Nonlinear Systems

MECH 6313
BMEN 6388
EECS 6336
SYSE 6324

- Class Format
- Asking Questions
- Elearning & MSTeams
- Quiz / Exam / HW
- Office Hours
- Book

Lecture 1

01/18/2021

Why do we need to learn about nonlinear systems?

Most real systems are nonlinear!

↳ their behavior is described by a differential eq'n or discrete eq'n that is nonlinear.

Ex. ✓ the Lagrange eq'n governing the motion of a robotic arm is nonlinear

✓ most biological systems are nonlinear

Analysis & Control of Nonlinear Systems

Analysis vs Simulation?

- ☐ computers are becoming more powerful
- ☐ simulations + good intuition $\Longrightarrow$ useful insight into systems behavior

However:

- ☐ impossible to rely on just simulations when proving stability or reachability

Why?

- Analysis tools provide formal mathematical proofs about a systems behavior.

$\boxed{\text{This class}}$

Nonlinear vs Linear ?

Linear system: $\dot{x} = A(t)x + B(t)u$

$x(t_0) = x_0$; $x \in \mathbb{R}^n$

$A(t) \in \mathbb{R}^{n \times n}$

- ☐ Linear approximations work well for important classes of systems → Linear systems MECH 6300
- ☐ Most real systems are Nonlinear. What does this mean?

Notation

Analysis: $\dot{x} = f(x)$

$\dot{x} = f(x,t)$

Control design: $\dot{x} = f(x,u)$

$\dot{x} = f(x,u,t)$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \in \mathbb{R}^n \text{ state; } \qquad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \in \mathbb{R}^m$$

control

disturbance

Ex.

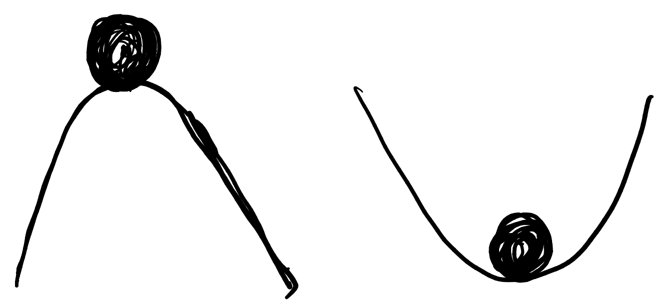
$\dot{x} = \sin(x) \qquad x(0) = x_0 \in \mathbb{R}$

$x(t) \in \mathbb{R}$ 1st-order system (scalar state)

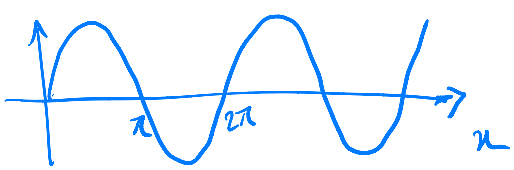
Equilibrium points $\bar{x}$: points that system would stay in if left on its own.

Solutions to $f(\bar{x}) = 0$

$\dot{x}\big|_{x=\bar{x}} = 0 = f(x)\big|_{x=\bar{x}} = 0$



$f(\bar{x}) = \sin(\bar{x}) = 0 \Rightarrow \bar{x} = k\pi; \quad k = 0, \pm 1, \pm 2, \ldots$



$\bar{x} = K\pi \quad ; \quad K = 0, \pm 1, \pm 2, \ldots$

Recall from linear systems

aside

e.p. are sol'ns to $A\bar{x} = 0$

$A \in \mathbb{R}^{n \times n}$

$\left\{ \begin{array}{l} \bar{x} = 0 \\ \bar{x} \in \text{Null}(A) \end{array} \right.$

if A is not invertible

so for A noninvertible

we have infinitly many e.p.'s $(\det(A) = 0)$

We can use linearization around equilibrium points to study their <u>local</u> stability properties.

$$\dot{x} = f(x) \xrightarrow{x(t) = \bar{x} + \tilde{x}(t)} \underbrace{\dot{\bar{x}}}_{0} + \dot{\tilde{x}} = f(\bar{x}+\tilde{x}) \quad \text{Taylor series}$$

state = e.p. + <u>fluctuation</u> around e.p.

$$\dot{\tilde{x}} \neq 0$$

$$\underbrace{f(\bar{x})}_{0} + \underbrace{\left.\frac{\partial f}{\partial x}\right|_{\bar{x}}}_{\text{Jacobian}} \tilde{x} + \text{H.O.T.}$$

$$\dot{\tilde{x}} = \underbrace{\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}}}_{A} \tilde{x} \implies \boxed{\dot{\tilde{x}} = A\tilde{x}}$$

a) $\mathrm{Re}(\lambda_i(A)) < 0 \; ; \forall i \rightarrow$ $\bar{x}$ is locally asymptotically stable

b) If there is an index i such that $\mathrm{Re}(\lambda_i(A)) > 0$

$\implies \bar{x}$ is unstable

linearization of $\dot{x} = \sin x$ around $\bar{x} = K\pi$

$$\left.\frac{\partial f}{\partial x}\right|_{\bar{x}} = \left.\cos x\right|_{x=\bar{x}} = \begin{cases} \cos(2n\pi) = 1 & ; \ K \text{ even} \\ \cos((2n+1)\pi) = -1 & K \text{ odd} \end{cases}$$

phase plots

$$A x = \dot{x}$$

$>$ $<$ $<$

$<$