

$$3) \quad A = \begin{bmatrix} -3 & -1 & -2 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 6 & 4 & 2 \\ -16 & -14 & -8 \end{bmatrix} \quad \checkmark \quad \rho(V) = 3$$

observable

$$\text{Let } F = \begin{bmatrix} -2 & -3 & 0 \\ 0 & -4 \end{bmatrix} \quad \text{and } G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -3 & 9 \\ 1 & -4 & 16 \end{bmatrix} \quad \checkmark \quad \rho(u) = 3$$

$$-FT + AT = GC \rightarrow \text{lyap}(-F, A, -GC)$$

$$T = (\text{in MATLAB}) \quad |T| = 10^{15}$$

non-singular ✓

$$L = T^{-1}G = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} -3 & -1 & -2 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{bmatrix} \hat{x} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \left( y - \begin{bmatrix} -2 & -1 & 0 \end{bmatrix} \hat{x} \right)$$

$$4) \lambda = -2, -4$$

$$\text{Let } F = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & -2 \\ 1 & -4 \end{bmatrix} \quad p(u) = 2 \quad \checkmark \text{ controllable}$$

$$-FA + TA = GC \rightarrow \text{lap}(-F, A, -GC)$$

$$T = \begin{bmatrix} 1 & 1 & -1 \\ -1.67 & -1.33 & 0.33 \end{bmatrix} \rightarrow H = TB = \begin{bmatrix} 1 \\ -3.67 \end{bmatrix}$$

$$P = \begin{bmatrix} C \\ T \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & -1 \\ -\frac{5}{3} & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} z + \begin{bmatrix} 1 \\ 1 \end{bmatrix} y + \begin{bmatrix} 1 \\ -\frac{11}{3} \end{bmatrix} u$$

$$\hat{x} = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & -1 \\ -\frac{5}{3} & -\frac{4}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}$$

---

```
% MECH 6300 - HW 8
```

```
% Problem 1/2
```

```
-----  
n = 4;  
A = [0  1  0  0;  
      0  0  1  0;  
     -2  1  3  1;  
      1  2  0  0];  
B = [0  0;  
      0  0;  
      2  1;  
      0  1];
```

```
lambda_0 = eig(A)
```

```
syms s
```

```
s_I_A_inv = inv(s * eye(n) - A)
```

```
charPoly = factor(det(s * eye(n) - A), 'FactorMode', 'real')
```

```
U = ctrb(A,B)
```

```
rank(U)
```

```
p = [-1+j*2, -1-j*2, -2+3*j, -2-3*j]
```

```
% Lyap Method
```

```
F = blkdiag([-1, -2; 2, -1], [-2, -3; 3, -2])
```

```
K_hat = [eye(2), eye(2)]
```

```
obsv_rank = rank(observ(F, K_hat))
```

```
T = lyap(A, -F, B*K_hat)
```

```
det_T = det(T)
```

```
K_lyap = K_hat * inv(T)
```

```
eig_A_BK_lyap = eig(A+B*K_lyap)
```

```
% Place Method
```

```
K_place = place(A, -B, p)
```

```
eig_A_BK_place = eig(A + B * K_place)
```

```
%-----
```

```
% Problem 3/4 -----
```

```
A = [-3 -1 -2;  
      0 -2  2;  
      1  0 -2];  
B = [2; 0; 1];
```



---

```

C = [-2,-1,0];

V = obsv(A,C)
rank_V = rank(V)

% Full Order
F = diag([-2,-3,-4])
G = ones(3,1)

u = ctrb(F,G)
rank_ctrl_FG = rank(u)

T = lyap(-F,A,-G*C)
det_T = det(T)

H = T * B

L = inv(T) * G

% Reduced Order
F = diag([-2,-4])
G = [1;1]

u = ctrb(F,G)
rank_u_FG = rank(u)

T = lyap(-F,A,-G*C)

H = T*B

lambda_0 =

    3.3277 + 0.0000i
    0.2093 + 0.5991i
    0.2093 - 0.5991i
   -0.7463 + 0.0000i

s_I_A_inv =

[(- s^3 + 3*s^2 + s + 2)/(- s^4 + 3*s^3 + s^2 + 1),      -(s*(s -
3))/(- s^4 + 3*s^3 + s^2 + 1),      -s/(- s^4 + 3*s^3 + s^2 + 1),
      -1/(- s^4 + 3*s^3 + s^2 + 1)]
[      (2*s - 1)/(- s^4 + 3*s^3 + s^2 + 1),      -(s^2*(s -
3))/(- s^4 + 3*s^3 + s^2 + 1),      -s^2/(- s^4 + 3*s^3 + s^2 + 1),
      -s/(- s^4 + 3*s^3 + s^2 + 1)]
[      -(- 2*s^2 + s)/(- s^4 + 3*s^3 + s^2 + 1),      -(s^2 +
1)/(- s^4 + 3*s^3 + s^2 + 1),      -s^3/(- s^4 + 3*s^3 + s^2 + 1),
      -s^2/(- s^4 + 3*s^3 + s^2 + 1)]
[      (- s^2 + 3*s + 5)/(- s^4 + 3*s^3 + s^2 + 1), (- 2*s^2 + 5*s +
3)/(- s^4 + 3*s^3 + s^2 + 1), -(2*s + 1)/(- s^4 + 3*s^3 + s^2 + 1),
      (- s^3 + 3*s^2 + s - 2)/(- s^4 + 3*s^3 + s^2 + 1)]

```

---

---

`charPoly =`

```
[s + 0.74625545091336047778605769765349,  
s - 3.327650852977051290247810325707, s^2  
- 0.41860459793630918753824737194646*s +  
0.40269360337510953075288601599424]
```

`U =`

0	0	0	0	2	1	6	4
0	0	2	1	6	4	20	13
2	1	6	4	20	13	66	43
0	1	0	0	4	2	14	9

`ans =`

4

`p =`

-1.0000 + 2.0000i   -1.0000 - 2.0000i   -2.0000 + 3.0000i   -2.0000 -  
3.0000i

`F =`

-1	-2	0	0
2	-1	0	0
0	0	-2	-3
0	0	3	-2

`K_hat =`

1	0	1	0
0	1	0	1

`obsv_rank =`

4

`T =`

0.0523	0.0585	0.0132	0.0222
0.0646	-0.1631	0.0403	-0.0839
-0.3908	0.0338	-0.3323	0.0470
-0.3292	-0.0738	-0.2116	-0.1098

---

$\det_T =$

$-2.3800e-04$

$K_{\text{lyap}} =$

-5.2374	-2.2019	-3.0500	-0.6816
-7.9972	-6.2908	2.1123	-5.0124

$\text{eig\_A\_BK\_lyap} =$

-2.0000 + 3.0000i
-2.0000 - 3.0000i
-1.0000 + 2.0000i
-1.0000 - 2.0000i

$K_{\text{place}} =$

10.4518	0.2611	-1.1681	1.2051
-29.5067	-12.1276	-4.8645	-1.7993

$\text{eig\_A\_BK\_place} =$

-2.0000 + 3.0000i
-2.0000 - 3.0000i
-1.0000 + 2.0000i
-1.0000 - 2.0000i

$V =$

-2	-1	0
6	4	2
-16	-14	-8

$\text{rank}_V =$

3

$F =$

-2	0	0
0	-3	0
0	0	-4

$G =$

---

```

      1
      1
      1

u =

      1      -2      4
      1      -3      9
      1      -4     16

rank_ctrl_FG =

      3

T =

1.0e+15 *

      0.0000      0.0000     -0.0000
     -3.0024     -3.0024     -0.0000
     -0.0000     -0.0000     -0.0000

det_T =

1.0008e+15

H =

1.0e+15 *

      0.0000
     -6.0048
     -0.0000

Warning: Matrix is close to singular or badly scaled. Results may be
inaccurate.
RCOND =  5.551115e-17.

L =

     -2.0000
      2.0000
     -1.0000

F =

     -2      0

```

---

---

```

      0      -4

G =

      1
      1

u =

      1      -2
      1      -4

rank_u_FG =

      2

T =

      1.0000      1.0000     -1.0000
     -1.6667     -1.3333     -0.3333

H =

      1.0000
     -3.6667

```

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