MECH 6300 - HW 3

3)

a)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 10 & 2 & 0 \\ 0 & 5 & 4 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\lambda_1 = 1$
$\lambda_2 = 2$
$\lambda_3 = 4$

i) Cayley - Hamilton Therem Method

$$\beta_0 I = \begin{bmatrix} \frac{8}{3}e^{t} - 2e^{2t} + \frac{1}{3}e^{4t} & 0 & 0 \\ 0 & \frac{8}{3}e^{t} - 2e^{2t} + \frac{1}{3}e^{4t} & 0 \\ 0 & 0 & \frac{8}{3}e^{t} - 2e^{2t} + \frac{1}{3}e^{4t} \end{bmatrix} \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 30 & 4 & 0 \\ 50 & 30 & 16 \end{bmatrix}$$

$$\beta_1 A = \begin{bmatrix} -2e^{t} + \frac{5}{2}e^{2t} - \frac{1}{2}e^{4t} & 0 & 0 \\ -20e^{t} + 25e^{2t} - 5e^{4t} & -4e^{t} + 5e^{2t} - e^{4t} & 0 \\ 0 & -10e^{t} + \frac{25}{2}e^{2t} - \frac{5}{2}e^{4t} & -8e^{t} + 10e^{2t} - 2e^{4t} \end{bmatrix}$$

$$\beta_2 A^2 = \begin{bmatrix} \frac{1}{3}e^{t} - \frac{1}{2}e^{2t} + \frac{1}{6}e^{4t} & 0 & 0 \\ 10e^{t} - 15e^{2t} + 5e^{4t} & \frac{4}{3}e^{t} - 2e^{2t} + \frac{2}{3}e^{4t} & \\ \frac{50}{3}e^{t} - 25e^{2t} + \frac{25}{3}e^{4t} & 10e^{t} - 15e^{2t} + 5e^{4t} & \frac{16}{3}e^{t} - 8e^{2t} + \frac{8}{3}e^{4t} \end{bmatrix}$$