MECH 6300 / EECS 6331 / SYSM 6307 LINEAR SYSTEMS
Design Application Problem Set C:
Controllability and Observability
Due: Monday, November 2, 2020 (noon, 12:00PM CST)

You should show work by hand for these problems, and only use Matlab for the final step if applicable (such as for computing rank); of course, as usual, you can use Matlab to verify your answers, if applicable.

1. Consider **Design Application #2**, the inverted pendulum on a cart, driven by a DC motor. Using the notation and numerical values of **Problem Set #B**, let the state vector and control input be defined as before,

$$x = \begin{bmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad , \quad u = e$$

where z is the position of the cart. Now using the (more) general expression for the output equation as

$$y = Cx$$

where y could be a vector or scalar, determine whether or not the system is observable with the following "measurements":

(a) Cart displacement only: $y = [1 \quad 0 \quad 0 \quad 0]x$

(b) Pendulum angle only: $y = [0 \quad 0 \quad 1 \quad 0]x$

(c) Cart velocity only: $y = [0 \quad 1 \quad 0 \quad 0]x$

(d) Cart velocity and pendulum angle: $y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x$

2. Consider the coupled cart problem (**Design Application #3**). Using the numerical values from **Problem Set #B**, answer the following:

(a) Is the system controllable using only one motor? (set $\frac{k_2}{rR_2M_2} = 0$)

(b) Is it controllable using both motors?

(c) Is it observable if only the position of the first cart is measured?

(d) Is it observable if only the velocity of the first cart is measured?

(e) Is it observable if the velocities of both carts are measured?