MECH 6313 - Homework5

Jonas Wagner

2021, April 14

1 Problem 1

The standard mass-spring-damper system described by

$$m\ddot{y} + \beta\dot{y} + ky = u \tag{1}$$

Then

1.1 Part a

Problem: Design a gradient algorithm to estimate the unknown parameters m, β, and k from known inputs and outputs $u(t)$ and $y(t)$.

Solution:

1.1.1 Part b

Problem: Simulate the algorithm for $m = 20$, $\beta = 0.1$, and $k = 5$ for different choices of $u(t)$ and resulting parameter convergence properties.

Solution:

2 Problem 2

Considering the reference model

$$\dot{y}_m = -ay_m + r(t),\ a > 0 \tag{2}$$

and the plant given as

$$\dot{y} = a^* y + b^* u,\ b^* \neq 0 \tag{3}$$

2.1 Part a

Problem: Show that a controller of form

$$u = \theta_1 y + \theta_2 r(t)$$

with gains θ_1^* and θ_2^* stabilizes the tracking error $e := y - y_m$ asymptotically to zero.

Solution:

2.1.1 Part b

Problem: Suppose a^* and $b*$ are unknown but the sign of b^* is known. Show that the adaptive implementation of the controller can achieve tracking when the gains are updated according to the following rule:

$$\dot{\theta}_1 = -\mathrm{sign}(b^*)\gamma_1 y e \tag{4}$$

$$\dot{\theta}_2 = -\mathrm{sign}(b^*)\gamma_2 r e \tag{5}$$

with $\gamma_1, \gamma_2 > 0$.

Solution:

2.1.2 Part c

Problem: Provide conditions that also guarantee that $\theta_1(t) \to \theta_1^*$ and $\theta_2(t) \to \theta_2^*$ as $t \to \infty$.

Solution:

3 Problem 3

A simplified model of an axial compressor, used in jet engine control studies, is given by the following second order system:

$$\begin{aligned}\dot{\phi} &= -\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \psi \\ \dot{\psi} &= \frac{1}{\beta^2}(\phi + 1 - u)\end{aligned} \tag{6}$$

This model captures the main surge instability between the mass flow and the pressure rise. Here, ϕ and ψ are deviations of the mass flow and the pressure rise from their set points, the control input u is the ow through the throttle, and β is a positive constant.

3.1 Part a

Problem: Use backstopping to obtain a control law to stabilize the origin.

Solution:

3.1.1 Part b

Problem: Use Sontag's Formula and the Control Lyapunov Function obtained previously to obtain an alternative control law.

Solution:

A MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6313