University of Texas at Dallas

Assignment #5

Last Name:	**First Name and Initial:** ....
Course Name: Real Analysis 1	**Number:** MATH 6301
Instructor: Wieslaw Krawcewicz	**Due Date:** December 1, 2022
E-mail Address:	**Student's Signature:** ....

Instructions:

1. Print this booklet
2. Use the space provided to write your solutions in this booklet
3. Hand in your assignment to your instructor on the due date during the class time.

Question	Weight	Your Score	Comments
1.	10		
2.	10		
3.	10		
4.	10		
5.	10		
Total:	50		

Problem 1. Let $(X, \mathscr{S}, \mu)$ be a measure space, $E \in \mathscr{S}$ and $f : E \to \overline{\mathbb{R}}$ a summable function. Show that

$$\mu\{x \in E : |f(x)| = \infty\} = 0.$$

SOLUTION:

Problem 2: Let $(X, \mathscr{S}, \mu)$ be a measure space, $E \in \mathscr{S}$ and $f, f_n : E \to \overline{\mathbb{R}}$, $n = 1, 2, \ldots$, summable functions. Show that

$$\lim_{n \to 0} \int_E |f - f_n| d\mu = 0 \quad \Rightarrow \quad f_n \xrightarrow{\mu} f.$$

Verify if the reverse implication is also true. Justify your answer.

SOLUTION:

Problem 3: Let (X, d) be a metric space and $A \subset X$. Define $f : X \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

Find the set

$$B := \{x_o \in X : \lim_{x \to x_o} f(x) = f(x_o)\}.$$

SOLUTION:

Problem 4: Let $(X, \mathscr{S}, \mu)$ be a measure space, $E \in \mathscr{S}$, $f, f_n : E \to \overline{\mathbb{R}}$, $n = 1, 2, \dots$, summable functions such that

$$\lim_{n\to\infty} \int_E |f_n - f| d\mu = 0,$$

and $\varepsilon_k > 0$ a given sequence such that $\lim\limits_{k\to\infty} \varepsilon_k = 0$. Show that there exists a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ such that

$$\forall_{k\in\mathbb{N}} \quad \int_E |f_{n_{k+1}} - f_{n_k}| d\mu < \varepsilon_k.$$

SOLUTION:

Problem 5: Let $(X, \mathscr{S}, \mu)$ be a complete measure space, $E \in \mathscr{S}$, $f, f_n : E \to \overline{\mathbb{R}}$, $n = 1, 2, \ldots$, summable functions such that

$$\lim_{n\to\infty} \int_E |f_n - f| d\mu = 0.$$

Show that there exists a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ such that

$$\lim_{k\to\infty} f_{n_k}(x) = f(x) \quad \text{a.e.} \quad x \in E.$$

SOLUTION: