MECH 6300 - HW 3

3) b)

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -75 & -45 \\ 0 & 100 & 60 \end{bmatrix} \qquad \begin{matrix} \lambda_1 = 0,\ m_1 = 2 \\ \lambda_2 = -15 \end{matrix} \quad J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

i) Cayley-Hamilton Method:

$$f(\lambda) = e^{\lambda t} \qquad h(\lambda) = \beta_0 + \beta_1 \lambda + \beta_2 \lambda^2$$

$$f'(\lambda) = t e^{\lambda t} \qquad h'(\lambda) = \beta_1 + 2\beta_2 \lambda$$

$$f(0) = 1 = h(0) = \beta_0 \Rightarrow \boxed{\beta_0 = 1}$$

$$f'(0) = t = h'(0) = \beta_1 \Rightarrow \boxed{\beta_1 = t}$$

$$f(-15) = e^{-15t} = h(-15) = \cancel{\beta_0}^{\,1} \cancel{-15\beta_1}^{\,0} + 15^2 \beta_2$$

$$\boxed{\beta_2 = \frac{e^{-15t} - 1}{15^2}}$$

$$h(\lambda) = 1 + (t)\lambda + \frac{e^{-15t} - 1}{15^2} (\lambda)^2$$

$$\boxed{e^{At} = I + (e^{-15t} - 1) \frac{A^2}{15^2}} \qquad \frac{A^2}{15^2} = \begin{bmatrix} 0 & \frac{-2}{9} & \frac{-2}{15} \\ 0 & 5 & 3 \\ 0 & \frac{-20}{3} & -4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 1 & \frac{2(1 - e^{-15t})}{9} & \frac{2(1 - e^{-15t})}{15} \\ 0 & -4 + 5e^{-15t} & -3(1 - e^{-15t}) \\ 0 & \frac{20(1 - e^{-15t})}{3} & 5 - 4e^{-15t} \end{bmatrix}$$