MECH 6318_Final-exam-Fall 2021

Tuesday, December 7, 2021 12:03 PM

Fall 2021 | MECH 6318
Engineering Optimization
Final Exam
12:00 pm-6:00pm Tuesday, December 07, 2021
(100 points)

NAME: Jonas Wagner **NetID:** JRW 200000

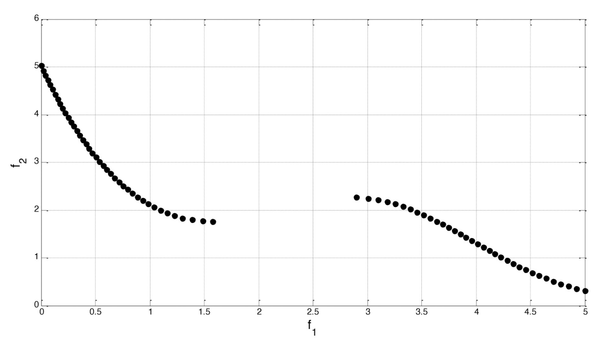
Any (i) communication of any kind with classmate, or (ii) plagiarization shall result in failing of the course!

-5 to 0 points reserved for Neatness and Professional Presentation. Show your work in detail and clearly. Scan your answers and save it as a PDF.
Matlab can be used for verification. Using matlab without showing the processes gets no points. (Only presenting the final results or outputs also gets no points.)
Please rename the file with the format: NetID_Lastname_firstname.pdf, e.g., yx1100000_liu_yuanzhi.pdf

Problem 1: True or False (20 *pts* x *1 = 20 pts*) (-1.5 points for choosing a wrong statement or missing a correct statement)

Please choose all the correct statement(s) from the following:

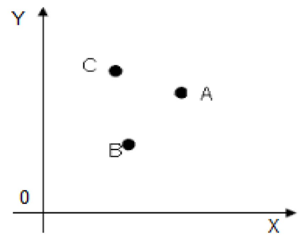
(A) Simplex search method is a first order method.
(B) When a linear programming problem has more than one optimal solution, then it will always have infinitely many optimal solutions.
(C) The sequential quadratic programming method linearizes both the objective function and constraints of an optimization problem.
(D) All the points on the curve in the following figure are Pareto solutions to a bi-objective minimization problem in f1-f2 space.



(E) For a triple-objective optimization problem, the Pareto frontier is a surface, if there exists.
(F) For pure integer programming problems, some design variables are allowed to assume only integer values, while others are allowed to assume continuous values.

(G) For f(x), suppose that ∇2f(x) is continuous in an open neighborhood of x∗, and that ∇f(x∗) = 0. If ∇2f(x∗) is positive definite, x∗ is a strict local minimum of f(x).

For subproblem: H & J X and Y are the two objectives of a bi-objective **maximization** problem as shown in the figure below. A, B and C are three points in the *X*-*Y* design objective space. Based on their positions, we can find:



(H) Solution-A dominates solution-B but not solution-C.
(I) Solution-C dominates both solution-A and solution-B.
(J) For design of experiments, the full factorial design method and Latin Hypercube method have the same accuracy in establishing a surrogate model.
(K) A solution x0 is a global optimal solution of problem P, if x0 is feasible and f(x0) ≤ f(x) for all x ∈ X.
(L) We can obtain a unique deterministic optimal solution using genetic algorithm.
(M) For the encoding of genetic algorithm, we know the range and the tolerance for the optimization variable are [0, 5] and 0.1, respectively. A "chromosome" with a length of 5 is enough for the given problem.
(N) The weighted sum method works for all types of multi-objective optimization.
(O) The KKT method works for both linear and nonlinear optimization.
(P) Tom is asked to calculate the mean absolute error of a radial basis function (RBF) surrogate model. The student first built up a model using a dataset [Xorg, Yorg], and the surrogate model is expressed as Yhat=f(x). The student then obtained the estimated set via Yest=f(Xorg). Finally, the MAE error is calculated by

$$MAE = \frac{1}{n}\sum_{1}^{n} \text{abs}\left(Yest_n - Yorg_n\right)$$

Do you agree with Tom? (True for yes, False for no)

Problem 2: Formulate and solve the following problem.

It is required to minimize the objective, f(x_1, x_2), which represents the sum of the two design variables, x_1 and x_2, but with consideration that two constraints, $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$, should be satisfied. The first constraint, $g_1(x_1, x_2)$, ensures that the two design variables are within, or on the boundary, of a circle, where the radius of the circle is 2 m. The second constraint ensures that the first design variable, x_1, is greater than or equal to -1.

a) Formulate the optimization problem mathematically. The mathematical problem statement must include objective function, design variable(s), and constraint function(s) in the standard form of an optimization problem. **(4 points)**

b) Solve the optimization problem using Karush-Kuhn Tucker conditions. **(8 points)**

c) Now solve this problem graphically. **(6 points)**

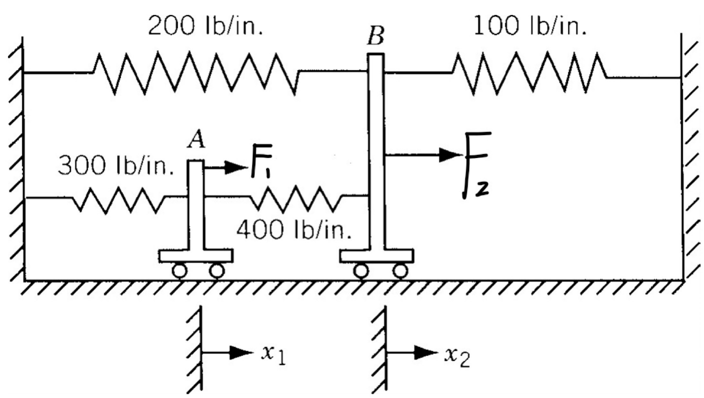
Problem 3: The figure below shows two bodies, A and B, connected by four linear springs. The springs are at their natural positions when there is no force applied to the bodies. The displacements x_1 and x_2 of the bodies under any applied force can be found by minimizing the potential energy of the system. Find the displacements of the bodies when forces of F1=2000 lb and F2=3000 lb are applied to bodies A and B, respectively.

a) Formulate / explain (but do not solve) the optimization problem. The problem must include objective function, design variable(s), and constraint function(s) (if any) in the standard form of an optimization problem. **(8 points)**

Hint: Potential energy of the system = strain energy of springs − potential of applied loads

where the strain energy of a spring of stiffness k and end displacements x_1 and x_2 is given by $\frac{1}{2}k(x_2 - x_1)^2$ and the potential of the applied force, F_i, is given by x_iF_i.

b) Solve the problem using Newton's method. Use $\varepsilon = 0.6$ for checking the convergence, $|\nabla f(x_k)| < \varepsilon$. Use the starting vector, $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. **(10 points)**



Problem 4: Solve the following integer problem.

$$\max_{X}\ 7x_1 + 9x_2$$

subject to

$$-2x_1 + 6x_2 \leq 12$$
$$7x_1 + x_2 \leq 35$$
$${x_1}^2 \leq 40$$
$$x_2 \leq 5$$
$$x_1, x_2 \geq 0$$
$$x_1, x_2 \in Z$$

(1) True or False: 1. This problem can be solved using simplex method. 2. This problem can be solved using graphic method**. (3 points)**

(2) Formulate a relaxed continuous linear programming problem and solve it using Karush-Kuhn-Tucker method. **(8 points)**

(3) Use branch and bound method to solve the integer optimization problem. **(6 points)**

Problem 5: You have installed a wind turbine at the University of Texas at Dallas. In the next 36 hours, it can be operated for only 4 hours 30 minutes (4.5 hours). The turbine can be switched on and off twice over the connected 36 hours (i.e., it can be operated only twice). It cannot be operated when *the chance of precipitation* (*C*) is 50% or more. The power generated (*P*) by the turbine in watts depends on the wind speed (*U*), as given by

$$P = 3.0 \times U^3$$

The energy generated (*E*) over a period of *T* minutes can be estimated as:

$$E = \sum_{i=1}^{T} P_i \times 60$$

where P_i is the power generated by the turbine in the i^{th} minute.

(1) Model the variations of wind speed (U) and chance of precipitation (C) with time (T) over the concerned 36 hours, using a surrogate model or response surface. You may use a quadratic or cubic polynomial function for this purpose. Please only write the formulation **(don't solve)**. **(4 points)**

(2) Formulate and model (don't solve) an optimization problem to maximize the total energy produced ($\boldsymbol{E_T}$) by the turbine over the next 36 hours, starting from a point of data recording. The design variables are the two starting times, $\boldsymbol{T_{T1}}$ and $\boldsymbol{T_{T2}}$, and the two stopping times, $\boldsymbol{T_{P1}}$ and $\boldsymbol{T_{P2}}$. All four design variables are to be expressed in minutes. **(4 points)**

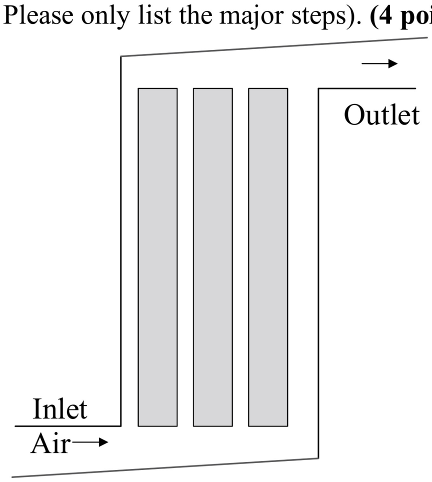
(3) What is the type of the optimization problem in Part (2), i.e., linear programming, non-linear programming, mix-integer programming, etc.? List **2 optimization algorithms (Hint: not Matlab solvers)** that can solve this type of optimization problem and explain the efficiency and advantages/disadvantages of each algorithm. **(6 points)**

Problem 6: you are assigned with a task to optimize a 2D cooling flow field, as shown in the below figure. The colored components can be regarded as isotropic homogeneity solid material with a uniform heat generation rate (Hint: the colored component is a heat source and the component itself has a same temperature). Your task is to minimize **the temperature differences** among the three components. Based on your understanding of this course, please provide your solutions for this task.

a) What are the parameters you think that could be considered for optimization? You can rename the parameters and label them on the figure. **(5 points)**

b) Please formulate the objective function (you can label or rename the temperature or other physical parameters if necessary). To establish the objective function, what approaches do you think are suitable for this problem? (Hint: how to form the mathematical objective function (analytically or numerically)? You only need to list one of them. If you need to use data-oriented method, for data collections, you can consider performing physical experiment or simulations.) **(4 points)**

c) Based on the idea you proposed in part (b), please briefly discuss the major steps you would like to perform for this structural optimization. (Hint: this is an open question, try to recall how you did for your course project. Please only list the major steps). **(4 points)**



The cooling air flow rate, the sizes of the colored components, and the component heat generation rate are fixed.

Problem 1

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Problem 1: True or False (20 *pts* x *1* = *20 pts*) (-1.5 points for choosing a wrong statement or missing a correct statement)

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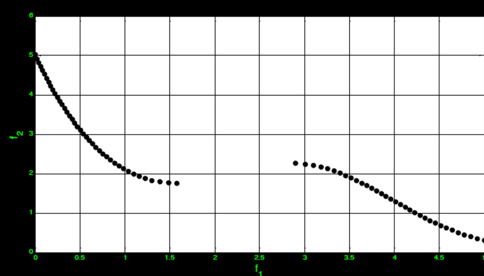

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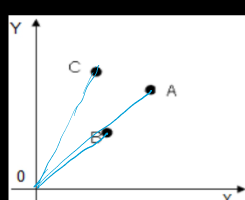

at is more important...

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Do you agree with Tom? (True for yes, False for no)

A, B*, C*, E*, G

* Assuming Real-valued unconstrained...

A, B$^{☆}$, C$^{☆}$, E$^{☆}$, G

$(H, I)^{+}$

K,

☆ Assuming Real-valued unconstrained...

☆ or at least it could be, it is labeled well...

☆ Technically they are always a surface... just not in 2-D

+ depending on the importance of each objective... Both true if caring about y the most...

Problem 2

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Problem 2: Formulate and solve the following problem.

It is required to minimize the objective, $f(x_1, x_2)$, which represents the sum of the two design variables, x_1 and x_2, but with consideration that two constraints, $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$, should be satisfied. The first constraint, $g_1(x_1, x_2)$, ensures that the two design variables are within, or on the boundary, of a circle, where the radius of the circle is 2 m. The second constraint ensures that the first design variable, x_1, is greater than or equal to -1.

a) Formulate the optimization problem mathematically. The mathematical problem statement must include objective function, design variable(s), and constraint function(s) in the standard form of an optimization problem. **(4 points)**

b) Solve the optimization problem using Karush-Kuhn Tucker conditions. **(8 points)**

c) Now solve this problem graphically. **(6 points)**

Let $x_1, x_2 \in \mathbb{R}$

$f, g_1, g_2 : \mathbb{R}^2 \to \mathbb{R}$

$f(x_1, x_2) = x_1 + x_2$

$g_1(x_1, x_2) = x_1^2 + x_2^2 - r_1^2 \quad , \quad r_1 = 2\,m$

$g_2(x_1, x_2) = -x_1 - 1$

$x_1^2 + x_2^2 \le r_1^2$

$x_1 \ge -1$

a)

$$\min_{x_1, x_2} \; f(x_1, x_2)$$

$$\text{s.t} \quad g_1(x_1, x_2) \le 0$$

$$g_2(x_1, x_2) \le 0$$

$$\min \; x_1 + x_2$$

$$\text{s.t.} \quad x_1^2 + x_2^2 \le 4^2$$

$$-x_1 \le 1$$

$$\min_{x_1, x_2} \; x_1 + x_2$$

$$\text{s.t.} \quad x_1^2 + x_2^2 - 4 \le 0$$

$$-x_1 - 1 \le 0$$

$$\text{s.t.} \quad x_1^2 + x_2^2 - 4 \leq 0$$
$$-x_1 - 1 \leq 0$$

b) <u>KKT</u>

$$g_1(x_1, x_2) = x_1^2 + x_2^2 - 4$$
$$g_2(x_1, x_2) = -x_1 - 1$$

$$L(x_1, x_2, \lambda_1, \lambda_2) = f(x_1, x_2) + \lambda_1 g_1(x_1, x_2) + \lambda_2 g_2(x_1, x_2)$$
$$= x_1 + x_2 + \lambda_1 (x_1^2 + x_2^2 - 4) + \lambda_2 (-x_1 - 1)$$
$$= \lambda_1 x_1^2 + (1 - \lambda_2) x_1 + \lambda_2 x_2^2 + x_2 - 4\lambda_1 - \lambda_2$$

$$\nabla L = 0$$
$$\lambda_1^* g_1(x_1^*, x_2^*) = 0$$
$$\lambda_2^* g_2(x_1^*, x_2^*) = 0$$
$$\lambda_1 \geq 0$$
$$\lambda_2 \geq 0$$

$$\nabla L = \boxed{\begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \end{bmatrix}} = \begin{bmatrix} 2\lambda_1 x_1 + 2 - \lambda_2 \\ 2\lambda_2 x_2 + 1 \end{bmatrix} = 0$$

$$\lambda_1 \underbrace{(x_1^2 + x_2^2 - 4)}_{\text{active}} \leq 0$$
$$\lambda_2 (-x_1 - 1) \leq 0$$
$$\lambda_1 \geq 0$$

active

$\lambda_1 \geq 0$ active (on boundary)

$\lambda_2 \geq 0$

$x_1^2 + x_2^2 - 4 = 0$

$-x_1 - 1 = 0$

$\Downarrow$

$x_1 = -1$

$x_1^2 + x_2^2 = 4 \Rightarrow x_2^2 = 4 - 1 = 3$

$x_2 = \pm\sqrt{3}$

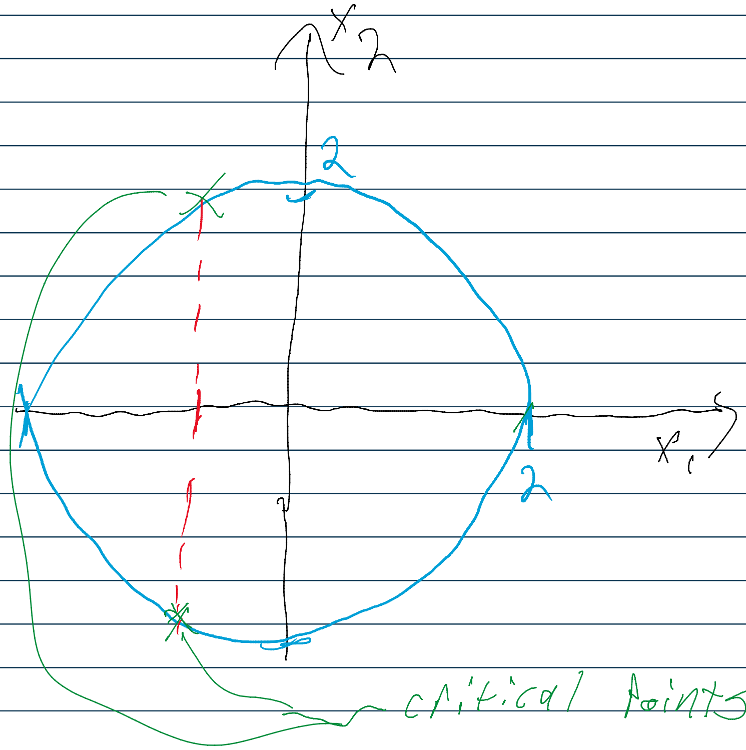
min @ $x_2 < 0$

$$\boxed{\begin{aligned} x_1^* &= -1 \\ x_2^* &= -\sqrt{3} \\ f(x_1^*, x_2^*) &= -(1+\sqrt{3}) \end{aligned}}$$

$\lambda_1 = 0$

$\lambda_2 = 2$

c)



critical points: $\begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}, \begin{bmatrix} -1 \\ -\sqrt{3} \end{bmatrix}$

A* a $\begin{bmatrix} x_1 = -1 \\ x_2 = -\sqrt{3} \end{bmatrix} = x_1 + x_2 = -1 - \sqrt{3}$

Problem 3

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Problem 3: The figure below shows two bodies, A and B, connected by four linear springs. The springs are at their natural positions when there is no force applied to the bodies. The displacements x_1 and x_2 of the bodies under any applied force can be found by minimizing the potential energy of the system. Find the displacements of the bodies when forces of F1=2000 lb and F2=3000 lb are applied to bodies A and B, respectively.

Lagrangian Mechanics... but no KE...

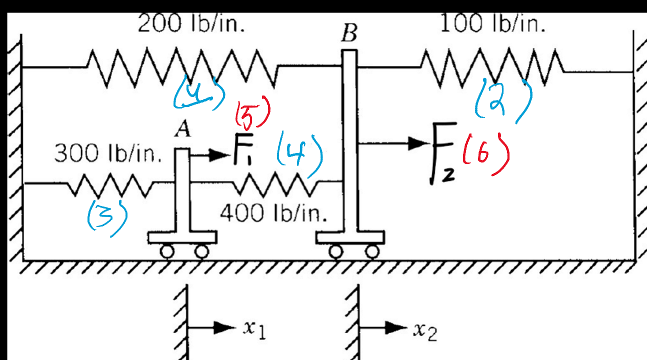
a) Formulate / explain (but do not solve) the optimization problem. The problem must include objective function, design variable(s), and constraint function(s) (if any) in the standard form of an optimization problem. **(8 points)**

Hint: Potential energy of the system = strain energy of springs − potential of applied loads

where the strain energy of a spring of stiffness k and end displacements x_1 and x_2 is given by $\frac{1}{2}k(x_2 - x_1)^2$ and the potential of the applied force, F_i, is given by x_iF_i.

work

b) Solve the problem using Newton's method. Use $\varepsilon = 0.6$ for checking the convergence, $|\nabla f(x_k)| < \varepsilon$. Use the starting vector, $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. **(10 points)**



PE:

(1) $\frac{1}{2}(200)(x_2 - 0)^2$

(2) $+\frac{1}{2}(100)(0 - X_2)^2$

(3) $+\frac{1}{2}(300)(X_1 - 0)^2$

(4) $+\frac{1}{2}(400)(X_2 - X_1)^2$

(5) $-2000(X_1)$

(6) $-3000(X_2)$

$\Downarrow$

$$PE = 100 x_2^2$$
$$+ 50 x_2^2$$
$$+ 150 x_1^2$$
$$+ 200(x_1^2 + 2x_1x_2 + x_2^2)$$
$$- 2000 x_1$$
$$- 3000 x_2$$

$$PE = 100\left[3.5x_1^2 - 20x_1 + 4x_1x_2 - 30x_2 + 3.5x_2^2\right]$$

---

a)
$$\min_{x_1, x_2} f(x_1, x_2) = 350x_1^2 - 2000x_1 + 400x_1x_2 - 3000x_2 + 350x_2^2$$

$$\min_{X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} 100\left(X^T \begin{bmatrix} 3.5 & 2 \\ 2 & 3.5 \end{bmatrix} X - \begin{bmatrix} 20 \\ 30 \end{bmatrix} X\right)$$

$$A = \begin{bmatrix} 350 & 200 \\ 200 & 350 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \times 10^3$$

$$\min_{x} x^T A x + bx$$

b) $\varepsilon = 0.6$ So small... assuming 0.6×10^3

converge @ $|\nabla f(x_k)| < \varepsilon$

$$X_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$f(x) = x^T A x + bx = 350x_1^2 - 2000x_1 + 400x_1x_2 - 3000x_2 + 350x_2^2$$
$$f'(x) = 2Ax + b = \begin{bmatrix} 700x_1 - 2000 + 400x_2 \\ 700x_2 - 3000 + 400x_1 \end{bmatrix}$$
$$f''(x) = 2A = \begin{bmatrix} 700 & 400 \\ 400 & 700 \end{bmatrix}$$

K	X_K	$f(X_K)$	$f'(X_K)$	$f''(X_K)$	$P_K = \frac{f'(X_K)}{f''(X_K)}$	$X_{K+1} = X_K - P_K$	$\lvert f'(X_{K+1})\rvert$	$< \varepsilon$?
0	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0	$-b$	$2A$	$\begin{matrix} -606 \\ -3934 \end{matrix}$	$\begin{bmatrix} 0.6061 \\ 0.6061 \end{bmatrix}$	3605	NO
1	0.6	-2 [illegible]	-1333	2A	0	0.6061 [illegible]	[illegible]	NO

0	0	0	-6	2A	-3934	0.6061	3605	NO
1	0.6 0.6	-2.6	-1333 -2333	2A	0 -3333	0.6061 3.9394	2626	NO
2	0.6 3.9	-6.5	0 0	2A	0 0	~	~	YES

$$\boxed{\begin{matrix}0.606\\3.939\end{matrix}}$$

Problem 4

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Problem 4: Solve the following integer problem.

$$\max_{x} \; 7x_1 + 9x_2$$

subject to

$$-2x_1 + 6x_2 \leq 12$$
$$7x_1 + x_2 \leq 35$$
$$x_1^2 \leq 40$$
$$x_2 \leq 5$$
$$x_1, x_2 \geq 0$$
$$x_1, x_2 \in Z$$

(1) True or False: 1. This problem can be solved using simplex method. 2. This problem can be solved using graphic method. **(3 points)**

(2) Formulate a relaxed continuous linear programming problem and solve it using Karush-Kuhn-Tucker method. **(8 points)**

(3) Use branch and bound method to solve the integer optimization problem. **(6 points)**

1) False , Maybe... depends on what visualization you use

2)

$$\min_{x} \; f(x_1, x_2) = -7x_1 + 9x_2$$

$$\text{s.t.} \; g_1(x_1, x_2) = -2x_1 + 6x_2 - 12 \leq 0$$

$$g_2(x_1, x_2) = 7x_1 + x_2 - 35 \leq 0$$

$$g_3(x_1, x_2) = x_1^2 - 40 \leq 0$$

$$g_4(x_1,x_2) = x_2 - 5 \le 0$$

$$g_5(x_1,x_2) = -x_1 \le 0$$

$$g_6(x_1,x_2) = -x_2 \le 0$$

relaxed....

$$L(x_1,x_2,\lambda) = f(x_1,x_2) + \sum_{i=1}^{n} \lambda_i g_i(x_1,x_2)$$

$$= -7x_1 + 9x_2$$

$$+ \lambda_1(-2x_1 + 6x_2 - 12)$$

$$+ \lambda_2(7x_1 + x_2 - 35)$$

$$+ \lambda_3(x_1^2 - 40)$$

$$+ \lambda_4(x_2 - 5)$$

$$+ \lambda_5(-x_1)$$

$$+ \lambda_6(-x_2)$$

$$\nabla_{x_1} L(x_1, x_2, \lambda) = 2\lambda_3 x_1 - 7 - 2\lambda_1 + 7\lambda_2 - \lambda_5$$

$$\nabla_{x_2} L(x_1, x_2, \lambda) = 9 + 6\lambda_1 + \lambda_2 + \lambda_4 - \lambda_6$$

$$\nabla_{x_2} L(x_1, x_2, \lambda) = 9 + 6\lambda_1 + \lambda_2 + \lambda_4 - \lambda_6$$

$$\lambda_i g_i(x_1, x_2) = 0 \quad \forall i = 1, \dots, 6$$

$$g_i(x_1, x_2) \le 0 \quad \forall i = 1, \dots, 6$$

$$\lambda_i \ge 0 \quad \forall i = 1, \dots, 6$$

math don in MATLAB...

(attached... I did code earlier and copied myself...)

```
results = solve([...
    D_L1(x1, x2) == 0;...
    D_L2(x1, x2) == 0;...
    lambda(1) * g1(x1, x2) == 0; ...
    lambda(2) * g2(x1, x2) == 0; ...
    lambda(3) * g3(x1, x2) == 0; ...
    lambda(4) * g4(x1, x2) == 0; ...
    lambda(5) * g5(x1, x2) == 0; ...
    lambda(6) * g6(x1, x2) == 0; ...
    g1(x1, x2) <= 0;...
    g2(x1, x2) <= 0;...
    g3(x1, x2) <= 0;...
    g4(x1, x2) <= 0;...
    g5(x1, x2) <= 0;...
    g6(x1, x2) <= 0
    ], ...
    [x1, x2, lambda]...
);
```

```
x1 = 5

x2 = 0

lambda_1 = 0

lambda_2 = 1

lambda_3 = 0

lambda_4 = 0

lambda_5 = 0

lambda_6 = 10

Lmin = -35

fmin = -35
```

Problem 5

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Problem 5: You have installed a wind turbine at the University of Texas at Dallas. In the next 36 hours, it can be operated for only 4 hours 30 minutes (4.5 hours). The turbine can be switched on and off twice over the connected 36 hours (i.e., it can be operated only twice). It cannot be operated when *the chance of precipitation* (C) is 50% or more. The power generated (P) by the turbine in watts depends on the wind speed (U), as given by

$$P = 3.0 \times U^3$$

The energy generated (E) over a period of T minutes can be estimated as:

$$E = \sum_{i=1}^{T} P_i \times 60$$

where P_i is the power generated by the turbine in the i^{th} minute.

(1) Model the variations of wind speed (U) and chance of precipitation (C) with time (T) over the concerned 36 hours, using a surrogate model or response surface. You may use a quadratic or cubic polynomial function for this purpose. Please only write the formulation **(don't solve)**. **(4 points)**

(2) Formulate and model (don't solve) an optimization problem to maximize the total energy produced ($\boldsymbol{E_T}$) by the turbine over the next 36 hours, starting from a point of data recording. The design variables are the two starting times, $\boldsymbol{T_{T1}}$ and $\boldsymbol{T_{T2}}$, and the two stopping times, $\boldsymbol{T_{P1}}$ and $\boldsymbol{T_{P2}}$. All four design variables are to be expressed in minutes. **(4 points)**

(3) What is the type of the optimization problem in Part (2), i.e., linear programming, non-linear programming, mix-integer programming, etc.? List **2 optimization algorithms (Hint: not Matlab solvers)** that can solve this type of optimization problem and explain the efficiency and advantages/disadvantages of each algorithm. **(6 points)**

1) Let

$$U_i = u_3 t_i^3 + u_2 t_i^2 + u_1 t_i + u_0 \quad (\text{wind speed})$$

$$C_i = c_3 t_i^3 + c_2 t_i^2 + c_1 t_i + c_0 \quad (\text{chance of rain})$$

$$\forall t_i \in \{1, \dots, 2163\} \ (\text{minutes})$$

Note:
It would also be possible to generate a table of values Initially to eliminate non linerities from P_i to solve as a linear problem from rounding at this problem formulation stage.

t_i	U_i	C_i	P_i

Personally I like the system model way I have it as it may be harder to solve with small models, but the same method can be used for more complicated/accurate models.

2)
$$\max_{T_{T1}, T_{T2}, T_{S1}, T_{S2}} \ E = 360 \sum_{i \in \{T_{T1}, \dots, T_{S1}, T_{T2}, \dots, T_{S2}\}} U_i^3$$

s.t.
$$U_i = u_3 t_i^3 + u_2 t_i^2 + u_1 t_i + u_0 \quad \forall i \in \{1, \dots, 2163\}$$

$$C_i = c_3 t_i^3 + c_2 t_i^2 + c_1 t_i + c_0 \quad \forall i \in \{1, \dots, 2163\}$$

$$C_i \leq 50 \quad \forall i \in \{T_{T1}, \dots, T_{S1}, T_{T2}, \dots, T_{S2}\}$$

3) Non linear Mixed Integer Programing ☆ see Note

Potential Algorithms:

- Various Hueristic methods... like GAs
  - Faster and more effiant at solving then a full search to something close to optimal
  - Hueristics can't gaurentee a solution optimality and can not converge all together.
- Relaxed solution method... like Branch and Bound
  - A robust solution method that is a good way of using relaxed solution solver methods into the mixed-integer

a good way of using relaxed solution solver methods into the mixed-integer parameters...

• Takes a long time, and depending on the problem, can scale bad w/ bigger problems

Problem 6

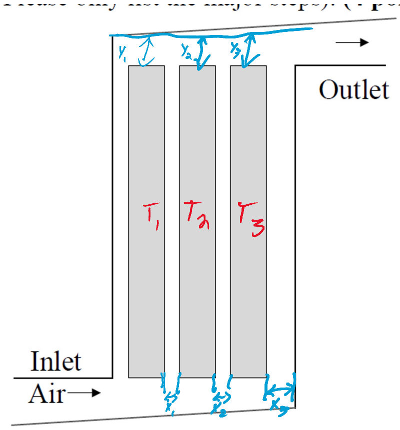
Tuesday, December 7, 2021 12:07 PM

Problem 6: you are assigned with a task to optimize a 2D cooling flow field, as shown in the below figure. The colored components can be regarded as isotropic homogeneity solid material with a uniform heat generation rate (Hint: the colored component is a heat source and the component itself has a same temperature). Your task is to minimize **the temperature differences** among the three components. Based on your understanding of this course, please provide your solutions for this task.

a) What are the parameters you think that could be considered for optimization? You can rename the parameters and label them on the figure. **(5 points)**

b) Please formulate the objective function (you can label or rename the temperature or other physical parameters if necessary). To establish the objective function, what approaches do you think are suitable for this problem? (Hint: how to form the mathematical objective function (analytically or numerically)? You only need to list one of them. If you need to use data-oriented method, for data collections, you can consider performing physical experiment or simulations.) **(4 points)**

c) Based on the idea you proposed in part (b), please briefly discuss the major steps you would like to perform for this structural optimization. (Hint: this is an open question, try to recall how you did for your course project. Please only list the major steps). **(4 points)**



The cooling air flow rate, the sizes of the colored components, and the component heat generation rate are fixed.

a) The seperation of each object... x_1, x_2, x_3
vertical positioning ... y_1, y_2, y_3

b) Assuming that a physics model exists to model the physical systems: $\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = f(\text{parameters})$
an objective function can be constructed as

$$\min \; |T_1 - T_2| + |T_2 - T_3| + |T_1 - T_3|$$

$$\text{s.t.} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = f(x_1, x_2, x_3, y_1, y_2, y_3)$$

This approch uses the parameter-dependent physics model of static behavior to construct slack variables of the temperature.

The optimization could then be solved using constrained nonlinear methods that can solve it easily.

c) well this is a prolem I'm unfamilur with, so the genearl steps would be

- Reaserch and understand the physics/dynamics of the system itself
- Using the most general model, construct an optimization that optimizes whatever cost function desired
- If nessicary, simulate specific system and solve for the ideal parameters

- Build and test the physical system w/ variability built in so experiments can be done to evaluate it... (with a DOE problem or similar)
- Continue to iterate model and design until good enough