```
% MECH 6300 - HW 6

A = blkdiag([2,1;0,2],2,2,[1,1;0,1],1)
B = [2  1   0
    2   1   1
    1   1   1
    3   2   1
    -1  0   1
    1   0   1
    1   0   0]
C = [2  2   1   3   -1  1   1
    1   1   1   2   0   0   0
    0   1   1   1   1   1   0]

U = ctrb(A,B)

ctrlRank = rank(U)

V = obsv(A,C)

obsvRank = rank(V)
```

```
A =

     2     1     0     0     0     0     0
     0     2     0     0     0     0     0
     0     0     2     0     0     0     0
     0     0     0     2     0     0     0
     0     0     0     0     1     1     0
     0     0     0     0     0     1     0
     0     0     0     0     0     0     1


B =

     2     1     0
     2     1     1
     1     1     1
     3     2     1
    -1     0     1
     1     0     1
     1     0     0


C =

     2     2     1     3    -1     1     1
     1     1     1     2     0     0     0
     0     1     1     1     1     1     0
```

```
U =

  Columns 1 through 13

    2     1     0     6     3     1    16     8     4    40    20
12    96
    2     1     1     4     2     2     8     4     4    16     8
 8    32
    1     1     1     2     2     2     4     4     4     8     8
 8    16
    3     2     1     6     4     2    12     8     4    24    16
 8    48
   -1     0     1     0     0     2     1     0     3     2     0
 4     3
    1     0     1     1     0     1     1     0     1     1     0
 1     1
    1     0     0     1     0     0     1     0     0     1     0
 0     1

  Columns 14 through 21

   48    32   224   112    80   512   256   192
   16    16    64    32    32   128    64    64
   16    16    32    32    32    64    64    64
   32    16    96    64    32   192   128    64
    0     5     4     0     6     5     0     7
    0     1     1     0     1     1     0     1
    0     0     1     0     0     1     0     0

ctrlRank =

     7


V =

    2     2     1     3    -1     1     1
    1     1     1     2     0     0     0
    0     1     1     1     1     1     0
    4     6     2     6    -1     0     1
    2     3     2     4     0     0     0
    0     2     2     2     1     2     0
    8    16     4    12    -1    -1     1
    4     8     4     8     0     0     0
    0     4     4     4     1     3     0
   16    40     8    24    -1    -2     1
    8    20     8    16     0     0     0
    0     8     8     8     1     4     0
   32    96    16    48    -1    -3     1
   16    48    16    32     0     0     0
    0    16    16    16     1     5     0
   64   224    32    96    -1    -4     1
   32   112    32    64     0     0     0
```

```
     0    32    32    32     1     6     0
   128   512    64   192    -1    -5     1
    64   256    64   128     0     0     0
     0    64    64    64     1     7     0
```

```
obsvRank =

     6
```

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1/2) a) $A = \begin{bmatrix} 0 & 2 \\ 0 & e^{2t} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 2 \end{bmatrix}$

$$V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & e^{2t} \end{bmatrix} \quad \det(V) = 2 \rightarrow \text{full Rank}$$

1a) This system is controllable.//

$$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & e^{2t} \end{bmatrix} \quad \det(V) = 0 \rightarrow \text{not full rank}$$

2a) This system is not observable.//

1/2) b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 3 & 0 \\ -2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

$$V = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 & 0 & -2 & 1 \\ 3 & 0 & -2 & 1 & -6 & -3 \\ -2 & 1 & -6 & -3 & 20 & 5 \end{bmatrix}$$

$$\text{rref}(V) = \begin{bmatrix} 1 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - \end{bmatrix} \rightarrow \rho(V) = 3$$

$-6 \quad -1R$
$+8 \quad +6$
$+18 \quad 5$
20

1b) The system is controllable.//

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & 0 \\ -6 & -12 & -8 \\ 0 & 0 & 2 \end{bmatrix}$$

2b) The system is Observable.//

$$\text{rref} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ : & : & : \\ : & : & : \end{bmatrix} \quad \rho(V) = 3$$

4/2)c) $A = \begin{bmatrix} 0 & 4 & 5 \\ 0 & 8 & 9 \\ 0 & -1 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 3 & 2 \end{bmatrix}$

$V = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 13 & 75 \\ 2 & 25 & 155 \\ 1 & -5 & -10 \end{bmatrix}$ $\text{rref}(V) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\rho(V) = 3$

4c) The system is controllable.//

$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ 0 & 22 & 21 \\ 0 & 155 & 135 \end{bmatrix}$ $\text{rref}(V) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(V) = 2$

The system is not observable.//

3/4) $A = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \rightarrow \rho(V) = 1$ (Not controllable)

$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \rightarrow \rho(V) = 1$ (Not observable)

3) cont.

Let $P^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow P = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$\bar{A} = P^{-1}AP = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & -5 \end{bmatrix}$$

$$\bar{B} = P^{-1}B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\bar{C} = CP = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

5) Controllability:

$\lambda = 2$:

$$U_2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \rightarrow \det(U_2) = -1 \rightarrow \text{Full Rank}$$

$\lambda = 2$ is controllable

$\lambda = 1$:

$$U_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \rho(U_1) = 2 \rightarrow \lambda = 1 \text{ is controllable}$$

Observability:

$\lambda = 2$

$$V_2 = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \det(V_2) = 0 \rightarrow \text{Not Full Rank}$$

$\lambda = 2$ is Not observable

$$V_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \rho(V_1) = 2 \rightarrow \lambda = 2 \text{ is observable}$$