MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems
Homework Assignment #2
Due: Friday, September 11, 2020 (by 10:00PM CDT)

In some of these problems you may find Matlab useful for verification of your results, but you should still do all calculations by hand. If you use Matlab, you should include your Matlab print-outs with your responses.

[1] Which of the following sets of vectors are linearly independent? Show your work.

(a) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\3\\2\end{bmatrix}, \begin{bmatrix}1\\-4\\1\end{bmatrix}$ in $(\mathbb{R}^3, \mathbb{R})$

(b) $\begin{bmatrix}2+i\\4+3i\end{bmatrix}, \begin{bmatrix}-1-i\\2-i\end{bmatrix}, \begin{bmatrix}i\\-1+5i\end{bmatrix}$ in $(\mathbb{C}^2, \mathbb{R})$

(c) te^{-t}, $2e^{-t}$, te^{-2t} in C , where C is the set of all piecewise continuous functions defined on $[0, \infty)$.

(d) $2s-1$, $2s-3$, s^2+2 in $(\mathbb{R}_3[s], \mathbb{R})$ (recall that in class we defined $\mathbb{R}_3[s]$ as the set of polynomials in *s* of degree less than 3).

[2] Show that the set of all 2 x 2 matrices with real coefficients forms a linear space over $\mathbb{R}$ with dimension 4.

[3] Show that similar matrices have the same characteristic polynomial, and consequently, the same set of eigenvalues. Hint: you may find it useful to utilize the identity det(AB)=det(A)det(B).

[4] What are the ranks and nullities of the following matrices?

(a) $\begin{bmatrix}4 & 3 & 1\\1 & -1 & -3\\3 & 0 & 2\end{bmatrix}$

(b) $\begin{bmatrix}1 & 0 & 1\\1 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3 & 4 & 5\\5 & 4 & 3 & 2 & 1\\2 & 1 & 0 & -1 & -2\end{bmatrix}$

[5] Does there exist a solution to the following linear equations? If so, find one.

$$\begin{bmatrix}4 & -1 & 0\\-2 & 8 & -4\\-5 & 5 & -2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\2\\0\end{bmatrix}$$