PSA: 1)

$$L = \frac{1}{2}(M+m)\dot{z}^2 + ml\dot{z}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$$

$$\frac{\partial L}{\partial \dot{z}} = (M+m)\dot{z} + ml\dot{\theta}\cos\theta \qquad \frac{\partial L}{\partial z} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = (M+m)\ddot{z} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$$

$$\boxed{f = (M+m)\ddot{z} + ml\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)}$$

$$\frac{\partial L}{\partial \dot{\theta}} = ml\dot{z}\cos\theta + ml^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta} + ml\ddot{z}\cos\theta - ml\dot{z}\dot{\theta}\sin\theta$$

$$\frac{\partial L}{\partial \theta} = -ml\dot{z}\dot{\theta}\sin\theta + mgl\sin\theta$$

$$0 = ml^2\ddot{\theta} + ml\ddot{z}\cos\theta - \cancel{ml\dot{z}\dot{\theta}\sin\theta} - \left(mgl\sin\theta - \cancel{ml\dot{z}\dot{\theta}\sin\theta}\right)$$

$$\boxed{0 = ml^2\ddot{\theta} - mgl\sin\theta + ml\ddot{z}\cos\theta}$$