Due Wednesday 04/14/2021 (midnight)

1. Consider the mass-spring-damper system described by

$$m\,\ddot{y} \;+\; \beta\,\dot{y} \;+\; k\,y \;=\; u$$

   (a) If $y(t)$ and $u(t)$ are available for measurement, design a gradient algorithm to estimate constant but unknown parameters m, β, and k.

   (b) Simulate your algorithm in (a) assuming that true values are $m = 20$, $\beta = 0.1$, and $k = 5$. Repeat your simulation for different choices of $u(t)$ and observe the resulting parameter convergence properties.

2. Consider the reference model

$$\dot{y}_m \;=\; -a\,y_m \;+\; r(t), \quad a > 0$$

and the plant

$$\dot{y} \;=\; a^\star y \;+\; b^\star u, \quad b^\star \;\neq\; 0.$$

   (a) Show that a controller of the form $u = \theta_1\,y \,+\, \theta_2\,r(t)$ with an appropriate choice of gains $\theta_1^\star$ and $\theta_2^\star$, drives the tracking error $e := y - y_m$ asymptotically to zero.

   (b) Now suppose $a^\star$ and $b^\star$ are unknown parameters, but the sign of $b^\star$ is known. Show that the adaptive implementation of the controller above achieves tracking when the gains are updated according to the rule

$$\dot{\theta}_1 \;=\; -\mathrm{sign}(b^\star)\,\gamma_1\,y\,e, \qquad \dot{\theta}_2 \;=\; -\mathrm{sign}(b^\star)\,\gamma_2\,r\,e$$

   where $\gamma_1 > 0$ and $\gamma_2 > 0$.

   (c) Provide a condition that also guarantees $\theta_1(t) \to \theta_1^\star$ and $\theta_2(t) \to \theta_2^\star$ as $t \to \infty$.

3. A simplified model of an axial compressor, used in jet engine control studies, is given by the following second order system

$$\begin{aligned}\dot{\phi} \;&=\; -\frac{3}{2}\phi^2 \;-\; \frac{1}{2}\phi^3 \;-\; \psi\\ \dot{\psi} \;&=\; \frac{1}{\beta^2}\,(\phi \;+\; 1 \;-\; u)\,.\end{aligned}$$

This model captures the main surge instability between the mass flow and the pressure rise. Here, ϕ and ψ are deviations of the mass flow and the pressure rise from their set points, the control input u is the flow through the throttle, and β is a positive constant.

   (a) Use backstopping to obtain a control law that stabilizes the origin $(\phi, \psi) = 0$.

   (b) Use Sontag's Formula and the Control Lyapunov Function obtained in part (a) to obtain an alternative control law.