MECH 6318 - Exam 2

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```
clear
close all
```

Problem 1

```
f = @(x1, x2) x1 + x1
```

```
f = function_handle with value:
    @(x1,x2)x1+x1
```

```
g1 = @(x1, x2) x1^2 + x2^2 - 4
```

```
g1 = function_handle with value:
    @(x1,x2)x1^2+x2^2-4
```

```
g2 = @(x1, x2) -x1 - 1
```

```
g2 = function_handle with value:
    @(x1,x2)-x1-1
```

```
L = @(x1, x2,lambda) ...
    f(x1, x2) ...
    + lambda(1) * g1(x1, x2) ...
    + lambda(2) * g2(x1, x2)
```

```
L = function_handle with value:
    @(x1,x2,lambda)f(x1,x2)+lambda(1)*g1(x1,x2)+lambda(2)*g2(x1,x2)
```

```
lambda = sym('lambda',[1,2]);
assume(lambda >= 0)

D_L1 = @(x1, x2) diff(L(x1, x2,lambda),x1);
D_L2 = @(x1, x2) diff(L(x1, x2,lambda),x2);

syms x1 x2
D_L_1 = D_L1(x1, x2)
```

D_L_1 = $2\lambda_1 x_1 - \lambda_2 + 2$

```
D_L_2 = D_L2(x1, x2)
```

D_L_2 = $2\lambda_1 x_2$

```
results = solve([...
    D_L1(x1, x2) == 0;...
    D_L2(x1, x2) == 0;...
```

```
    lambda(1) * g1(x1, x2) == 0;...
    lambda(2) * g2(x1, x2) == 0
    g1(x1, x2) == 0;...
    g2(x1, x2) == 0
    ], ...
    [x1, x2, lambda]...
);

x1 = double(results.x1)
```

```
x1 = 2×1
    -1
    -1
```

```
x2 = double(results.x2)
```

```
x2 = 2×1
    1.7321
   -1.7321
```

```
lambda_1 = double(results.lambda1)
```

```
lambda_1 = 2×1
     0
     0
```

```
lambda_2 = double(results.lambda2)
```

```
lambda_2 = 2×1
     2
     2
```

```

```

Problem 3

```
A = [350, 200;
     200, 350];
b = [-2000;
     -3000];

f = @(x) x'*A*x + b'*x;
df = matlabFunction(gradient(f([sym('x1');sym('x2')])));
ddf = matlabFunction(hessian(f([sym('x1');sym('x2')])));

f_x = f(sym('x',[2,1]))
```

```
f_x =
```
$$x_1\,(350\,\overline{x_1}+200\,\overline{x_2})-3000\,x_2-2000\,x_1+x_2\,(200\,\overline{x_1}+350\,\overline{x_2})$$

```
df_x = df(sym('x1'),sym('x2'))
```

```
df_x =
```
$$\begin{pmatrix}350\,x_1+200\,x_2+350\,\overline{x_1}+200\,\overline{x_2}-2000\\200\,x_1+350\,x_2+200\,\overline{x_1}+350\,\overline{x_2}-3000\end{pmatrix}$$

```
ddf_x = ddf()
```

```
ddf_x = 2×2
   700   400
   400   700
```

Part b

```
p = @(x) 2*A%ddf([x(1);x(2)]) \ df([x(1);x(2)]);
```

```
p = function_handle with value:
    @(x)2*A
```

```
p_x = p(sym('x',[2,1]))
```

```
p_x = 2×2
   700   400
   400   700
```

```
N = 4;
X = ones(2,N);
F = zeros(1,N);
DF = zeros(2,N);
% DDF = zeros(1,N);
P = zeros(2,N);
DF_norm = zeros(1,N);


x_0 = [0;0]
```

```
x_0 = 2×1
     0
     0
```

```
F(1,1) = f(x_0);
DF(:,1) = df(x_0(1),x_0(2));
% DDF(:,1) = ddf();
P(:,1) = ddf() \ df(x_0(1),x_0(2));
X(:,1) = x_0 - P(1,1);
DF_norm(1,1) = norm(DF(:,1));

for k = 2:N
    F(1,k) = f(X(:,k-1));
    DF(:,k) = df(X(1,k-1),X(2,k-1));
%     DDF(1,k) = ddf();
    P(:,k) = ddf()\df(X(1,k-1),X(2,k-1));
    X(:,k) = X(:,k-1) - P(:,k);
    DF_norm(1,k) = norm(F(:,k)-F(:,k-1));
end

X
DF
P
```

```
DF_norm
```

Problem 4

```
f = @(x1, x2) -7*x1 + 9*x2
```

```
f = function_handle with value:
    @(x1,x2)-7*x1+9*x2
```

```
g1 = @(x1, x2) -2*x1 + 6*x2 - 12
```

```
g1 = function_handle with value:
    @(x1,x2)-2*x1+6*x2-12
```

```
g2 = @(x1, x2) 7*x1 + x2 - 35
```

```
g2 = function_handle with value:
    @(x1,x2)7*x1+x2-35
```

```
g3 = @(x1, x2) x1^2 - 40
```

```
g3 = function_handle with value:
    @(x1,x2)x1^2-40
```

```
g4 = @(x1, x2) x2 - 5
```

```
g4 = function_handle with value:
    @(x1,x2)x2-5
```

```
g5 = @(x1, x2) -x1
```

```
g5 = function_handle with value:
    @(x1,x2)-x1
```

```
g6 = @(x1, x2) -x2
```

```
g6 = function_handle with value:
    @(x1,x2)-x2
```

```
L = @(x1, x2,lambda) ...
    f(x1, x2) ...
    + lambda(1) * g1(x1, x2) ...
    + lambda(2) * g2(x1, x2) ...
    + lambda(3) * g3(x1, x2) ...
    + lambda(4) * g4(x1, x2) ...
    + lambda(5) * g5(x1, x2) ...
    + lambda(6) * g6(x1, x2)
```

```
L = function_handle with value:
    @(x1,x2,lambda)f(x1,x2)+lambda(1)*g1(x1,x2)+lambda(2)*g2(x1,x2)+lambda(3)*g3(x1,x2)+lambda(4)*g4(x1,x2)+lambda(5
```

```
lambda = sym('lambda',[1,6]);
assume(lambda >= 0)

D_L1 = @(x1, x2) diff(L(x1, x2,lambda), x1);
```

```
D_L2 = @(x1, x2) diff(L(x1, x2,lambda), x2);

syms x1 x2
D_L_1 = D_L1(x1, x2)
```

D_L_1 = $7\,\lambda_2 - 2\,\lambda_1 - \lambda_5 + 2\,\lambda_3\,x_1 - 7$

```
D_L_2 = D_L2(x1, x2)
```

D_L_2 = $6\,\lambda_1 + \lambda_2 + \lambda_4 - \lambda_6 + 9$

```
results = solve([...
    D_L1(x1, x2) == 0;...
    D_L2(x1, x2) == 0;...
    lambda(1) * g1(x1, x2) == 0; ...
    lambda(2) * g2(x1, x2) == 0; ...
    lambda(3) * g3(x1, x2) == 0; ...
    lambda(4) * g4(x1, x2) == 0; ...
    lambda(5) * g5(x1, x2) == 0; ...
    lambda(6) * g6(x1, x2) == 0; ...
    g1(x1, x2) <= 0;...
    g2(x1, x2) <= 0;...
    g3(x1, x2) <= 0;...
    g4(x1, x2) <= 0;...
    g5(x1, x2) <= 0;...
    g6(x1, x2) <= 0
    ], ...
    [x1, x2, lambda]...
);

x1 = double(results.x1)
```

x1 = 5

```
x2 = double(results.x2)
```

x2 = 0

```
lambda_1 = double(results.lambda1)
```

lambda_1 = 0

```
lambda_2 = double(results.lambda2)
```

lambda_2 = 1

```
lambda_3 = double(results.lambda3)
```

lambda_3 = 0

```
lambda_4 = double(results.lambda4)
```

lambda_4 = 0

```
lambda_5 = double(results.lambda5)
```

```
lambda_5 = 0
```

```
lambda_6 = double(results.lambda6)
```

```
lambda_6 = 10
```

```
Lmin = L(x1,x2, [lambda_1,lambda_2,lambda_3,lambda_4,lambda_5,lambda_6]')
```

```
Lmin = -35
```

```
fmin = f(x1,x2)
```

```
fmin = -35
```