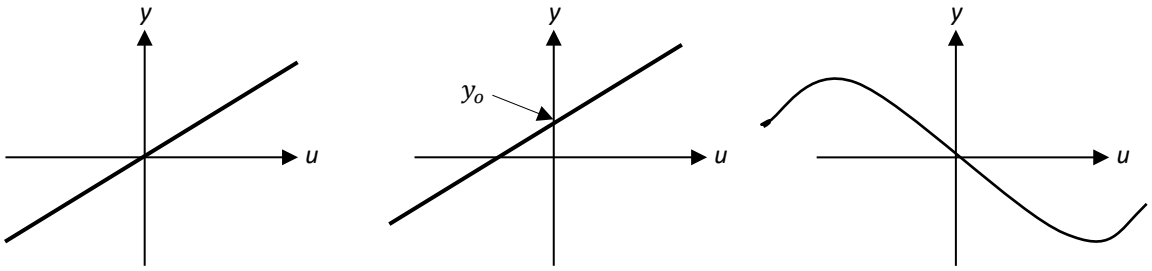
MECH 6300/EECS 6331/SYSM 6307 Linear Systems
Homework Assignment #1
Due: Monday, August 31, 2020 by 10:00pm (US Central time)

1. Consider the systems shown below, all without memory, where *u* is the input and *y* is the output. Discuss linearity for each system in mathematical terms.



2. Consider a system whose input *u* and output *y* are related by

$$y(t) = (P_\alpha u)(t) = \begin{cases} u(t) & \text{for } t \le \alpha \\ 0 & \text{for } t > \alpha \end{cases}$$

where α is a constant. This system relationship is called a "truncation operation", which truncates the input after a specified time. Provide mathematical arguments to answer the following questions: (a) Is the system linear? (b) Is the system time-invariant? (c) Is the system causal?

3. Provide a mathematical argument to determine whether the systems with output *y* and input *u* described by the following equations are linear:

a) $y(t) = t^2 u(t)$
b) $y(t) = 2u^2(t)$

4. Provide a mathematical argument to determine whether the systems with output *y* and input *u* described by the following equations are time invariant:

c) $y(t) = \sin(u(t))$
d) $\dot{y} = -ty(t) + u(t), t \ge 0, y(0) = 0.$

5. Consider a linear system with output *y* and input *u* described the following differential equation:

$$\ddot{y} - 4y = \dot{u} - 2u$$

Derive the transfer function and the impulse response.