EESC 6349 / MECH 6312

Supplementary Test, Oct. 14, 2020
Instructor: Dr. Minn

1. The joint probability density function (pdf) of random variables X and Y is given by

$$f_{XY}(x,y) = \begin{cases} \frac{3\ e^{-x}\ (\pi^2 - y^2)}{2\pi^3}, & x > 0,\ 0 < y < \pi \\ 0, & \text{else.} \end{cases}$$

If $U = X\sin(Y)$ and $V = \sqrt{X}$, find the joint pdf of U and V. [50 points]

2. An air-borne virus detection sensor network consists of 100 sensors which give the output signals $Y_i = X + W_i,\ i = 1, 2, \cdots, 100$, where X, representing either the presence (1) or absence (0) of the virus, has the probability mass function

$$P_X(x) = \begin{cases} 0.01, & x = 1 \\ 0.99, & x = 0 \\ 0, & \text{else} \end{cases}$$

and $\{W_i\}$, independent from X, are independent and identically distributed noises, each with mean 0 and variance 1. Suppose the sensing network computes

$$Z = \frac{1}{100}\sum_{i=1}^{100} Y_i$$

and makes decision as follows:

$$\begin{aligned} \text{If } Z > 0.6, &\ \ \text{Decision = virus present} \\ \text{otherwise}, &\ \ \text{Decision = virus absent.} \end{aligned}$$

Compute the approximate expression in terms of Gaussian tail probability (Q function) for the following probabilities:
(a) the conditional probability of decision error given the presence of the virus. [20 points]
(b) the conditional probability of decision error given the absence of the virus. [20 points]
(c) the average probability of decision error. [10 points]