MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems
Homework Assignment #4
Due: Friday, October 9, 2020 (10:00PM CDT)

In some of these problems you may find Matlab useful for verification of your results. Also, except for Problem 4, you may use Matlab for matrix inversion (for matrices larger than 2x2), but you should do other calculations by hand. If you use Matlab, include Matlab print-outs with your responses.

[1] Find the solution of

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix} x(t)$$

for $t \geq 0$, with

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad u(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[2] For the following system, find the *companion form* (recall we did an example in class; see also Section 3.4 in the text).

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 & -1 \\ -16 & -2 & 7 \\ 0 & -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x(t)$$

[3] For the system of Problem [2], follow the example on canonical forms in the text to get the *modal form* $\bar{A}$ for the system matrix A. Confirm your answer by using the Matlab command **canon** and then also to obtain $\bar{B}$, $\bar{C}$, and $\bar{D}$.

[4] Find e^{At} for the following matrix using $\mathcal{L}\{e^{At}\} = (sI - A)^{-1}$. Note: Although this is a simple (sparse) matrix, it is still a little tedious, but very mechanical; working through it by hand will give you an appreciation for matrix inverse operations and the complexity of the problem.

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[5] Find a realization for the proper rational matrix

$$\hat{G}(s) = \begin{bmatrix} \dfrac{s+5}{s+2} & \dfrac{2}{s+1} \\ \dfrac{4s+1}{s+3} & \dfrac{1}{s+2} \end{bmatrix}$$

[6] Determine if the following two systems are (a) equivalent, or (b) zero-state equivalent. For part (b), you can make use of the Matlab command **ss2tf** in your solution.

$$\dot{x}(t)=\begin{bmatrix} 2 & -1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \quad , \quad y(t) = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x(t)$$

$$\dot{x}(t)=\begin{bmatrix} 2 & 3 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \quad , \quad y(t) = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x(t)$$

[7] Find the fundamental matrix and the state transition matrix of the following homogeneous equation, using the two simple initial condition choices given:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

[8] Find the fundamental matrix and the state transition matrix of the following homogeneous equation, using the two simple initial condition choices given:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & e^{-t} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$