MECH 6313 - Term Exam

Name: Jonas Wagner
UTD ID: 2021531784

2021, April 30

Contents

1 Problem 1

Consider the system:

$$
\begin{aligned}
\tau \dot{x} &= x - \frac{1}{3}x^3 - y \\
\dot{y} &= x + \mu
\end{aligned}
\tag{1}
$$

where $\tau > 0$ and $\mu \geq 0$ are constants.

1.1 Part a

Problem: Determine the equilibrium points and classify their stability properties depending on the values of parameter μ.

Solution:

1.1.1 Equilibrium Point Identification

The equilibrium points exist whenever $\dot{x} = \dot{y} = 0$ and can be identified as follows:

$$
\begin{aligned}
\tau(0) &= x - \frac{1}{3}x^3 - y \\
(0) &= x + \mu
\end{aligned}
\tag{2}
$$

which becomes:

$$
\begin{aligned}
y &= x - \frac{1}{3}x^3 \\
x &= -\mu
\end{aligned}
\tag{3}
$$

and can then substituted in as:

$$
\begin{aligned}
x_{eq} &= -\mu \\
y_{eq} &= -\mu - \frac{1}{3}(-\mu)^3
\end{aligned}
\tag{4}
$$

This results in the equilibrium points being defined in terms of μ as:

$$
\boxed{
\begin{aligned}
x_{eq} &= -\mu \\
y_{eq} &= \frac{1}{3}\mu^3 - \mu
\end{aligned}
}
\tag{5}
$$

1.1.2 Stability of Equilibrium Points

The stability around an equalibrium point can be evaluated by looking at the linearized model, which can be found as follows:

Let the state-variables be defined as:

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

The nonlinear state equation would then be defined as:

$$\dot{X} = f(x) = \begin{bmatrix} \dfrac{x_1 - \frac{1}{3}x_1^3 - x_2}{\tau} \\ x_1 + \mu \end{bmatrix} \tag{6}$$

Then the equilibrium point is described as

$$X_{eq} = \begin{bmatrix} -\mu \\ \frac{1}{3}\mu^3 - \mu \end{bmatrix}$$

and the jacobian can be computed as:

$$A = \frac{\mathrm{d}f}{\mathrm{d}X} = \begin{bmatrix} \frac{\mathrm{d}f_1}{\mathrm{d}x_1} & \frac{\mathrm{d}f_1}{\mathrm{d}x_2} \\ \frac{\mathrm{d}f_2}{\mathrm{d}x_1} & \frac{\mathrm{d}f_2}{\mathrm{d}x_2} \end{bmatrix} \tag{7}$$

$$jj \tag{8}$$

1.2 Part b

Problem: At which value of μ does a bifurcation occur and what type of bifurcation is it?

Solution:

A MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6313