1)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 1 & 3 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} u$$

$\lambda_{as} = \{-1 \pm j2, -2 \pm j3\}$

$$\Delta_{as}(s) = (s+1-j2)(s+1+j2)(s+2-j3)(s+2+j3)$$
$$= (s^2 + 2s + 5)(s^2 + 4s + 13)$$
$$= s^4 + \underbrace{6}_{\alpha_1} s^3 + \underbrace{26}_{\alpha_2} s^2 + \underbrace{46}_{\alpha_3} s + \underbrace{65}_{\alpha_4}$$

$$\Delta(s) = |sI - A| = \begin{vmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 2 & -1 & s+3 & -1 \\ -1 & -2 & 0 & s \end{vmatrix}$$

$= s[s(s(s+3)(s) + 1(-s-2)) + 1(+1(2s-1))$

$= s^4 - \underbrace{3s^3}_{\alpha_1} - \underbrace{s^2}_{\alpha_2} - \underbrace{1}_{\alpha_4}$

$\alpha_3 = 1$

$$U = \begin{bmatrix} b & Ab & A^2b & A^3b \\ 0 & 0 & 0 & 0 & 2 & 1 & 6 & 4 \\ 0 & 0 & 2 & 1 & 6 & 4 & 20 & 13 \\ 2 & 1 & 6 & 4 & 20 & 13 & 66 & 43 \\ 0 & 1 & 0 & 0 & 4 & 2 & 14 & 9 \end{bmatrix}$$

✓

$\rho(U) = 4$

1) cont.

③ Let $F = \begin{bmatrix} -1 & -2 & 0 \\ 2 & -1 & \\ & & -2 & -3 \\ 0 & & 3 & -2 \end{bmatrix}$ ② and $\bar{K} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$p(U) \neq 4$ ← from MATLAB

Observable ← $u = -Kx$ vs $u = Kx$

③ $AT - TF = -B\bar{K} = B$

$AT + T(-F) = (B\bar{K}) \rightarrow \text{lyap}(A, -F, +B\bar{K})$

$T =$ (in MATLAB...)

$|T| = -2.38 \times 10^{-4} \neq 0 \rightarrow$ non-singular

(it is close though...)

④ $K = \bar{K} T^{-1} = \begin{bmatrix} 5.2 & 2.2 & 3.1 & 0.68 \\ 8.0 & 6.3 & -2.1 & 5.0 \end{bmatrix}$

2) see MATLAB

- They are not unique. K is not unique for any A+BK and the Place() command selects the most robust K, but in this case, the K I chose was not it. (numerically/computationally doesn't always have the same result)

3)

$$A = \begin{bmatrix} -3 & -1 & -2 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 6 & 4 & 2 \\ -16 & -14 & -8 \end{bmatrix} \quad \rho(V) = 3 \checkmark$$

observable

Let $F = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ and $G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$U = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -3 & 9 \\ 1 & -4 & 16 \end{bmatrix} \quad \rho(U) = 3 \checkmark$$

$-FT + AT = GC \rightarrow$ lyap(-F, A, -GC)

T = (in MATLAB) $|T| = 10^{15}$

non-singular ✓

$$L = T^{-1}G = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} -3 & -1 & -2 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{bmatrix} \hat{x} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} \left(y - \begin{bmatrix} -2 & -1 & 0 \end{bmatrix} \hat{x} \right)$$

4) $\lambda = -2, -4$

Let $F = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$ $G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 1 & -2 \\ 1 & -4 \end{bmatrix}$ $\rho(u) = 2$ ✓ Controllable

$-FT + TA = GL \rightarrow \text{lyap}(-F, A, -GC)$

$T = \begin{bmatrix} 1 & 1 & -1 \\ -1.67 & -1.33 & -0.33 \end{bmatrix} \rightarrow H = TB = \begin{bmatrix} 1 \\ -3.66 \end{bmatrix}$

$P = \begin{bmatrix} C \\ T \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & -1 \\ -\frac{5}{3} & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix}$

$\dot{z} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} z + \begin{bmatrix} 1 \\ 1 \end{bmatrix} y + \begin{bmatrix} 1 \\ -\frac{11}{3} \end{bmatrix} u$

$\hat{x} = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & -1 \\ -\frac{5}{3} & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}$