LINEAR SYSTEMS (MECH 6300 / EECS 6331 / SYSM 6307)
Design Application Problem Set E:
Observer Design
Due: Monday, November 23, 2020 (10:00PM CDT)

For the first two problems you should show work by hand for these problems, and only use Matlab for the intermediate steps if applicable (such as for computing inverses and verifying matrix operations); of course, as usual, you can use Matlab to verify your answers, if applicable.

1. Consider **Design Application #2**. Using the notation, numerical values, and development of previous problem sets, an observer is to be designed using the cart displacement measurement. Determine the observer gain matrix for which the observer poles satisfy the following characteristic equation:

$$\left(\frac{s}{5}\right)^4 + 2.613\left(\frac{s}{5}\right)^3 + \left(2+\sqrt{2}\right)\left(\frac{s}{5}\right)^2 + 2.613\left(\frac{s}{5}\right) + 1 = 0$$

The observer poles thus lie in a "4th order Butterworth pattern of radius 5." To see what this looks like, plot the observer poles in the complex *S*-plane.

2. Repeat (1), except design a 3rd-order observer with poles in a 3rd-order Butterworth pattern of radius 5; that is, the characteristic equation is to be

$$\left(\frac{s}{5}\right)^3 + 2\left(\frac{s}{5}\right)^2 + 2\left(\frac{s}{5}\right) + 1 = 0$$

3. ***OPTIONAL*** *(for those students who need more points, and who have good Matlab and control systems background)*. A compensator based on the full-order observer you designed in (1) above is to be constructed. You may use Matlab extensively (whenever needed) in this problem.

(a) Using the state feedback gains you obtained in **Problem Set D**, problem (1), determine the transfer function *G(s)* of the compensator (where $U(s) = -G(s)Y(s)$) from

$$G(s) = K(sI - A - BK - LC)^{-1}L$$

Note that this comes from the general observer equation where $u = K\hat{x}$ and the compensator "input" is $-y$.

(b) Supposing that the compensator is not exactly as designed, assume a gain variation α at the control input so that the closed-loop "return difference" is

$$T(s) = 1 + \alpha G(s)P(s)$$

where *P*(*s*) is the plant transfer function (cart and pendulum); this is essentially the root locus equation for parameter α. Find the range of α for which the closed-loop system is stable (for example, use the Routh criterion or root locus analysis).

(c) Is the resulting compensator practical? Why or why not? In answering this, consider the "sensitivity" of the system to small variations in α, and how such variations might affect stability.