EE 6312 - Exam 2 | Jonas Wagner | 2020-10

4)

$$\phi_{XY}(\omega_1, \omega_2) = \frac{1}{(1-j\omega_1)^2(1-j\omega_2)^2}$$

a) Correllation:

$$\rho = \frac{m_{11}}{\sigma_X \sigma_Y}$$

$$\sigma_X^2 = m_{20} = (\cancel{-j})^{\cancel{2}\to 1}{}^2 \frac{\partial^2}{\partial \omega_1^2}\left(\frac{1}{(1-j\omega_1)^2(1-j\omega_2)^2}\right)\Bigg|_{0,0}$$

$$\frac{1}{(1-j\omega_2)^2}\frac{\partial^2}{\partial \omega_1^2}(1-j\omega_1)^{-2} \to \frac{\partial}{\partial \omega_1} -2(-j)(1-j\omega_1)$$

$$6(\cancel{-j})^{\cancel{2}\to 1}(1-j\omega_1)^{-4}$$

$$m_{20} = \cancel{\frac{6(-1)(1)}{(1-j\omega_1)^4(1-j\omega_2)^2}}\Bigg|_{0,0} = 6$$

$$\sigma_Y^2 = m_{02} = 6$$

$$m_{11} = (\cancel{-j})^{\cancel{2}\to 1}\frac{\partial^2}{\partial \omega_1 \partial \omega_2}\left[(1-j\omega_1)^{-2}(1-j\omega_2)^{-2}\right]\Bigg|_{0,0}$$

$$= -\left((-2)(-j)(1-j\omega_1)^{-3}(-2)(-j)(1-j\omega_2)^{-3}\right)$$

$$= \frac{\cancel{-}4}{(1-j\omega_1)^3(1-j\omega_2)^3}\Bigg|_{0,0} = 4$$

$$\rho = \frac{4}{\sqrt{6}\sqrt{6}} = \frac{4}{6} = \frac{2}{3}$$

$$\boxed{\rho = \frac{2}{3}}$$

b) Yes, $\rho \neq 0$

2)

$$f_{XY}(x,y) = \begin{cases} \frac{1}{\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}\right), & x \ge 0, y \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

$g(x,y) = U = x^2 + y^2$

$h(x,y) = V = \left|\tan^{-1}\left(\frac{y}{x}\right)\right|$

$\frac{\partial h}{\partial x} = ???$

$\frac{y}{x} = \tan(V) \quad V \ge 0$

$Y = X\tan(V)$

single set of roots

R_i: $U = x^2 + x^2\tan^2(V)$

$x = \sqrt{1+\tan^2(V) - U}$

$Y = \sqrt{1+\tan^2(V) - U}\,\tan(V)$

↓??

$$|J| = \begin{vmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial g}{\partial g} \end{vmatrix} = \begin{vmatrix} 2x & \\ 2y & \end{vmatrix}$$

$$f_{UV}(u,v) = \frac{f_{XY}(\ ,\ )}{|J|}, \quad v > 0$$

3) $R = X + \sum_{i=1}^{100} W_i$

$P_X(x) = \begin{cases} 0.1, & x=2 \\ 0.9, & x=0 \\ 0, & \text{otherwise} \end{cases}$

$x=2$: $\mu = 0.01$
$\sigma = 0.01$

$x=0$: $\mu = 0$
$\sigma = 0.01$

$P[R>1] = P[R>1 \mid x=0]\, P[x=0]\,(0.9)$
$\quad + P[R>1 \mid x=2]\, P[x=2]\,(0.1)$

$P[R>1 \mid x=0]$ $\quad \mu = 0$, $\sigma = 0.01$ $\quad \cancel{0 + \frac{E[x]}{1}} \approx 0$

~~Markov Inequality:~~

$P[R>1 \mid x=2]$ $\quad \mu = 0.01$, $\sigma = 0.01$ $\quad \frac{2 \cdot 0.1}{1}$

looking back I should have used
LLN w/ $\Theta(t, w)$