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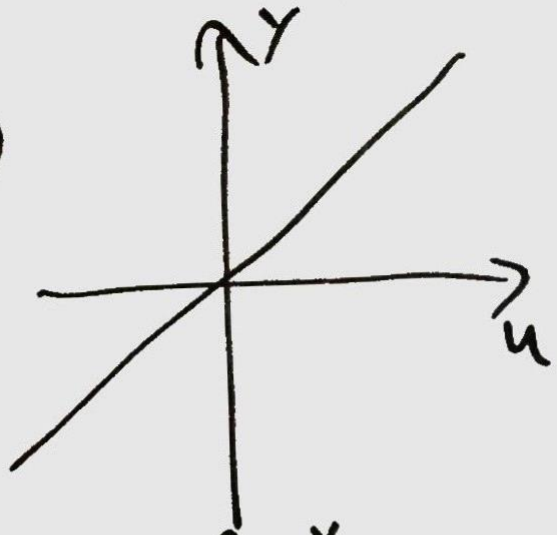
MECH 6300 - Fall 2020
Assignment 1
2020-08-21

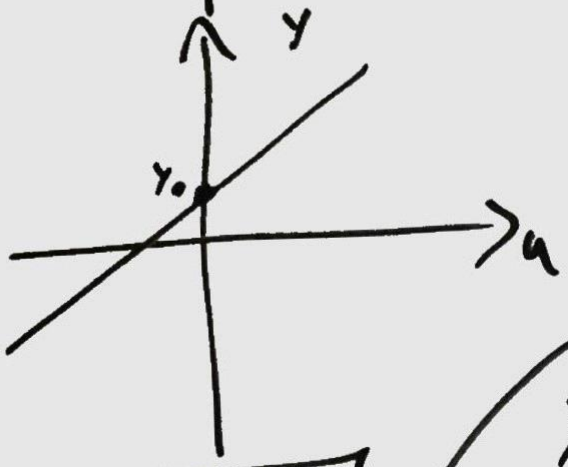
1) a)




$$y(u) = \alpha u$$

1st order Linear

b)




$$y(u) = \alpha u + y_0$$

Includes DC Gain...

Non-Linear

$$y(u_1) = \alpha u_1 + y_0$$

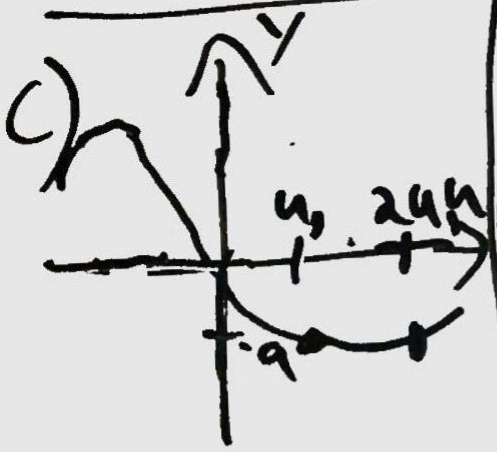
$$y(u_2) = \alpha u_2 + y_0$$

$$y(u_1 + u_2) = \alpha(u_1 + u_2) + y_0$$

$$\boxed{y(u_1 + u_2) \neq y(u_1) + y(u_2)}$$

Not Additive

c)




Non-Linear:

$$y(u_1) = -a$$

$$y(2u_1) = -a$$

$$\boxed{y(2u_1) \neq 2y(u_1)}$$

Not Homogeneous

2)

$$y(t) = (P_\alpha u)(t) = \begin{cases} u(t), & t \le \alpha \\ 0, & t > \alpha \end{cases}$$

a) The system is Linear w/in two Pts/rns

For $t \le \alpha$:

$y(u(\alpha - t_1)) = u(\alpha - t_1)$

$y(\beta_1 u_1(\alpha - t_1) + \beta_2 u_2(\alpha - t_1)) = \beta_1 u_1(\alpha - t_1) + \beta_2 u_2(\alpha - t_1)$

y(t) satisfies Superposition

For $t > \alpha$:

$y(u(\alpha + t_1)) = 0$

$y(\beta_1 u_1 + \beta_2 u_2) = 0$ } satisfies Superposition

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b) The System is Time-Varient

Let $u(t) = \beta t$, where $\beta$ is a Constant

$y(u(\alpha - t_1)) = \beta(\alpha - t_1)$

$y(u(\alpha + t_1)) = 0$ } Time Varient

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c) The System is Causal

y(u) is not dependent on any future values of u(t). //

3) a) $y(t) = t^2 u(t)$ → Linear

$y_1 = y(u_1(t)) = t^2 u_1(t)$

$y_2 = y(u_2(t)) = t^2 u_2(t)$ → satisfies Superposition

$y(\alpha u_1 + \beta u_2) = t^2(\alpha u_1 + \beta u_2)$

$\alpha y_1 + \beta y_2 = t^2(\alpha u_1 + \beta u_2)$

$\alpha y_1 + \beta y_2 \overset{\checkmark}{=} y(\alpha u_1 + \beta u_2)$

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b) $y(t) = 2(u(t))^2$

Let $u(t) = \alpha t$

$y(u(t)) = 2(\alpha t)^2 = 2\alpha^2 t^2$

$y(2u(t)) = 2(2\alpha t)^2 = 8\alpha^2 t^2$

$2y(u(t)) \neq y(2u(t))$

Non-Linear Does not Satisfy Homogeneity

4) c) $y(t) = \sin(u(t))$

Let $u(\tau) = \alpha t$

$y_1 = y(u(t)) = \sin(\alpha t)$

$y(u(t-t_0)) = \sin(\alpha(t-t_0)) = \sin(\alpha t - \alpha t_0)$

$y_1(t-t_0) = \sin(\alpha t - t_0)$

$y(u(t-t_0)) \neq y(u(t)-t_0)$

Time Invariant $\forall t_0 \neq \pm n2\pi$, $n = 0, 1, \ldots$

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d) $\dot{y} = -t\,y(t) + u(t)$, $t \geq 0$, $y(0) = 0$

state coefficient is not constant

$A(t) = t \Rightarrow$ Time-Invariant

b) $\ddot{y} - 4y = \dot{u} - 2u$

$s^2 Y(s) - 4 Y(s) = s U(s) - 2 U(s)$

$(s^2 - 4) Y(s) = (s - 2) U(s)$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{(s-2)}{(s-2)(s+2)}$$

$$\boxed{H(s) = \frac{1}{s+2}}$$

$\downarrow \mathcal{L}^{-1}$

$$\boxed{h(t) = e^{-2t}}$$