1) $\dot{x} = Ax + Bu$

$y = Cx \quad , t \geq 0$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix} \quad x(0) = \begin{bmatrix} 1 \\ 8 \end{bmatrix} \quad u(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 2 & 4 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+1)(s^2+2s+2)} \begin{bmatrix} s^2+3s+4 & s+3 & 1 \\ -2 & s(s+3) & s \\ -2s & -2(2s+1) & s^2 \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B + D$$

$$= \frac{1}{(s+1)(s^2+2s+2)} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} s^2+3s+3 & s+4 \\ -(s+2) & s(s+4) \\ -s(s+2) & s^2-4s-2 \end{bmatrix} + 0$$

$$G(s) = \begin{bmatrix} \frac{s^2+s-2}{(s+1)(s^2+2s+2)} & \frac{8s+2}{(s+1)(s^2+2s+2)} \\ \frac{s^2+s-1}{(s+1)(s^2+2s+2)} & \frac{2s^2+9s+4}{(s+1)(s^2+2s+2)} \end{bmatrix}$$

1) cont.

A in companion Form

$$e^{At} = \begin{bmatrix} 1 & e^{t} & 1 \\ 1 & 1 & e^{t} \\ e^{-2t} & e^{-4t} & e^{-3t} \end{bmatrix} \qquad x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 1 \\ 1 \\ e^{-2t} \end{bmatrix} \qquad Ce^{At} = \begin{bmatrix} 1-e^{-2t} \\ 3 \end{bmatrix}$$

$$Y(s) = G(s)\,U(s)$$

$$= \begin{bmatrix} \dfrac{s^2+s-2+8s+2}{(s+1)(s^2+2s+2)} \\ \dfrac{s^2+s-1+2s^2+9s+4}{(s+1)(s^2+2s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s(s+9)}{(s+1)(s^2+2s+2)} \\ \dfrac{3s^2+10s+3}{(s+1)(s^2+2s+2)} \end{bmatrix} = \begin{bmatrix} \dfrac{9s+16}{s^2+2s+2} - \dfrac{8}{s+1} \\ \dfrac{7s+11}{s^2+2s+2} - \dfrac{4}{s+1} \end{bmatrix}$$

$$\mathcal{L}^{-1}\{\ \}$$

$$y(t) = \begin{bmatrix} \cancel{1-e^{-2t}} \\ 3 \end{bmatrix} + \begin{bmatrix} \cancel{e^{-2t}} + e^{-4t} \cancel{-1} \\ -e^{t} - 2 \end{bmatrix}$$

$$\boxed{y(t) = \begin{bmatrix} e^{-4t} \\ 1-e^{t} \end{bmatrix}}$$

2)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -16 & -2 & 7 \\ 0 & -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$$

$$Ab = \begin{bmatrix} 10 \\ -9 \\ -2 \end{bmatrix} \quad A^2 b = \begin{bmatrix} 2 \\ 4 \\ 13 \end{bmatrix} \quad A^3 b = \begin{bmatrix} -11 \\ 51 \\ -30 \end{bmatrix}$$

$$|sI - A| = \begin{vmatrix} s-1 & 0 & 1 \\ 16 & s+2 & -7 \\ 0 & 1 & s+2 \end{vmatrix} = s^3 + 3s^2 + 7s + 5$$
$$= (s+1)(s+(1-j2))(s+(1+j2))$$

$$\boxed{\bar{A} = \begin{bmatrix} 0 & 0 & -5 \\ 1 & 0 & -7 \\ 0 & 1 & -3 \end{bmatrix}}$$

$$A^3 b = -5b - 7Ab - 3A^2 b$$

$$\begin{bmatrix} -11 \\ 51 \\ -39 \end{bmatrix} = \begin{bmatrix} -5 + 0 - 6 \\ 0 + 63 - 12 \\ -5 + 6 - 39 \end{bmatrix} \checkmark$$

3)

$$\lambda_1 = -1 \qquad q_1 = \alpha \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$$

$$\lambda_{2,3} = -1 \pm j2 \qquad q_{2,3} = \alpha \begin{bmatrix} -3 \pm j1 \\ 20 \\ -4 \pm j8 \end{bmatrix}$$

$$\hat{A} = PAP^{-1}$$

$$\boxed{\hat{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -2 & -1 \end{bmatrix}}$$

$$P^{-1} = \begin{bmatrix} q_1 & Re(q_2) & Im(q_2) \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -1 & -3 & 1 \\ 2 & 20 & 0 \\ -2 & -4 & 8 \end{bmatrix}$$

$$\hat{B} = PB$$

$$\boxed{\hat{B} = \begin{bmatrix} -1.75 \\ 0.175 \\ -0.225 \end{bmatrix}}$$

$$P = \frac{1}{-80} \begin{bmatrix} 160 & 20 & -20 \\ -16 & -6 & 2 \\ 32 & 2 & -14 \end{bmatrix}$$

$$\hat{C} = C P^{-1}$$

$$P = \begin{bmatrix} -2 & -0.25 & 0.25 \\ 0.2 & 0.075 & -0.025 \\ -0.4 & -0.025 & 0.175 \end{bmatrix}$$

$$\boxed{\hat{C} = \begin{bmatrix} -3 & -23 & 1 \end{bmatrix}}$$

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4)

$$\Delta(s) = (s-1)^4$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s-1 & 0 & -1 & -1 \\ 0 & s-1 & 0 & 0 \\ 0 & 0 & s-1 & 1 \\ 0 & 0 & 0 & s-1 \end{bmatrix}$$

$$\operatorname{adj}(sI-A) = \begin{bmatrix} (s-1)^3 & 0 & (s-1)^2 & -(s-1)(s-2) \\ 0 & (s-1)^3 & 0 & 0 \\ 0 & 0 & (s-1)^3 & -(s-1)^2 \\ 0 & 0 & 0 & (s-1)^3 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{(s-1)^4} \operatorname{adj}(sI-A)$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 & \frac{1}{(s-1)^2} & \frac{(s-2)}{(s-1)^3} \\ 0 & \frac{1}{s-1} & 0 & 0 \\ 0 & 0 & \frac{1}{s-1} & \frac{-1}{(s-1)^2} \\ 0 & 0 & 0 & \frac{1}{s-1} \end{bmatrix}$$

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4) cont.

$$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = e^{-t}$$

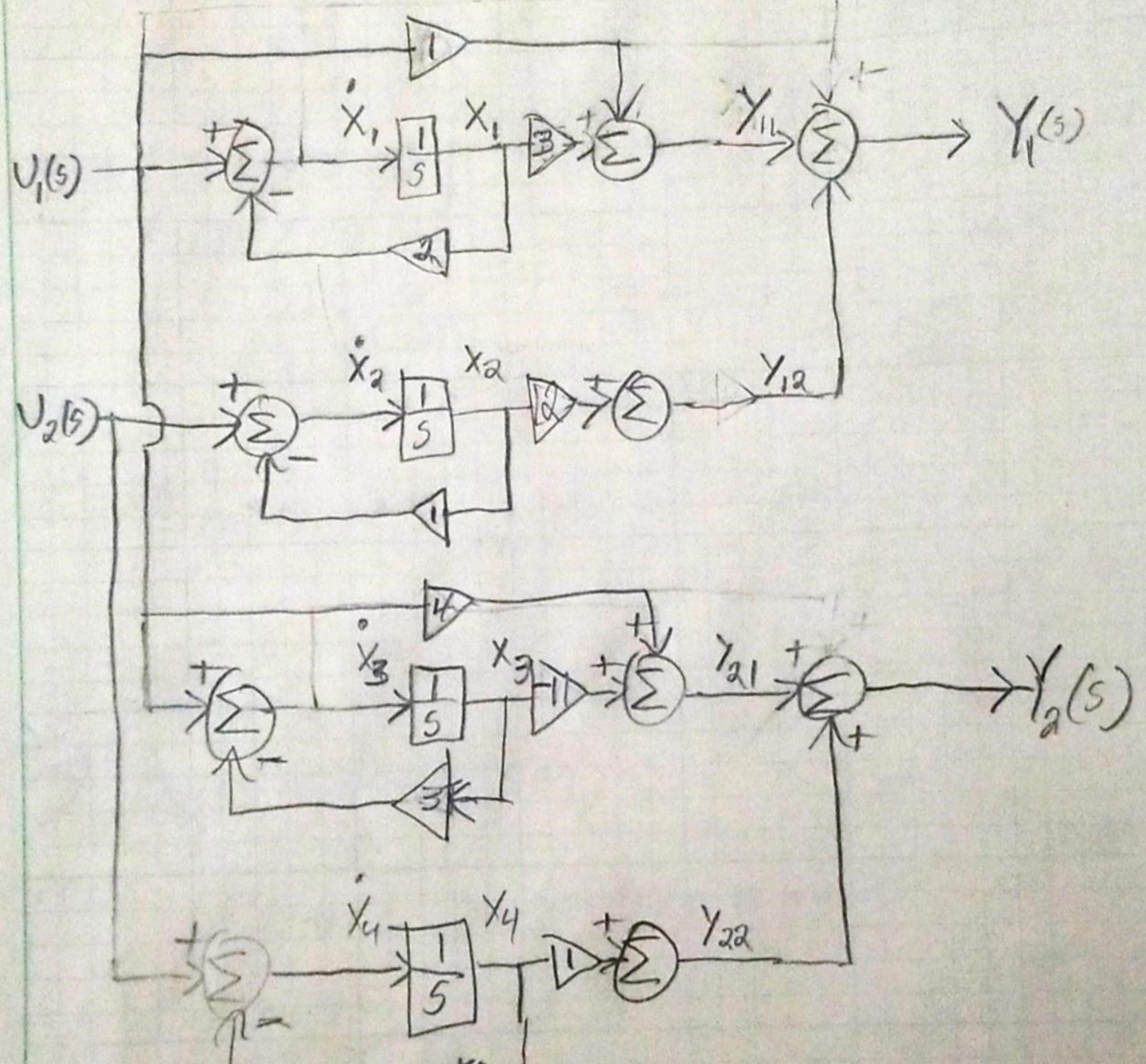
$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} = te^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{s-2}{(s-1)^3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3} - \frac{1}{(s-1)^2}\right\} = \frac{t^2}{2}e^{-t} - te^{-t}$$

$$e^{At} = \begin{bmatrix} e^{-t} & 0 & te^{-t} & \frac{t^2}{2}e^{-t} - te^{-t} \\ 0 & e^{-t} & 0 & 0 \\ 0 & 0 & e^{-t} & -te^{-t} \\ 0 & 0 & 0 & e^{-t} \end{bmatrix}$$

5)

$$\hat{G}(s) = \begin{bmatrix} \frac{s+5}{s+2} & \frac{2}{s+1} \\ \frac{4s+1}{s+3} & \frac{1}{s+2} \end{bmatrix} = \begin{bmatrix} 1 + \frac{3}{s+2} & \frac{2}{s+1} \\ 4 - \frac{11}{s+3} & \frac{1}{s+2} \end{bmatrix}$$



5) cont.

$$\dot{x} = Ax + Bu \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$y = Cx + Du$$

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 0 & 0 & -11 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$$

---

6)

a) $\Delta_1(s) = (s-2)(s+1)(s-1)$

$\Delta_2(s) = (s-2)(s+1)^2$

They are not equivent!

b) $G_1(s) = \dfrac{2}{(s-2)(s+1)}$

$G_2(s) = \dfrac{6}{(s-2)(s+1)}$

They are not zero-state equivelent!

7) $A(t) = \begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix} \qquad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = t x_2 \longrightarrow \boxed{x_2(t) = x_2(0) + \frac{t^2}{2} x_2(t)}$

$\frac{d x_1}{dt} = x_2(0) + \frac{t^2}{2} x_2(t) \qquad x_2(0) = \frac{t^2+2}{2} x_2(0)$

$\int dx_1 = \int_0^t \frac{\tau^2}{2} x_2(0) \, d\tau + x_1(0)$

$\boxed{x_1(t) = \frac{t^3}{6} x_2(0) + x_1(0)}$

$\boxed{\Psi(t) = \begin{bmatrix} 1 & \frac{t^3}{6} \\ 0 & \frac{t^2+2}{2} \end{bmatrix}}$

$\Psi^{-1}(t) = \frac{2}{t^2+2} \begin{bmatrix} \frac{t^2+2}{2} & \frac{-t^3}{6} \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{-t^3}{3(t^2+2)} \\ 0 & \frac{2}{t^2+2} \end{bmatrix}$

$\Phi(t,\tau) = \Psi(t) \Psi^{-1}(\tau)$

$= \begin{bmatrix} 1 & \frac{t^3}{6} \\ 0 & \frac{t^2+2}{2} \end{bmatrix} \begin{bmatrix} 1 & \frac{-\tau^3}{3(\tau^2+2)} \\ 0 & \frac{2}{\tau^2+2} \end{bmatrix}$

$\boxed{\Phi(t,\tau) = \begin{bmatrix} 1 & \frac{t^3 - \tau^3}{3(\tau^2+2)} \\ 0 & \frac{t^2+2}{\tau^2+2} \end{bmatrix}}$

8) $A(t) = \begin{bmatrix} -2 & e^{-t} \\ 0 & -1 \end{bmatrix}$ $\quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\dot{x}_1 = -2x_1 + e^{-t}x_2$

$\dot{x}_2 = -x_2 \longrightarrow \boxed{x_2(t) = x_2(0)e^{-t}}$

$\dot{x}_1 = -2x_1 + e^{-t}(x_2(0)e^{-t})$

$\dot{x}_1 + 2x_1 = x_2(0)e^{-2t}$

$\dot{x}_1 e^{2t} + 2x_1 e^{2t} = x_2(0)e^{-2t}e^{2t}$

$\frac{d}{dt}(x_1 e^{2t}) = x_2(0)$

$\int x_1 e^{2t} = \int_0^t x_2(0)\,d\tau$

$x_1 e^{2t} = t\,x_2(0) + \overset{x_1(0)}{c}$

$\boxed{x_1(t) = x_1(0)e^{-2t} + x_2(0) + e^{-2t}}$

$\Psi(t) = \begin{bmatrix} e^{-2t} & te^{-2t} \\ 0 & e^{-t} \end{bmatrix}$

$\Psi^{-1}(t) = \frac{1}{e^{-3t}}\begin{bmatrix} e^{-t} & -te^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$

$\Psi^{-1}(t) = \begin{bmatrix} e^{2t} & -te^{-t} \\ 0 & e^{t} \end{bmatrix}$

$\Phi(t,\tau) = \Psi(t)\Psi^{-1}(\tau)$

$\Phi(t,\tau) = \begin{bmatrix} e^{-2t} & te^{-2t} \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} e^{2\tau} & -\tau e^{-\tau} \\ 0 & e^{\tau} \end{bmatrix} = \begin{bmatrix} e^{-2(t-\tau)}, & -\tau e^{-2t-\tau} + te^{-2t+\tau} \\ 0, & e^{-(t-\tau)} \end{bmatrix}$

$$\boxed{\Phi(t,\tau) = \begin{bmatrix} e^{-2(t-\tau)} & (t)\left(e^{-(2t-\tau)} - e^{-(2t+\tau)}\right) \\ 0 & e^{-(t-\tau)} \end{bmatrix}}$$

MECH 6300 - Homework 4

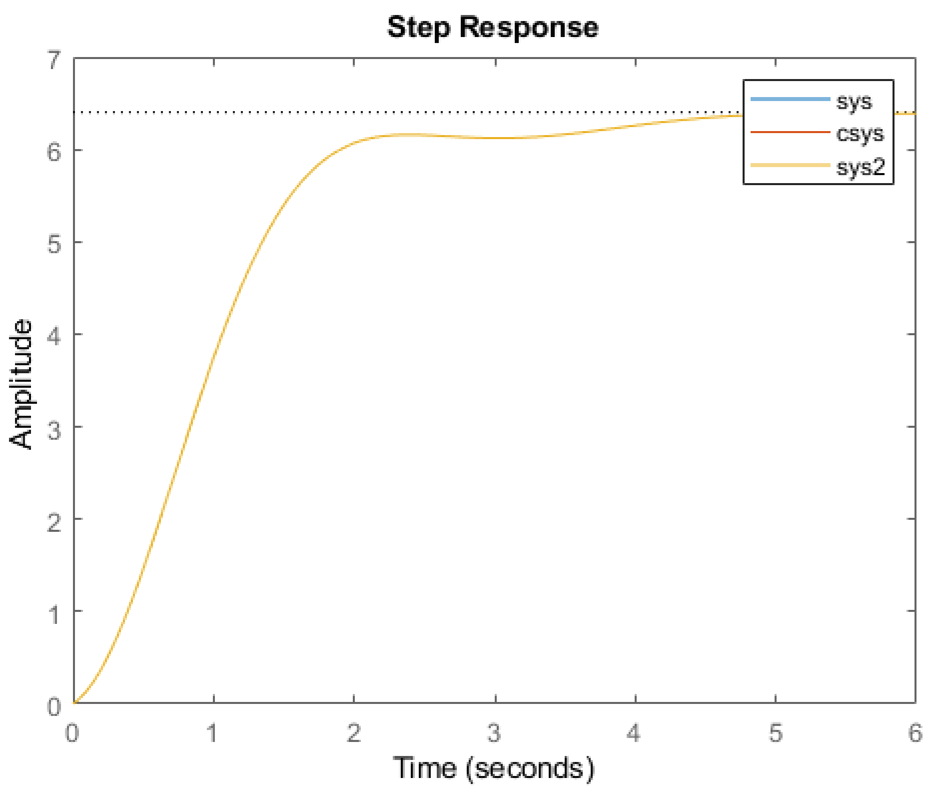
```
close all

% Problem 1

A = [0  1   0
     0  0   1
     -2 -4  -3];
B = [1  0
     0  1
     -1 1];
C = [0  1   -1
     1  2   0];
D = 0;

sys = ss(A,B,C,D)

% Matrix Inversions
syms s
sI_A = (s * eye(3) - A);
poles = factor(det(sI_A),s);
sI_A_inv = sI_A^-1


% Problem 2 and 3
A = [1  0   -1
    -16 -2  7
    0   -1  -2];
B = [1;0;1];
C = [1  -1  0];
D = 0;

sys = ss(A,B,C,D);
[csys,T] = canon(sys)

P_inv = [-1 -3  1
        2   20  0
        -2  -4  8];
P = inv(P_inv);

A_hat = P*A*P_inv;
B_hat = P*B;
C_hat = C*P_inv;
D_hat = D;

sys2 = ss(A_hat,B_hat,C_hat,D_hat)

figure()
step(sys)
hold on
step(csys)
```

```
step(sys2)
legend()
hold off


%Problem 4 Verification
A = [1  0   1   1
    0   1   0   0
    0   0   1   -1
    0   0   0   1];

s*eye(4)-A;
adjoint(s*eye(4)-A);
inv(s*eye(4)-A)
e_At = ilaplace(inv(s*eye(4)-A))
exp(A*t)


%Problem 6
A1 = [2 -1  2
      0 -1  -1
      0 0   1];
B1 = [1; 1; 0];
C1 = [1 -1 0];
D1 = 0;
sys1 = ss(A1,B1,C1,D1);
tf1 = tf(sys1)


A2 = [2 3   2
      0 -1  1
      0 0   -1];
B2 = [1; 1; 0];
C2 = [1 -1  0];
D2 = 0;
sys2 = ss(A2,B2,C2,D2);
tf2 = tf(sys2)


sys =

  A =
        x1  x2  x3
   x1    0   1   0
   x2    0   0   1
   x3   -2  -4  -3

  B =
        u1  u2
   x1    1   0
   x2    0   1
   x3   -1   1

  C =
```

```
       x1  x2  x3
   y1   0   1  -1
   y2   1   2   0

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.


sI_A_inv =

[ (s^2 + 3*s + 4)/(s^3 + 3*s^2 + 4*s + 2),        (s + 3)/(s^3 + 3*s^2
 + 4*s + 2),   1/(s^3 + 3*s^2 + 4*s + 2)]
[              -2/(s^3 + 3*s^2 + 4*s + 2),    (s*(s + 3))/(s^3 + 3*s^2
 + 4*s + 2),   s/(s^3 + 3*s^2 + 4*s + 2)]
[          -(2*s)/(s^3 + 3*s^2 + 4*s + 2), -(2*(2*s + 1))/(s^3 + 3*s^2
 + 4*s + 2), s^2/(s^3 + 3*s^2 + 4*s + 2)]


csys =

  A =
       x1  x2  x3
   x1  -1   0   0
   x2   0  -1   2
   x3   0  -2  -1

  B =
           u1
   x1   5.026
   x2  0.3246
   x3   1.484

  C =
           x1      x2      x3
   y1   1.044   2.617  -3.436

  D =
       u1
   y1   0

Continuous-time state-space model.


T =

    5.7446    0.7181   -0.7181
    0.9561   -0.1534   -0.6315
    2.1836    0.3925   -0.6994
```

```
sys2 =

  A =
       x1  x2  x3
   x1  -1   0   0
   x2   0  -1   2
   x3   0  -2  -1

  B =
           u1
   x1   -1.75
   x2   0.175
   x3  -0.225

  C =
        x1   x2   x3
   y1   -3  -23    1

  D =
       u1
   y1   0

Continuous-time state-space model.


ans =

[ 1/(s - 1),         0, 1/(s - 1)^2, (s - 2)/(s - 1)^3]
[         0, 1/(s - 1),           0,                 0]
[         0,         0,   1/(s - 1),      -1/(s - 1)^2]
[         0,         0,           0,         1/(s - 1)]


e_At =

[ exp(t),      0, t*exp(t), t*exp(t) - (t^2*exp(t))/2]
[      0, exp(t),       0,                          0]
[      0,      0,   exp(t),                 -t*exp(t)]
[      0,      0,       0,                     exp(t)]


ans =

[ exp(t),      1, exp(t),  exp(t)]
[      1, exp(t),      1,       1]
[      1,      1, exp(t), exp(-t)]
[      1,      1,      1,  exp(t)]


tf1 =

       2
  -----------
  s^2 - s - 2
```

```
Continuous-time transfer function.


tf2 =

         6
  -----------
  s^2 - s - 2

Continuous-time transfer function.
```



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