1) $f_{XY} = \begin{cases} \frac{3e^{-x}(x^2-y^2)}{2x^3}, & x>0,\ 0<y<x \\ 0, & \text{otherwise} \end{cases}$

$g(x,y): V = X\sin(Y)$
$h(x,y): V = \sqrt{X}$

$\longrightarrow X = V^2,\quad Y = \sin^{-1}\left(\frac{V}{\sqrt{X}}\right)$ $\quad V \neq 0,\ V \geq 0$

$$|J| = \begin{vmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \end{vmatrix} = \begin{vmatrix} \sin(Y) & X\cos(Y) \\ \frac{X^{-1/2}}{2} & 0 \end{vmatrix} = \sqrt{X}\cos(Y)$$

$$|J| = V\cos\left(\sin^{-1}\left(\frac{V}{\sqrt{V^2}}\right)\right)$$

$$f_{UV}(u,v) = \begin{cases} \frac{3e^{-v^2}\left(u^2 - \left(\sin^{-1}\left(\frac{v}{\sqrt{v^2}}\right)\right)^2\right)}{2u^3\, V\cos\left(\sin^{-1}\left(\frac{v}{\sqrt{v^2}}\right)\right)}, & v \neq 0,\ u>0 \\ 0, & \text{otherwise} \end{cases}$$

$0 < u < v^2$

2) $N = 100$

$x = \begin{cases} 1 = \text{Presence} \\ 0 = \text{absence} \end{cases}$

$Y_i = X + W_i \quad \forall i = 1, 2, \dots, N$

$P_X(x) = \begin{cases} 0.01, & x = 1 \\ 0.99, & x = 0 \\ 0, & \text{otherwise} \end{cases}$

$W_i \sim (0, 1)$ — μ, σ^2

$Z = \frac{1}{N} \sum_{i=1}^{N} Y_i$

$P[Z > 0.6]$ — N users present

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a) $P[Z \leq 0.6 \mid X = 1]$ ~~$\Phi(0.6) = 1 - Q(0.6)$~~

$R = \frac{Z - \mu_X}{\sigma_X} \Rightarrow -1 \leq R \leq -0.4$

$P[R \leq -0.4 \mid X = 1] \sim 1 - Q(0.4) = \boxed{Q(0.4)}$

b) $P[Z > 0.6 \mid X = 0] \sim \boxed{1 - Q(0.6)}$

c) $P[\text{Error}] = P[Z \leq 0.6 \mid X = 1] P[X = 1] + P[Z > 0.6 \mid X = 0] P[X = 0]$

$\boxed{P[\text{Error}] = 0.01[Q(0.4)] + 0.99[1 - Q(0.6)]}$