1/2) a) $A = \begin{bmatrix} 0 & 2 \\ 0 & e^{2t} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 2 \end{bmatrix}$

$$V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & e^{2t} \end{bmatrix} \quad \det(V) = 2 \rightarrow \text{full Rank}$$

1a) This system is controllable.//

$$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & e^{2t} \end{bmatrix} \quad \det(V) = 0 \rightarrow \text{not full rank}$$

2a) This system is not observable.//

1/2) b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 3 & 0 \\ -2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

$$V = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 & 0 & -2 & 1 \\ 3 & 0 & -2 & 1 & -6 & -3 \\ -2 & 1 & -6 & -3 & 20 & 5 \end{bmatrix}$$

$$\text{rref}(V) = \begin{bmatrix} 1 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - \end{bmatrix} \rightarrow \rho(V) = 3$$

$-6 \quad -1R$
$+8 \quad +6$
$+18 \quad 5$
20

1b) The system is controllable.//

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & 0 \\ -6 & -12 & -8 \\ 0 & 0 & 2 \end{bmatrix}$$

2b) The system is Observable.//

$$\text{rref} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ : & : & : \\ : & : & : \end{bmatrix} \quad \rho(V) = 3$$

4/2)c) $A = \begin{bmatrix} 0 & 4 & 5 \\ 0 & 8 & 9 \\ 0 & -1 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 3 & 2 \end{bmatrix}$

$V = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 13 & 75 \\ 2 & 25 & 155 \\ 1 & -5 & -10 \end{bmatrix}$ $\text{rref}(V) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\rho(V) = 3$

4c) The system is controllable.//

$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ 0 & 22 & 21 \\ 0 & 155 & 135 \end{bmatrix}$ $\text{rref}(V) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(V) = 2$

The system is not observable.//

3/4) $A = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \rightarrow \rho(V) = 1$ (Not controllable)

$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \rightarrow \rho(V) = 1$ (Not observable)

3) cont.

Let $P^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow P = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$\bar{A} = P^{-1}AP = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & -5 \end{bmatrix}$$

$$\bar{B} = P^{-1}B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\bar{C} = CP = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

5) Controllability:

$\lambda = 2$:

$$U_2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \rightarrow \det(U_2) = -1 \rightarrow \text{Full Rank}$$

$\lambda = 2$ is controllable

$\lambda = 1$:

$$U_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \rho(U_1) = 2 \rightarrow \lambda = 1 \text{ is controllable}$$

Observability:

$\lambda = 2$

$$V_2 = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \det(V_2) = 0 \rightarrow \text{Not Full Rank}$$

$\lambda = 2$ is Not observable

$$V_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \rho(V_1) = 2 \rightarrow \lambda = 2 \text{ is observable}$$