MECH 6300-Exam1 | Jonas Wagner | 2020-09-23

1)

I am in agreement with the pledge stated in problem 1.

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2)

$$\dot{x}_1 = x_1 - x_1^2 + \alpha u^2$$
$$\dot{x}_2 = x_1 + x_2^3 \qquad x_e = \begin{bmatrix} -1 \\ \beta \end{bmatrix} \quad u_e = 1$$

a) $\alpha = 2$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -(-1)^2 \\ \beta^3 \end{bmatrix} + \begin{bmatrix} 2(1)^2 \\ 0(1)^2 \end{bmatrix}$$

$$0 = \cancel{-1 - 1 + 2} \xrightarrow{-2} 0$$
$$0 = -1 + \beta^3 \longrightarrow \boxed{\beta = 1}$$

b) $h = \begin{bmatrix} x_1 - x_1^2 + \alpha u^2 \\ x_1 + x_2^3 \end{bmatrix} \qquad x_e = \begin{bmatrix} -1 \\ \beta \end{bmatrix} \quad u_e = 1$

$$\frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} \end{bmatrix}_{\substack{x=x_e \\ u=u_e}} = \begin{bmatrix} 1-2x_1 & 0 \\ 1 & 3x_2^2 \end{bmatrix}_{x=x_e}$$

$$\boxed{A = \begin{bmatrix} 3 & 0 \\ 1 & 3\beta^2 \end{bmatrix}}$$

2) b) cont.

$$\frac{\partial h}{\partial u} = \begin{bmatrix} \frac{\partial h_1}{\partial u} \\ \frac{\partial h_2}{\partial u} \end{bmatrix}_{\substack{x=x_e \\ u=u_e}} = \begin{bmatrix} 2\alpha u \\ 0 \end{bmatrix}_{u=u_e} \rightarrow \boxed{B = \begin{bmatrix} 2\alpha \\ 0 \end{bmatrix}}$$

$$\dot{x} = Ax + Bu$$

$$\boxed{A = \begin{bmatrix} 3 & 0 \\ 1 & 3\beta^2 \end{bmatrix} \qquad B = \begin{bmatrix} 2\alpha \\ 0 \end{bmatrix}}$$

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3)

a)

$$W = \begin{bmatrix} 1 & 1 & 2 \\ \alpha & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$$

$$|W| = 1(0-2) - \alpha(4-2) + 3(2-0) = -2 - 2\alpha + 6 = 4 - 2\alpha$$

if $\alpha = 2$
$\rightarrow |W| = 0$

$$\boxed{\text{L.I. } \forall \alpha \neq 2}$$

b)

$$V = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & \beta \end{bmatrix}$$

$$|V| = 2(2\beta - 0) + 0 + 0 = 4\beta$$

if $\beta = 0$
$\rightarrow |V| = 0$

$$\boxed{\text{L.I. } \forall \beta \neq 0}$$

3) c)

$$V = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & \beta \end{bmatrix}$$

Cofactors:

$$\begin{bmatrix} (\lambda-2)(\lambda-\beta) & 0 & 0 \\ 0 & (\lambda-2)(\lambda-\beta) & 0 \\ 0 & (\lambda-2)(\lambda-\beta) & (\lambda-2)^2 \end{bmatrix}$$

$$(\lambda I - V) = \begin{bmatrix} \lambda-2 & 0 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & -3 & \lambda-\beta \end{bmatrix}$$

$$\boxed{adj(\lambda I - V) = \begin{bmatrix} (\lambda-2)(\lambda-\beta) & 0 & 0 \\ 0 & (\lambda-2)(\lambda-\beta) & (\lambda-2)(\lambda-\beta) \\ 0 & 0 & (\lambda-2)^2 \end{bmatrix}}$$

d) $\beta = 1$

$$d(\lambda) = (\lambda-2)$$

$$\Delta(s) = (s-2)^2(s-1)$$

$$\psi(s) = \frac{\Delta(s)}{d(s)} -$$

$$\boxed{\psi(s) = (s-2)(s-1)}$$

e) $\Delta(s) = (s-2)^3$

$$d(\lambda) = (\lambda-2)^2$$

$$\boxed{\psi(s) = (s-2)}$$

I noticed I switched to s by default... same as λ

4) $\dot{x} = Ax$

$$A = \begin{bmatrix} -3 & -4 \\ \alpha & 1 \end{bmatrix}$$

a) $$(sI - A) = \begin{bmatrix} s+3 & 4 \\ -\alpha & s-1 \end{bmatrix}$$

$$\Delta(s) = (s+3)(s-1) - -4\alpha$$
$$= s^2 + 3s - s - 3 + 4\alpha$$
$$= s^2 + 2s + 4\alpha - 3$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4(1)(4\alpha - 3)}}{2(1)}$$

$$= -1 \pm \sqrt{\frac{4 - 16\alpha + 12}{4}} = -1 \pm \sqrt{\frac{16 - 16\alpha}{4}}$$

$$\boxed{\lambda_{1,2} = -1 \pm \sqrt{\beta} \quad , \quad \beta = 4(1-\alpha)}$$

4) b)

$$A = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

$$|sI - A| = s^2 + 2s + 1 = (s+1)^2 \quad \lambda_{1,2} = -1$$

$$(A - \lambda I) = \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\left.\begin{aligned} -2a - 4b &= 0 \\ a + 2b &= 0 \end{aligned}\right] \rightarrow \quad a = -2b \quad \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} -2\alpha \\ \alpha \end{pmatrix}$$

$$(A - \lambda I)x_2 = x_1$$

$$\left.\begin{aligned} -2(a) - 4b &= -2 \\ a + 2b &= 1 \end{aligned}\right] \rightarrow \left.\begin{aligned} a - 2b &= 1 \\ a + 2b &= 1 \end{aligned}\right] \rightarrow \begin{aligned} a &= 1 \\ b &= 0 \end{aligned}$$

$$\boxed{x_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad x_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}} \qquad x_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

4) c) $J = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ $M = \begin{bmatrix} 2 & 0 \\ -1 & -\frac{1}{2} \end{bmatrix}$

$$\boxed{e^{Jt} = \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{bmatrix}}$$

$M^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & -2 \end{bmatrix}$

d) $e^{At} = M e^{Jt} M^{-1}$

$= \begin{bmatrix} 2 & 0 \\ -1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2e^{-t} & 2te^{-t} \\ -e^{-t} & -te^{-t} - \frac{1}{2}e^{-t} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & -2 \end{bmatrix}$

$e^{At} = e^{-t} - 2te^{-t} \quad -4te^{-t}$

$-\frac{1}{2}e^{-t} + te^{-t} + \frac{1}{2}e^{-t} \quad 2te^{-t} + e^{-t}$

$e^{At} =$

$$\boxed{\begin{bmatrix} e^{-t} - 2te^{-t} & -4te^{-t} \\ te^{-t} & 2te^{-t} + e^{-t} \end{bmatrix}}$$

5) $A = \begin{bmatrix} 1 & \alpha \\ -2 & \beta \end{bmatrix}$ $\bar{A} = \begin{bmatrix} -3 & 1 \\ 0 & -3 \end{bmatrix}$

$(sI - A) = \begin{bmatrix} s-1 & -\alpha \\ 2 & s-\beta \end{bmatrix}$

$\Delta(s) = (s+3)^2$
$= s^2 + 6s + 9$

$\Delta(s) = (s-1)(s-\beta) - -2\alpha$
$= s^2 - s - \beta s + \beta + 2\alpha$

$$\boxed{\begin{aligned} \beta &= -7 \\ \alpha &= 8 \end{aligned}}$$

$9 = -7 + 2\alpha$
$16 = 2\alpha$

6.1) True: c

6.2) True: a, b, f

6.3) True: d

A Typo on d?

how many degrees less...

6.4) True: a, c, f

6.5) True: a