

**(MECH 6300 / EECS 6331 / SYSM 6307) LINEAR SYSTEMS**  
**Design Application Problem Set D**  
**Due: Friday, November 13, 2020 (10:00PM CDT)**

You should show work by hand for these problems, and only use Matlab for the intermediate steps if applicable (such as for computing inverses and verifying matrix operations); of course, as usual, you can use Matlab to verify your answers, if applicable.

1. Consider **Design Application #2**, the inverted pendulum on a cart, driven by a DC motor. Using the notation, numerical values, and development of **Problem Sets B and C** it is desired to place the dominant poles at

$$s = -4 \quad \text{and} \quad s = -2 \pm j2\sqrt{3}$$

(note that this is known as a Butterworth configuration) and to leave the pole at  $s = -800$  unchanged.

- (a) Find the gain matrix that produces this set of closed-loop poles.
- (b) Movement of the cart from one position to another without causing the pendulum to fall involves the introduction of a reference input in order to move the cart to some desired position  $z_d$ . How is the control law of part (a) modified to account for this case?

2. Consider the coupled cart problem (**Design Application #3**), using the notation, numerical values, and development from **Problem Sets B and C**. It is desired to bring the system to rest at a particular point using only the motor on cart 1. For the sake of computations, let that point be the origin (in the state space). Find the gain matrix in the control law  $u = -kx$  which places the poles at  $s = -1 \pm j1$  and  $s = -100 \pm j100$ .

**EXTRA CREDIT for Pendulum on the Cart Problem** (this is optional, not required)

- 1) Give the pendulum an initial condition (theta nonzero) and start the cart at some nonzero initial condition. Simulate, plotting the input and output as well as the states.
- 2) Suppose we wish the system to go 30% “faster” (that is, settle faster). Do a re-design and simulate, plotting the input and output as well as the states.