

2)

$$y(t) = (P_\alpha u)(t) = \begin{cases} u(t), & t \leq \alpha \\ 0, & t > \alpha \end{cases}$$

a) The system is Linear w/in two P/ths
For $t \leq \alpha$:

$$y(u(\alpha-t_1)) = u(\alpha-t_1)$$

$$y(\beta_1 u_1(\alpha-t_1) + \beta_2 u_2(\alpha-t_1)) = \beta_1 u_1(\alpha-t_1) + \beta_2 u_2(\alpha-t_1)$$

$y(t)$ satisfies Superposition

For $t > \alpha$:

$$\left. \begin{aligned} y(u(\alpha+t_1)) &= 0 \\ y(\beta_1 u_1 + \beta_2 u_2) &= 0 \end{aligned} \right\} \text{satisfies Superposition}$$

b) The system is Time-Variant

Let $u(t) = \beta t$, where β is a constant

$$y(u(\alpha-t_1)) = \beta(\alpha-t_1) \quad \left. \begin{aligned} y(u(\alpha+t_1)) &= 0 \end{aligned} \right\} \text{Time Variant}$$

c) The system is Causal

$y(u)$ is not dependent on any future values of $u(t)$.