MECH 6313 - Homework 1

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1 Problem 1 - Hopf Bifurcation

1.1 Part a

Problem: Simulate the following system for various parameters to determine the type of bifurcation that occurs:

$$\begin{aligned} \dot{x} &= \alpha x + y \\ \dot{y} &= -x + \alpha y - x^2 y \end{aligned} \tag{1}$$

Solution: The system was simulated for various parameter values and initial conditions using MATLAB's built-in ode45 solver. The MATLAB code for this assignment is available in Appendix1

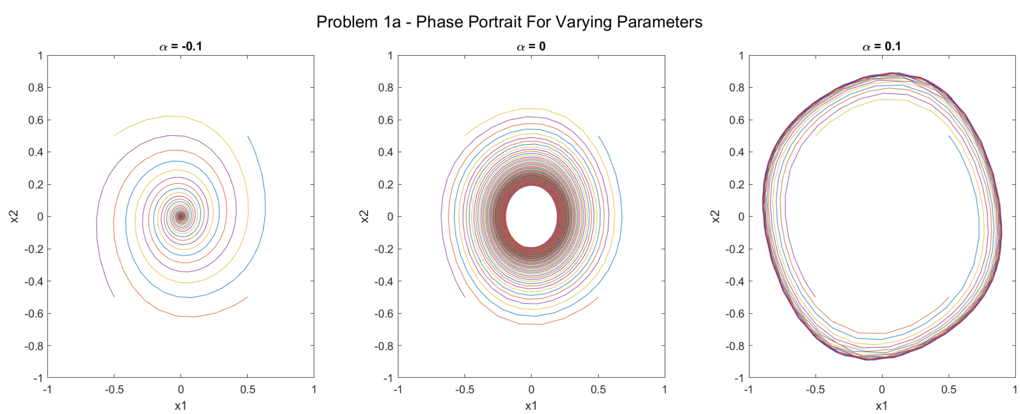

Figure 1: Phase Plot of the system for various values of α and initial conditions

The numerical solution clearly shows a stable limit cycle for $\alpha > 0$, thus it exhibits supercritical hopf bifucation.

1.2 Part b

Problem: Simulate the following system for various parameters to determine the type of bifurcation that occurs:

$$\begin{aligned} \dot{x} &= \alpha x + y - x^3 \\ \dot{y} &= -x + \alpha y + 2y^3 \end{aligned} \tag{2}$$

Solution: The system was simulated for various parameter values and initial conditions using MATLAB's built-in ode45 solver. The MATLAB code for this assignment is available in Appendix1

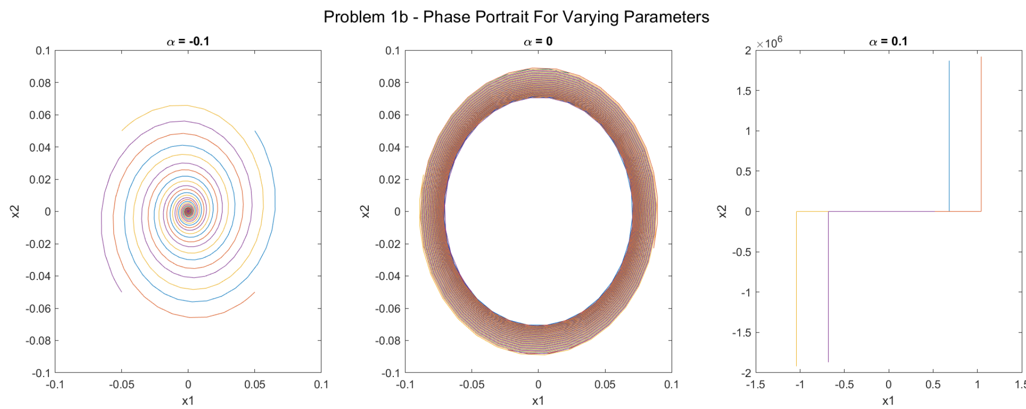

Figure 2: Phase Plot of the system for various values of α and initial conditions

The numerical solution clearly shows a very unstable system for $\alpha > 0$, thus it exhibits subcritical hopf bifucation. One interesting occurrence though is the limit cycle that is occurring for $\alpha \approx 0$.

1.3 Part c

Problem: Simulate the following system for various parameters to determine the type of bifurcation that occurs:

$$\begin{aligned}\dot{x} &= \alpha x + y - x^2 \\ \dot{y} &= -x + \alpha y + 2x^3\end{aligned} \tag{3}$$

Solution: The system was simulated for various parameter values and initial conditions using MATLAB's built-in ode45 solver. The MATLAB code for this assignment is available in Appendix1

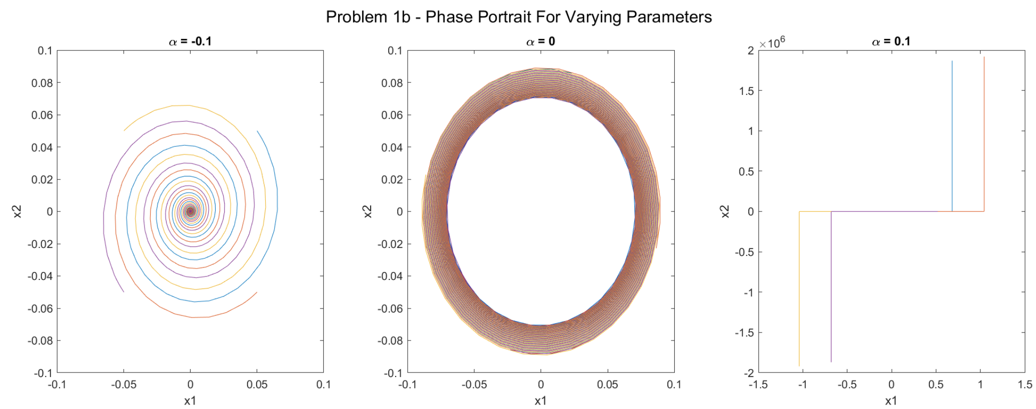

Figure 3: Phase Plot of the system for various values of α and initial conditions

The numerical solution clearly shows unstable behavior for $\alpha > 0$, thus it exhibits subcritical hopf bifucation.

2 Problem 2

2.1 Part a

2.1.1 System Linearization

Problem: Linearize and analyze the following system.

$$
\begin{aligned}
\dot{x}_1 &= x_2 + x_1 x_2^2 \\
\dot{x}_2 &= -x_1 + x_1^2 x_2
\end{aligned}
\tag{4}
$$

Solution: The linearized solution can be calculated by determining the first-order taylor expansion of the nonlinear system:

$$
f(x) \approx f(x_0) + \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0} \bar{x} + H.O.T., \quad \bar{x} = x - x_0 \tag{5}
$$

In this case, the A matrix is calculated as the Jacobian of the system dynamics evaluated at $x = x_0 = 0$:

$$
\begin{aligned}
A = \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0} &= \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{x=x_0}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} && (6) \\
&= \left.\begin{bmatrix} x_2^2 & 2x_1x_2 + 1 \\ 2x_1x_2 - 1 & x_1^2 \end{bmatrix}\right|_{x_1=x_2=0} && (7) \\
&= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} && (8)
\end{aligned}
$$

The linear dynamics for the equilibrium point is therefore given as:

$$
\dot{\mathbf{x}} = A\mathbf{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x} \tag{9}
$$

The characteristics roots are therefore calculated as the eigenvalues of A:

$$
\lambda_{1,2} = \pm j
$$

From this we can conclude the linear system is a harmonic oscillator that is marginally stable.

2.1.2 Bendixon's Criteria

Problem: Show that the system has no closed orbits.

Solution: Bendixon's Criterion states that if the divergence of $f(x)$ is not equivalently zero and does not change sign with a simply connected region then there are no periodic orbits in that region.

The divergence of the system can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= x_1^2 + x_2^2\end{aligned} \tag{10}$$

Let D be defined as the region satisfying $0 < x_1^2 + x_2^2 \leq r^2$, then there does exist an $r > 0$ in which Bendixon's Criterion applies. (In this case r can be ∞). This is sufficient to say that there are no periodic orbits exist (even though the linear system at the equilibrium point suggests this is the case).

2.2 Part b

2.2.1 Bendixon's Criteria

Problem: Show that the following system has no closed orbits:

$$\begin{aligned}\dot{x}_1 &= x_1 x_2^3 \\ \dot{x}_2 &= x_1\end{aligned} \tag{11}$$

Solution: Bendixon's Criterion states that if the divergence of $f(x)$ is not equivalently zero and does not change sign with a simply connected region then there are no periodic orbits in that region.

The divergence of the system can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= x_2^3 + 0 \\ &= x_2^3\end{aligned} \tag{12}$$

First, let $D1$ and $D2$ be defined as the entire upper and lower planes respectively:

$$\begin{aligned}D1 &:= \{x \in \Re^2 \mid x_2 < 0\} \\ D2 &:= \{x \in \Re^2 \mid x_2 > 0\}\end{aligned} \tag{13}$$

Within the regions $D1$ and $D2$, the divergence is strictly negative and positive respectively. Thus for each region the Bendixon Criteria applies and they independently contain no periodic orbits. Additionally, whenever $x_2 = 0$ an equilibrium point exists. This is sufficent to say that the entire domain contains no periodic orbits.

3 Problem 3

Problem: For each of the following systems demonstrate that no limit cycles exist:
Solution:

3.1 2.20.1

Let the following system be defined

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= g(x_1) + ax_2 \end{aligned} \tag{14}$$

where $g(x)$ is an arbitrary function and $a \neq 1$.

The divergence of the system can be calculated as:

$$\begin{aligned} \boldsymbol{\nabla} \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= -1 + a & (15) \\ &= a - 1 & (16) \end{aligned}$$

Given $a \neq 1$, it is true that the divergence of the system is always a constant not equal to zero. This satisfies Bendixon's Criteron for the entire domain. This is sufficient to prove no periodic orbits exist, and thus no limit cycles can exist.

3.2 2.20.2

Let the following system be defined

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_1^3 + x_1 x_2^2 \\ \dot{x}_2 &= -x_2 + x_2^3 + x_1^2 x_2\end{aligned} \tag{17}$$

The divergence of the system can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= 3x_1^2 + x_2^2 - 1 + 3x^2 + x_1^2 - 1 && (18) \\ &= 4x_1^2 + 4x_2^2 - 2 && (19)\end{aligned}$$

For the region

$$D = \{x \in \Re^2 \mid 4x_1^2 + 4x_2^2 < 2\}$$

, the Bendixon Criteron applies as the divergence is always positive. This is also true for the compliment of D.

This is not true for the border region,

$$B = \{x \in \Re^2 \mid 4x_1^2 + 4x_2^2 = 2\} \tag{20}$$

however the system is unstable for the entire boarder. This can be seen by analyzing the Jacobian with $x_0 \in B$:

$$\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{x=x_0} \tag{21}$$

$$= \left.\begin{bmatrix} 3x_1^2 + x_2^2 - 1 & 2x_1 x_2 \\ 2x_1 x_2 & x_1^2 + 3x_2^2 - 1 \end{bmatrix}\right|_{x=x_0} \tag{22}$$

The eigenvalues of the Jacobian matrix can be calculated to be:

$$\begin{aligned}\lambda_1 &= x_1^2 + x_2^2 - 1 \\ \lambda_2 &= 3x_1^2 + 3x_2^2 - 1\end{aligned} \tag{23}$$

This can then be evaluated for the boarder region B to be

$$\lambda_{1,2} = \pm\frac{1}{2} \tag{24}$$

Due to the unstable pole at $\frac{1}{2}$ this disqualifies a limit cycle from occurring at the boarder.

Given both the conclusion of the Bendixon Criterion and the unstable boundary, it can be said that no limit cycles exist.

3.3 2.20.3

Let the following system be defined

$$\begin{aligned} \dot{x}_1 &= 1 - x_1 x_2^2 \\ \dot{x}_2 &= x_1 \end{aligned} \tag{25}$$

The divergence of the system can be calculated as:

$$\begin{aligned} \nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= -x_2 + 1 && (26) \\ &= 1 - x_2 && (27) \end{aligned}$$

For the regions defined as

$$\begin{aligned} D_1 &:= \{x \in \Re^2 \mid x_2 \leq 1\} \\ D_2 &:= \{x \in \Re^2 \mid x_2 \geq 1\} \end{aligned} \tag{28}$$

divergence remains positive and negative for D_1 and D_2 respectively.

From this, this can be said to not contain a periodic obit using the Bendixon's Criterion.

3.4 2.20.4

Let the following system be defined

$$\begin{aligned}\dot{x}_1 &= x_1 x_2 \\ \dot{x}_2 &= x_2\end{aligned} \tag{29}$$

The divergence of the system can be calculated as:

$$\begin{aligned}\boldsymbol{\nabla} \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= 1 + x_2\end{aligned} \tag{30}$$

For the region

$$D = \{x \in \Re^2 \mid x_2 > -1\}$$

, the Bendixon Criteron applies as the divergence is always positive. This is also true for the compliment of D as the divergence is always negative.

This is not true for the border region,

$$B = \{x \in \Re^2 \mid x_2 = -1\} \tag{31}$$

however the system is unstable for the entire boarder. This can be seen by analyzing the Jacobian with $x_0 \in B$:

$$\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{x=x_0} \tag{32}$$

$$= \left.\begin{bmatrix} x_2 & x_1 \\ 0 & 1 \end{bmatrix}\right|_{x=x_0} \tag{33}$$

The eigenvalues of the Jacobian matrix can be calculated to be:

$$\lambda_1 = 1\lambda_2 \quad = x_2 \tag{34}$$

This can then be evaluated for the boarder region B to be

$$\lambda_{1,2} = \pm 1 \tag{35}$$

Due to the unstable pole at $\lambda = 1$ this disqualifies a limit cycle from occurring at the boarder (plus its a line...).

Given both the conclusion of the Bendixon Criterion and the unstable boundary, it can be said that no limit cycles exist.

3.5 2.20.5

Let the following system be defined

$$\begin{aligned}\dot{x}_1 &= x_2\cos(x_1)\\ \dot{x}_2 &= \sin(x_1)\end{aligned} \tag{36}$$

The divergence of the system can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2}\\ &= -x_2\sin(x_1)\end{aligned} \tag{37}$$

Let the following regions be defined:

$$\begin{aligned}D_1 &:= \{x \in \Re^2 \mid n\pi < x_1 < (n\pi + \frac{\pi}{2})\ \forall n = 0, 1, \ldots \text{ and } x_2 < 0\}\\ D_2 &:= \{x \in \Re^2 \mid n\pi < x_1 < (n\pi + \frac{\pi}{2})\ \forall n = 0, 1, \ldots \text{ and } x_2 > 0\}\\ D_3 &:= \{x \in \Re^2 \mid (n\pi + \frac{\pi}{2}) < x_1 < n\pi\ \forall n = 0, 1, \ldots \text{ and } x_2 < 0\}\\ D_4 &:= \{x \in \Re^2 \mid (n\pi + \frac{\pi}{2}) < x_1 < n\pi\ \forall n = 0, 1, \ldots \text{ and } x_2 > 0\}\end{aligned} \tag{38}$$

Each of the regions individualy satisfy Bendixon's criterion as D_1 and D_4 are always positive while D_2 and D_3 are always negative.

For the points not included in the the 4 regions,

$$\begin{aligned}B_1 &:= \{x \in \Re^2 \mid x_1 = n\pi\ \forall n = 0, 1, \ldots\}\\ B_2 &:= \{x \in \Re^2 \mid x_1 = \frac{\pi}{2} + n\pi\ \forall n = 0, 1, \ldots\}\end{aligned} \tag{39}$$

it can be shown that each is an equilibrium point.

This can be seen by analyzing the Jacobian with $x_0 \in B_1$ and B_2:

$$\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=x_0} = \left.\begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{bmatrix}\right|_{x=x_0} \tag{40}$$

$$= \left.\begin{bmatrix}-x_2 sin(x_1) & cos(x_1)\\ cos(x_1) & 0\end{bmatrix}\right|_{x=x_0} \tag{41}$$

The eigenvalues of the Jacobian matrix for B_1 can be calculated and evaluated as

$$\lambda_{1,2} = \pm 1 \tag{42}$$

Similarily,the eigenvalues of the Jacobian matrix for B_2 can be calculated and evaluated as

$$\lambda_{1,2} = 0 \tag{43}$$

Given both the conclusion of the Bendixon Criterion and the unstable boundary, it can be said that no limit cycles exist.

4 Problem 4

A nonlinear system is defined as:

$$\begin{aligned}\dot{x_1} &= x_2 \\ \dot{x_2} &= -[2b - g(x_1)]ax_2 - a^2x_1\end{aligned} \tag{44}$$

where $a, b > 0$ and

$$g(x_1) = \begin{cases} 0 & |x_1| > 1 \\ k & |x_1| \leq 1 \end{cases} \tag{45}$$

4.1 Bendixson's Criterion

Problem: Use Bendixson's Criterion to prove no periodic orbits exists if $k < 2b$.
Solution:
Fist let the following domains be defined:

$$\begin{aligned}D_1 &:= \{x \in \Re^2 \mid |x_1| \leq 1\} \\ D_2 &:= \{x \in \Re^2 \mid |x_1| > 1\}\end{aligned} \tag{46}$$

The divergence of the system in D_1 can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= 0 - (2b - k)a && (47) \\ &= (k - 2b)a && (48)\end{aligned}$$

In the case that $k < 2b$, it can be seen that the divergence will always be negative.

Similarly the divergence of the system in D_2 can be calculated as:

$$\begin{aligned}\nabla \cdot f &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \\ &= 0 - (2b - 0)a && (49) \\ &= -2ab && (50)\end{aligned}$$

In this case the divergence will also always be negative.

From this it can be concluded using Bendixson's Criterion that no periodic orbits exist.

4.2 Poincare-Bendixon Criterion

Problem: Use Poincare-Bendixon Criteron to show that these is a periodic orbit if $k > 2b$.
Solution: Using the same domain and divergence calculations from before, the divergence within D_1 is now seen to always be positive while in D_2 it remains negative. This does not explictly mean that a periodic orbit exists, but it does disqualify it from meeting the Bendixon Criterion.

Let a boundary be defined as the circle of arbitrary radius.The boundary at which the system points purely tangentially can be found by solving for where r causes the following to be equally zero, but we only need to show that it this is a value less then zero.

$$\begin{aligned} f(x)^T \cdot \boldsymbol{\nabla}(x) &= (f_1)(2x_1) + (f_2)(2x_1) && (51) \\ &= (x_2)(2x_1) + ((k-2b)ax_2 - a^2x_1)(2x_1) && (52) \\ &= 2(1 + ka - 2ab)x_1x_2 - 2a^2x_1^2 && (53) \end{aligned}$$

From the fact $-2x_1x_2 \leq x_1^2 + x_2^2$

$$\begin{aligned} &\leq -(1 + ka - 2ab)(x_1^2 + x_2^2) - 2a^2x_1^2 && (54) \\ &\leq -(1 + ka - 2ab)r^2 - 2a^2 \operatorname{atan}(\theta)r^2 && (55) \end{aligned}$$

Since $|\operatorname{atan}(\theta)| < 1$,

$$\leq -(1 + ka - 2ab - 2a^2)r^2 \quad (56)$$

From this we know that $\exists r > 0$ such that M is positively invariant. Since no equilibrium points exists within M, this means that the system must contain a periodic orbit.

A MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6313

Script 1: MECH6313_HW1

```
%% MECH6313 - HW 2
clear
close all

pblm1 = false;
pblm2 = false;
pblm3 = true;

if pblm1
%% Problem 1
% using ode 45 instead....
parta = true;
partb = true;
partc = true;

if parta
%% Problem 1a
% System Def
sys_func = @pblm1a;
Parms = 0.1 * [-1, 1e-10, 1];

% Simulation Setup
T = [0 100];
X_0 = 0.5 * [1, 1, -1, -1; 1, -1, 1, -1];
% X_0 = [ -0.5,0.8, -1.5, 3;
% 0.5, -0.5, 2.7, -1.9];

% Sim Phase Plots
fig = figure('position',[0,0,1500,500]);
N1 = size(Parms,2);
N2 = size(X_0,2);
simNum = 1;
for i = 1:N1
    ax(i) = subplot(1,N1,i);
    parms = Parms(i);
    for j = 1:N2
        [t,y] = ode45(@(t,y) sys_func(t,y,parms),T,X_0(:,j));
        plot(y(:,1),y(:,2));
```

```
        xlabel('x1')
        ylabel('x2')
        title(['\alpha = ', num2str(round(parms,3))])
        hold on
        simNum = simNum + 1;
    end
end
linkaxes(ax,'xy')

sgtitle('Problem 1a - Phase Portrait For Varying Parameters')
saveas(fig,fullfile([pwd '\\' 'HW2' '\\' 'fig'],'pblm1a.png'))

end

if partb
%% Problem 1b
% System Def
sys_func = @pblm1b;
Parms = 0.1 * [-1, 1e-10, 1];

% Simulation Setup
T = [0 100];
X_0 = 0.05 * [1, 1, -1, -1; 1, -1, 1, -1];

% Sim Phase Plots
fig = figure('position',[0,0,1500,500]);
N1 = size(Parms,2);
N2 = size(X_0,2);
simNum = 1;
for i = 1:N1
    ax(i) = subplot(1,N1,i);
    parms = Parms(i);
    for j = 1:N2
        [t,y] = ode45(@(t,y) sys_func(t,y,parms),T,X_0(:,j));
        plot(y(:,1),y(:,2));
        xlabel('x1')
        ylabel('x2')
        title(['\alpha = ', num2str(round(parms,3))])
        hold on
        simNum = simNum + 1;
    end
end
linkaxes([ax(1),ax(2)],'xy')
```

```
sgtitle('Problem 1b - Phase Portrait For Varying Parameters')
saveas(fig,fullfile([pwd '\\' 'HW2' '\\' 'fig'],'pblm1b.png'))

end

if partc
%% Problem 1c
% System Def
sys_func = @pblm1c;
Parms = 0.5 * [-1, 1];

% Simulation Setup
T = [0 10];
X_0 = 0.5 * [1, 1, -1, -1; 1, -1, 1, -1];

% Sim Phase Plots
fig = figure('position',[0,0,1000,500]);
N1 = size(Parms,2);
N2 = size(X_0,2);
simNum = 1;
for i = 1:N1
    ax(i) = subplot(1,N1,i);
    parms = Parms(i);
    for j = 1:N2
        [t,y] = ode45(@(t,y) sys_func(t,y,parms),T,X_0(:,j));
        plot(y(:,1),y(:,2));
        xlabel('x1')
        ylabel('x2')
        title(['\alpha = ', num2str(round(parms,3))])
        hold on
        simNum = simNum + 1;
    end
    if ax(i).XLim(1) < -5
        ax(i).XLim(1) = -5;
    end
    if ax(i).XLim(2) > 5
        ax(i).XLim(2) = 5;
    end
    if ax(i).YLim(1) < -5
        ax(i).YLim(1) = -5;
    end
```

```
    if ax(i).YLim(2) > 30
        ax(i).YLim(2) = 30;
    end
end


sgtitle('Problem 1c - Phase Portrait For Varying Parameters')
saveas(fig,fullfile([pwd '\\' 'HW2' '\\' 'fig'],'pblm1c.png'))
end
end

if pblm2
%% Problem 2
parta = true;

if parta
%% Problem 2a
disp('____________________ Problem 2: ____________________')
% sys def
sys2a = nlsys(@pblm2a)

syms x1 x2
linsys2a_sym = sys2a.linearize([x1;x2])
linsys2a = sys2a.linearize([0;0])

end
end

if pblm3
%% Problem 3
% Problem 2.20.2
syms x1 x2
A2 = [3 * x1^2 + x2^2 - 1, 2 * x1 * x2;
    2 * x1 * x2, 3 * x2^2 + x1^2 - 1]
eigA2 = eig(A2)
% x2 = sqrt((2 - 4 * x1^2)/4);
eigA2_B = subs(eigA2, x2, sqrt((2 - 4 * x1^2)/4))

% Problem 2.20.3
syms x1 x2
A3 = [-x2^2, -2 * x1 * x2; 1, 0]
eigA3 = eig(A3)
eigA3_B = subs(eigA3, x2, 1)
```

```matlab

% Problem 2.20.4
syms x1 x2
A4 = [x2, x1; 0, 1]
eigA4 = eig(A4)
eigA4_B = subs(eigA4, x2, -1)

% Problem 2.20.4
syms x1 x2
A5 = [-x2 * sin(x1), cos(x1); cos(x1), 0]
eigA5 = eig(A5)
eigA5_B0 = subs(eigA5, [x1,x2],[0,0])
eigA5_B1 = subs(eigA5, [x1,x2],[pi/2,0])

end
%% Local Functions
function dx = pblm1a(t, x, parms)
    % pblm1a function
    arguments
        t (1,1) = 0;
        x (2,1) = [0; 0];
        parms = false;
    end

    if parms == false
        alpha = 1;
    else
        alpha = parms(1);
    end

    % State Upadate Eqs
    dx(1,1) = alpha * x(1) + x(2);
    dx(2,1) = - x(1) + alpha*x(2) - x(1)^2 * x(2);
end

function y = pblm1b(t,x,parms)
    % pblm1b function
    arguments
        t (1,1) = 0;
        x (2,1) = [0; 0];
        parms = false;
    end

```

```
    if parms == false
        alpha = 1;
    else
        alpha = parms(1);
    end

    % State Upadate Eqs
    y(1,1) = alpha * x(1) + x(2) - x(1)^3;
    y(2,1) = - x(1) + alpha*x(2) + 2 *x(2)^3;
end

function y = pblm1c(t,x,parms)
    % pblm1c function
    arguments
        t (1,1) = 0;
        x (2,1) = [0; 0];
        parms = false;
    end

    if parms == false
        alpha = 1;
    else
        alpha = parms(1);
    end
    % State Upadate Eqs
    y(1,1) = alpha * x(1) + x(2) - x(1)^2;
    y(2,1) = - x(1) + alpha*x(2) + 2 * x(1)^2;
end

function y = pblm2a(x,u)
    % pblm2 function
    arguments
        x (2,1) = [0; 0];
        u (1,1) = 0;
    end

    % Array Sizes
    n = 2;
    p = 1;

    % State Upadate Eqs
    y(1,1) = x(2) + x(1) * x(2)^2;
    y(2,1) = - x(1) + x(1)^2 * x(2);
```

```

    if nargin == 0
        y = [n;p];
    end
end
```