EESC 6349 / MECH 6312

Test 3 (Question Set A), Oct. 19, 2020
Instructor: Dr. Minn

1. Two sensing devices provide the following 2×1 real-valued random vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$:

$$\boldsymbol{X} = \boldsymbol{S} + \boldsymbol{V}$$
$$\boldsymbol{Y} = \boldsymbol{S} + \boldsymbol{W}$$

where $\boldsymbol{S} = [S_1, S_2]^T$, $\boldsymbol{V} = [V_1, V_2]^T$, $\boldsymbol{W} = [W_1, W_2]^T$, $E[\boldsymbol{S}] = [0.1, 0.1]^T$, $E[S_1^2] = E[S_2^2] = 1$, and $E[S_1 S_2] = 0.01$. V_1, V_2, W_1, and W_2 are mutually independent random variables, each with mean 0 and variance 0.1, and they are uncorrelated from S_1 and S_2. If $\boldsymbol{Z} = \boldsymbol{X} + \boldsymbol{Y}$, find the covariance matrix of $\boldsymbol{Z}$. [40 points]

2. The received real-valued signal vector $\boldsymbol{X} = [X_1, X_2]^T$ from a WiFi receiver has the following statistics: $E[\boldsymbol{X}] = \boldsymbol{0}$, $E[\boldsymbol{X}\boldsymbol{X}^T] = \begin{bmatrix} 4, & 1 \\ 1, & 2 \end{bmatrix}$. For data detection, a whitening transformation is done on $\boldsymbol{X}$, yielding $\boldsymbol{Y}$ with $E[\boldsymbol{Y}] = \boldsymbol{0}$ and $E[\boldsymbol{Y}\boldsymbol{Y}^T] = \boldsymbol{I}$. Find the required transformation based on the eigen-decomposition approach. [60 points]

Note: The roots of $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.