Due end of the day, Wednesday 04/12/2021 (11:59pm)

1. In class, we used the PR Lemma to show that a positive real linear systems,

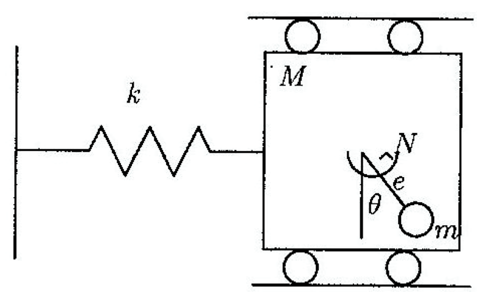
$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

has relative degree of one ($CB \neq 0$). Show that positive realness also implies a minimum phase phase property. (Hint: Write the system equations in normal form and apply PR Lemma.)

2. The dynamics of the translational oscillator with rotating actuator (TORA) are descibed by:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-x_1 + \epsilon x_4^2 \sin x_3}{1 - \epsilon^2 \cos^2 x_3} + \frac{-\epsilon \cos x_3}{1 - \epsilon^2 \cos^2 x_3} u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{1 - \epsilon^2 \cos^2 x_3}\left(\epsilon \cos x_3 (x_1 - \epsilon x_4^2 \sin x_3) + u\right)\end{aligned}$$

where x_1 and x_2 are the displacement and the velocity of the platform, x_3 and x_4 are the angle and angular velocity of the rotor carrying the mass m, and u is the control torque applied to the rotor. the parameter $\epsilon > 1$ depends on the eccentricity e and the masses m and M. With $y = x_3$ as the output, determine the relative degree and the zero dynamics. Provide a physical interpretation of the zero dynamics.



3. Khalil, Problem 13.27. In part (b), do the simulations, but skip the performance comparison equation.

13.24 Consider the system (13.44)–(13.45), where $A - BK$ is Hurwitz, the origin of $\dot{\eta} = f_0(\eta, 0)$ is asymptotically stable with a Lyapunov function $V_0(\eta)$ such that $[\partial V_0/\partial \eta] f_0(\eta, 0) \leq -W(\eta)$ for some positive definite function $W(\eta)$. Suppose $\|\delta\| \leq k[\|\xi\| + W(\eta)]$. Using a composite Lyapunov function of the form $V = V_0(\eta) + \lambda\sqrt{\xi^T P \xi}$, where P is the solution of $P(A - BK) + (A - BK)^T P = -I$, show that, for sufficiently small k, the origin $z = 0$ is asymptotically stable

13.25 Consider the system

$$\dot{x}_1 = x_2 + 2x_1^2, \quad \dot{x}_2 = x_3 + u, \quad \dot{x}_3 = x_1 - x_3, \quad y = x_1$$

Design a state feedback control law such that the output y asymptotically tracks the reference signal $r(t) = \sin t$.

13.26 Repeat the previous exercise for the system

$$\dot{x}_1 = x_2 + x_1 \sin x_1, \quad \dot{x}_2 = x_1 x_2 + u, \quad y = x_1$$

13.27 The magnetic suspension system of Exercise 1.18 is modeled by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= g - \frac{k}{m}x_2 - \frac{L_0 a x_3^2}{2m(a + x_1)^2} \\
\dot{x}_3 &= \frac{1}{L(x_1)}\left[-Rx_3 + \frac{L_0 a x_2 x_3}{(a + x_1)^2} + u\right]
\end{aligned}$$

where $x_1 = y$, $x_2 = \dot{y}$, $x_3 = i$, and $u = v$. Use the following numerical data: $m = 0.1$ kg, $k = 0.001$ N/m/sec, $g = 9.81$ m/sec^2, $a = 0.05$ m, $L_0 = 0.01$ H, $L_1 = 0.02$ H, and $R = 1\ \Omega$.

(a) Show that the system is feedback linearizable.

(b) Using feedback linearization, design a state feedback control law to stabilize the ball at $y = 0.05$ m. Repeat parts (d) and (e) of Exercise 12.8 and compare the performance of this controller with the one designed in part (c) of that exercise.

(c) Show that, with the ball position y as the output, the system is input–output linearizable.

(d) Using feedback linearization, design a state feedback control law so that the output y asymptotically tracks $r(t) = 0.05 + 0.01 \sin t$. Simulate the closed-loop system.