MECH 6318 - Homework 10

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```
clear
close all
```

Problem Definition

```
c = [5;
     8];
A = [1,  1;
     5,  9];
b = [ 6;
     45];
```

Problem 14.3 - Branch and Bound

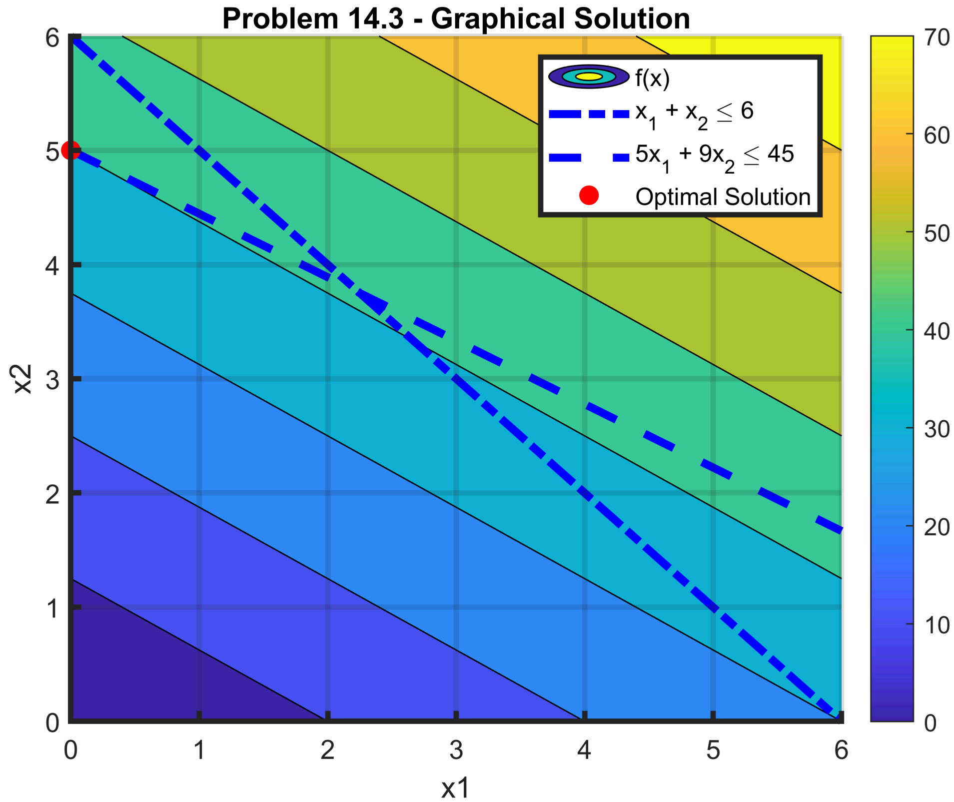
```
x1 = linspace(0, 6);
x2 = linspace(0, 6);
[X1, X2] = meshgrid(x1, x2);

F = c(1) * X1 + c(2) * X2;
c1 = @(x1) (b(1) - A(1,1)*x1)/A(1,2);
c2 = @(x1) (b(2) - A(2,1)*x1)/A(2,2);

xstar = [0; 5];

figure()
hold on
grid on
contourf(X1,X2,F,...
    'DisplayName', 'f(x)')
plot(x1, c1(x1), 'b-.',...
    'LineWidth', 3,...
    'DisplayName', 'x_1 + x_2 \leq 6')
plot(x1, c2(x1), 'b--',...
    'LineWidth', 3,...
    'DisplayName', '5x_1 + 9x_2 \leq 45')
plot(xstar(1), xstar(2), 'r.', ...
    'MarkerSize', 25,...
    'DisplayName','Optimal Solution')
legend
colorbar
ax = gca;
ax.Layer = 'top';
ax.LineWidth = 2;
title('Problem 14.3 - Graphical Solution')
xlabel('x1')
```

```
ylabel('x2')
```



Problem 14.6 - Relaxed Constraint Method

```
opt = @(x) - (c(1) * x(1) + c(2) * x(2))
```

```
opt = function_handle with value:
    @(x)-(c(1)*x(1)+c(2)*x(2))
```

```
x0 = [0; 0];
% A = A;
% b = b;
Aeq = [];
beq = [];
lb = [0; 0];
ub = [];

[xstar, fstar] = fmincon(opt,x0,A,b,Aeq,beq,lb,ub)
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
xstar = 2×1
```

```
    2.2500
    3.7500
fstar = -41.2500
```

```
xstar_int = [floor(xstar), ceil(xstar)];
```

Branch Testing

```
% Opt Problem Solution
opt_pblm = @(x0,lb,ub) fmincon(opt,x0,A,b,Aeq,beq,lb,ub);
```

Branch 1 - $x_1 \leq 2$

```
x0 = [0, 0];
lb = [0, 0];
ub = [2, inf];
[Xstar(:,1), Fval(1)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 2 - $x_1 \leq 2$ & $x \leq 3$

```
x0 = [2, 3];
lb = [0, 0];
ub = [2, 3];
[Xstar(:,2), Fval(2)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 3 - $x_1 \leq 2$ & $x \geq 4$

```
x0 = [2, 4];
lb = [0, 4];
ub = [2, inf];
[Xstar(:,3), Fval(3)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 4 - $x_1 \leq 1$ **&** $x \geq 4$

```
x0 = [2, 4];
lb = [0, 4];
ub = [1, inf];
[Xstar(:,4), Fval(4)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 5 - $x_1 \leq 1$ **&** $x \geq 5$

```
x0 = [2, 4];
lb = [0, 5];
ub = [1, inf];
[Xstar(:,5), Fval(5)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 6 - $x_1 \geq 3$

```
x0 = [3, 0];
lb = [3, 0];
ub = [inf, inf];
[Xstar(:,6), Fval(6)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

Branch 7 - $x_1 \geq 3$ **&** $x \geq 3$ **(not needed...)**

```
x0 = [3, 4];
lb = [3, 3];
ub = [inf, inf];
[Xstar(:,7), Fval(7)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.
```

```
<stopping criteria details>
```

Branch 7 - $x_1 \geq 3$ & $x \leq 3$ (not needed...)

```
x0 = [3, 3];
lb = [3, 0];
ub = [inf, 3];
[Xstar(:,8), Fval(8)] = opt_pblm(x0,lb,ub);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
results = table(Xstar(1,:)',Xstar(2,:)', Fval',...
    'VariableNames',{'x_1','x_2','f(x_1,x_2)'})
```

results = 8×3 table

	x_1	x_2	f(x_1,x_2)
1	2.0000	3.8889	-41.1111
2	2.0000	3.0000	-34.0000
3	1.8000	4.0000	-41.0000
4	1.0000	4.4444	-40.5556
5	0.0000	5.0000	-40.0000
6	3.0000	3.0000	-39.0000
7	3.0000	3.0000	-39.0000
8	3.0000	3.0000	-39.0000

Clearly this implies the solution is branch 5 which is the smallest function value with a feasabel integer solution. Therefore, the solution to the original maximization problem is:

```
xstar = [0; 5]
```

```
xstar = 2×1
     0
     5
```

```
fstar = -opt(xstar)
```

```
fstar = 40
```

Problem 14.9 - Cutting Plane

Simplex Table

```
syms f
T = [[A, eye(size(A,1)), b];...
    [-c',zeros(1,size(A,1)),f]]
```

T =

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 6 \\ 5 & 9 & 0 & 1 & 45 \\ -5 & -8 & 0 & 0 & f \end{pmatrix}$$

Simplex Relaxed Solution

```
R1 = T(1,:);
R2 = T(2,:);
R3 = T(3,:);

R2 = R2/R2(2);
T1 = [R1; R2; R3]
```

T1 =

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 6 \\ \frac{5}{9} & 1 & 0 & \frac{1}{9} & 5 \\ -5 & -8 & 0 & 0 & f \end{pmatrix}$$

```
R1 = R1 - R2(2)*R2;
R3 = R3 - R3(2)*R2;
T2 = [R1; R2; R3]
```

T2 =

$$\begin{pmatrix} \frac{4}{9} & 0 & 1 & -\frac{1}{9} & 1 \\ \frac{5}{9} & 1 & 0 & \frac{1}{9} & 5 \\ -\frac{5}{9} & 0 & 0 & \frac{8}{9} & f+40 \end{pmatrix}$$

```
R1 = R1/R1(1);
T3 = [R1; R2; R3]
```

T3 =

$$\begin{pmatrix} 1 & 0 & \frac{9}{4} & -\frac{1}{4} & \frac{9}{4} \\ \frac{5}{9} & 1 & 0 & \frac{1}{9} & 5 \\ -\frac{5}{9} & 0 & 0 & \frac{8}{9} & f+40 \end{pmatrix}$$

```
R2 = R2 - R2(1)*R1;
R3 = R3 - R3(1)*R1;
```

```
T4 = [R1; R2; R3]
```

T4 =

$$\begin{pmatrix} 1 & 0 & \frac{9}{4} & -\frac{1}{4} & \frac{9}{4} \\ 0 & 1 & -\frac{5}{4} & \frac{1}{4} & \frac{15}{4} \\ 0 & 0 & \frac{5}{4} & \frac{3}{4} & f+\frac{165}{4} \end{pmatrix}$$

Cutting Plane

We have

$$x_1 = \frac{1}{4}(9 - 9s_1 + s_2)$$

Which implies

$$x_1 + 2s_1 - 2 = \frac{1}{4}(1 - s_1 + s_2)$$

Therefore

$$\frac{1}{4}(-s_1 + s_2) \le -\frac{1}{4}$$

which implies

$$-s_1 + s_2 \le -1$$

Adding in the new slack variable s_3 we have:

$$-s_1 + s_2 + s_3 \le -1$$

New Table

```
R1 = [T4(1,1:4),  0, T4(1,end)];
R2 = [T4(2,1:4),  0, T4(2,end)];
R3 = [0, 0, 1, 1, 1, 1];
R4 = [T4(3,1:4),  0, T4(end,end)];
T5 = [R1; R2; R3; R4]
```

T5 =

$$\begin{pmatrix} 1 & 0 & \frac{9}{4} & -\frac{1}{4} & 0 & \frac{9}{4} \\ 0 & 1 & -\frac{5}{4} & \frac{1}{4} & 0 & \frac{15}{4} \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \frac{5}{4} & \frac{3}{4} & 0 & f+\frac{165}{4} \end{pmatrix}$$

Final Table

```
R1 = R1 - R1(3)*R3;
```

```
R2 = R2 - R2(3)*R3;
R4 = R4 - R4(3)*R3;
T6 = [R1; R2; R3; R4]
```

T6 =

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{5}{2} & -\frac{9}{4} & 0 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{5}{4} & 5 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{5}{4} & f+40 \end{pmatrix}$$

Which results in the following solution:

```
xstar = [0, 5]'
```

```
xstar = 2×1
     0
     5
```

```
fval = -opt(xstar)
```

```
fval = 40
```