```
% MECH 6318 - HW 2
% Jonas Wagner
% 2021-09-07

clear
close all
```

Problem 11.3

```
f = @(x) 8*x(1,:,:) + 10*x(2,:,:) + 4
```

```
f = function_handle with value:
    @(x)8*x(1,:,:)+10*x(2,:,:)+4
```

```
A = [-1,-1;
      1, 1]
```

```
A = 2×2
    -1    -1
     1     1
```

```
b = [4;
      6]
```

```
b = 2×1
     4
     6
```

```
LB = [0;
      0]
```

```
LB = 2×1
     0
     0
```

```
UB = []
```

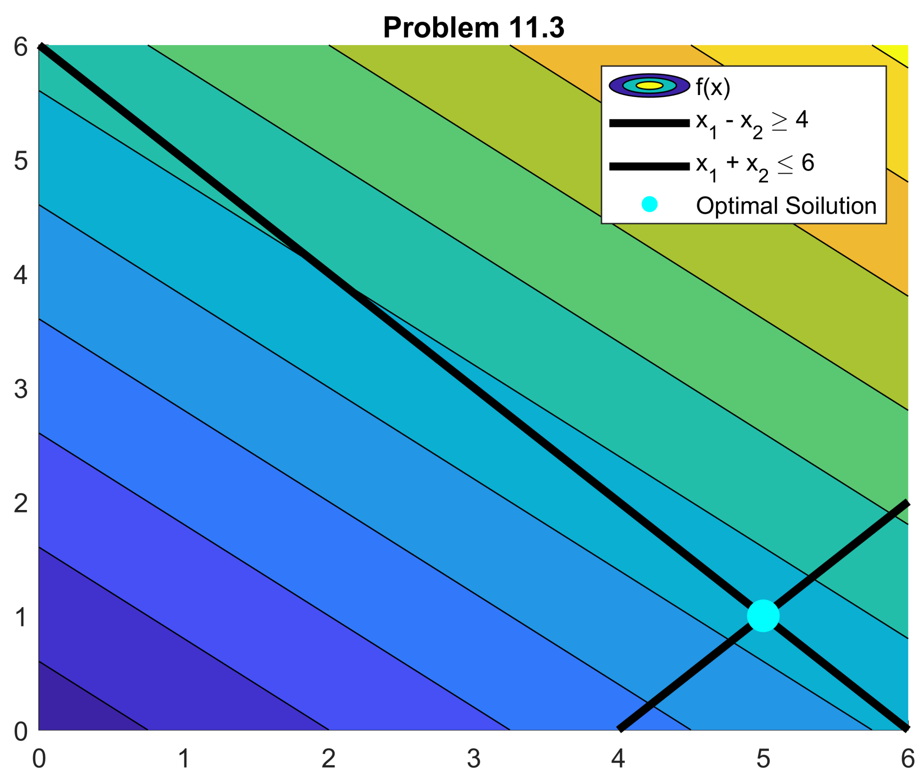
```
UB =

     []
```

```
% Plot F(x)
lb = 0;
ub = 6;
[X1,X2] = meshgrid(lb:0.1:ub, lb:0.1:ub);%3.5:0.1:6,0:0.1:2);
F = 8*X1 + 10*X2 + 4
```

```
F = 61×61
    4.0000    4.8000    5.6000    6.4000    7.2000    8.0000    8.8000    9.6000 ···
    5.0000    5.8000    6.6000    7.4000    8.2000    9.0000    9.8000   10.6000
    6.0000    6.8000    7.6000    8.4000    9.2000   10.0000   10.8000   11.6000
    7.0000    7.8000    8.6000    9.4000   10.2000   11.0000   11.8000   12.6000
    8.0000    8.8000    9.6000   10.4000   11.2000   12.0000   12.8000   13.6000
    9.0000    9.8000   10.6000   11.4000   12.2000   13.0000   13.8000   14.6000
```

```
   10.0000   10.8000   11.6000   12.4000   13.2000   14.0000   14.8000   15.6000
   11.0000   11.8000   12.6000   13.4000   14.2000   15.0000   15.8000   16.6000
   12.0000   12.8000   13.6000   14.4000   15.2000   16.0000   16.8000   17.6000
   13.0000   13.8000   14.6000   15.4000   16.2000   17.0000   17.8000   18.6000
      ⋮
```

```
figure()
hold on
contourf(X1, X2, F, 'DisplayName', 'f(x)')
fimplicit(@(x1,x2) x1 - x2 - 4, [lb,ub], ...
    'k', 'LineWidth', 3, ...
    'DisplayName', 'x_1 - x_2 \geq 4')
fimplicit(@(x1,x2) x1 + x2 - 6, [lb,ub], ...
    'k', 'LineWidth', 3, ...
    'DisplayName', 'x_1 + x_2 \leq 6')
scatter(5,1, 150, 'filled', 'c',...
    'DisplayName', 'Optimal Soilution')
legend
title('Problem 11.3')
```



Problem 11.4

```
F = [1;
     5;
     3]
```

```
F = 3×1
```

```
     1
     5
     3
```

```
A = [-1, 5, -3;
      5, 1, 2;
     -2, 1, 2;
      3, 8, 3]
```

```
A = 4×3
    -1     5    -3
     5     1     2
    -2     1     2
     3     8     3
```

```
b = [-1;
      5;
      4;
      3]
```

```
b = 4×1
    -1
     5
     4
     3
```

```
LB = [0; 0; -inf]
```

```
LB = 3×1
     0
     0
  -Inf
```

```
UB = []
```

```
UB =

     []
```

Problem 11.5

```
A = [1, 2, 2, 0;
     1, 1, 1 -1;
     0, 3, 1, 2]
```

```
A = 3×4
     1     2     2     0
     1     1     1    -1
     0     3     1     2
```

```
b = [4;
     1;
     6]
```

```
b = 3×1
```

```
     4
     1
     6
```

```
% Part a
n = size(A,1)
```

```
n = 3
```

```
m = rank(A)
```

```
m = 3
```

```
max_basic_sol = nchoosek(n,m)
```

```
max_basic_sol = 1
```

```
% Part b
rref_11_5 = rref([A,b])
```

```
rref_11_5 = 3×5
    1.0000         0         0   -2.0000   -2.0000
         0    1.0000         0    0.5000    1.5000
         0         0    1.0000    0.5000    1.5000
```

Problem 11.7

```
c = [1;
    -2]
```

```
c = 2×1
     1
    -2
```

```
A = [-4, 6;
       1, 1]
```

```
A = 2×2
    -4     6
     1     1
```

```
b = [9;
      4]
```

```
b = 2×1
     9
     4
```

```
syms f
T = [[A,eye(2),b];[c',zeros(1,2),f]]
```

T =

$$\begin{pmatrix} -4 & 6 & 1 & 0 & 9 \\ 1 & 1 & 0 & 1 & 4 \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

```
R1 = T(1,:)
```

R1 = $\begin{pmatrix} -4 & 6 & 1 & 0 & 9 \end{pmatrix}$

```
R2 = T(2,:)
```

R2 = $\begin{pmatrix} 1 & 1 & 0 & 1 & 4 \end{pmatrix}$

```
R3 = T(3,:)
```

R3 = $\begin{pmatrix} 1 & -2 & 0 & 0 & f \end{pmatrix}$

```
R1 = R1/6
```

R1 =

$$\begin{pmatrix} -\frac{2}{3} & 1 & \frac{1}{6} & 0 & \frac{3}{2} \end{pmatrix}$$

```
T = [R1;R2;R3]
```

T =

$$\begin{pmatrix} -\frac{2}{3} & 1 & \frac{1}{6} & 0 & \frac{3}{2} \\ 1 & 1 & 0 & 1 & 4 \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

```
R2 = (3/5)*(R2 - R1)
```

R2 =

$$\begin{pmatrix} 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \end{pmatrix}$$

```
T = [R1;R2;R3]
```

T =

$$\begin{pmatrix} -\frac{2}{3} & 1 & \frac{1}{6} & 0 & \frac{3}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

```
R1 = R1 + (2/3)*R2
```

R1 =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \end{pmatrix}$$

```
T = [R1;R2;R3]
```

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

```
R3 = R3 + 2*R1
```

R3 =

$$\begin{pmatrix} 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

```
T = [R1;R2;R3]
```

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

```
R3 = R3 - R2
```

R3 =

$$\begin{pmatrix} 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

```
T = [R1;R2;R3]
```

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

```
final_simplex_tbl = [T(2,:);T(1,:);T(3,:)]
```

final_simplex_tbl =

$$\begin{pmatrix} 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

```
final_simplex_soln = final_simplex_tbl(1:2,5)
```

final_simplex_soln =

$$\begin{pmatrix} \frac{3}{2} \\ \frac{5}{2} \end{pmatrix}$$

```
final_simplex_value = c'*final_simplex_soln
```

final_simplex_value =

$$-\frac{7}{2}$$

Problem 11.8

```
c = [1;
      2;
     -7]
```

```
c = 3×1
     1
     2
    -7
```

```
A = [2, 1, 1;
    -1, 2,-1;
     1, 5, 5]
```

```
A = 3×3
     2     1     1
    -1     2    -1
     1     5     5
```

```
b = [15;
       7;
      25]
```

```
b = 3×1
    15
     7
```

25

```
syms f
T = [[A,eye(3),b];[c',zeros(1,3),f]]
```

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ 1 & 5 & 5 & 0 & 0 & 1 & 25 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

```
R1 = T(1,:)
```

R1 = $\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \end{pmatrix}$

```
R2 = T(2,:)
```

R2 = $\begin{pmatrix} -1 & 2 & -1 & 0 & 1 & 0 & 7 \end{pmatrix}$

```
R3 = T(3,:)
```

R3 = $\begin{pmatrix} 1 & 5 & 5 & 0 & 0 & 1 & 25 \end{pmatrix}$

```
R4 = T(4,:)
```

R4 = $\begin{pmatrix} 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$

```
T = [R1; R2; R3; R4]
```

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ 1 & 5 & 5 & 0 & 0 & 1 & 25 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

```
R3 = R3/5
```

R3 =

$$\begin{pmatrix} \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \end{pmatrix}$$

```
T = [R1; R2; R3; R4]
```

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

```
R1 = R1 - R3
```

R1 =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \end{pmatrix}$$

```
R2 = R2 + R3
```

R2 =

$$\begin{pmatrix} -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \end{pmatrix}$$

```
R4 = R4 + 7*R3
```

R4 =

$$\begin{pmatrix} \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

```
T = [R1; R2; R3; R4]
```

T =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

```
R1 = (5/9) * R1
```

R1 =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \end{pmatrix}$$

```
T = [R1;R2;R3;R4]
```

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

```
R2 = (1/3)*( R2 + (4/5)*R1)
```

R2 =

$$\begin{pmatrix} 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \end{pmatrix}$$

```
T = [R1;R2;R3;R4]
```

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

```
R3 = R3 - R1/5 -R2
```

R3 =

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \end{pmatrix}$$

```
R4 = R4 - (12/5) * R1 - 9*R2
```

R4 =

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

```
T = [R1;R2;R3;R4]
```

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

```
final_simplex_tbl = T
```

final_simplex_tbl =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

```
final_simplex_soln = T(1:3,7)
```

final_simplex_soln =

$$\begin{pmatrix} \frac{50}{9} \\ \frac{148}{27} \\ -\frac{43}{27} \end{pmatrix}$$

```
final_simplex_value = c'*final_simplex_soln
```

final_simplex_value =

$$\frac{83}{3}$$

MECH6318 HW03

Monday, September 20, 2021 1:13 PM

Fall 2021 | MECH 6318
Engineering Optimization – Prof. Jie Zhang
HOMEWORK #3
September 14, 2021

DUE: Tuesday, Sep 21, 2021 **5pm (central time)**
Submit the HW to eLearning

Points Distribution
30 points maximum
-5 to 0 points reserved for **Neatness and Professional Presentation**
(legible, stapled, show key Matlab commands, properly labeled plots, etc.)

Book Problems:

Problems 11.3 (Provide the Matlab plot), 11.4, 11.5, 11.7, 11.8 (Upload the M files to eLearning)

11.3 You are given the following optimization problem. Solve the following problem graphically.

$$\min_x \quad 8x_1 + 10x_2 + 4 \qquad (11.118)$$

subject to

$$x_1 - x_2 \geq -4 \qquad (11.119)$$
$$x_1 + x_2 \leq 6 \qquad (11.120)$$
$$x_1, x_2 \geq 0 \qquad (11.121)$$

Plot the objective function and the constraint equations. Identify the feasible design space and the optimal solution.

$$\min_x f(x) = 8x_1 + 10x_2 + 4$$

s.t.

$$x_1 - x_2 \geq -4 \longrightarrow -x_1 + x_2 \leq 4$$

s.t.

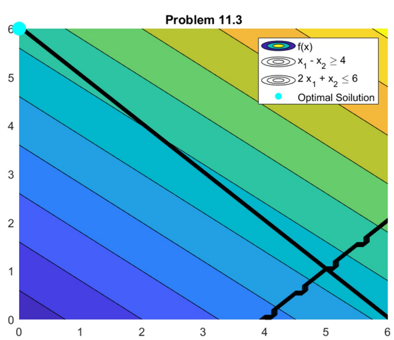
$$x_1 - x_2 \geq -4 \longrightarrow -x_1 + x_2 \leq 4$$

$$x_1 + x_2 \leq 6 \qquad A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$x_1, x_2 \geq 0$$

Problem 11.3

```
f = @(x) 8*x(1,:,:) + 10*x(2,:,:) + 4

A = [-1,-1;
     1, 1]
b = [4;
     6]

LB = [0;
     0]
UB = []

% Plot F(x)
lb = 0;
ub = 6;
[X1,X2] = meshgrid(lb:0.1:ub,lb:0.1:ub);
F = 8*X1 + 10*X2 + 4

figure()
hold on
contourf(X1, X2, F, 'DisplayName', 'f(x)')

contour(X1, X2, X1 - X2 >=  4, 'k',...
    'DisplayName', 'x_1 - x_2 \geq 4')
contour(X1, X2, X1 + X2 <=  6, 'k',...
    'DisplayName', '2 x_1 + x_2 \leq 6')
scatter(0,6, 150, 'filled', 'c',...
    'DisplayName', 'Optimal Soilution')
legend
title('Problem 11.3')
```



11.4 Transform the following problem into the standard LP formulation.

$$\min_{x} \; z = x_1 + 5x_2 + 3x_3 \tag{11.122}$$

subject to

$$x_1 - 5x_2 + x_3 \geq 1 \tag{11.123}$$
$$5x_1 + x_2 + 2x_3 \leq 5 \tag{11.124}$$
$$-2x_1 + 3x_2 + 3x_3 \leq 4 \tag{11.125}$$
$$3x_1 + 8x_2 + 5x_3 \leq 3 \tag{11.126}$$
$$x_1, x_2 \geq 0, x_3 \text{ unbounded} \tag{11.127}$$

$$\min_{x} \quad x_1 + 5x_2 + 3x_3 = F'x$$

$$F = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$$

s.t.

$$-x_1 + 5x_2 - x_3 \leq -1$$
$$5x_1 + x_2 + 2x_3 \leq 5$$
$$-2x_1 + 3x_2 + 3x_3 \leq 4$$
$$3x_1 + 8x_2 + 5x_3 \leq 3$$
$$x_1, x_2 \geq 0$$

$$Ax \leq b$$

$$A = \begin{bmatrix} -1 & 5 & -3 \\ 5 & 1 & 2 \\ -2 & 3 & 3 \\ 3 & 8 & 3 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

11.5 Consider the following set of equations.

(1) How many basic solutions are possible for this set of constraints?

(2) Transform them into the reduced row echelon form with respect to the basic variables x_1, x_2, and x_3.

$$x_1 + 2x_2 + 2x_3 = 4 \tag{11.128}$$
$$x_1 + x_2 + x_3 - x_4 = 1 \tag{11.129}$$
$$3x_2 + x_3 + 2x_4 = 6 \tag{11.130}$$

Problem 11.5

```
A = [1, 2, 2, 0;
     1, 1, 1 -1;
     0, 3, 1, 2]
b = [4;
     1;
     6]

% Part a
n = size(A,1)
m = rank(A)
max_basic_sol = nchoosek(n,m)

% Part b
rref_11_5 = rref([A,b])
```

```
A = 3x4
       1     2     2     0
       1     1     1    -1
       0     3     1     2

b = 3x1
       4
       1
       6
```

```
n = 3
m = 3
max_basic_sol = 1

rref_11_5 = 3x5
      1.0000         0         0   -2.0000   -2.0000
           0    1.0000         0    0.5000    1.5000
           0         0    1.0000    0.5000    1.5000
```

11.7 Consider the following LP problem from Sec. 11.3.1. Solve it using the Simplex method.

$$\min_x \; x_1 - 2x_2 \tag{11.131}$$

subject to

$$-4x_1 + 6x_2 \le 9 \tag{11.132}$$
$$x_1 + x_2 \le 4 \tag{11.133}$$
$$x_1, x_2 \ge 0 \tag{11.134}$$

1) $c = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$A = \begin{bmatrix} -4 & 6 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$

$x_1, x_2 \ge 0$

2) simplex form:

$-4x_1 + 6x_2 + s_1 = 9$

$x_1 + x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2 \ge 0$

simplex table

$x_1 \quad x_2 \quad s_1 \quad s_2$

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\
\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \left[\begin{array}{cccc|c} -4 & 6 & 1 & 0 & 9 \\ 1 & 1 & 0 & 1 & 4 \\ 1 & -2 & 0 & 0 & f \end{array}\right]
\end{array}
$$

3) pivital collumn

4) Pivital Row $\left(\frac{9}{6}\right) < \left(\frac{4}{1}\right)$

5) Pivitol Element

Problem 11.7

```
c = [1;
    -2]

A = [-4, 6;
      1, 1]
b = [9;
     4]


syms f
T = [[A,eye(2),b];[c',zeros(1,2),f]]

R1 = T(1,:)
R2 = T(2,:)
R3 = T(3,:)

R1 = R1/6
T = [R1;R2;R3]

R2 = (3/5)*(R2 - R1)
T = [R1;R2;R3]

R1 = R1 + (2/3)*R2
T = [R1;R2;R3]

R3 = R3 + 2*R1
T = [R1;R2;R3]

R3 = R3 - R2
T = [R1;R2;R3]

final_simplex_tbl = T
final_simplex_soln = T(1:2,5)
final_simplex_value = c'*final_simplex_soln
```

T =

$$\begin{pmatrix} -\frac{2}{3} & 1 & \frac{1}{6} & 0 & \frac{3}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

final_simplex_tbl =

$$\begin{pmatrix} 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

final_simplex_soln =

$$\begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$$

final_simplex_value =

$$-\frac{1}{2}$$

11.8 Solve the following problem using the Simplex method. Verify the correctness of your solution using `linprog`.

$$\min_{x} \; x_1 + 2x_2 - 7x_3 \tag{11.135}$$

subject to

$$2x_1 + x_2 + x_3 \le 15 \tag{11.136}$$

$$-x_1 + 2x_2 - x_3 \le 7 \tag{11.137}$$

$$x_1 + 5x_2 + 5x_3 \le 25 \tag{11.138}$$

$$x_1, x_2, x_3 \ge 0 \tag{11.139}$$

$$c = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & 5 & 5 \end{bmatrix} \qquad b = \begin{bmatrix} 15 \\ 7 \\ 25 \end{bmatrix}$$

$$X_1, X_2, X_3 \ge 0$$

$$\begin{array}{c|ccc|ccc|c|c} & X_1 & X_2 & X_3 & S_1 & S_2 & S_3 & & \\ \hline R_1 & 2 & 1 & 1 & 1 & 0 & 0 & 15 & \frac{15}{1} \\ R_2 & -1 & 2 & -1 & 0 & 1 & 0 & 7 & \cancel{\frac{7}{-1}} \\ R_3 & 1 & 5 & 5 & 0 & 0 & 1 & 25 & \frac{25}{5} \\ \hline R_4 & 1 & 2 & \underline{-7} & 0 & 0 & 0 & f & \end{array}$$

Problem 11.8

```
c = [1;
     2;
    -7]

A = [2, 1, 1;
    -1, 2,-1;
     1, 5, 5]
b = [15;
      7;
     25]


syms f
T = [[A,eye(3),b];[c',zeros(1,3),f]]

R1 = T(1,:)
R2 = T(2,:)
R3 = T(3,:)
R4 = T(4,:)
T = [R1; R2; R3; R4]

R3 = R3/5
T = [R1; R2; R3; R4]

R1 = R1 - R3
R2 = R2 + R3
R4 = R4 + 7*R3
T = [R1; R2; R3; R4]

R1 = (5/9) * R1
T = [R1;R2;R3;R4]

R2 = (1/3)*( R2 + (4/5)*R1)
T = [R1;R2;R3;R4]

R3 = R3 - R1/5 -R2
R4 = R4 - (12/5) * R1 - 9*R2
T = [R1;R2;R3;R4]

final_simplex_tbl = T
final_simplex_soln = T(1:3,7)
final_simplex_value = c'*final_simplex_soln
```

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ 1 & 5 & 5 & 0 & 0 & 1 & 25 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

R3 =

$$\begin{pmatrix} \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \end{pmatrix}$$

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

R1 =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \end{pmatrix}$$

R2 =

$$\begin{pmatrix} -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \end{pmatrix}$$

R4 =

$$\begin{pmatrix} \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

T =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R2 =

$$\begin{pmatrix} 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

final_simplex_tbl =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

final_simplex_soln =

$$\begin{pmatrix} \frac{50}{9} \\ \frac{148}{27} \\ -\frac{43}{27} \end{pmatrix}$$

final_simplex_value =

$$\frac{83}{3}$$

Not Feasible….