MECH 6300 HW2

1)

a) $\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\3\\2\end{pmatrix}, \begin{pmatrix}1\\-4\\1\end{pmatrix}$

$$\begin{vmatrix}1 & 1 & 1\\1 & 3 & -4\\1 & 2 & 1\end{vmatrix} = \begin{matrix}1(3 - -8)\\ -1(1 - -5)\\ +1(2-3)\end{matrix} \begin{matrix}\\ = 11-5-1 \\ = 5 \neq 0\end{matrix}$$

Full Rank

Linearly Independent

b) $\begin{pmatrix}2+j\\4+j3\end{pmatrix}, \begin{pmatrix}-1-j\\2-j\end{pmatrix}, \begin{pmatrix}j\\-1+j5\end{pmatrix}$

$(2+j)(2-j) - (-1-j)(4+j3)$

$(4+1) - (-4-j3-j4+3)$

$5 - (-1-j7) = 4+j7 \neq 0$

Full Rank

Linearly Independent

Since the first 2 define a basis in $\mathbb{C}^2$, this is dependent on the first 2...

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c) $f(t) = \{te^{-t}, 2e^{-2t}, te^{-2t}\}$

$$W(f(t)) = \begin{bmatrix} te^{-t} & 2e^{-2t} & te^{-2t} \\ e^{-t}-te^{-t} & -4e^{-2t} & e^{-2t}-2te^{-2t} \\ -e^{-t}-e^{-t}+te^{-t} & 8e^{-2t} & -e^{-2t}-2e^{-2t}+4te^{-2t} \end{bmatrix}$$

$$|W| = 2(2t^2-1)e^{-5t} \neq 0$$

$\rho(W) = 3$

$f(t)$ spans C

L.I.

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1 d) $f(t) = \{2s-1,\ 2s-3,\ s^2+2\}$

$$W = \begin{bmatrix} 2s-1 & 2s-3 & s^2+2 \\ 2 & 2 & 2s \\ 0 & 0 & 2 \end{bmatrix}$$

$$|W| = (2s-1)(4-0) - (2s-3)(4-0) + 0$$

$$= (8s-4) - (8s-12)$$

$$= 8 \neq 0$$

L.I.

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2) $\Omega = \mathbb{R}^2 = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}, \dots \right\}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & c & 0 & b \\ 0 & d & 0 & 0 \end{bmatrix} \qquad E_{ref} = \begin{bmatrix} a & & & 0 \\ & b & & \\ & & c & \\ 0 & & & d \end{bmatrix}$$

E

let $a, b, c, d \neq 0$

$|E| = abcd$

$\neq 0 \rightarrow$ Full Rank

$\rho(E) = 4$

Ω spans $\mathbb{R}^4$

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3) $A' = P^{-1}AP$ $\quad P \equiv$ invertable

$$|sI - A| = |sI - A'|$$
$$= |sI - P^{-1}AP|$$
$$= |P^{-1}sIP - P^{-1}AP|$$
$$= |P^{-1}(sI - A)P|$$
$$= |P^{-1}||sI - A)|P|$$
$$= \cancel{|P^{-1}||P|}^{\,1}|sI - A|$$

$$|sI - A| = |sI - A|$$

$\vee$

same charectoristic Polynomial...

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4) a) $A = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -1 & -3 \\ 3 & 0 & 2 \end{bmatrix}$

$|A| = 4\cancel{(-2-0)} \rightarrow -8$
$- 3\cancel{(2+6)} \rightarrow 24$
$+ 1\cancel{(0+3)} \rightarrow 3$

$|A| = -29 \neq 0$
Full Rank

$\boxed{\begin{aligned} \rho[A] &= 3 \\ \gamma[A] &= 0 \end{aligned}}$

b) $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Independent...

Dependent...

$\boxed{\begin{aligned} \rho[B] &= 2 \\ \gamma[B] &= 1 \end{aligned}}$

c) $C = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \\ 2 & 1 & 0 & -1 & -2 \end{bmatrix}$

Full Rank

$\boxed{\rho(C) = 3}$

$\rho(C) + \gamma(C) = 5$

$\boxed{\gamma(C) = 2}$

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5) $\begin{bmatrix} 4 & -1 & 0 \\ -2 & 8 & -4 \\ -5 & 5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$A \quad x = y$

$$|A| = 4(\cancel{-16}+20) + 1(4-20) + 0 = 0 \ldots$$

$$A_{ref} = \begin{bmatrix} 1 & -1 & \frac{2}{5} \\ 0 & 1 & \frac{2}{15} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \underbrace{\rho(A)=3}_{\text{solution exists...}}$$

$$[A|y] = \left[\begin{array}{ccc|c} 4 & -1 & 0 & 1 \\ -2 & 8 & -4 & 2 \\ -5 & 5 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & \frac{-5}{2} \end{array}\right]$$

$$\boxed{x = \begin{bmatrix} 0 \\ -1 \\ \frac{-5}{2} \end{bmatrix}}$$