

MECH 6300-HW 3

3) b) $A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -75 & -45 \\ 0 & 100 & 60 \end{bmatrix}$ $\lambda_1 = 0, m_1 = 2$ $\lambda_2 = -15$ $J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -15 \end{bmatrix}$

ii) Jordan Form Method: $Q = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -45 \\ 0 & -5 & 60 \end{bmatrix}$

$$e^{Jt} = \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-15t} \end{bmatrix}$$
$$Q^{-1} = \begin{bmatrix} 1 & \frac{2}{9} & \frac{2}{15} \\ 0 & \frac{4}{3} & -1 \\ 0 & \frac{1}{9} & \frac{1}{15} \end{bmatrix}$$

$$e^{At} = Q e^{Jt} Q^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -45 \\ 0 & -5 & 60 \end{bmatrix} \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-15t} \end{bmatrix} \begin{bmatrix} 1 & \frac{2}{9} & \frac{2}{15} \\ 0 & \frac{4}{3} & -1 \\ 0 & \frac{1}{9} & \frac{1}{15} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 1 & \frac{2}{9} - \frac{4}{3}t + \frac{2}{9}e^{-15t} & \frac{2}{15} - t - \frac{2}{15}e^{-15t} \\ 0 & -4 + 5e^{-15t} & -3 + 3e^{-15t} \\ 0 & \frac{20}{3} - \frac{20}{3}e^{-15t} & 5 - 4e^{-15t} \end{bmatrix}$$