MECH6318 HW02

Tuesday, September 7, 2021 3:43 PM

Fall 2021 | MECH 6318
Engineering Optimization – Prof. Jie Zhang
HOMEWORK #2
September 07, 2021

DUE: Tuesday, Sep 14, 2021 **5pm (central time)**
Submit the HW to eLearning

Points Distribution
30 points maximum
-5 to 0 points reserved for **Neatness and Professional Presentation**
(legible, stapled, show key Matlab commands, properly labeled plots, etc.)

Book Problems:

Problems 5.1, 5.3, 5.4 (please also submit the Matlab code files)

5.1 Formulate the following problem (by hand) to represent the generic optimization format shown in Eq. (5.15) to (5.20) of the textbook.

$$\min_{x} f(x) = 5x_1^2 - 23x_1x_2 - 9x_2^2 + 9x_3^2 + 15 \quad (5.32)$$

subject to

$$x_1^2 + x_2^2 = 10 \quad (5.33)$$

$$x_1x_3 \leq 100 \quad (5.34)$$

$$4x_1 - 19x_2 = 50$$

$$3x_1 + 4x_3 = 100 \quad (5.35)$$

$$x_1 + x_2 + x_3 \geq -10$$

$$-500 \leq x_1, x_2, x_3 \leq 500$$

(5.36)

(5.37)

(5.38)

$$\min_{x} \ f(x) \quad (5.15)$$

subject to

$$c(x) \leq 0 \quad (5.16)$$

$$ceq(x) = 0 \quad (5.17)$$

$$A\,x \leq b \quad (5.18)$$

$$Aeq\ x = beq \quad (5.19)$$

$$LB \leq x \leq UB \quad (5.20)$$

$$\min_{x} \ f(x) = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 5 & -23 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 15$$

s.t.

$$c(x) = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - 100 \leq 0$$

$$c_{eq}(x) = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 10 = 0$$

$$Ax = -I_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq 10 = b$$

$$A_{eq}x = \begin{bmatrix} 4 & -19 & 0 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 100 \end{bmatrix} = b$$

$$LB = -500\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \leq \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq 500\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = UB$$

5.3 Consider the following problem.

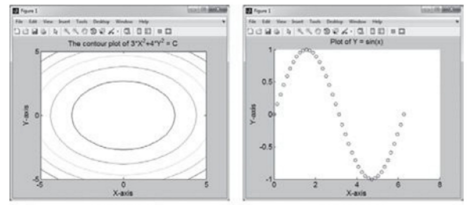
$$\min_x f(x) = x_1^2 + 10x_2^2 - 3x_1x_2 \tag{5.42}$$

subject to

$$2x_1 + x_2 \geq 4 \tag{5.43}$$
$$x_1 + x_2 \geq -5 \tag{5.44}$$
$$-5 \leq x_1, x_2 \leq 5 \tag{5.45}$$

(a) Solve the above problem using MATLAB. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(x)$.

```
[X,Y] = meshgrid(-5:.5:5,-5:.5:5);
  Z = (3*X.^2+4*Y.^2);
  [C,h] = contour(X,Y,Z,5);
  xlabel('X-axis')
  ylabel('Y-axis')
  title('The contour plot of 3*X^2+4*Y^2 = C')
```



(a) Contour Plot (b) Scatter Plot

Figure 1.11. Contour and Scatter Plots

(b) Solve the above problem by removing the first constraint $2x_1 + x_2 \geq 4$. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(x)$.

(c) Now, solve the same problem by removing **all** the constraints. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(x)$.

(d) Create a contour plot similar to Fig. 1.11 (a) showing the contours of the objective function. Show on it the locations of the optima obtained in Parts (a) - (c).

```
% MECH 6318 - HW 2
% Jonas Wagner
% 2021-09-07

clear
close all

% Problem 5.3 ----------------------------------------
H = [ 2, -3;
     -3, 20]
```

```
H = 2x2
     2    -3
    -3    20
```

```
f = zeros(2,1);

A = [-2, -1;
     -1, -1]
```

```
A = 2x2
    -2    -1
    -1    -1
```

```
b = [-4;
      5]
```

```
b = 2x1
    -4
     5
```

```
lb = -5
```

```
lb = -5
```

```
ub = 5
```

```
ub = 5
```

```
x0 = ones(2,1)
```

```
x0 = 2x1
     1
     1
```

```
% Part a
[x_opt,f_min] = quadprog(H,f,A,b,[],[],lb,ub)
```

```
x_opt = 2x1
    1.8298
    0.3404
f_min = 2.6383
```

```
% Part b
[x_opt_b,f_min_b] = quadprog(H,f,A(1,:),b(1,:),[],[],lb,ub)
```

```
x_opt_b = 2x1
    1.8298
    0.3404
```

```
[x_opt_b,f_min_b] = quadprog(H,f,A(1,:),b(1,:),[],[],lb,ub)
```

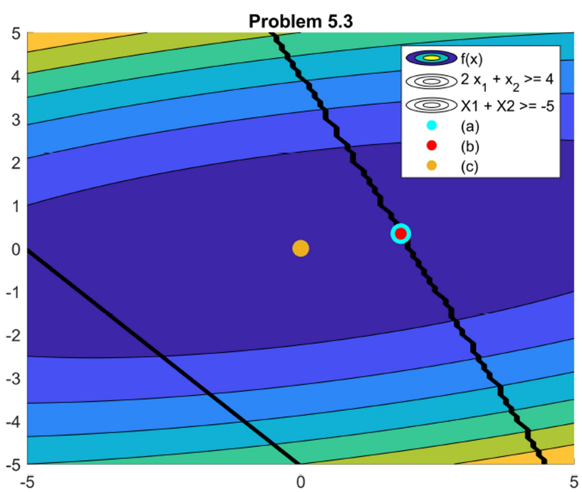
```
x_opt_b = 2x1
    1.8298
    0.3404
f_min_b = 2.6383
```

```
% Part c
[x_opt_c,f_min_c] = quadprog(H,f,[],[],[],[],lb,ub)
```

```
x_opt_c = 2x1
     0
     0
f_min_c = 0
```

```
% Part d
[X1,X2] = meshgrid(lb:0.1:ub,lb:0.1:ub);
F = X1.^2 + 10.*X2.^2 - 3.*X1.*X2;

figure()
hold on
contourf(X1, X2, F, 'DisplayName', 'f(x)')
contour(X1, X2, 2 * X1 + X2 >=  4, 'k',...
    'DisplayName', '2 x_1 + x_2 >= 4')
contour(X1, X2,     X1 + X2 >= -5, 'k',...
    'DisplayName', 'X1 + X2 >= -5')
scatter(x_opt(1) ,x_opt(2), 150, 'filled', 'c',...
    'DisplayName', '(a)')
scatter(x_opt_b(1) ,x_opt_b(2), 50, 'filled', 'r',...
    'DisplayName', '(b)')
scatter(x_opt_c(1) ,x_opt_c(2), 100, 'filled', 'o',...
    'DisplayName', '(c)')
legend
title('Problem 5.3')
```



5.4 The following is a linear programming problem.

$$\min_x f(x) = 20x_1 + 64x_2 \tag{5.46}$$

subject to

$$25x_1 + 70x_2 \geq 2,100 \tag{5.47}$$
$$0 \leq x_1 \leq 70 \tag{5.48}$$
$$0 \leq x_2 \leq 50 \tag{5.49}$$

(a) Solve the above linear optimization problem using the linprog command in MATLAB.

(b) Solve the above problem using the fmincon command in MATLAB. Compare the results with the results obtained in (a).

```
% Problem 5.4 ---
f = [20; 64]
```

```
f = 2×1
    20
    64
```

```
A = [-25, -70]
```

```
A = 1×2
   -25   -70
```

```
b = -2100
```

```
b = -2100
```

```
lb = [ 0;  0]
```

```
lb = 2×1
     0
     0
```

```
ub = [70; 50]
```

```
ub = 2×1
    70
    50
```

```
% Part a
[x_opt, f_opt] = linprog(f, A, b, [], [], lb, ub)
```

```
x_opt = 2×1
    70
     5
f_opt = 1720
```

```
% Part b
[x_opt_b, f_obt_b] = fmincon(...
    @(x) 20 * x(1) + 64 * x(2),...
    [0; 0], A, b, [], [], lb, ub)
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
x_opt_b = 2×1
   70.0000
    5.0000
f_obt_b = 1.7200e+03
```