1)
$$A = \begin{bmatrix} -1 & -2 & -3 \\ 0 & -1 & 3 \\ 1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \quad \Delta(s) = |sI - A| = s^3 + 3s^2 + 6s + 10$$

$$C = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix}$$

$$H(s) = C(sI-A)^{-1}B = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} s+1 & 2 & 3 \\ 0 & s+1 & -3 \\ -1 & 0 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \left(\frac{1}{s^3+3s^2+6s+10} \right) \begin{bmatrix} (s+1)^2 & -2(s+1) & -3(s+3) \\ -3 & s^2+2s+4 & 3(s+1) \\ s+1 & -2 & (s+1)^2 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{\Delta(s)} \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3(s+1)^2 - 6(s+3) \\ 9 + 6(s+1) \\ 3(s+1) + 2(s+1)^2 \end{bmatrix} \rightarrow \begin{bmatrix} 3(s^2+2s+1) - 6s - 18 \\ 6s+15 \\ 3s+3+2(s^2+2s+1) \end{bmatrix}$$

$$= \frac{3(3s^2 + \cancel{6s} + 3 - \cancel{6s} - 18) + 3(6s+15)}{\Delta(s)}$$

$$= \frac{9s^2 \cancel{-45} + 18s \cancel{+45}}{\Delta(s)}$$

$$\boxed{H(s) = \frac{9s^2 + 18s}{s^3 + 3s^2 + 6s + 10}} = \frac{9s(s+2)}{s^3+3s^2+6s+10}$$

$$P^{-1} = u\bar{u}^{-1} = \begin{bmatrix} 3 & -9 & -6 \\ 0 & 6 & -3 \\ 2 & 1 & -10 \end{bmatrix} \begin{bmatrix} 6 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \begin{bmatrix} -15 & 0 & 3 \\ 15 & 6 & 0 \\ 5 & 7 & 2 \end{bmatrix} \end{array}$$

1) cont.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

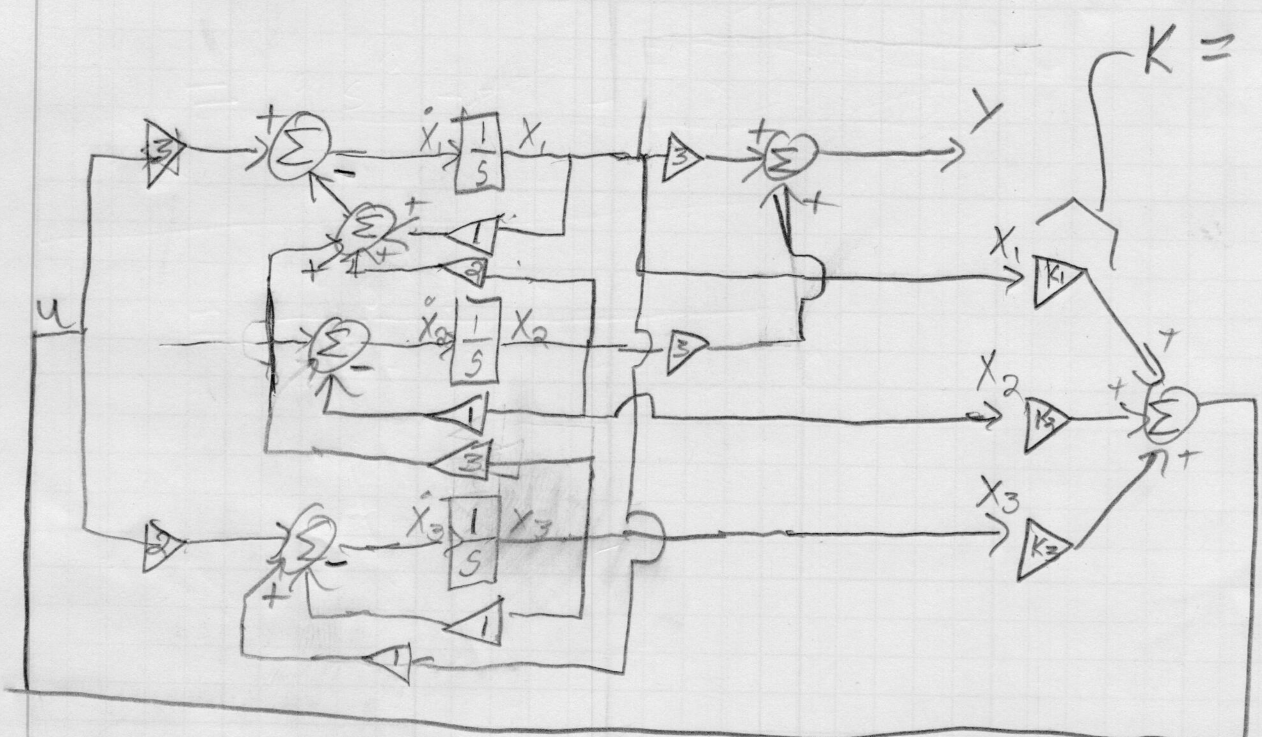
$$y = \begin{bmatrix} 0 & 18 & 9 \end{bmatrix} \bar{x}$$

2)

$$\dot{x} = \begin{bmatrix} 0 & 0 & -10 \\ 1 & 0 & -6 \\ 0 & 1 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 18 \\ 9 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{x}$$

3) $\lambda_{cls} = \{-1, -2, -2\}$



3) cont.

$$\Delta(s) = (s+1)(s+2)(s+3) = (s^2+2s+2)(s+3)$$

$$= s^3 + 2s^2 + 2s + 3s^2 + 6s + 6$$

$$\boxed{\Delta_{cl}(s) = s^3 + \underbrace{5}_{\bar{\alpha}_1} s^2 + \underbrace{8}_{\bar{\alpha}_2} s + \underbrace{6}_{\bar{\alpha}_3}}$$

$$\hat{K} = \begin{bmatrix} \alpha_3 - \bar{\alpha}_3 & \alpha_2 - \bar{\alpha}_2 & \alpha_1 - \bar{\alpha}_1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-6 & 6-8 & 3-5 \end{bmatrix}$$

$$\boxed{\hat{K} = \begin{bmatrix} 4 & -2 & -2 \end{bmatrix}}$$

$$K = \hat{K} P^{-1} = \begin{bmatrix} 4 & -2 & -2 \end{bmatrix} \begin{bmatrix} -15 & 0 & 3 \\ 15 & 6 & 0 \\ 5 & 7 & 2 \end{bmatrix}$$

$$\boxed{K = \begin{bmatrix} -100 & -26 & 8 \end{bmatrix}}$$

4) $A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 3 \\ 2 \end{bmatrix} u \qquad u = K^T x \qquad K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$

set $\lambda_{cls} = \{-2, -3\}$

$$A + bK^T = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix}}_{\begin{bmatrix} 3K_1 & 3K_2 \\ 2K_1 & 2K_2 \end{bmatrix}}$$

$$= \begin{bmatrix} 1+3K_1 & 4+3K_2 \\ -2+2K_1 & 3+2K_2 \end{bmatrix}$$

$$\left[sI - (A + bK^T)\right] = \begin{bmatrix} s-(1+3K_1) & -(4+3K_2) \\ -(-2+2K_1) & s-(3+2K_2) \end{bmatrix}$$

$$\Delta_{cls}(s) = (s-(1+3K_1))(s-(3+2K_2)) - + (-2+2K_1)(4+3K_2)$$

$$= s^2 - (1+3K_1)s - (3+2K_2)s - (-8 - \cancel{6K_2} + 8K_1 \cancel{+6K_2})$$
$$+ ^+ (1+3K_1)(3+2K_2)$$

$$= s^2 + (-4-3K_1-2K_2)s + (3+2K_2+9K_1+6K_1K_2)$$
$$+ (8-8K_1)$$

$$\Delta_{cls}(s) = s^2 + (-4-3K_1-2K_2)s + (11+K_1+2K_2+6K_1K_2)$$

$$\Delta_{cls}(s) = (s+2)(s+3) = s^2 + 5s + 6$$

$$\begin{cases} 5 = -4 - 3K_1 - 2K_2 \\ 6 = 11 + K_1 + 2K_2 + 6K_1K_2 \end{cases} \qquad K = \begin{bmatrix} -0.144 \\ -4.283 \end{bmatrix}$$

$$\left.\begin{aligned} -3K_1 - 2K_2 &= 9 \\ K_1 + 2K_2 + 6K_1K_2 &= -5 \end{aligned}\right] \rightarrow \begin{aligned} K_1 &= -0.144 \\ K_2 &= -4.283 \end{aligned}$$

5)

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

From the Jordan Form, it can be seen that the Jordan block w/ $\lambda_1 = 2$ is controllable. Aditionally, the two Jordan blocks w/ $\lambda_2 = -1$ are not both controllable by Corrallary 6.8. (They are stabalizable, and one of the λ_2 modes is controllable. //

a) $\Lambda = \{\underbrace{-2,-2}_{\substack{\lambda_1 = 2 \\ \text{Controllable}}}, \underbrace{-1,-1}_{\substack{\lambda_2 = -1 \\ \text{unmoved}}}\}$ $\quad$ $\underline{K^T \text{ exists}}$

b) $\Lambda = \{\underbrace{-2,-2}_{\substack{\lambda_1 = 2 \\ \text{Controllable}}}, \underbrace{-2}_{\text{one } \lambda_2 = -1 \text{ is controllable}}, \underbrace{-1}_{\text{unmoved}}\}$ $\quad$ $\underline{K^T \text{ exists}}$

c) $\Lambda = \{\underbrace{-2,-2}_{\text{Controllable}}, -2, \underbrace{-2}_{\lambda_2 = -1}\}$

second one is uncotrollable

K^T does not exist. //

```
% MECH 6300 - HW 7

% Problem 1/2/3
n = 3;
A = [-1 -2  -3;
    0   -1  3;
    1   0   -1];
B = [3; 0; 2];
C = [3  3   0];

syms s

s_I_A_inv = inv(s * eye(n) - A);

charPoly = factor(det(s * eye(n) - A))%,'FactorMode', 'real')


% Problem 4
syms k1 k2
eq1 = 5 == -4 -3 * k1 -2 * k2;
eq2 = 6 == 11 + k1 + 2 * k2 + 6 * k1 * k2;

[k1,k2] = solve([eq1,eq2],[k1,k2]);
k1 = double(k1(1))
k2 = double(k2(1))


% Problem 5
A = blkdiag([2,1;0,2],-1,-1);
B = [0; 1; 1; 1];

% Part a
K = [1 1 1 1]
jordan(A + B*K)
K = [-1 3 5 1]
jordan(A + B*K)


charPoly =

s^3 + 3*s^2 + 6*s + 10


k1 =

   -0.1444


k2 =

   -4.2834
```

```
K =

     1     1     1     1


ans =

    1.0000         0         0         0
         0   -1.0000         0         0
         0         0    0.6972         0
         0         0         0    4.3028


K =

    -1     3     5     1


ans =

   -1.0000         0         0         0
         0    9.0713         0         0
         0         0    0.6210         0
         0         0         0    2.3077
```

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