1)

a) $g(t) = \frac{3}{4t+1}$

$$\int_0^\infty |g(t)| dt \leq M < \infty$$

$$\int_0^\infty \left|\frac{3}{4t+1}\right| dt = \frac{3}{4} \ln(4t+1)\Big|_0^\infty = \frac{3}{4}\left(\infty - \ln(1)\right) \to \infty$$

$$\boxed{\int_0^\infty |g(t)| dt \not< \infty}$$

This system is <u>not</u> BIBO stable

b) $g(t) = 2te^{-4t}$

$$\int_0^\infty |g(t)| dt = \int_0^\infty |2te^{-4t}| dt = 2\int_0^\infty te^{-4t} dt$$

$$= -\frac{1}{8} e^{-4t}(4t+1)\Big|_0^\infty = 0 - -\frac{1}{8}(1)$$

$$\int_0^\infty |g(t)| dt = \frac{1}{8} < \infty$$

This system is BIBO stable

2)

$$A = \begin{bmatrix} -3 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Upper Triangular} \quad \lambda_1 = -3 \quad m_1 = 1, \quad \lambda_2 = 0 \quad m_2 = 2$$

$$(sI - A) = \begin{bmatrix} s+3 & 0 & -2 \\ 0 & s & -1 \\ 0 & 0 & s \end{bmatrix} \qquad |sI - A| = s^2(s+3)$$

The System is niether Asymptotically or Marginally stable I.S.L. as $Re\{\lambda_i\} \not< 0 \ \forall i$ and $m_i \not= 1 \ \forall i : Re\{\lambda_i\} = 0$

---

3)

$$\Delta(s) = s^5 + 6s^4 + 13s^3 + 2s^2 + 4s + 1$$

$$H(s) = 0$$

The system is not Asymptotically stable due to the Poles:

$$\lambda = 0.11 \pm j0.57$$

which are on the RHP.

The system is BIBO stable due to the output $y = 0 \ \forall t \geq 0$ //

4) $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 4 & 2 \end{bmatrix}$ $D = 0$

Upper Triangular:

$\left.\begin{array}{ll} \lambda_1 = -3 & m_1 = 1 \\ \lambda_2 = -2 & m_2 = 1 \\ \lambda_3 = 0 & m_3 = 1 \end{array}\right\} H(s) = \dfrac{9(s+3.11)}{(s+3)(s+2)}$

Asmptotic Stability:

The system is not Asymptotically stable due to $\lambda_3 = 0$, which is not stricty on the LHP.

BIBO Stability:

The system is BIBO stable because the Poles of the transfer function $\lambda(H(s)) = -3, -2$ which both reside on the LHP.//

5)

a) $$A = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} \qquad (sI - A) = \begin{bmatrix} s-2 & -3 & -2 \\ -3 & s-1 & 0 \\ -2 & 0 & s-2 \end{bmatrix}$$

$$|sI - A| = (s-2)(s-1)(s-2) + [+3(-3(s-2))] + (-2)(+2(s-1)) = s^3 - 5s^2 - 5s + 18$$

$\lambda_1 = -2$

$\lambda_2 = 1.70$

$\lambda_3 = 5.31$

3.7: Condition 1 Not True

$Re\{\lambda_i\} \not> 0 \ \forall i$

3.7: Condition 2

$M_1 = 2$

$M_2 = 2 - 9 = -7$

$$M_3 = 2(2) - 3(6) + 2(-2) = 4 - 18 - 4 = -18$$

Condition 2 does not hold...

The Matrix is Not Positive definite

5)
b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$\lambda_1 = -0.41$
$\lambda_2 = 0$
$\lambda_3 = 2.41$

Condition 1: Does not hold

Condition 2:
$M_1 = 0$
$M_2 = 0$
$M_3 = 0$

The Matrix is ~~semi~~ positive semi-Definite.

---

c) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\lambda_1 = 0, m_1 = 2$
$\lambda_2 = \phi, m_1 = 1$

Condition 1: positive semi-definite

Condition 2: positive semi-definite
$M_1 = 0$
$M_2 = 0$
$M_3 = 0$

MECH 6300 - Homework 5

```
% Problem 3
charEq = [1 6   13  2   4   1]
poles3 = roots(charEq)


% Problem 4
A = [-3 1   0
    0   -2   0
    0   0   0];
B = [1
    2
    0];
C = [1  4   2];
D = 0;
sys = ss(A,B,C,D);
zpk4 = zpk(sys)


% Problem 5
A = [2  3   2
    3   1   0
    2   0   2];
eig_5a = eig(A)
syms s
delta_s = det(s*eye(3) - A)

A = [0  0   1
    0   0   0
    1   0   2];
Eig_5b = eig(A)
```

```
charEq =

     1     6    13     2     4     1


poles3 =

  -2.9902 + 1.9135i
  -2.9902 - 1.9135i
   0.1096 + 0.5658i
   0.1096 - 0.5658i
  -0.2389 + 0.0000i


zpk4 =

  9 (s+3.111)
  -----------
```

```
  (s+3) (s+2)

Continuous-time zero/pole/gain model.


eig_5a =

   -2.0000
    1.6972
    5.3028


delta_s =

s^3 - 5*s^2 - 5*s + 18


Eig_5b =

   -0.4142
         0
    2.4142
```

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