1) I am in agreement with the pledge as stated in problem 1.

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2)
$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 16 \end{bmatrix} x + \begin{bmatrix} -3 \end{bmatrix} u$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 16 \end{bmatrix} \quad D = \begin{bmatrix} -3 \end{bmatrix}$$

$$\mathcal{U} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$\rho(\mathcal{U}) = 2 = n \rightarrow$ controllable

$$\mathcal{V} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 16 \\ 0 & 16 \end{bmatrix}$$

$\rho(\mathcal{V}) = 1 \neq n \rightarrow$ <u>Not Observable</u>

b) Yes, controllable implies stabalizable

2) cont.

c) $A_{22} = A_c = 1$

$A_{11} = A_{\bar{c}} = 1$ → The uncontllable subsystem is <u>not</u> stable ⇒ Not Detectable

d) $u = KX$ $\Lambda = \{\lambda_1, \lambda_2\}$ $\boxed{\lambda_1 = \lambda_2 = \lambda}$

$$\Delta_{cl} = (s-\lambda_1)(s-\lambda_2) = s^2 - 2\lambda s + \lambda^2$$

$$\bar{\alpha}_1 = -2\lambda \quad \bar{\alpha}_2 = \lambda^2$$

ope-loop

$$\Delta(s) = (s-1)(s-1) = s^2 - 2s + 1$$

$$\alpha_1 = -2 \quad \alpha_2 = 1$$

$$-K = \begin{array}{cc} K_1 & K_2 \\ \left[\bar{\alpha}_2 - \alpha_2\right. & \left.\bar{\alpha}_1 - \alpha_1\right] \end{array}$$

$$= \left[-2\lambda - -2 \quad \lambda^2 - 1\right]$$

$$= \left[2(\lambda - 1) \quad \lambda^2 - 1\right]$$

$$-K_1 = -2(\lambda-1)$$

$$-K_2 = \lambda^2 - 1$$

→ $$\boxed{\begin{aligned} K_1 &= -2(1-\lambda) \\ K_2 &= 1 - \lambda^2 \end{aligned}}$$

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2) cont.

$$k_1 = -2(1-\lambda) \qquad k_2 = 1-\lambda^2$$

$$\lambda^2 = 1-k_2$$

$$k_1 = -2\left(1-\sqrt{1-k_2}\right) \qquad \lambda = \sqrt{1-k_2}$$

$$\boxed{k_1 = 2\left(\sqrt{1-k_2} - 1\right)}$$

3)
$$\dot{x} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u$$

$$y = \begin{bmatrix} \beta & 0 \end{bmatrix} x$$

a) $\mathcal{U} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ \alpha & 0 \end{bmatrix}$ $\quad \rho(\mathcal{U}) = 2 \Rightarrow$ Controllable

$\mathcal{V} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} \beta & 0 \\ 0 & 2\beta \end{bmatrix}$ $\quad \rho(\mathcal{V}) = 2 \Rightarrow$ observable

3) cont.

$u = r - Kx$

b) $\lambda_{1,2(cl)} = -\frac{1}{2} \pm j\frac{1}{2}$ $\quad K = \begin{bmatrix} \frac{9}{4} & 1 \end{bmatrix}$

$$A_{cl} = (A - BK) = \left(\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \alpha \end{bmatrix} \begin{bmatrix} \frac{9}{4} & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 2 \\ 2 - \frac{9}{4}\alpha & -\alpha \end{bmatrix}$$

$$\Delta(s) = |sI - A_{cl}| = \begin{bmatrix} s & -2 \\ -2 + \frac{9}{4}\alpha & s+\alpha \end{bmatrix} = s \cdot (s+\alpha) + (+2)(-2 + \frac{9}{4}\alpha)$$

$$= s^2 + \alpha s + (-4 + \frac{9}{2}\alpha)$$

$$\Delta(s) = s^2 + \alpha s + (\frac{9}{2}\alpha - 4)$$

$$\lambda_{1,2}(s) = \frac{-\alpha \pm \sqrt{\alpha^2 - 4(1)(\frac{9}{2}\alpha - 4)}}{2} \quad \sqrt{16 - 18\alpha - \alpha^2}$$

$$\boxed{\alpha = 1}$$

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3 c) $(A - LC)$ $\lambda_{1,2(s)} = -2 \pm j2$

Let, $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

$$(A^T - C^T L^T) = \left(\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} \beta \\ 0 \end{bmatrix} \begin{bmatrix} L_1 & L_2 \end{bmatrix} \right)$$

$$\begin{bmatrix} L_1\beta & L_2\beta \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -L_1\beta & 2 - L_2\beta \\ 2 & 0 \end{bmatrix}$$

$$\Delta(s) = \left| sI - (A^T - C^T L^T) \right| = \begin{bmatrix} s + L_1\beta & L_2\beta - 2 \\ -2 & s \end{bmatrix}$$

$$= (s)(s + L_1\beta) + (+2)(L_2\beta - 2)$$

$$= s^2 + L_1\beta s + (2L_2\beta - 4)$$

$$= \frac{-L_1\beta \pm \sqrt{(L_1\beta)^2 - 4(1)(2L_2\beta - 4)}}{2}$$

$$= \boxed{\frac{-L_1\beta}{2}} \pm j\sqrt{\frac{8L_2\beta - 16 + (L_1\beta)^2}{4}}$$

$$2 = \frac{-L_1\beta}{2}$$

$$\boxed{\frac{-4}{\beta} = L_1}$$

$$\sqrt{2L_2\beta - \cancel{4} + \cancel{\frac{\left(\frac{-4}{\beta}\beta\right)^2}{4}}} = 2$$

$$4^2 = L_2\beta \rightarrow L_2 = \frac{2}{\beta}$$

3)c) cont.

$$L_1 = -\frac{4}{\beta} \qquad L_2 = \frac{2}{\beta}$$

$$\boxed{L = \begin{bmatrix} -\frac{4}{\beta} \\ \frac{2}{\beta} \end{bmatrix}}$$

3)d) $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}$ $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$C = \begin{bmatrix} \beta & 0 \end{bmatrix}$

$u = r - K\hat{x}$ $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu$

$y = C\cancel{x}$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} & \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} \end{bmatrix} r$$

$$\dot{\hat{x}} = \left(\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} \beta & 0 \end{bmatrix} \right) \hat{x} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} \beta & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} \left(r - \begin{bmatrix} K_1 & K_2 \end{bmatrix} \hat{x} \right)$$

$$\begin{matrix} L_1\beta & 0 \\ L_2\beta & 0 \end{matrix} \qquad \begin{matrix} L_1\beta & 0 \\ L_2\beta & 0 \end{matrix} \qquad \begin{bmatrix} 0 \\ \alpha(r - K_1\hat{x}_1 - K_2\hat{x}_2) \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} -L_1\beta & 2 \\ 2 - L_2\beta & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} L_1\beta & 0 \\ L_2\beta & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} r + \begin{bmatrix} -\alpha K_1 & 0 \\ 0 & -\alpha K_2 \end{bmatrix} \hat{x}$$

3) d) cont.

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} 0 & 2 & -K_1 & 0 \\ 2 & 0 & 0 & -K_2 \\ L_1 \beta & 0 & -L_1\beta - \alpha K_1 & 2 \\ L_2 \beta & 0 & 2 - L_2 \beta & -\alpha K_2 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \\ 0 \\ \alpha \end{bmatrix} r$$

4) a)

$$\dot{x} = \begin{bmatrix} \frac{-2}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} x + \begin{bmatrix} \frac{1}{RC} \\ \frac{1}{L} \end{bmatrix} u \qquad R, C, L \in \mathbb{R}_{>0}$$

$$y = \begin{bmatrix} -1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \end{bmatrix} u$$

b)

$$\mathcal{U} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} \frac{1}{RC} & \frac{-2}{(RC)^2} + \frac{1}{LC} \\ \frac{1}{L} & -\frac{1}{LRC} \end{bmatrix}$$

$\underline{\rho(\mathcal{U}) = 2}$

$\frac{1}{L} \neq \frac{-1}{LRC}$ makes it always L.I. $\rightarrow \rho(\mathcal{U}) = 2$

The ~~sy~~ Steem is Always Controllable given the definition of the problem

4) c)

$$\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

Yes, controllable

$$V = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \rightarrow$$ Yes, observable

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$$\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ Yes, controllable

$$\begin{bmatrix} 0 & 5 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & 5 \\ 5 & -25 \end{bmatrix} \leftarrow$$ Yes, observable

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Yes, they are both controllable and observable

5) $L = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $\quad \lambda_{obs} = -1$

$$\dot{\hat{\bar{X}}} = \underbrace{\left(\bar{A}_{22} - L\bar{A}_{12}\right)}_{\text{Dynamics}} \hat{\bar{X}}_2 + Lw + \bar{u}$$

$\bar{A}_{12} = \begin{bmatrix} 2 & -3 \end{bmatrix}$

$\bar{A}_{22} = \begin{bmatrix} -4 & 2 \\ \alpha & 6 \end{bmatrix}$

$$\left(\bar{A}_{22} - L\bar{A}_{12}\right) = \begin{bmatrix} -4 & 2 \\ \alpha & 6 \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \end{bmatrix}\begin{bmatrix} 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{-4+2}^{-2} & \cancel{2+-3}^{-1} \\ \underbrace{\alpha+4} & \cancel{6+-6}^{0} \end{bmatrix}$$

$\rightarrow \alpha + 4 = -1 \rightarrow \boxed{\alpha = -5}$

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6)

6.1) a, b, f

6.2) a, c, f

6.3) d

6.4) b, c, f

6.5) a, b, c, e