1)

$$A = \begin{bmatrix} -1 & -2 & -3 \\ 0 & -1 & 3 \\ 1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \quad \Delta(s) = |sI - A| = s^3 + 3s^2 + 6s + 10$$

$$C = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix}$$

$$H(s) = C(sI-A)^{-1}B = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} s+1 & 2 & -3 \\ 0 & s+1 & -3 \\ -1 & 0 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \left(\frac{1}{s^3 + 3s^2 + 6s + 10} \right) \begin{bmatrix} (s+1)^2 & -2(s+1) & -3(s+3) \\ -3 & s^2 + 2s + 4 & 3(s+1) \\ s+1 & -2 & (s+1)^2 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{\Delta(s)} \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3(s+1)^2 - 6(s+3) \\ 9 + 6(s+1) \\ 3(s+1) + 2(s+1)^2 \end{bmatrix} \rightarrow \begin{bmatrix} 3(s^2+2s+1) - 6s - 18 \\ 6s + 15 \\ 3s + 3 + 2(s^2+2s+1) \end{bmatrix}$$

$$= \frac{3(3s^2 + \cancel{6s} + 3 - \cancel{6s} - 18) + 3(6s + 15)}{\Delta(s)}$$

$$= \frac{9s^2 \cancel{-45} + 18s \cancel{+45}}{\Delta(s)}$$

$$\boxed{H(s) = \frac{9s^2 + 18s}{s^3 + 3s^2 + 6s + 10}} = \frac{9s(s+2)}{s^3 + 3s^2 + 6s + 10}$$

$$P^{-1} = U \bar{U}^{-1} = \begin{bmatrix} 3 & -9 & -6 \\ 0 & 6 & -3 \\ 2 & 1 & -10 \end{bmatrix} \begin{bmatrix} 6 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \begin{bmatrix} -15 & 0 & 3 \\ 15 & 6 & 0 \\ 5 & 7 & 2 \end{bmatrix} \end{array}$$

1) cont.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

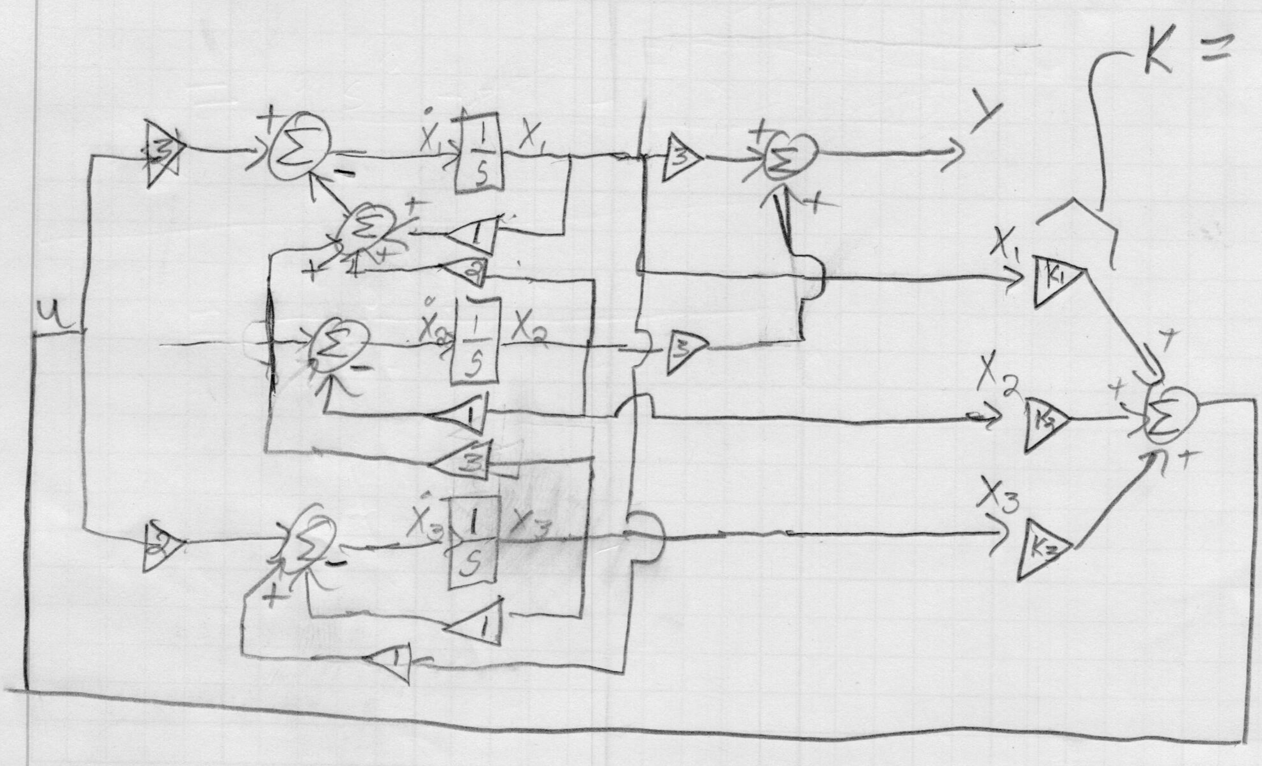
$$y = \begin{bmatrix} 0 & 18 & 9 \end{bmatrix} \bar{x}$$

2)

$$\dot{x} = \begin{bmatrix} 0 & 0 & -10 \\ 1 & 0 & -6 \\ 0 & 1 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 18 \\ 9 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{x}$$

3) $\lambda_{cls} = \{-1, -2, -2\}$



3) cont.

$$\Delta(s) = (s+1)(s+2)(s+3) = (s^2+2s+2)(s+3)$$
$$= s^3 + 2s^2 + 2s + 3s^2 + 6s + 6$$

$$\boxed{\Delta_{cl}(s) = s^3 + 5s^2 + 8s + 6}$$
$$\bar{\alpha}_1 \quad \bar{\alpha}_2 \quad \bar{\alpha}_3$$

$$\hat{K} = \begin{bmatrix} \alpha_3 - \bar{\alpha}_3 & \alpha_2 - \bar{\alpha}_2 & \alpha_1 - \bar{\alpha}_1 \end{bmatrix}$$
$$= \begin{bmatrix} 10-6 & 6-8 & 3-5 \end{bmatrix}$$

$$\boxed{\hat{K} = \begin{bmatrix} 4 & -2 & -2 \end{bmatrix}}$$

$$K = \hat{K} P^{-1} = \begin{bmatrix} 4 & -2 & -2 \end{bmatrix} \begin{bmatrix} -15 & 0 & 3 \\ 15 & 6 & 0 \\ 5 & 7 & 2 \end{bmatrix}$$

$$\boxed{K = \begin{bmatrix} -100 & -26 & 8 \end{bmatrix}}$$

4) $A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 3 \\ 2 \end{bmatrix} u \qquad u = K^T x \qquad K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$

set $\lambda_{cls} = \{-2, -3\}$

$$A + bK^T = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}}_{\begin{bmatrix} 3k_1 & 3k_2 \\ 2k_1 & 2k_2 \end{bmatrix}}$$

$$= \begin{bmatrix} 1+3k_1 & 4+3k_2 \\ -2+2k_1 & 3+2k_2 \end{bmatrix}$$

$$[sI - (A + bK^T)] = \begin{bmatrix} s-(1+3k_1) & -(4+3k_2) \\ -(-2+2k_1) & s-(3+2k_2) \end{bmatrix}$$

$$\Delta_{cls}(s) = (s-(1+3k_1))(s-(3+2k_2)) - (-2+2k_1)(4+3k_2)$$

$$= s^2 - (1+3k_1)s - (3+2k_2)s - (-8 \cancel{-6k_2} + 8k_1 \cancel{+6k_2})$$
$$+ (1+3k_1)(3+2k_2)$$

$$= s^2 + (-4-3k_1-2k_2)s + (3+2k_2+9k_1+6k_1k_2)$$
$$+ (8-8k_1)$$

$$\Delta_{cls}(s) = s^2 + (-4-3k_1-2k_2)s + (11+k_1+2k_2+6k_1k_2)$$

$$\Delta_{cls}(s) = (s+2)(s+3) = s^2 + 5s + 6$$

$$\begin{cases} 5 = -4 - 3k_1 - 2k_2 \\ 6 = 11 + k_1 + 2k_2 + 6k_1k_2 \end{cases} \qquad K = \begin{bmatrix} -0.144 \\ -4.283 \end{bmatrix}$$

$$\left.\begin{aligned} -3k_1 - 2k_2 &= 9 \\ k_1 + 2k_2 + 6k_1k_2 &= -5 \end{aligned}\right] \rightarrow \begin{aligned} k_1 &= -0.144 \\ k_2 &= -4.283 \end{aligned}$$

5)

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

From the Jordan Form, it can be seen that the Jordan block w/ $\lambda_1 = 2$ is controllable. Aditionally, the two Jordan blocks w/ $\lambda_2 = -1$ are not both controllable by Corrallary 6.8. (They are stabalizable, and one of the λ_2 modes is controllable. //

a) $\Lambda = \{\underbrace{-2, -2}_{\substack{\lambda_1 = 2 \\ \text{Controllable}}}, \underbrace{-1, -1}_{\substack{\lambda_2 = -1 \\ \text{unmoved}}}\}$ $\quad$ K^T exists

b) $\Lambda = \{\underbrace{-2, -2}_{\substack{\lambda_1 = 2 \\ \text{Controllable}}}, \underbrace{-2}_{\text{one } \lambda_2 = -1 \text{ is controllable}}, \underbrace{-1}_{\text{unmoved}}\}$ $\quad$ K^T exists

c) $\Lambda = \{\underbrace{-2, -2}_{\text{Controllable}}, \underbrace{-2, -2}_{\lambda_2 = -1}\}$

second one is uncotrollable

K^T does not exist. //