MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems
Homework Assignment #8
Due: Monday, November 23, 2020 (10:00PM CDT)

[1] Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 1 & 3 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} u$$

Find a gain matrix *K* for the control law $u = Kx$, using the method of your choice, to place the closed-loop poles at $-1 \pm 2j$ and $-2 \pm 3j$. You may use Matlab for intermediate calculations in the method you choose, but not for the direct calculation of *K*. Verify your result by computing (in Matlab) the eigenvalues of $A + BK$. ***Hint:*** Because of the special structure of *A*, and because *BK = (A+BK) – A,* calculation of *K* becomes simpler (by hand) because you end up with an expression that looks like

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ * & * \\ * & * \end{bmatrix} K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

and you can essentially ignore the top two rows (in solving for *K*), thereby resulting in a 2×2 invertible matrix on the left side of *K*.

[2] For the system in Problem [1] above , use the Matlab command `place` to find the gain matrix *K* for the same desired closed-loop eigenvalues. Is the result the same as what you obtained in Problem [1]? If not, why not? Caution: You must understand what the `place` command does to answer this correctly.

[3] Test the system below for observability. Using a method of your choice, but using Matlab only for intermediate calculations, find a three-dimensional state estimator with eigenvalues –2, –3, –4 for the following dynamical system:

$$\dot{x} = \begin{bmatrix} -3 & -1 & -2 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix} x$$

Write down the final expression for the estimated state equations in the form of

$$\frac{d\hat{x}}{dt} = \mathrm{A}\hat{x} + bu + L(y - c\hat{x})$$

[4] Using a method of your choice, but using Matlab only for intermediate calculations, find a two-dimensional state estimator with eigenvalues –2 and –4 for the system of Problem [3]. Write down the final expression for the estimated state equations in the form discussed in class.