

4)

$$a) g(t) = \frac{3}{4t+1}$$

$$\int_0^{\infty} |g(t)| dt \leq M < \infty$$

$$\int_0^{\infty} \left| \frac{3}{4t+1} \right| dt = \frac{3}{4} \ln(4t+1) \Big|_0^{\infty} = \frac{3}{4} (\infty - \ln(1)) \rightarrow \infty$$

$$\boxed{\int_0^{\infty} |g(t)| dt \not< \infty}$$

This system is not BIBO stable

$$b) g(t) = 2 + e^{-4t}$$

$$\int_0^{\infty} |g(t)| dt = \int_0^{\infty} |2 + e^{-4t}| dt = 2 \int_0^{\infty} 1 + e^{-4t} dt$$

$$= -\frac{1}{8} e^{-4t} (4t + 1) \Big|_0^{\infty} = 0 - \left(-\frac{1}{8}\right)(1)$$

$$\int_0^{\infty} |g(t)| dt = \frac{1}{8} < \infty$$

This system is BIBO stable



2)  $A = \begin{bmatrix} -3 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} \text{upper} \\ \text{Triangular} \end{matrix} \quad \begin{matrix} \lambda_1 = -3 & m_1 = 1 \\ \lambda_2 = 0 & m_2 = 2 \end{matrix}$

$(sI - A) = \begin{bmatrix} s+3 & 0 & -2 \\ 0 & s & -1 \\ 0 & 0 & s \end{bmatrix} \quad |sI - A| = s^2(s+3)$

The system is neither Asymptotically or Marginally stable I.e. as  $\text{Re} \lambda_i \not\leq 0 \forall i$  and  $m_i \neq 1 \forall \lambda_i: \text{Re} \lambda_i = 0$

3)  $A(s) = s^5 + 6s^4 + 13s^3 + 2s^2 + 4s + 1$   
 $H(s) = 0$

The system is not Asymptotically stable due to the poles:

$\lambda = 0.11 \pm j0.57$   
 which are on the RHP.

The system is BIBO stable due to the output  $y = 0 \forall t > 20s$



$$4) A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$C = [1 \ 4 \ 2] \quad D = 0$$

Upper Triangular:

$$\lambda_1 = -3 \quad m_1 = 1$$

$$\lambda_2 = -2 \quad m_2 = 1$$

$$\lambda_3 = 0 \quad m_3 = 1$$

$$\left. \begin{array}{l} \lambda_1 = -3 \quad m_1 = 1 \\ \lambda_2 = -2 \quad m_2 = 1 \\ \lambda_3 = 0 \quad m_3 = 1 \end{array} \right\} H(s) = \frac{9(s+3.11)}{(s+3)(s+2)}$$

Asymptotic Stability:

The system is not Asymptotically stable due to  $\lambda_3 = 0$ , which is not strictly on the LHP.

BIBO stability:

The system is BIBO stable because the poles of the transfer function  $\lambda(H(s)) = -3, -2$  which both reside on the LHP. //



5)

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s-2 & -3 & -2 \\ -3 & s-1 & 0 \\ -2 & 0 & s-2 \end{bmatrix}$$

$$\lambda_1 = -2$$

$$\lambda_2 = 1.70$$

$$\lambda_3 = 5.31$$

$$\begin{aligned} |sI - A| &= (s-2)(s-1)(s-2) \\ &\quad + [3(-3(s-2))] \\ &\quad + (-2)(+2(s-1)) \\ &= s^3 - 5s^2 - 5s + 18 \end{aligned}$$

3.7: condition 1 Not TrueRe  $\lambda_i \neq 0 \forall i$ 3.7.2 Condition 2

$$M_1 = 2$$

$$M_2 = 2 - 9 = -7$$

$$\begin{aligned} M_3 &= 2(2) - 3(6) + 2(-2) \\ &= 4 - 18 - 4 \\ &= -18 \end{aligned}$$

Condition 2 does not hold...  
The Matrix is Not Positive definite



$$b) \quad A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\lambda_1 = -0.41$$

$$\lambda_2 = 0$$

$$\lambda_3 = 2.41$$

condition 1: does not hold

condition 2:

$$M_1 = 0$$

$$M_2 = 0$$

$$M_3 = 0$$

The Matrix is

Positive

Semi-Definite.

$$c) \quad A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_1 = 0, m_1 = 2$$

$$\lambda_2 = 1, m_2 = 1$$

condition 1: Positive semi-definite

condition 2: Positive semi-definite

$$M_1 = 0$$

$$M_2 = 0$$

$$M_3 = 0$$