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4)
$$\Phi_{XY}(w_1, w_2) = \frac{1}{(1-jw_1)^2(1-jw_2)^2}$$

a) <u>Correllation:</u> $\rho = \frac{m_{11}}{\sigma_X \sigma_Y}$

$$\sigma_X^2 = m_{20} = \cancel{(-j)^2}^{-1} \frac{\partial^2}{\partial w_1^2}\left(\frac{1}{(1-jw_1)^2(1-jw_2)^2}\right)\Big|_{(0,0)}$$

$$\frac{1}{(1-jw_2)^2}\frac{\partial^2}{\partial w_1^2}(1-jw_1)^{-2} \Rightarrow \frac{\partial}{\partial w_1} -2(-j)(1-jw_1)^{-3}$$

$$6\cancel{(-j)^2}^{-1}(1-jw_1)^{-4}$$

$$m_{20} = \frac{6\cancel{(-1)(-1)}}{\cancel{(1-jw_1)^4(1-jw_2)^2}}\Big|_{0,0} = 6$$

$$\sigma_Y^2 = m_{02} = 6$$

$$m_{11} = \cancel{(-j)^2}^{-1} \frac{\partial^2}{\partial w_1 \partial w_2}\left[(1-jw_1)^{-2}(1-jw_2)^{-2}\right]\Big|_{0,0}$$

$$= -\left((-2)(-j)(1-jw_1)^{-3}(-2)(-j)(1-jw_2)^{-3}\right)$$

$$= \frac{\cancel{-}\,4}{(1-jw_1)^3(1-jw_2)^3}\Big|_{0,0} = 4$$

$$\rho = \frac{4}{\sqrt{6}\sqrt{6}} = \frac{4}{6} = \frac{2}{3} \qquad \boxed{\rho = \frac{2}{3}}$$

b) Yes, $\rho \neq 0$

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2) $f_{XY}(x,y) = \begin{cases} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right), & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$g(x,y) = U = x^2 + y^2$

$h(x,y) = V = \left|\tan^{-1}\left(\frac{y}{x}\right)\right|$

$\frac{dh}{dx} = ???$

$\frac{y}{x} = \tan(v)$ $\quad v \geq 0$

$y = x\tan(v)$

single set of roots

R_i: $v = x^2 + x^2\tan^2(v)$

$x = \sqrt{1+\tan^2(v) - v}$

$y = \sqrt{1+\tan^2(v) - v}\,\tan(v)$

↓??

$J = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{vmatrix} 2x & \\ 2y & \end{vmatrix}$

$f_{UV}(u,v) = \frac{f_{XY}(\quad , \quad)}{|J|}$, $v > 0$

3) $R = X + \sum_{i=1}^{100} W_i$ $\qquad P_X(x) = \begin{cases} 0.1, & x=2 \\ 0.9, & x=0 \\ 0, & \text{otherwise} \end{cases}$

$x=2$: $\mu = 0.01$
$\sigma = 0.01$
$x=0$: $\mu = 0$
$\sigma = 0.01$

$P[R>1] = P[R>1 \mid x=0]\, P[x=0] \,(0.9)$
$+ P[R>1 \mid x=2]\, P[x=2] \,(0.1)$

$P[R>1 \mid x=0]$ $\quad \mu = 0$, $\sigma = 0.01$ $\rightarrow \frac{0 + \cancel{E[x]}}{1} \approx 0$

Markov Inequality:

$P[R>1 \mid x=2]$ $\quad \mu = 0.01$, $\sigma = 0.01$ $\rightarrow \frac{2 \cdot 0.1}{1}$

looking back I should have used LLN w/ $\Theta(+, w)$