PSA: 2)

$$\ddot{z} = \frac{f - m\ell\ddot{\theta}\cos\theta + \dot{\theta}^2 \sin\theta}{M+m}$$

$$\ddot{\theta} = \frac{mg\ell\sin\theta - m\ell\ddot{z}\cos\theta}{m\ell^2}$$

$$\ddot{z} = \frac{f - m\ell\ddot{\theta}}{M+m}$$

$$\ddot{\theta} = \frac{-\ddot{z}}{\ell}$$

$$\ddot{z} = \frac{f + m\ell\left(\frac{\ddot{z}}{\ell}\right)}{M+m}$$

$$\ddot{z}\left(1 - \frac{m}{M+m}\right) = \frac{f}{M+m}$$

$$\ddot{z} = \frac{f}{(M+m)\left(1 - \frac{m}{M+m}\right)} = \frac{f}{M}$$

$$\boxed{\ddot{\theta} = \frac{-f}{\ell M}} \qquad \boxed{\ddot{z} = \frac{f}{M}}$$