MECH 6300 / EESC 6331 / SYSM 6307 Linear Systems
Homework Assignment #5
Due: Monday, October 19, 2020 (10:00PM CDT)

You should work all of these problems by hand except where noted, but you are welcome to use Matlab to verify your answers, if applicable.

[1] Consider the systems whose impulse responses are given for $t \geq 0$ in the following. Discuss BIBO stability for these systems.

(a) $g(t) = \frac{3}{4t+1}$ (b) $g(t) = 2te^{-4t}$

[2] Discuss Lyapunov stability (internal stability) for the following system (that is, investigate whether or not the system is stable, asymptotically stable, both or neither):

$$\dot{x} = \begin{bmatrix} -3 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x$$

[3] For the following system, write down the characteristic equation. Then use the Matlab **roots** command to determine the system poles. Determine the system transfer function (do not be intimidated….no computations are necessary, but you use could use Matlab to confirm). Is the origin asymptotically stable (for the case $u \equiv 0$)? Is the zero-state response BIBO stable?

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & -4 & -2 & -13 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [\,0 \quad 0 \quad 0 \quad 0 \quad 0\,]x$$

[4] For the following system, determine if the origin is asymptotically stable, and if the zero-state response is BIBO stable. Feel free to use Matlab for all aspects of this problem, but be sure to document your steps.

$$\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 4 \quad 2]x$$

[5] Use conditions (1) and (2) of Theorem 3.7 (p. 74) to determine which of the following hermitian (symmetric) matrices are positive definite or positive semi-definite, or neither. You may use Matlab for condition (1), but you must do condition (2) by hand.

(a) $\begin{bmatrix} 2 & 3 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$