MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems
Design Application Problem Set B
Due: Monday, September 21, 2020 (10:00PM CDT)
State Variable Representations and Transfer Functions

1. Consider **Design Application #2**, the inverted pendulum on a cart, driven by a DC motor. Let the state vector, control, and output be defined as

$$x = \begin{bmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad , \qquad u = e \qquad , \qquad y = \begin{bmatrix} z \\ \theta \end{bmatrix}$$

where z is the position of the cart. Note that this definition of the state vector is different than what was specified for Problem Set A. Use the following numerical values:
m = 0.2kg, M = 1.0kg, ℓ= 1.0m, g = 9.8m/s^2, k = 2.0v-s, R = 50Ω, and r = 0.01m.

(a) Find the state variable description for this system.
(b) Find $(sI - A)^{-1}$ (optional: find the state transition matrix, e^{At}, too)
(c) Find the transfer functions from the input u to the two outputs.
(d) Discuss system stability in terms of the system poles (roots of the characteristic equation).

2. Consider the coupled cart problem (**Design Application #3**), where again a motor drives the wheels. Let $M_1 = M_2 = 2$kg, K = 40N/m (spring constant for the carts), k = 2.0v-s (torque constant), R = 100Ω, and r = 0.01m.

(a) Find the transfer functions from the input voltages to the cart positions.
(b) Find the system poles (roots of the characteristic equation).
(c) (optional) Simulate the system for some initial positions and some inputs (torques produced by the motors); this will take some creativity and trial-and-error.