MECH 6300-HW3

1)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\lambda_1 = 1$
$\lambda_2 = 2$
$\lambda_3 = 4$

i) Cayley-Hamilton Therom Method

$$f(\lambda) = e^{\lambda t} \quad h(\lambda) = \beta_0 + \beta_1 \lambda + \beta_2 \lambda^2$$

$$f(1) = e^{t} = h(1) = \beta_0 + \beta_1 + \beta_2$$
$$f(2) = e^{2t} = h(2) = \beta_0 + 2\beta_1 + 4\beta_2$$
$$f(4) = e^{4t} = h(4) = \beta_0 + 4\beta_1 + 16\beta_2$$

$$\beta_0 = \frac{8}{3}e^{t} - 2e^{2t} + \frac{1}{3}e^{4t}$$

$$\beta_1 = -2e^{t} + \frac{5}{2}e^{2t} - \frac{1}{2}e^{4t}$$

$$\beta_2 = \frac{1}{3}e^{t} - \frac{1}{2}e^{2t} + \frac{1}{6}e^{4t}$$

$$e^{At} = \beta_0 I + \beta_1 A + \beta_2 A^2$$