MECH 6318 - Homework 9

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```
clear
close all
```

Problem 13.1

```
% f = @(x) x(1)^2 - x(2)
% h1 = @(x) x(1)^2 - x(2) - 1
```

Part 1

```
% syms x1 x2
% eq1 = -x2^2 + x2 - 1 == 0;
% eq2 = x1^2 - x2^2 - 1 == 0;
% eq = [eq1; eq2]
%
% [X1, X2] = solve(eq, [x1, x2])
```

Problem 13.2

```
A = [2, 0, 0, 1, 2;
     0, 2, 0, 2, 2;
     0, 0, 3, 3, 1;
     1, 2, 3, 0, 0;
     2, 2, 1, 0, 0;
    ]
```

```
A = 5×5
     2     0     0     1     2
     0     2     0     2     2
     0     0     3     3     1
     1     2     3     0     0
     2     2     1     0     0
```

```
b = [0, 0, 0, 7, 9/2]'
```

```
b = 5×1
         0
         0
         0
    7.0000
    4.5000
```

```
syms x1 x2 x3 v1 v2
x_sym = [x1, x2, x3, v1, v2]'
```

```
x_sym =
```

$$\begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \\ \overline{v_1} \\ \overline{v_2} \end{pmatrix}$$

```
results = solve(A * x_sym == b, x_sym');

x1 = double(results.x1)
```

```
x1 = 0.3723
```

```
x2 = double(results.x2)
```

```
x2 = 1.1596
```

```
x3 = double(results.x3)
```

```
x3 = 1.4362
```

```
v1 = double(results.v1)
```

```
v1 = -1.5745
```

```
v2 = double(results.v2)
```

```
v2 = 0.4149
```

```

```

Problem 13.3

```
f = @(H,D) pi * D * H + (pi * D^2)/2
```

```
f = function_handle with value:
    @(H,D)pi*D*H+(pi*D^2)/2
```

```
g1 = @(H,D) pi * (1 - D^2 * H)
```

```
g1 = function_handle with value:
    @(H,D)pi*(1-D^2*H)
```

```
g2 = @(H,D) 4.5 - D
```

```
g2 = function_handle with value:
    @(H,D)4.5-D
```

```
g3 = @(H,D) D - 12
```

```
g3 = function_handle with value:
    @(H,D)D-12
```

```
g4 = @(H,D) 10 - H
```

```
g4 = function_handle with value:
```

```
    @(H,D)10-H
```

```
g5 = @(H,D) H - 18
```

```
g5 = function_handle with value:
    @(H,D)H-18
```

```
L = @(H,D,lambda) (...
    f(H,D) ...
    + lambda(1) * g1(H,D) ...
    + lambda(2) * g2(H,D) ...
    + lambda(3) * g3(H,D) ...
    + lambda(4) * g4(H,D) ...
    + lambda(5) * g5(H,D)...
)
```

```
L = function_handle with value:
    @(H,D,lambda)(f(H,D)+lambda(1)*g1(H,D)+lambda(2)*g2(H,D)+lambda(3)*g3(H,D)+lambda(4)*g4(H,D)+lambda(5)*g5(H,D))
```

```
lambda = sym('lambda',[1,5]);
assume(lambda >= 0)

D_H_L = @(D,H) diff(L(H,D,lambda),H);
D_D_L = @(D,H) diff(L(H,D,lambda),D);

syms D H
L_H = D_H_L(D,H)
```

L_H = $-\lambda_1 \pi \, \mathrm{D}^2 + \pi \, \mathrm{D} - \lambda_4 + \lambda_5$

```
L_D = D_D_L(D,H)
```

L_D = $\lambda_3 - \lambda_2 + \pi \, \mathrm{D} + \pi \, H - 2 \, \pi \, \mathrm{D} \, H \, \lambda_1$

```
results = solve([...
    D_H_L(D,H) == 0,...
    D_D_L(D,H) == 0,...
    lambda(1) * g1(H,D) == 0,...
    lambda(2) * g2(H,D) == 0,...
    lambda(3) * g3(H,D) == 0,...
    lambda(4) * g4(H,D) == 0,...
    lambda(5) * g5(H,D) == 0,...
    g1(H,D) <= 0,...
    g2(H,D) <= 0,...
    g3(H,D) <= 0,...
    g4(H,D) <= 0,...
    g5(H,D) <= 0
    ], ...
    [D, H, lambda]...
);

D = double(results.D)
```

```
D = 4.5000
```

```
H = double(results.H)
```

```
H = 10
```

```
lambda_1 = double(results.lambda1)
```

```
lambda_1 = 0
```

```
lambda_2 = double(results.lambda2)
```

```
lambda_2 = 45.5531
```

```
lambda_3 = double(results.lambda3)
```

```
lambda_3 = 0
```

```
lambda_4 = double(results.lambda4)
```

```
lambda_4 = 14.1372
```

```
lambda_5 = double(results.lambda5)
```

```
lambda_5 = 0
```

```

```

Problem 13.4

```
clear
f = @(x) (x(1) - 1.5)^2 + (x(2) - 1.5)^2;
g1 = @(x) x(1) + x(2) - 2;

x = sym('x',[2,1],'real')
```

```
x =
```

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

```
fx = f(x)
```

```
fx =
```

$$\left(x_1 - \frac{3}{2}\right)^2 + \left(x_2 - \frac{3}{2}\right)^2$$

```
g1x = g1(x)
```

```
g1x = x_1 + x_2 - 2
```

```

P = @(x,R) f(x) + R/(g1(x));
P_xR = P(x,sym('R'))
```

```
P_xR =
```

$$\left(x_1-\frac{3}{2}\right)^2+\left(x_2-\frac{3}{2}\right)^2+\frac{R}{x_1+x_2-2}$$

```
D_x_P = @(x,R) gradient(P(x,R),x);
D_x_P_R = D_x_P(x,sym('R'))
```

D_x_P_R =

$$\begin{pmatrix} 2\,x_1-\frac{R}{(x_1+x_2-2)^2}-3 \\ 2\,x_2-\frac{R}{(x_1+x_2-2)^2}-3 \end{pmatrix}$$

```
R = [1; 0.1; 0.01; 0.001];
X1 = arrayfun(@(R) double(solve(D_x_P(x,R),x).x1),R);
X2 = arrayfun(@(R) double(solve(D_x_P(x,R),x).x2),R);
G = arrayfun(@(x1,x2) g1([x1,x2]), X1, X2);
F = arrayfun(@(x1,x2) f([x1,x2]), X1, X2);
Omega = R./G;

Results = table(R,X1,X2,G,F,Omega)
```

Results = 4×6 table

	R	X1	X2	G	F	Omega
1	1.0000	1.7328	1.7328	1.4656	0.1084	0.6823
2	0.1000	1.5425	1.5425	1.0850	0.0036	0.0922
3	0.0100	1.5049	1.5049	1.0098	0.0000	0.0099
4	0.0010	1.5005	1.5005	1.0010	0.0000	0.0010