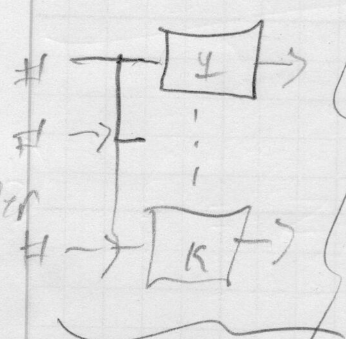
4) K independent process

2 per $\lambda = 2$

Failure → F when 3+ request for all

Find K $P[F] < 0.1$



$$P_i = P[\# > 2] = 1 - P[\# \le 2] = 1 - 4e^{-2} = 0.459$$

$$P[\# \le 2] = \sum \frac{\lambda^k}{k!} e^{-\lambda} = \frac{2^1}{1} e^{-2} + \frac{2^2}{2} e^{-2} = (2+2)e^{-2} = 4e^{-2} \approx 0.541$$

$$P[F] = \prod_{i=1}^{K} (1 - P_i) = \prod_{i=1}^{K} (0.541) = (0.541)^K$$

$$P[F] < 0.1 = (0.541)^K \Rightarrow K = 3.748 \rightarrow 4$$

$$\boxed{K = 4}$$

2) $f_{XY}(x,y) = \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{2|x| + y^2}{2}\right), & -\infty < x < \infty, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$

a) $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right) \underbrace{\int_{-\infty}^{\infty} e^{\frac{-2|x|}{2}}\,dx}$

$\int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx$

$(e^{0} - e^{-\infty}) + (-e^{-\infty} + e^{0})$

2

$$\boxed{f_Y(y) = \frac{2}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right)}$$

b) (g) $V = X^2 + Y^2$ (h) $W = \frac{X}{Y}$

$v - (x^2 + y^2) = 0$

$x^2 = v - y^2$ ψ $\boxed{y = wx}$

$x^2 = v - (wx)^2$

$x^2 = v - w^2x^2$

$(1+w^2)x^2 = v$

$x^2 = \frac{v}{1+w^2}$

1: + +
2: + -
3: - +
4: - -

φ $\boxed{x = \pm\sqrt{\frac{v}{1+w^2}}}$

ψ $\boxed{y = \pm w\sqrt{\frac{v}{1+w^2}}}$

$|J_i| = \begin{vmatrix} \frac{dg}{dx_i} & \frac{dg}{dy_i} \\ \frac{dh}{dx_i} & \frac{dh}{dy_i} \end{vmatrix} = \begin{vmatrix} 2x_i & 2y_i \\ \frac{1}{y_i} & \frac{-x_i}{y_i^2} \end{vmatrix}$

$= 2x_i\left(\frac{-x_i}{y_i^2}\right) - \cancel{2y_i}\frac{1}{\cancel{y_i}}$

$= 2\frac{x_i^2}{y_i^2} - 2$

$|J_i|_{v_i} = 2\frac{\left(\sqrt{\frac{v}{1+w^2}}\right)^2}{\left(w\sqrt{\frac{v}{1+w^2}}\right)^2} - 2 = \frac{2}{w^2} - 2$

S_{VW} $v = (-\infty, \infty)$
$w = (-\infty, \infty)$

$|J_i|^{-1} = \frac{1}{\frac{2}{w^2} - 2}$

$$f_{VW}(v,w) = 4\left(\frac{1}{\frac{2}{w^2} - 2}\right)\left(\frac{1}{\sqrt{2\pi}}\right) \exp\left(-\frac{2\sqrt{\frac{v}{1+w^2}} + w^2\frac{v}{1+w^2}}{2}\right)$$

3) $Y = X + \sum_{i=1}^{N} V_i$

$X \sim \begin{cases} 0.2, & x=1 \\ 0.8, & x=0 \end{cases}$

$\begin{cases} \hat{X} = 1, & Y \geq 0.4 \\ \hat{X} = 0, & \text{otherwise} \end{cases}$

$V_i \sim iid(0, 0.001)$

$P_N[n] = \begin{cases} 0.6, & n=100 \\ 0.4, & n=400 \end{cases}$

Let $Z_n = \frac{1}{s_n} \sum_{i=1}^{N} V_i$

$s_n^2 = \sum_{i=1}^{N} (0.01)^2 = N(0.01)^2$

$s_n = \sqrt{N}(0.01)$

$Y = X + s_n Z_n$

a) For $X = 1$: $P[\hat{X} \neq 1 | X=1] = P[s_n Z_n < -0.6]$

$Y < 0.4$

@ $n=100$

$s_n = \sqrt{100}(0.01) = 0.1 \rightarrow P[Z_n < \frac{-0.6}{0.1}] = -6$

$= 1 - P[\cancel{Z_n > 6}] \approx 1$

$= 0$

@ $n=400$

$s_n = \sqrt{400}\, 0.1 = 2$

$P[Z_n < \frac{-0.6}{2}] = 1 - P[Z_n < 0.3] = 1 - 0.6179$

$= 0.3821$

$P[\hat{X} \neq 1 | X=1] = 0.6(0) + 0.4(0.3821)$

$\boxed{P[\hat{X} \neq 1 | X=1] = 0.1528}$

3) cont.

b) For $X=0$: $P[\hat{X} \neq 0 | X=0] = P[s_n Z_n > 0.4]$

for $n=100$:

$s_n = 0.1$ $\quad P[Z_n > 10(0.4)] = P[Z_n > 4] \approx 0$

~~for n=400:~~

$s_n = 2$ $\quad P[Z_n > \frac{1}{2}(0.4)] = P[Z_n > 0.2]$

$= 1 - Z_n[Z < 0.2]$

$= 1 - 0.5793$

$= 0.4207$

$P[\hat{X} \neq 0 | X=0] = (0.6)(0) + (0.4)(0.42)$

$$\boxed{P[\hat{X} \neq 0 | X=0] = 0.1683}$$

c) $P[\hat{X} \neq X] = 0.2(0.1528) + 0.8(0.1683)$

$= 0.0306 + 0.1346$

$$\boxed{P[\hat{X} \neq X] = 0.1652}$$

MECH 6325-Final Exam Jonas Wagner 2020-12-07

4) $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \underset{X}{h}\,\underset{A}{s} + n$ $n = \begin{bmatrix} n_0 \\ n_2 \end{bmatrix}$, $n = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$

$\sim (0, 0.1 I)$

Transmission Signal (deterministic)

All independent

n_1, n_2 Gaussian w/ $\rho = 0.7$

$K_n = 0.1 I$

Independent $\Rightarrow$ uncorrelated

$E[n_1] = E[n_2] = 0$

$E[n_1^2] = E[n_2]^2 = 1$

$\sigma_1^2 \quad \sigma_2^2$

$K_X = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$

$K_r = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

$K_r = \begin{bmatrix} 1.1 & 0.7 \\ 0.7 & 1.1 \end{bmatrix}$

$y = A r$ Whitening

$\lambda_i = \{1.8, 0.4\}$

$\Lambda^{-1/2} = \begin{bmatrix} 1.8^{-1/2} & 0 \\ 0 & 0.4^{-1/2} \end{bmatrix}$

$U = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$

$A = \Lambda^{-1/2} U^H = \begin{bmatrix} 0.74 & 0 \\ 0 & 1.58 \end{bmatrix} U^H$

$$\boxed{A = \begin{bmatrix} 0.527 & 0.527 \\ -1.118 & 1.118 \end{bmatrix}}$$

5) Rank Sum

$\alpha = 0.09$

$H_1 : F_x = F_y$

Var =

I did in excel... ordered them acording to formulas

$t = \Sigma$ Y ranks $= 190$

a.)

$\mu_T = 232$

$\sigma_T = 59$

$F(t_L) = 0.5\alpha = 0.045$ $t_L = 232 - 0.517(59) = 201$

$F(t_u) = 1 - 0.5\alpha = 1 - \frac{1}{\ }$ $t_u = 232 + 0.832(59) = 281$

190 is outside of [201, 281)

reject $H_1 \rightarrow F_x \neq F_y$

b)

~~ran~~ ran out of space...

see next page

c) $\hat{\mu} = 71.26$ $\delta = 0.5$

unkown σ... $\hat{\sigma} = 4.242$

$$\left[71.26 - (0.831)\frac{4.24}{\sqrt{30}},\ 71.26 + (0.831)\frac{4.24}{\sqrt{30}}\right]$$

$$\boxed{[70.61, 71.9]}$$

a) $\text{Median} = \dfrac{Y_{[0.5N]} + Y_{[0.5N+1]}}{2}$

$\boxed{X_{[50\%]} = 72}$

b) $H_1 : P[w] = \frac{1}{3} \ \forall \ [60,70), [70,75), [75,80)$

$\alpha = 0.025$

actual $\to$ 10   12   8

$\hat{X}_{[0.33]} = 70$ $\hat{X}_{[0.66]} = 75$

~~$\alpha = P[\text{Reject } H_1 \mid H_1 = \text{True}] = 0.025$~~

Pearson's Test: χ^2

~~GLRT : / Test:~~

P Actual = 69.33   P[74

$$V = \sum_{i=1}^{L=3} \left(\frac{Y_i - nP_{0i}}{\sqrt{nP_{0i}}} \right)^2$$

$P_{0i} = 0.33 \ \forall i = 1,2,3$

$nP_{0i} = 30\left(\frac{1}{3}\right) = 10$

$$= \cancel{\left(\frac{10-10}{\sqrt{10}}\right)^2}^{\,0} + \left(\frac{12-10}{\sqrt{10}}\right)^2 + \left(\frac{8-10}{\sqrt{10}}\right)^2$$

$$= \left(\frac{2}{\sqrt{10}}\right)^2 + \left(\frac{-2}{\sqrt{10}}\right)^2 = \frac{2(4)}{10} = \frac{8}{10} = \boxed{0.8 = V}$$

$1-\alpha = 0.975$, $L-1 = 2$

$F_{\chi^2}(X_{[0.975]}, 2) = 0.975$

$\boxed{X_{[0.975]} = 7.38}$

$V < X_{[0.975;2]}$

$\downarrow$

Accept Hypothesis