MECH6318 HW06

Wednesday, October 6, 2021 12:47 AM

Fall 2021 | MECH 6318
Engineering Optimization – Prof. Jie Zhang
HOMEWORK #6
October 5, 2021

DUE: Tuesday, October 12, 2021 **11PM**
Submit the HW to eLearning

Points Distribution
30 points maximum
-5 to 0 points reserved for **Neatness and Professional Presentation**
(legible, stapled, show key Matlab commands, properly labeled plots, etc.)

Book Problems:

Problems 12.4 parts 1(b), 1(d), 2, 3, and 4.

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Book Problems:
Problems 12.4 parts 1(b), 1(d), 2, 3, and 4.

12.4 Consider the following function:

$$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{x^2}, \quad \frac{1}{2} \le x \le 2$$

We wish to find the minimum of the above function in the given range of x.

1. Using the following listed methods, estimate the minimum of the above function using up to three iterations. Solve Parts (a) through (c) by hand, and please show your complete work.
   (a) Interval Halving Method (*i.e.*, Bisection method)
   (b) Newton-Raphson Method using $x_0 = 1$ as the starting point
   (c) Golden Section Method
   (d) Plot the function $f(x)$ using MATLAB, and identify the optimum solution in the range of x. For Parts (a) through (c), calculate the percentage error between the optimum x value and the value of x after three iterations. Present your results in a tabular form, as shown below, for methods in Parts (a), (b), and (c).
2. Write a MATLAB code for the Newton-Raphson method that evaluates the minimum of the function in Problem 1. Use an appropriate stopping criterion.
3. Verify your results using fmincon. Make sure your code gives you the same solution as that given by fmincon. You might have to change your stopping criterion.
4. How many function evaluations does your code need? How many function evaluations does fmincon need? Use the same starting point for your code and fmincon.

Table 12.6. *Results Summary for Given Search Methods*

Iteration	(a)		(b)		(c)	
	x	$f(x)$	x	$f(x)$	x	$f(x)$
1						
2						
3						

- Newton's method approximates the function f(x) with a quadratic function at each iteration. The approximated quadratic function is minimized exactly, and a descent direction is evaluated.

$$\tilde{f}(x) \approx f(x_k) + \nabla f(x_k)^T (x - x_k) + \frac{1}{2}(x - x_k)^T \nabla^2 f(x_k)(x - x_k)$$

- The first derivative should satisfy $\nabla \tilde{f}(x) = 0$

$$\nabla \tilde{f}(x) = \nabla f(x_k) + \nabla^2 f(x_k)(x - x_k) = 0$$

$$\nabla^2 f(x_k)(x - x_k) = -\nabla f(x_k).$$

$$x = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

- The second term is used as the descent direction to calculate x_{k+1}

$$x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

- The descent direction

$$p_k = [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

- Look for the search direction p_k that satisfies

$$\nabla^2 f(x_k) p_k = -\nabla f(x_k)$$

- P_k can be solved by Gaussian Elimination, or other suitable methods.

12.4. 1b)

(b) Newton-Raphson Method using $x_0 = 1$ as the starting point

$$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{x^2}, \quad \frac{1}{2} \le x \le 2$$

12.4. 1b)

(b) Newton-Raphson Method using $x_0 = 1$ as the starting point

$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{x^2}, \quad \frac{1}{2} \le x \le 2$

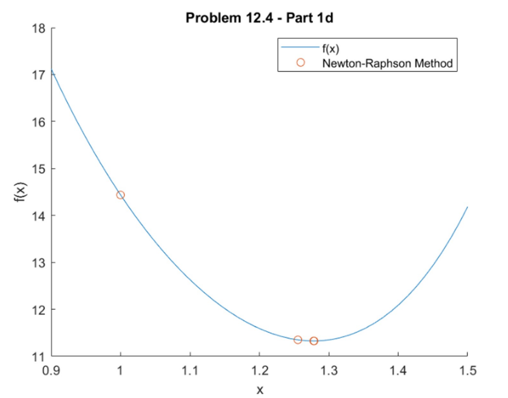
$f'(x) = -9x^2 - 24x^{-3} + 4xe^{x^2}$ $\qquad \varepsilon = 0.01$

$f''(x) = -18x + 72x^{-4} + 8x^2e^{x^2} + 4e^{x^2}$

K	X_K	$f(X_K)$	$f'(X_K)$	$f''(X_K)$	$P_K = \frac{f'(X_K)}{f''(X_K)}$	$X_{K+1} = X_K - P_K$	$\lvert f'(X_{K+1})\rvert$	$< \varepsilon$?				
0	1.0	14.4	-22.12	86.62	-3.915	1.2554	2	N				
1	1.255	11.35	-2.027	86.71	-0.0234	1.2788	0.055	N				
2	1.2788	11.33	0.055	91.564	6e-4	1.2782	4.1e-5	Y				
3												
4												

$\boxed{\begin{aligned} x^* &= 1.278 \\ f(x^*) &= 11.327 \end{aligned}}$

12.4.1d)



See attached code for the rest of the problems...

```
% MECH 6318 - Homework 6
% Jonas Wagner
% 2021-10-06

close all
clear
```

Problem 12.4

```
f = @(x) -3 .* x.^3 + 12*(x.^(-2)) + 2*exp(x.^2);
df = matlabFunction(diff(f(sym('x'))));
ddf = matlabFunction(diff(diff(f(sym('x')))));

f_x = f(sym('x'))
```

f_x =

$$2\,e^{x^2}+\frac{12}{x^2}-3\,x^3$$

```
df_x = df(sym('x'))
```

df_x =

$$4\,x\,e^{x^2}-9\,x^2-\frac{24}{x^3}$$

```
ddf_x = ddf(sym('x'))
```

ddf_x =

$$4\,e^{x^2}-18\,x+8\,x^2\,e^{x^2}+\frac{72}{x^4}$$

```
x_min = 0.5;
x_max = 2;
```

Part 1b: Newton-Raphson Method By Hand

```
p = @(x) df(x)/ddf(x);
p_x = p(sym('x'))
```

p_x =

$$-\frac{9\,x^2-4\,x\,e^{x^2}+\frac{24}{x^3}}{4\,e^{x^2}-18\,x+8\,x^2\,e^{x^2}+\frac{72}{x^4}}$$

```
x_0 = 1
```

x_0 = 1

```
f_1 = f(x_0)
```

```
f_1 = 14.4366
```

```
df_1 = df(x_0)
```

```
df_1 = -22.1269
```

```
ddf_1 = ddf(x_0)
```

```
ddf_1 = 86.6194
```

```
p_1 = p(x_0)
```

```
p_1 = -0.2554
```

```
x_1 = x_0 - p_1
```

```
x_1 = 1.2554
```

```

f_2 = f(x_1)
```

```
f_2 = 11.3498
```

```
df_2 = df(x_1)
```

```
df_2 = -2.0271
```

```
ddf_2 = ddf(x_1)
```

```
ddf_2 = 86.7119
```

```
p_2 = p(x_1)
```

```
p_2 = -0.0234
```

```
x_2 = x_1 - p_2
```

```
x_2 = 1.2788
```

```

f_3 = f(x_2)
```

```
f_3 = 11.3265
```

```
df_3 = df(x_2)
```

```
df_3 = 0.0550
```

```
ddf_3 = ddf(x_2)
```

```
ddf_3 = 91.5640
```

```
p_3 = p(x_2)
```

```
p_3 = 6.0025e-04
```

```
x_3 = x_2 - p_3
```

```
x_3 = 1.2782
```

```
f_4 = f(x_3)
```

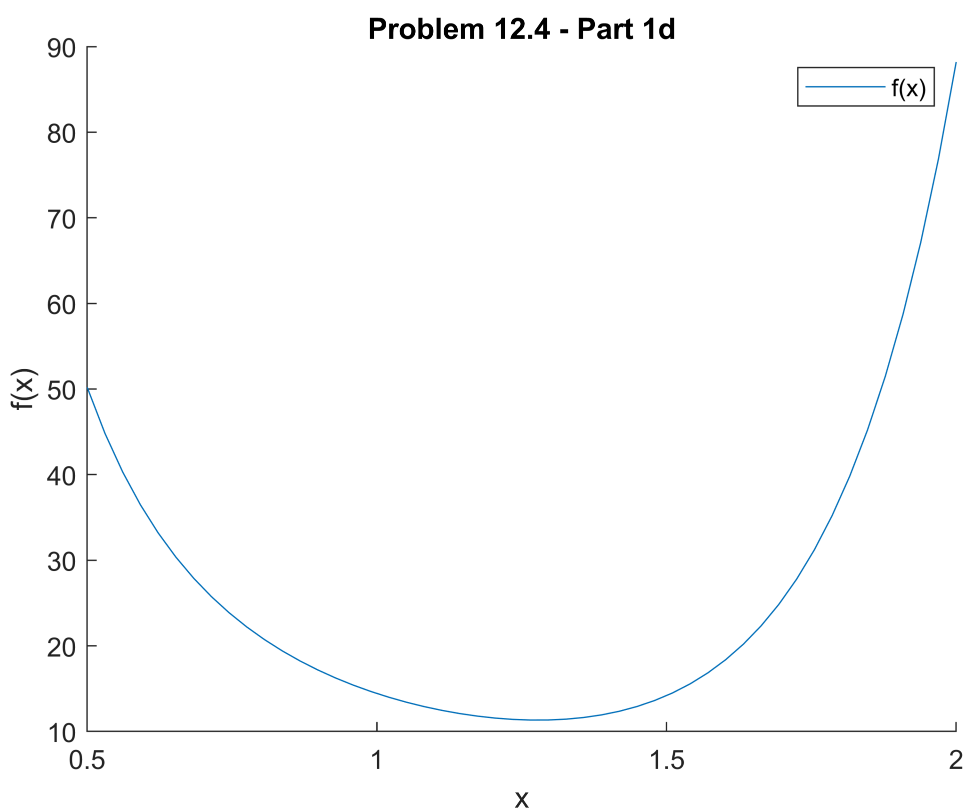
```
f_4 = 11.3265
```

```
df_4 = df(x_3)
```

```
df_4 = 4.0833e-05
```

Part 1d: Plot f(x)

```
X_all = linspace(x_min,x_max,50);
F_all = f(X_all);
figure()
hold on
plot(X_all,F_all, 'DisplayName', 'f(x)')
% scatter([x_0,x_1,x_2,x_3],...
%         [f_1,f_2,f_3,f_4],...
%         'DisplayName', 'Newton-Raphson Method')
% hold off
legend()
xlabel('x')
ylabel('f(x)')
title('Problem 12.4 - Part 1d')
```



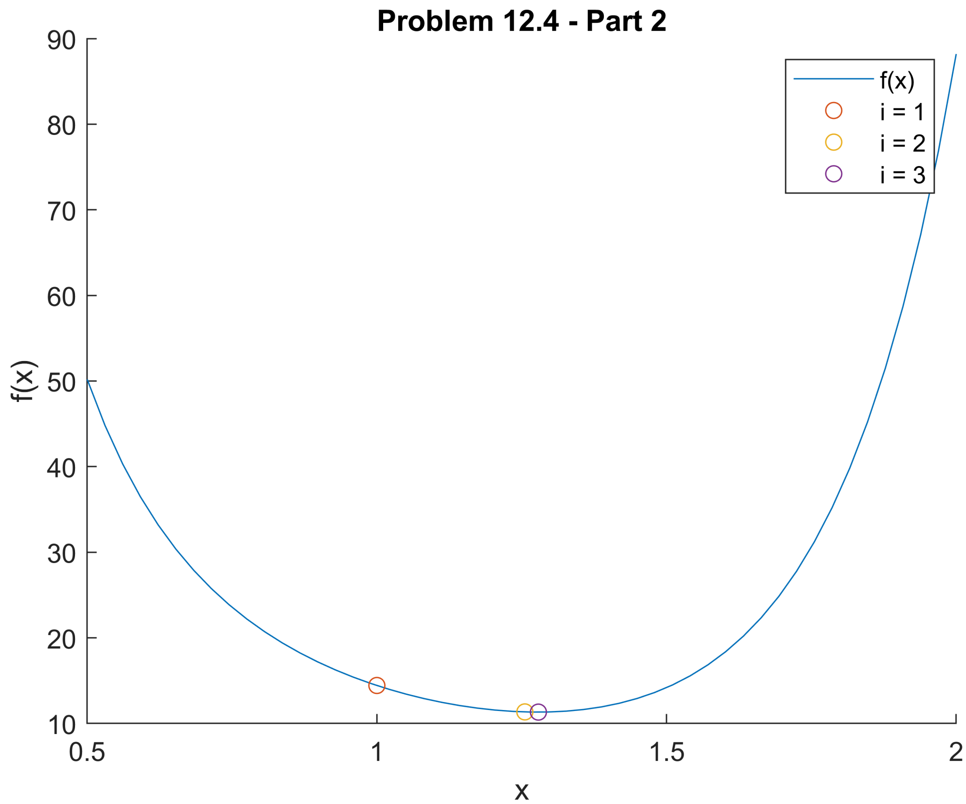
Part 2: Newton-Raphson in MATLAB

```
disp('Part 2 - Coded Newto-Raphson ------------------------------------')
```

```
Part 2 - Coded Newto-Raphson ------------------------------------
```

```
figure()
hold on
plot(X_all,F_all, 'DisplayName', 'f(x)')
legend()
xlabel('x')
ylabel('f(x)')
title('Problem 12.4 - Part 2')

x_0 = 1;
epsilon = 0.01;
max_itr = 10;
X(1) = x_0;
for i = 1:max_itr
    F(i) = f(X(i));
    DF(i) = df(X(i));
    DDF(i) = ddf(X(i));
    P(i) = p(X(i));
    X(i+1) = X(i) - P(i);
    scatter([X(i)],[F(i)],'DisplayName',['i = ',num2str(i)])
    if (abs(df(X(i+1))) < epsilon)
        disp(['Number of iterations: ', num2str(i)])
        disp(['Optimal Variable: ', num2str(X(i+1))])
        disp(['Optimal Value: ', num2str(f(X(i+1)))])
        break
    end
end
```




```
Number of iterations: 3
Optimal Variable: 1.2782
Optimal Value: 11.3265
```

Part 3 : fmincon validation

```
disp('Part 3 - fmincon validation ------------------------------------')
```

```
Part 3 - fmincon validation ------------------------------------
```

```
[x_star, f_star, exitflag, output] = fmincon(f,x_0,[],[],[],[],x_min,x_max)
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
x_star = 1.2782
f_star = 11.3265
exitflag = 1
output = struct with fields:
         iterations: 8
          funcCount: 19
    constrviolation: 0
           stepsize: 2.0168e-09
          algorithm: 'interior-point'
      firstorderopt: 2.7777e-07
       cgiterations: 0
            message: '↵Local minimum found that satisfies the constraints.↵↵Optimization completed because the objective
```

```
        bestfeasible: [1×1 struct]
```

Part 4: final notes

The cold I wrote goes through 3 iterations but fmincon goes through 8. fmincon is using the interior-point method, so more iterations makes sense.