MECH6318_Midterm-exam-Fall 2021

Tuesday, October 12, 2021 12:48 PM

Fall 2021| MECH 6318
Engineering Optimization
Mid-term Exam
12:00pm-5:00pm Tuesday, October 12, 2021
(100 points)

NAME: ______________________________ **NetID:** ______________________________

Any (i) communication of any kind with classmate, or (ii) plagiarization shall result in failing of the course!

-5 to 0 points reserved for Neatness and Professional Presentation. Show your work in detail and clearly.
Write your answers on papers; Scan your answers and save it as a PDF.
Please rename the file with the format: NetID_Lastname_firstname.pdf, e.g., yxl100000_liu_yuanzhi.pdf

Problem 1: True or False (*10pts* x *2* = *20 pts*) (You will lose 2 points for choosing a wrong statement or missing a true statement.)

1) Please choose all the correct statement(s) from the following:
   (A) For mixed-integer programming, the design variables are allowed to take only integer values, i.e., X∈Z.

   (B) For binary programming, -1 and 1 are the most commonly used binary number.
   (C) Newton method can always converge with different starting points.
   (D) If A is a symmetric matrix, then $A^T = A$.
   (E) For a convex function, the local minimum is also the global minimum.
   (F) $f(x) = \frac{1}{x-1},\ x > 1$ is convex.
   (G) x^a is convex for x > 0 if a ≥ 1 or a ≤ 0, and is concave for x > 0 if 0 ≤ a ≤ 1
   (H) Quasi-Newton method is a second-order method.
   (I) Golden section method is an interior point method.
   (J) A student realized something wrong when he was solving a convex problem using golden section search: the student received an interval of [1, 3], while the previous iteration had an iterval of [1, 2].
   (K) The system of constrain equations has a solution:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 3x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

2) Consider the optimization problem shown below:

$$\min_{X} f(X) = 5x_1^2 - 3x_1x_2 - 9x_2^2 + 9x_3^2 + 15$$
$$X = \{x_1, x_2, x_3\}$$

subject to

$$x_1^2 + x_2^2 = 10$$
$$x_1x_3 \le 100$$
$$4x_1 - 19x_2 = 50$$
$$3x_1 + 4x_3 = 100$$
$$x_1 + x_2 + x_3 \ge -10$$
$$-500 \le x_1, x_2, x_3 \le 500$$

When solving this problem in Matlab using ***fmincon***, please choose the correct statement(s) from the following:

(A) It is a nonlinear programming problem.
(B) The problem can be transformed into a one-variable optimization problem
(C) There are 4 linear and 1 nonlinear behavioral constraints.
(D) There are 2 inequality constraints and 3 equality constraints.
(E) All the linear constraints can be defined in the nonlinear constraint function.
(F) In Matlab code, the lower and upper bounds can be defined as

```
LB = [-500]; UB = [500];
```

(G) If the optimization tolerance is 1e-3, it is impossible to yield a solution with more than 4 decimal places, e.g., 0.6318.
(H) It has no impacts to the final solution if there is another unequal constraint:

$$x_1 + x_2 + x_3 \ge -12$$

(I) A different initial point can lead to a different solution.
(J) This problem can also be solved using ***linprog*** in Matlab.
(K) To solve this problem by hand, we can use the dual simplex algorithm.

Problem 2: Consider the following function and solve the questions below by hand.

$$f(\boldsymbol{x}) = 2x_1^3 + 2x_2^3 - 6x_1x_2$$

(1) What are the gradient and the Hessian of *f(x)*? *(6 pts)*

(2) What are the stationary point(s) of *f(x)*? (*6 pts)*

(3) Using the eigenvalues of the Hessian, can the stationary point(s) be classified into a minimum, maximum, or saddle point? *(6 pts)*

Problem 3: Linear regression can be formulated as:

$$y \approx a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_n x_p$$

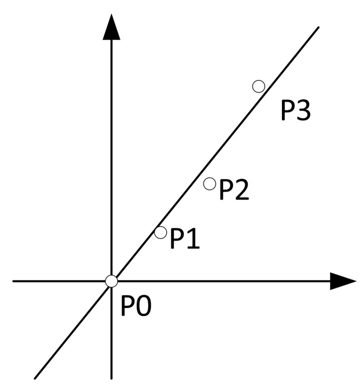
Assuming the i_{th} sample is $[x_1{}^{(i)}, x_2{}^{(i)} \ldots x_p{}^{(i)}, y^{(i)}]$ (the data has a dimension of P+1), the estimated i_{th} target variable $\hat{y}^{(i)}$ will be:

$$\hat{y}^{(i)} \approx a_0 + a_1 x_1{}^{(i)} + a_2 x_2{}^{(i)} + \ldots + a_n x_p{}^{(i)}$$

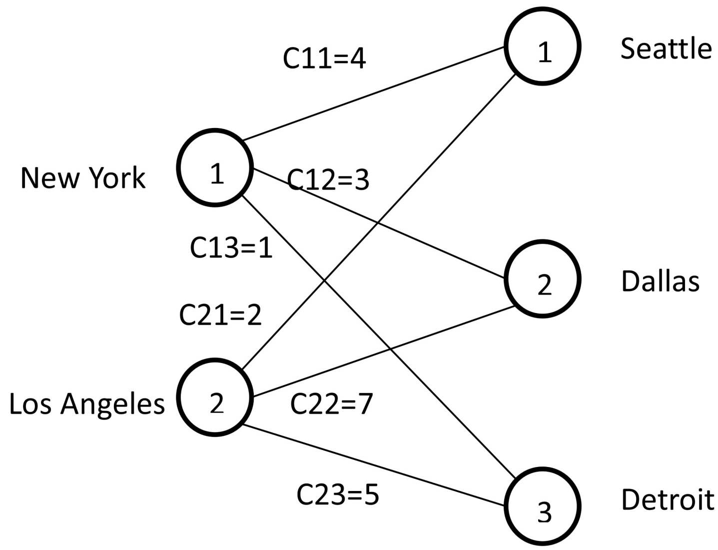
To find out the coefficient $[a_0, a_1 \ldots a_p]$ that best fits a set of data (n samples), it needs to minimize the mean squared error:

$$L(a_0, a_1 \ldots, a_p) = \frac{1}{n}\left[\left(y^{(1)} - \hat{y}^{(1)}\right)^2 + \left(y^{(2)} - \hat{y}^{(2)}\right)^2 + \ldots + \left(y^{(n)} - \hat{y}^{(n)}\right)^2\right]$$

1. Given three points in a two-dimension coordinate: P1(1,1), P2(2,2), P3(3,4), formulate the optimization problem for linear regression (You need to indicate the objective function, design variables, and constraints). *(6 pts)*
2. It is known that a given linear regression line passes through the origin P0(0,0), as shown in the figure below, try to solve the problem. *(6 pts)*
3. For a 2-dimensional quadratic regression, the formulation is $y \approx ax^2 + bx + c \;\; (a \neq 0)$, try to formulate the optimization problem by minimizing the mean squared error with the given points P1, P2, and P3 (you don't need to solve it). *(6 pts)*



Problem 4: A contractor has three sets of heavy construction equipment available at both New York and Los Angeles. He has construction jobs in Seattle, Dallas, and Detroit that require two, three, and one set of equipment, respectively. The shipping costs per set between cities i and j (Cij) are shown in the figure. Formulate and solve the problem of finding the shipping pattern that minimizes the cost. *(Hint: the problem is very straight forward, you don't need to use Simplex method here)(14 pts)*



Problem 5: A company manufactures glassware for wine, beer, champagne and whiskey. Each type of glassware requires time in the molding shop, time in the packaging shop, and a certain quantity of glass. The resources required to make each type of glass are given in the following table:

Design Variables	x_1 WINE GLASS	x_2 BEER GLASS	x_3 CHAMPAGNE GLASS	x_4 WHISKEY GLASS
Molding time	4 minutes	8 minutes	7 minutes	9 minutes
Packaging time	1 minute	1 minute	3 minutes	4 minutes
Glass	3 oz.	4 oz.	2 oz.	1 oz.
Selling price	\$6	\$10	\$9	\$22

At present, 610 minutes of molding time, 420 minutes of packaging time and 500 oz. of glass are available (x_i here represents the number of units of the ith type of glassware, and x_i is not necessarily to be an integer).

(1) Formulate / explain (but do not solve) an optimization problem to maximize the revenue of this company. The problem must include objective function, design variable(s), and constraint function(s) in the standard form of an optimization problem. *(7 pts)*

(2) If a 15% off discount is applied to the original selling price for all types of glassware, how would it affect the optimal solutions of this optimization problem? *(5 pts)*

(3) Transform the formulated problem into standard form in order to apply simplex method. *(4 pts)* (you don't need to solve it.)

Problem 6: Solve the following optimization problem using Golden Section method by hand. *(14 pts)*
(Assume convergence tolerance = 0.5)

$$\max_{x}\ (x - 4.2)^2$$

subject to

$$3 \leq x \leq 4$$

Problem 1

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1) Please choose all the correct statement(s) from the following:

(A) For mixed-integer programming, the design variables are allowed to take only integer values, i.e., X∈Z.

(B) For binary programming, -1 and 1 are the most commonly used binary number.

(C) Newton method can always converge with different starting points.

(D) If A is a symmetric matrix, then $A^T = A$.

(E) For a convex function, the local minimum is also the global minimum.

(F) $f(x) = \frac{1}{x-1}$, $x > 1$ is convex.

(G) x^a is convex for x > 0 if a ≥ 1 or a ≤ 0, and is concave for x > 0 if 0 ≤ a ≤ 1

(H) Quasi-Newton method is a second-order method.

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(J) A student realized something wrong when he was solving a convex problem using golden section search: the student received an interval of [1, 3], while the previous iteration had an iterval of [1, 2].

(K) The system of constrain equations has a solution:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 3x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

Fa se

Not all variables must be integers

[0, 1]

False, not always

Tr

T

Fa

True... must decrease by τ

True

$\text{Rank}\left(\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}\right) = 3$

$f(x) = \frac{1}{x-1}$, $x > 1$

$f(tx_1 + (1-t)x_2) \le t f(x_1) + (1-t) f(x_2)$

$\frac{1}{tx_1 + (1-t)x_2 - 1} \le \frac{t}{x_1 - 1} + \frac{1-t}{x_2 - 1}$

$\frac{(x_1-1)(x_2-1)(1)}{(x_1-1)(x_2-1)[tx_1+(1-t)x_2-1]} \le \frac{(t(x_2-1)+(1-t)(x_1-1))(tx_1+(1-t)x_2-1)}{(x_1-1)(x_2-1)[tx_1+(1-t)x_2-1]}$

$x_1x_2 - x_1 - y_2 + 1 \le x_1x_2 - x_1 - x_2 + 1 + (-t^2(x_1^2 - 2x_1x_2 + x_2^2) + t(x_1^2 - 2x_1x_2$

$x_1x_2 - x_1 - y_2 + 1 \le x_1x_2 - x_1 - x_2 + 1 + \underbrace{(t - t^2)}_{\ge 0 \ \forall t = [0,1]} \underbrace{(x_1^2 - 2x_1x_2 + x_2^2)}_{\ge 0 \ \forall x_1, x_2 \ge 1}$

∴ Convex $\forall x > 1$

Soln: D, E, F, G, H, J, K

2) Consider the optimization problem shown below:

$$\min_X f(X) = 5x_1^2 - 3x_1x_2 - 9x_2^2 + 9x_3^2 + 15$$
$$X = \{x_1, x_2, x_3\}$$

subject to

$$x_1^2 + x_2^2 = 10$$
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(I) A different initial point can lead to a different solution.

(J) This problem can also be solved using ***linprog*** in Matlab.

(K) To solve this problem by hand, we can use the dual simplex algorithm.

-500[1, 1, 1]

500[1, 1, 1]

False wording ignores all other settings...

it is convex

False

False

Soln: D, E, H

$+x_i^2))$

Problem 2

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Problem 2: Consider the following function and solve the questions below by hand.

$$f(\boldsymbol{x}) = 2x_1^3 + 2x_2^3 - 6x_1x_2$$

(1) What are the gradient and the Hessian of *f(x)*? *(6 pts)*

(2) What are the stationary point(s) of *f(x)*? *(6 pts)*

(3) Using the eigenvalues of the Hessian, can the stationary point(s) be classified into a minimum, maximum, or saddle point? *(6 pts)*

1)

$$f(x) = 2x_1^3 + 2x_2^3 - 6x_1x_2$$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 6x_1^2 - 6x_2 \\ -6x_1 + 6x_2^2 \end{bmatrix}$$

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 12x_1 & -6 \\ -6 & 12x_2 \end{bmatrix}$$

2) $\nabla f(x) = 0$

$$\frac{\partial f}{\partial x_1} = 6x_1^2 - 6x_2 = 0 \rightarrow x_2 = x_1^2$$

$$\frac{\partial f}{\partial x_2} = -6x_1 + 6x_2^2 = 0 \rightarrow x_1 = x_2^2$$

$$x_1 = (x_1^2)^2$$

wierd non-real roots

$$x_1^4 - x_1 = 0$$

$$\boxed{\begin{matrix} x_1 = 0,\ 1,\ -(-1)^{\frac{1}{3}},\ (-1)^{\frac{2}{3}}, \\ x_2 = 0,\ 1,\ (-1)^{\frac{2}{3}},\ (-1)^{\frac{4}{3}} \end{matrix}}$$

$x_2 = 0,\ 1,\ (-1)^{\cdot},\ (-1)^{\cdot}$

3) $H_f = \begin{bmatrix} 12x_1 & -6 \\ -6 & 12x_2 \end{bmatrix}$

i) $x_1 = 0$
$x_2 = 0$

$$H_f = \begin{bmatrix} 0 & -6 \\ -6 & 0 \end{bmatrix} \qquad \lambda_1 = 6,\ \lambda_2 = -6 \Rightarrow \text{Saddel Point}$$

ii) $x_1 = 1$
$x_2 = 1$

$$H_f = \begin{bmatrix} 12 & -6 \\ -6 & 12 \end{bmatrix} \qquad \lambda_1 = 18,\ \lambda_2 = 6 \Rightarrow \text{Minimum}$$

Problem 3

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Problem 3: Linear regression can be formulated as:

$$y \approx a_0 + a_1 x_1 + a_2 x_2 + ... + a_n x_p$$

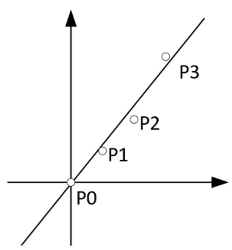
Assuming the i_{th} sample is $[x_1{}^{(i)}, x_2{}^{(i)} ... x_p{}^{(i)}, y^{(i)}]$ (the data has a dimension of P+1), the estimated i_{th} target variable $\hat{y}^{(i)}$ will be:

$$\hat{y}^{(i)} \approx a_0 + a_1 x_1{}^{(i)} + a_2 x_2{}^{(i)} + ... + a_n x_p{}^{(i)}$$

To find out the coefficient $[a_0, a_1 ... a_p]$ that best fits a set of data (n samples), it needs to minimize the mean squared error:

$$L(a_0, a_1 ..., a_p) = \frac{1}{n}[(y^{(1)} - \hat{y}^{(1)})^2 + (y^{(2)} - \hat{y}^{(2)})^2 + ... + (y^{(n)} - \hat{y}^{(n)})^2]$$

1. Given three points in a two-dimension coordinate: P1(1,1), P2(2,2), P3(3,4), formulate the optimization problem for linear regression (You need to indicate the objective function, design variables, and constraints). *(6 pts)*
2. It is known that a given linear regression line passes through the origin P0(0,0), as shown in the figure below, try to solve the problem. *(6 pts)*
3. For a 2-dimensional quadratic regression, the formulation is $y \approx ax^2 + bx + c$ $(a \neq 0)$, try to formulate the optimization problem by minimizing the mean squared error with the given points P1, P2, and P3 (you don't need to solve it). *(6 pts)*



Linear Regression:

$P = 3$ dim

$y^{(1)} = P_1(1,1)$

$y^{(2)} = P_2(2,2)$

$y^{(3)} = P_3(3,4)$

$\hat{y}^{(i)} = a_0 + a_1 x_1^{(i)} + a_2 x_2^{(i)}$

$L(a_0, a_1, a_2) = \frac{1}{n}\left[\left(y^{(1)} - \hat{y}^{(1)}\right)^2 + \left(y^{(2)} - \hat{y}^{(2)}\right)^2 + \left(y^{(3)} - \hat{y}^{(3)}\right)^2\right]$

$= \frac{1}{3}\Big[\left(P_1(1,1) - a_0 - a_1(1) - a_2(1)\right)^2$

$+ \left(P_2(2,2) - a_0 - a_1(2) - a_2(2)\right)^2$

$+ \left(P_3(3,4) - a_0 - a_1(3) - a_2(4)\right)^2\Big]$

1)

min $L(a_0, a_1, a_2) = \frac{1}{}\left[\left(y^{(1)} - \hat{y}^{(1)}\right)^2 + \left(y^{(2)} - \hat{y}^{(2)}\right)^2 + \left(y^{(3)} - \hat{y}^{(3)}\right)^2\right]$

1)

$$\min_{a_0, a_1, a_2} L(a_0, a_1, a_2) = \frac{1}{n}\left[\left(y^{(1)} - \hat{y}^{(1)}\right)^2 + \left(y^{(2)} - \hat{y}^{(2)}\right)^2 + \left(y^{(3)} - \hat{y}^{(3)}\right)^2\right]$$

$$y^{(1)} = P_1(1,1)$$
$$y^{(2)} = P_2(2,2)$$
$$y^{(3)} = P_3(3,4)$$
$$\hat{y}^{(i)} = a_0 + a_1 x_1^{(i)} + a_2 x_2^{(i)}, \quad i = 1, 2, 3$$

2) $P_0(0,0) = 0$

$$y^{(0)} = 0 \Rightarrow a_0 = 0$$

$$\min_{a_1, a_2} \left(\frac{1}{3}\right)\left[\left(P_1(1,1) - a_1 - a_2\right)^2 + \left(P_2(2,2) - 2a_1 - 2a_2\right)^2 + \left(P_3(3,4) - 3a_1 - 4a_2\right)^2\right]$$

$$\left(\frac{1}{3}\right)\Big[P_1(1,1)^2 + P_1(1,1)(-a_1 - a_2) + 2a_1a_2 + a_1^2 + a_2^2$$
$$+ P_2(2,2)^2 + P_2(2,2)(-4a_1 - 4a_2) + 8a_1a_2 + 4a_1^2 + 4a_2^2$$
$$+ P_3(3,4)^2 + P_3(3,4)(-6a_1 - 8a_2) + 24a_1a_2 + 9a_1^2 + 16a_2^2\Big]$$

$$\left(\frac{1}{3}\right)\Big[P_1(1,1)\left(P_1(1,1) - a_1 - a_2\right)$$
$$+ P_2(2,2)\left(P_2(2,2) - 4a_1 - 4a_3\right)$$
$$+ P_3(3,4)\left(P_3(3,4) - 6a_1 - 8a_3\right)$$
$$+ 34a_1a_2 + 14a_1^2 + 21a_2^2\Big]$$

Hard to solve by hand..

3)

$$\min_{a_1, a_2} \left(\frac{1}{3}\right)\left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}^T \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{bmatrix} -P_1(1,1) - 4P_2(2,2) - 6P_3(3,4) \\ -P_1(1,1) - 4P_2(2,2) - 8P_3(3,4) \end{bmatrix}^T \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + P_1(1,1)^2 + P_2(2,2)^2 + P_3(3,4)^2\right]$$

Problem 4

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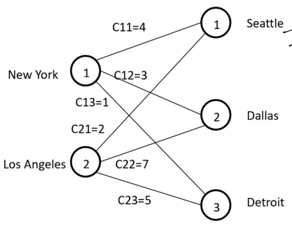
Problem 4: A contractor has three sets of heavy construction equipment available at both New York and Los Angeles. He has construction jobs in Seattle, Dallas, and Detroit that require two, three, and one set of equipment, respectively. The shipping costs per set between cities i and j (Cij) are shown in the figure. Formulate and solve the problem of finding the shipping pattern that minimizes the cost. *(Hint: the problem is very straight forward, you don't need to use Simplex method here)(14 pts)*



3 @ 2

2 @ 1

3 @ 2

1 @ 3

$$x_0 = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad x_f = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \\ 1 \end{bmatrix} \quad x_f = x_0 + Ax$$

$$C = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 7 & 5 \end{bmatrix}$$

$$A = \mathrm{diag}\left(a_{11}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + a_{12}\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + a_{13}\begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + a_{21}\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + a_{22}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + a_{23}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$X = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} -a_{11}-a_{12}-a_{13} & & & & 0 \\ & -a_{21}-a_{22}-a_{23} & & & \\ & & a_{11}+a_{21} & & \\ & & & a_{12}+a_{22} & \\ 0 & & & & a_{13}+a_{23} \end{bmatrix}$$

to lin Prog

$$c = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \\ 7 \\ 5 \end{bmatrix} \quad x = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}$$

$$A = \mathrm{diag}\begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} X$$

Mabye nessicary

Soln.

$$\boxed{\begin{aligned} &\min_{x} \; c^T x \\ &Ax = x_f - x_0 \end{aligned}}$$

Altednative:

$$\begin{bmatrix} a_{11}+a_{12}+a_{13} \\ a_{21}+a_{22}+a_{23} \\ a_{11}+a_{21} \\ a_{12}+a_{22} \\ a_{13}+a_{23} \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -3 \\ -3 \\ -3 \\ 2 \\ 3 \\ 1 \end{bmatrix}}_{b}$$

$$C^T = \begin{bmatrix} 4 & 3 & 1 & 2 & 7 & 5 \end{bmatrix}$$

$$\boxed{\begin{aligned} \min_{x} \quad & C^T x \\ & Ax = b \end{aligned}}$$

$\leftarrow$ ultimently this assumes that the "equipment" is interchangable and allows

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If Integer assumed and restricted to the simple, purely positive movement, a search of all feasable options can be done:

$$X = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$I_2 X = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$X I_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}$$

T

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad c^T x$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \qquad 24$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad 24$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad 30$$

$$\begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad 18$$

optimal solutions given assumptions

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix} \qquad 30$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \qquad 18$$

⋮ Infite more if allow -1
(which would break assumptions

Problem 5

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Problem 5: A company manufactures glassware for wine, beer, champagne and whiskey. Each type of glassware requires time in the molding shop, time in the packaging shop, and a certain quantity of glass. The resources required to make each type of glass are given in the following table:

Design Variables	x_1 WINE GLASS	x_2 BEER GLASS	x_3 CHAMPAGNE GLASS	x_4 WHISKEY GLASS
Molding time	4 minutes	8 minutes	7 minutes	9 minutes
Packaging time	1 minute	1 minute	3 minutes	4 minutes
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Selling price	\$6	\$10	\$9	\$22

At present, 610 minutes of molding time, 420 minutes of packaging time and 500 oz. of glass are available (x_i here represents the number of units of the ith type of glassware, and x_i is not necessarily to be an integer).

(1) Formulate / explain (but do not solve) an optimization problem to maximize the revenue of this company. The problem must include objective function, design variable(s), and constraint function(s) in the standard form of an optimization problem. *(7 pts)*

(2) If a 15% off discount is applied to the original selling price for all types of glassware, how would it affect the optimal solutions of this optimization problem? *(5 pts)*

(3) Transform the formulated problem into standard form in order to apply simplex method. *(4 pts)* (you don't need to solve it.)

1)

$$c = \begin{bmatrix} 6 \\ 10 \\ 9 \\ 22 \end{bmatrix}$$

Revenue from each type of class

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The number of each glass wear to be prodced (not restricting to integer)

Molding time

$$A = \begin{bmatrix} 4 & 8 & 7 & 9 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{bmatrix}$$

Packaging time

Glass

$$b = \begin{bmatrix} 610 \\ 420 \\ 500 \end{bmatrix}$$

$$\boxed{\begin{array}{l} \max\limits_{X} \; c^T X \\ \\ Ax \leq b \end{array}}$$

maximize revenue

Remain within limiting reacorces

2) The optimal solution value will decrease

2) The optimal solution value will decrease by 15% but the selections of production (optimal variables) will remain the same.

3) Aleady in standard form...

$$\max_{x} \begin{bmatrix} 6 & 10 & 9 & 22 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 & 7 & 9 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \leq \begin{bmatrix} 610 \\ 420 \\ 500 \end{bmatrix}$$

Problem 6

Tuesday, October 12, 2021 12:56 PM

Problem 6: Solve the following optimization problem using Golden Section method by hand. *(14 pts)* (Assume convergence tolerance = 0.5)

$$\max_{x} (x - 4.2)^2$$

subject to

$$3 \leq x \leq 4$$

$$\min_{x} f(x) = -(x^2 - 8.4x + 17.64)$$

$3 \leq x \leq 4$ $\quad \varepsilon = 0.5$ $\quad \tau \simeq 0.618$

N	a	b	b-a	l	r	$f(l) = -x^2 + 8.4x - 17.64$	$f(r) = -x^2 + 8.4x - 17.64$	$f(l) > f(r)$ $a = l, b = b, l = r, r = a + b(l-r)$	$f(l) < f r$ $a = a, b = r, r = l, l = b - \tau(b-a)$
1	3	4	1	3.382	3.618	-0.669	-0.338		X
2	3	3.618	0.618	3.236	3.382	-0.929	-0.669		X
3	3	3.382	0.382						
4									
5									
6									

$b - a < \varepsilon$

$x^* = \frac{a+b}{2}$ $\quad \boxed{x^* = 3.118}$

$f(x^*) = -x^{*2} + 8.4x^* - 17.64$ $\quad f(x^*) \simeq -1.17 \longrightarrow \max\ -f(x) \rightarrow \boxed{f^* = 1.17}$

$$\boxed{\begin{array}{l} \underline{\text{Soln.}} \\ x^* = 3.118 \\ f^* = 1.17 \end{array}}$$