MECH 6300 / EECS 6331 / SYSM 6307 LINEAR SYSTEMS
Assignment #6
Controllability and Observability
Due: Monday, November 2, 2020 (noon, 12:00PM CST)

You should <u>show work by hand for all of these problems</u>, and only use Matlab for the final step if applicable (such as for computing rank); of course, as usual, you can use Matlab to verify your answers, if applicable.

[1] Check the controllability of the following dynamical systems.

(a) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 2\\ 0 & e^{2t}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \quad , \quad y = \begin{bmatrix}0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

(b) $\dot{x} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -4 & -3\end{bmatrix}x + \begin{bmatrix}0 & 0\\ 3 & 0\\ -2 & 1\end{bmatrix}u \quad , \quad y = \begin{bmatrix}1 & 3 & 0\\ 2 & 0 & 0\end{bmatrix}x$

(c) $\dot{x} = \begin{bmatrix}0 & 4 & 5\\ 0 & 8 & 9\\ 0 & -1 & -3\end{bmatrix}x + \begin{bmatrix}0\\ 2\\ 1\end{bmatrix}u \quad , \quad y = \begin{bmatrix}0 & 3 & 2\end{bmatrix}x$

[2] Check the observability of the dynamical systems in Problem [1] above.

[3] Show that the following dynamical system is not controllable, and then reduce it to a controllable system:

$$\dot{x} = \begin{bmatrix}-3 & 2\\ 2 & -3\end{bmatrix}x + \begin{bmatrix}1\\ 1\end{bmatrix}u \quad , \quad y = \begin{bmatrix}1 & 1\end{bmatrix}x$$

[4] For the system in Problem [3], show that the system is not observable, and then reduce it to an observable system.

[5] Without the use of Matlab, follow the example in the text (Section 6.5) to determine if the following Jordan-form system is controllable and observable. Confirm your answer with Matlab.

$$\dot{x} = \begin{bmatrix}2 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}x + \begin{bmatrix}2 & 1 & 0\\ 2 & 1 & 1\\ 1 & 1 & 1\\ 3 & 2 & 1\\ -1 & 0 & 1\\ 1 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}u \quad , \quad y = \begin{bmatrix}2 & 2 & 1 & 3 & -1 & 1 & 1\\ 1 & 1 & 1 & 2 & 0 & 0 & 0\\ 0 & 1 & 1 & 1 & 1 & 1 & 0\end{bmatrix}x$$