```matlab
% MECH 6300 - HW 8

% Problem 1/2
 --------------------------------------------------------
n = 4;
A = [0   1    0    0;
     0   0    1    0;
     -2  1    3    1;
     1   2    0    0];
B = [0   0;
     0   0;
     2   1;
     0   1];


lambda_0 = eig(A)

syms s

s_I_A_inv = inv(s * eye(n) - A)

charPoly = factor(det(s * eye(n) - A),'FactorMode', 'real')

U = ctrb(A,B)
rank(U)

p = [-1+j*2,-1-j*2,-2+3*j,-2-3*j]

% Lyap Method
F = blkdiag([-1,-2;2,-1],[-2,-3;3,-2])
K_hat = [eye(2),eye(2)]

obsv_rank = rank(obsv(F,K_hat))

T = lyap(A,-F,B*K_hat)

det_T = det(T)

K_lyap = K_hat * inv(T)

eig_A_BK_lyap = eig(A+B*K_lyap)

% Place Method
K_place = place(A,-B,p)

eig_A_BK_place = eig(A + B * K_place)
%----------------------------------------------------------------------

% Problem 3/4 -------------------------------------------------------
A = [-3 -1  -2;
     0   -2  2;
     1   0   -2];
B = [2;0;1];
```

```matlab
C = [-2,-1,0];

V = obsv(A,C)
rank_V = rank(V)

% Full Order
F = diag([-2,-3,-4])
G = ones(3,1)

u = ctrb(F,G)
rank_ctrl_FG = rank(u)

T = lyap(-F,A,-G*C)
det_T = det(T)

H = T * B

L = inv(T) * G

% Reduced Order
F = diag([-2,-4])
G = [1;1]

u = ctrb(F,G)
rank_u_FG = rank(u)

T = lyap(-F,A,-G*C)

H = T*B
```

lambda_0 =

* 3.3277 + 0.0000i*
* 0.2093 + 0.5991i*
* 0.2093 - 0.5991i*
* -0.7463 + 0.0000i*

s_I_A_inv =

*[(- s^3 + 3*s^2 + s + 2)/(- s^4 + 3*s^3 + s^2 + 1), -(s*(s -*
* 3))/(- s^4 + 3*s^3 + s^2 + 1), -s/(- s^4 + 3*s^3 + s^2 + 1),*
* -1/(- s^4 + 3*s^3 + s^2 + 1)]*
*[(2*s - 1)/(- s^4 + 3*s^3 + s^2 + 1), -(s^2*(s -*
* 3))/(- s^4 + 3*s^3 + s^2 + 1), -s^2/(- s^4 + 3*s^3 + s^2 + 1),*
* -s/(- s^4 + 3*s^3 + s^2 + 1)]*
*[-(- 2*s^2 + s)/(- s^4 + 3*s^3 + s^2 + 1), -(s^2 +*
* 1)/(- s^4 + 3*s^3 + s^2 + 1), -s^3/(- s^4 + 3*s^3 + s^2 + 1),*
* -s^2/(- s^4 + 3*s^3 + s^2 + 1)]*
*[(- s^2 + 3*s + 5)/(- s^4 + 3*s^3 + s^2 + 1), (- 2*s^2 + 5*s +*
* 3)/(- s^4 + 3*s^3 + s^2 + 1), -(2*s + 1)/(- s^4 + 3*s^3 + s^2 + 1),*
* (- s^3 + 3*s^2 + s - 2)/(- s^4 + 3*s^3 + s^2 + 1)]*

```
charPoly =

[s + 0.74625545091336047778605769765349,
 s - 3.3276508529770512902478103325707, s^2
 - 0.41860459793630918753824737194646*s +
 0.40269360337510953075288601599424]


U =

     0      0      0      0      2      1      6      4
     0      0      2      1      6      4     20     13
     2      1      6      4     20     13     66     43
     0      1      0      0      4      2     14      9


ans =

     4


p =

  -1.0000 + 2.0000i  -1.0000 - 2.0000i  -2.0000 + 3.0000i  -2.0000 -
 3.0000i


F =

    -1     -2      0      0
     2     -1      0      0
     0      0     -2     -3
     0      0      3     -2


K_hat =

     1      0      1      0
     0      1      0      1


obsv_rank =

     4


T =

    0.0523     0.0585     0.0132     0.0222
    0.0646    -0.1631     0.0403    -0.0839
   -0.3908     0.0338    -0.3323     0.0470
   -0.3292    -0.0738    -0.2116    -0.1098
```

```
det_T =

  -2.3800e-04


K_lyap =

   -5.2374   -2.2019   -3.0500   -0.6816
   -7.9972   -6.2908    2.1123   -5.0124


eig_A_BK_lyap =

  -2.0000 + 3.0000i
  -2.0000 - 3.0000i
  -1.0000 + 2.0000i
  -1.0000 - 2.0000i


K_place =

   10.4518     0.2611   -1.1681    1.2051
  -29.5067   -12.1276   -4.8645   -1.7993


eig_A_BK_place =

  -2.0000 + 3.0000i
  -2.0000 - 3.0000i
  -1.0000 + 2.0000i
  -1.0000 - 2.0000i


V =

    -2    -1     0
     6     4     2
   -16   -14    -8


rank_V =

     3


F =

    -2     0     0
     0    -3     0
     0     0    -4


G =
```

```
        1
        1
        1


u =

        1     -2      4
        1     -3      9
        1     -4     16


rank_ctrl_FG =

        3


T =

    1.0e+15 *

     0.0000     0.0000    -0.0000
    -3.0024    -3.0024    -0.0000
    -0.0000    -0.0000    -0.0000


det_T =

    1.0008e+15


H =

    1.0e+15 *

     0.0000
    -6.0048
    -0.0000

Warning: Matrix is close to singular or badly scaled. Results may be
 inaccurate.
RCOND =  5.551115e-17.

L =

    -2.0000
     2.0000
    -1.0000


F =

    -2      0
```

```
     0      -4


G =

     1
     1


u =

     1     -2
     1     -4


rank_u_FG =

     2


T =

    1.0000    1.0000   -1.0000
   -1.6667   -1.3333   -0.3333


H =

    1.0000
   -3.6667
```

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