1) $X = S + V$
$Y = S + W$

$E[S] = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ $E[S_1^2] = E[S_2^2] = 1$
$E[S_1 S_2] = 0.01$

$V_{1,2} \sim (0, 0.1)$
$W_{1,2} = (0, 0.1)$
Mutually Independent

$R_S = \begin{bmatrix} 1 & 0.01 \\ 0.01 & 1 \end{bmatrix}$

$R_V = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

$R_{SW} = R_{WS} = R_{SV} = R_{VS} = 0$

$\mu_X = \mu_S + \mu_V = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} + 0 = \mu_S$

$R_V = R_W$ $\quad R_W = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

$\mu_Y = \mu_S + \mu_W = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} + 0 = \mu_S$

$R_X = R_S + \cancelto{0}{R_{SV}} + \cancelto{0}{R_{VS}} + R_V = \begin{bmatrix} 1 & 0.01 \\ 0.01 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

$R_X = \begin{bmatrix} 1.1 & 0.01 \\ 0.01 & 1.1 \end{bmatrix}$

$R_Y = R_S + \cancelto{0}{R_{SW}} + \cancelto{0}{R_{WS}} + R_W = R_X$

$R_Y = \begin{bmatrix} 1.1 & 0.01 \\ 0.01 & 1.1 \end{bmatrix}$

Assuming X and Y are uncorrelated

$R_Z = R_X + \cancelto{0}{R_{XY} + R_{YX}} + R_Y$
$= \begin{bmatrix} 1.1 & 0.01 \\ 0.01 & 1.1 \end{bmatrix} + \begin{bmatrix} 1.1 & 0.01 \\ 0.01 & 1.1 \end{bmatrix}$

$\mu_Z = \mu_X + \mu_Y = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$

$\mu_Z = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$

$R_Z = \begin{bmatrix} 2.2 & 0.02 \\ 0.02 & 2.2 \end{bmatrix}$

$\mu_Z \mu_Z^T = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}$

$K_Z = R_Z - \mu_Z \mu_Z^T = \begin{bmatrix} 2.2 & 0.02 \\ 0.02 & 2.2 \end{bmatrix} - 0.04 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.04 \end{bmatrix}$

1) cont.

$$\boxed{K_Z = \begin{bmatrix} 2.16 & -0.02 \\ -0.02 & 2.16 \end{bmatrix}}$$

2) $E[X] = 0 \longrightarrow \mu_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ zero mean

$R_x = E[X X^T] = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad \downarrow \quad \mu_Y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ zero mean

$\downarrow$

$R_Y = E[Y Y^T] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$K_x = R_x - \cancel{\mu \mu^T} \to 0 = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$

$|K_x - \lambda I| = \begin{vmatrix} 4-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda) - 1$

$\lambda^2 - 6\lambda + 8 - 1$

$\lambda^2 - 6\lambda + 7 = 0$

$\downarrow$

$\lambda_{1,2} = \dfrac{6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)}$

$\lambda_{1,2} = 3 \pm \sqrt{2}$

$(K - \lambda_1 I)\phi_1 = 0$

$\begin{bmatrix} [4-(3+\sqrt{2})]a + b = 0 \\ a + [2-(3+\sqrt{2})]b = 0 \end{bmatrix}$

$\phi_1 = \begin{bmatrix} 0.92 \\ 0.38 \end{bmatrix}$

$(K - \lambda_2 I)\phi_2 = 0$

$\begin{bmatrix} [4-(3-\sqrt{2})]a + b = 0 \\ a + [2-(3-\sqrt{2})]b = 0 \end{bmatrix}$

$\phi_2 = \begin{matrix} -0.38 \\ 0.92 \end{matrix}$

$U = \begin{bmatrix} 0.924 & -0.383 \\ 0.383 & 0.924 \end{bmatrix}$

$\Lambda = \begin{bmatrix} 3+\sqrt{2} & 0 \\ 0 & 3-\sqrt{2} \end{bmatrix}$

2) cont.

$$\Lambda^{-1/2} = \begin{bmatrix} 0.476 & 0 \\ 0 & 0.794 \end{bmatrix}$$

$$V^H = V^{-1}$$

$$VV^H = I \checkmark$$

$$A = \Lambda_X^{-1/2} V^H = \begin{bmatrix} 0.476 & 0 \\ 0 & 0.794 \end{bmatrix} \begin{bmatrix} 0.924 & 0.383 \\ -0.383 & 0.924 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 0.440 & 0.182 \\ -0.304 & 0.734 \end{bmatrix}}$$

$$Y = AX$$