

MECH 6300-HW 3

3) b)

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -75 & -45 \\ 0 & 100 & 60 \end{bmatrix} \quad \lambda_1 = 0, m_1 = 2 \quad \lambda_2 = -15 \quad J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

i) Cayley-Hamilton Method:

$$h(\lambda) = 1 + (\lambda) \lambda + (e^{-15\lambda} - 1) \frac{\lambda^2}{15^2}$$

$$h(A) = I + (t)A + (e^{-15t} - 1) \frac{\lambda^2}{15^2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2t & t \\ 0 & -75t & -45t \\ 0 & 100t & 60t \end{bmatrix} + \begin{bmatrix} 0 & -\frac{2}{9} & -\frac{2}{15} \\ 0 & 5 & \frac{2}{3} \\ 0 & -\frac{20}{3} & 4 \end{bmatrix} (e^{-15t} - 1)$$

$$e^{At} = \begin{bmatrix} 1 & \frac{2}{9} - \frac{4}{3}t - \frac{2}{9}e^{-15t} & \frac{2}{15} - t + \frac{2}{15}e^{-15t} \\ 0 & -4 + 5e^{-15t} & -3 + 3e^{-15t} \\ 0 & \frac{20}{3} - \frac{20}{3}e^{-15t} & 5 - 4e^{-15t} \end{bmatrix}$$