MECH 6300 / EECS 6331 / SYSM 6307 Linear Systems **Homework Assignment #3**

Due: Monday, September 21, 2020 (10:00PM CDT)

In some of these problems you may find Matlab useful for verification of your results, but you should still do all calculations by hand. If you use Matlab, you should include your Matlab print-outs with your responses.

[1] Find the Jordan-canonical-form representations of the following matrices.

(a)
$$\begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix}$$
 (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 1 \\ 0 & -75 & -45 \\ 0 & 100 & 60 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 8 & 6 \\ 0 & 40 & 32 \\ 0 & -50 & -40 \end{bmatrix}$

(d)
$$\begin{bmatrix} 0 & 8 & 6 \\ 0 & 40 & 32 \\ 0 & -50 & -40 \end{bmatrix}$$

(e)
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9 & 18 & 8 & -2 \end{bmatrix}$$
 (f)
$$\begin{bmatrix} \frac{7}{2} & \frac{21}{2} & 14 \\ -\frac{1}{2} & -\frac{3}{2} & -2 \\ -\frac{1}{2} & -\frac{3}{2} & -2 \end{bmatrix}$$
 (g)
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

f)
$$\begin{bmatrix} \frac{7}{2} & \frac{21}{2} & 14 \\ -\frac{1}{2} & -\frac{3}{2} & -2 \\ -\frac{1}{2} & -\frac{3}{2} & -2 \end{bmatrix}$$

$$(g) \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[2] Find the characteristic polynomials and the minimal polynomials of the following matrices.

$$\text{(a)} \quad \left[\begin{array}{ccccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{array} \right] \quad \text{(b)} \quad \left[\begin{array}{ccccc} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{array} \right]$$

(b)
$$\begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix}$$

$$\text{(c)} \ \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix} \ \text{(d)} \ \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 1 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix}$$

[3] Compute e^{At} for the following matrices by using Theorem 3.5 and by using the Jordan-form representation.

(a)
$$\begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix}$$
 (b)
$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & -75 & -45 \\ 0 & 100 & 60 \end{bmatrix}$$