MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems
Homework Assignment #7
Due: Friday, November 13, 2020 (10:00PM CDT)

[1] Transfer the following equation into the controllable canonical form. To do this, use the transformation matrix Q which transforms the system into controllable and uncontrollable subsystems. Do all your computation by hand, using Matlab only for calculating roots (of a third order polynomial) and for calculating Q^{-1}. Then, from your result, write down the transfer function.

$$\dot{x} = \begin{bmatrix} -1 & -2 & -3 \\ 0 & -1 & 3 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} x$$

[2] Transform the system in Problem [1] above to the observable canonical form (Note: there is no need to re-do all your calculations).

[3] Use state feedback to place the eigenvalues of the system in Problem [1] to {-1, -2, -2}. Do all your computation by hand, using Matlab only for calculating roots (of a third order polynomial) and for calculating matrix inverses (of course, you can check your final answer using Matlab). Draw a block diagram, as we did in class, for the original system and then add the required feedback to your diagram.

[4] The idea behind this problem is to have you experience how the calculations work for state feedback. So, you are to do this entirely by hand, using Matlab only to check your final answer. For the dynamical system below, find the gain matrix $k^T = [k_1 \quad k_2]$ such that the closed-loop system (using state feedback) has {-2,-3} as its eigenvalues. To do this, compute $\det[(sI - (A + bk^T)]$ and then compare that polynomial to the "desired" polynomial.

$$\dot{x} = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 3 \\ 2 \end{bmatrix} u \quad \text{where} \quad u = k^T x$$

[5] Consider the following uncontrollable system:

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

Is it possible to find a gain matrix k^T such that the system with state feedback $u = k^T x$ has eigenvalues (a) {-2,-2,-1,-1}, (b) {-2,-2,-2,-1}, or (c) {-2,-2,-2,-2}? Use Matlab only to verify your answers.