7) $\mu_2 = 3000$ $\mu_1 = 2700$

$\sigma_2 = 400$ $\sigma_1 = 400$

$\sigma_1 = \sigma_2 = \sigma$

$\alpha = 0.1$ $H_1 : \mu = \mu_1$ — $N(\mu_1, \sigma^2)\ \xi_1$

$H_2 : \mu = \mu_2$ — $N(\mu_2, \sigma^2)\ \xi_2$

$n = 25 \rightarrow \{x_i : i = 1, \dots, 25\}$

$$\Lambda = \frac{\Pi_{i=1}^{n} f_{x_i|\xi_1}(x_i)}{\Pi_{i=1}^{n} f_{x_i|\xi_2}(x_i)} > K$$

$$\Lambda = \exp\left(\frac{1}{2\sigma^2} \sum_{i=1}^{n} \left[(x_i - \mu_2)^2 - (x_i - \mu_1)^2\right]\right)$$

$$\ln[\Lambda] = \frac{n(\mu_1 - \mu_2)}{\sigma^2}\left(\hat{\mu} - \frac{1}{2}(\mu_1 + \mu_2)\right)$$

$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$

a) Accept H_1 if $\hat{\mu} < C$, otherwise Reject

w/ $\boxed{\hat{\mu} = \frac{1}{25} \sum x_i}$

$$C = \mu_1 + \frac{\sigma}{\sqrt{n}} Z_{[1-\alpha]}$$

$$= 2700 + \frac{400}{\sqrt{25}} Z_{[0.9]}$$

$$= 2700 + 80(1.28) \leftarrow \text{doblecheck}$$

$$\boxed{C = 2802.5}$$

b) Power of Test

$P[H_2 | H_2] = P[\hat{\mu} < C | H_2]$

$$F_{SN}\left(\frac{C_n - \mu_2}{\sqrt{\sigma^2/n}}\right)$$

$$F_{SN}\left(\frac{2802.5 - 2700}{\sqrt{400/25}}\right) = F_{SN}(25.6+)$$

$\boxed{1 - \beta = 1}$ Full Power.

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2) $\alpha = 0.05$

$H_1 : \sigma_1^2 = \sigma_2^2$

F-Test:

$\hat{\mu}_1 = 28.84$

$\hat{\mu}_2 = 30.62$

$$\left.\begin{aligned} \sigma_1^2 &= 7.47 \\ \sigma_2^2 &= 5.92 \end{aligned}\right\} \rightarrow V_R = \frac{\sigma_1^2}{\sigma_2^2} = 1.26$$

$t_l' = x_{[0.5\alpha]}$ w/ $F_F(x_{[0.5\alpha]}; \overset{12}{m-1}; \overset{12}{n-1})$

$\rightarrow t_l' = 0.305$

$t_u' = x_{[1-0.5\alpha]} = 3.27$

Critical Region: $\{0 < V_R < t_l'\} \vee \{t_u' < V_R < \infty\}$

since V_R is within (t_l', t_u'),

Accept H_1.//

The variances are the same w/ $\alpha = 0.05$.//