Lecture - 10

PRACTICING OPTIMIZATION

Reference: Book Chapter 10

- In Class Code Problem
  - Problem 8.4 (GA Solver)
  - Problem 19.3 (Multiobjective GA Solver)
  - Problem 6.1 (Bi-objective problem)
  - Problem 6.5 (Tri-objective problem)
  - Problem 6.11 (Practical two bar problem)
- Work on the HW #7 problems if you finished the problems

SAMPLE PROJECTS FROM PREVIOUS YEARS

- Power Optimization For A Wind Turbine
- Wind Turbine Blade Optimization
- Cost Optimization For 3D Printed Parts
- Smart Home Energy Optimization
- Energy Management in Smart Grid
- Optimization Of Onshore Wind Farm Layouts For Wake Loss And Cabling Costs
- Optimization Of Dynamical Systems
- Optimization Of Photovoltaic Cell
- Design And Optimization Of A Low Loss Optical Fiber Coupler
- Optimization of Drone Delivery System
- Heat Exchanger Optimization
- River Locks and Dams Optimization

ENERGY SYSTEMS EXAMPLES

- Wind Energy System Optimization
- Solar Energy System Optimization
- Building Energy System Optimization
- Smart Grid and Power Systems Problems

WIND ENERGY EXAMPLE: INDIVIDUAL TURBINE OUTPUT

- Effective velocity of wind approaching Turbine-*j*:*

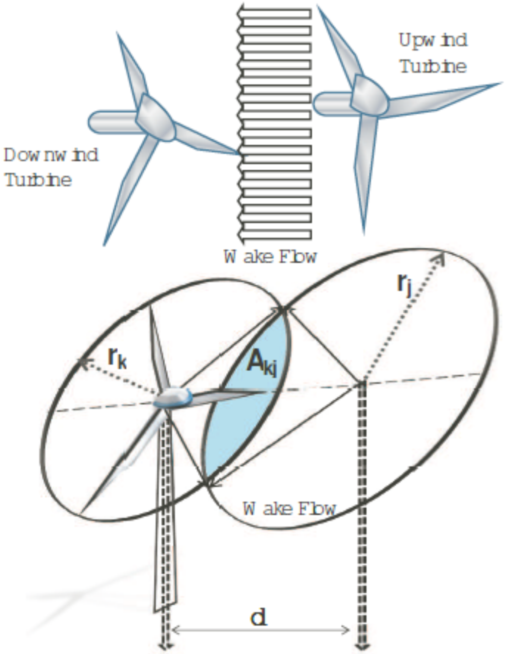
$$U_j = U_0 - \sqrt{\sum_k \frac{A_{kj}}{A_j}(U_0 - U_{kj})^2}$$

U_0: free-stream velocity

U_{kj}: velocity of wake created by turbine-*k* and approaching turbine-*j*

A_{kj}: overlapping area between the wake front and the downstream turbine swept area

Accounts for wake merging and partial wake-rotor overlap



Partial wake-rotor overlap

- The power generated by turbine-*j* is given by: $P_j = k_g k_b C_p' \left(\frac{1}{2}\rho\pi\frac{D_j^2}{4}U_j^3\right)$

or given by the power curve reported by the manufacturer

WIND ENERGY EXAMPLE: ENERGY PRODUCTION MODEL

- **Annual Energy Production** of a farm is given by:

Probability of wind condition

$$E_{farm} = (365 \times 24) \int_{0^\circ}^{360^\circ} \int_{0}^{U_{max}} \boxed{P_{farm}(U, \theta)}\, \boxed{p(U, \theta)}\, dU\, d\theta$$

- This integral equation can be numerically* e

Wind farm power generation

$$E_{farm} = (365 \times 24) \sum_{i=1}^{N_p} \boxed{P_{farm}(U^i, \theta^i)}\, \boxed{p(U^i, \theta^i)}\, \Delta U \Delta\theta,$$

$$\Delta U \Delta\theta = U_{max} \times 360^\circ / N_p$$

- **Capacity Factor**: Measure of overall efficiency of the wind farm

$$\boxed{CF = \frac{E_{farm}}{(365 \times 24) \sum_{j=1}^{N} P_{rj}} \times 100}$$

*Monte Carlo Integration

OFFSHORE WIND ENERGY HARVESTING

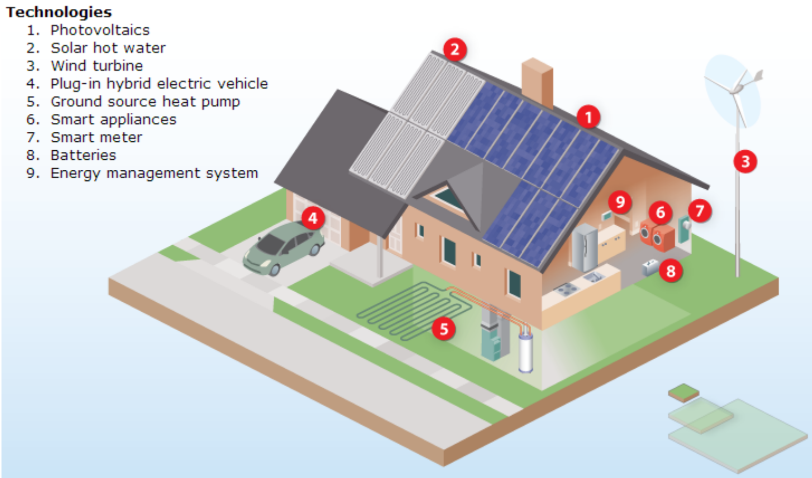
- Trans-disciplinary design exploration enabling economically-viable, grid-integrative, and environment-friendly offshore wind energy harvesting
  - Advance our fundamental understanding of the **trade-offs** between the key *engineering challenges, economic objectives, and environmental impact* in harvesting offshore wind energy, under different penetration scenarios
  - Develop a unique interdisciplinary multi-criteria design strategy for macro/micro-siting of offshore turbines and planning of associated infrastructure to accomplish an economically favorable **balance** between overall energy production, energy integration, and net impact on marine environment



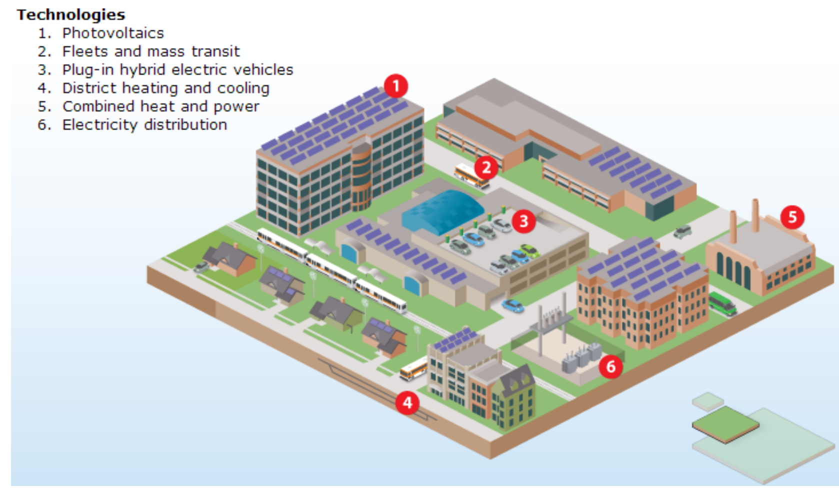
*Source: Google

SMART HOME, COMMUNITIES, AND CITIES

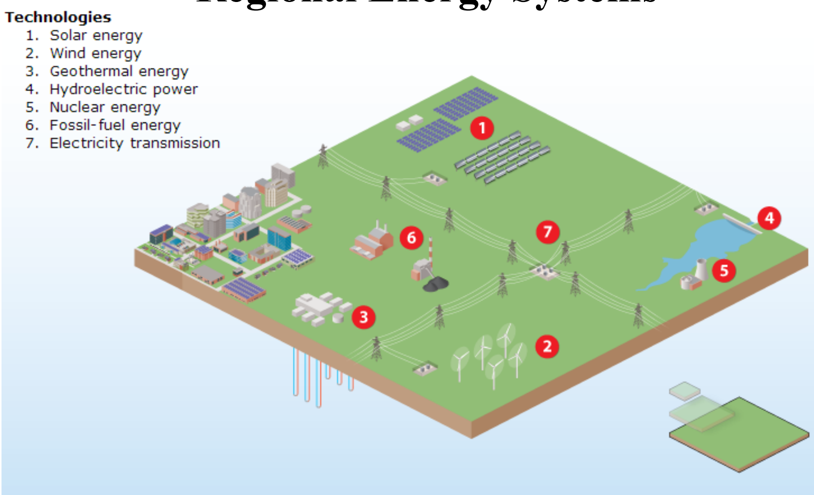
Smart Home Energy Systems



Smart Community Energy Systems



Regional Energy Systems

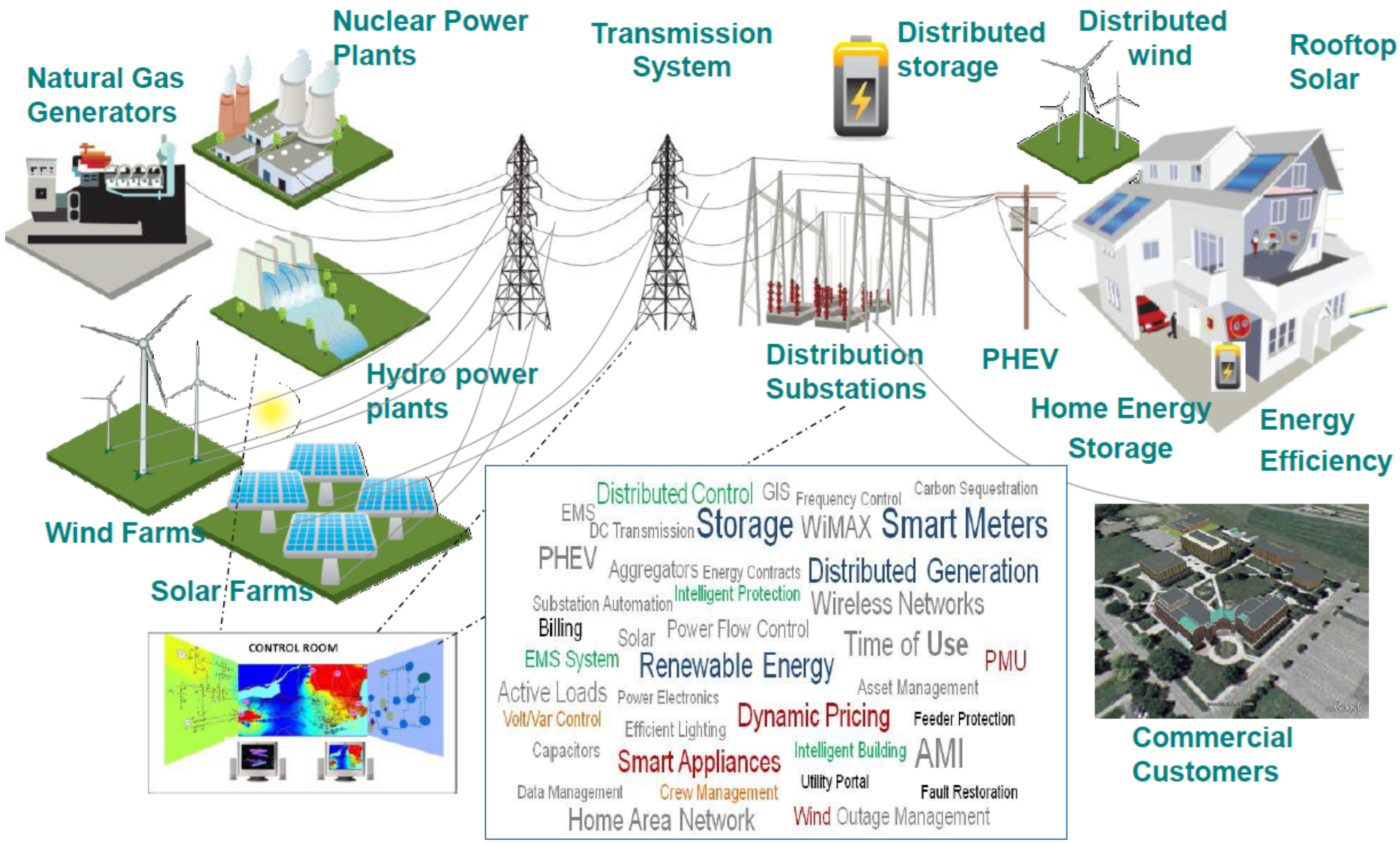


System-of-Systems

*Source: NREL

POWER SYSTEMS PROBLEMS

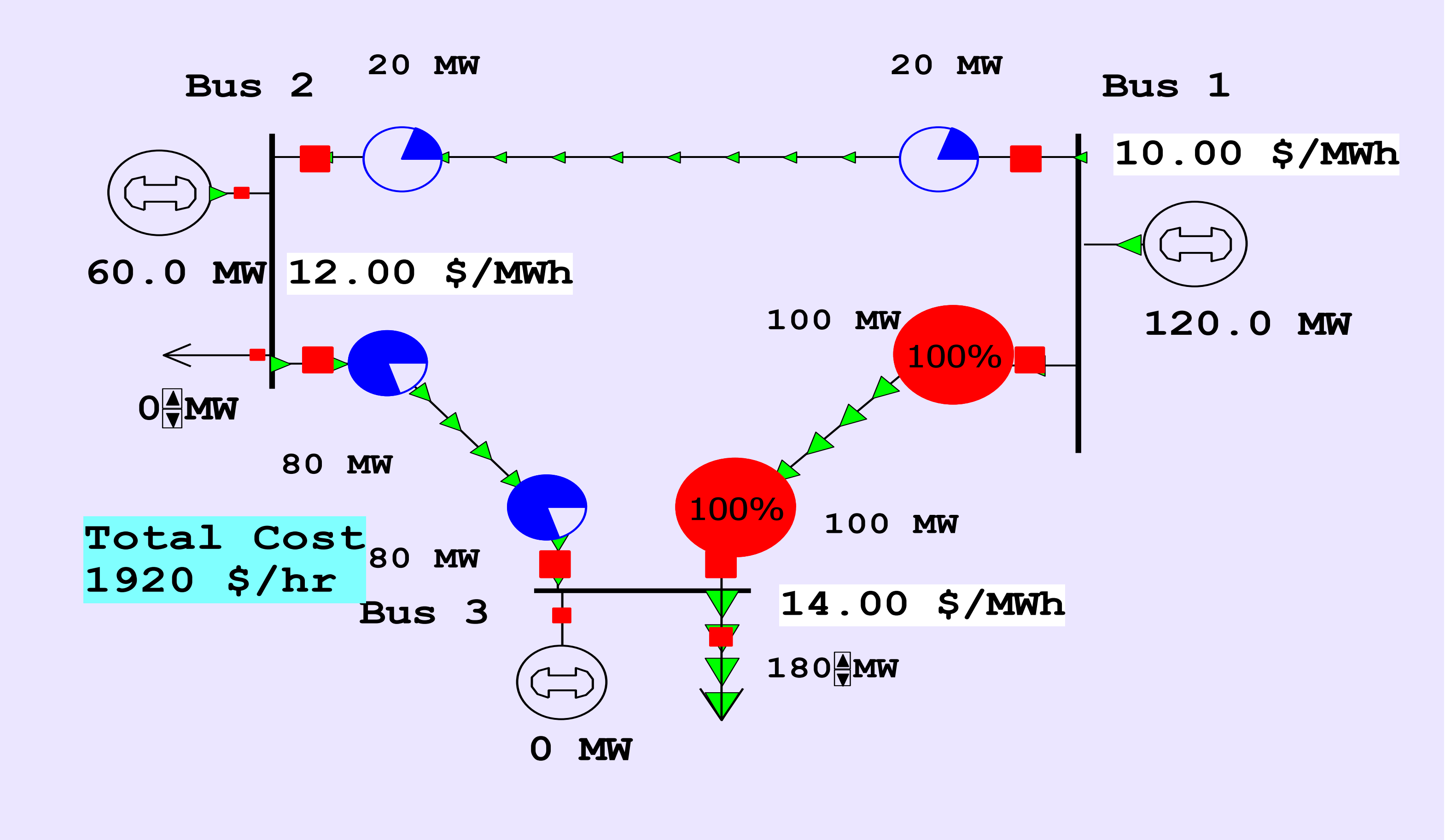
- Modernization of the electricity infrastructure and the transition of the electricity industry to a paradigm based on large penetrations of renewable energy and energy efficiency technologies.



*Source: NREL

OPTIMAL POWER FLOW

- The Optimal Power Flow (OPF) model represents the problem of determining the best operating levels for electric power plants in order to meet demands given throughout a transmission network, usually with the objective of minimizing operating cost.



OPTIMAL POWER FLOW

Three generator controls P_1, P_2, P_3

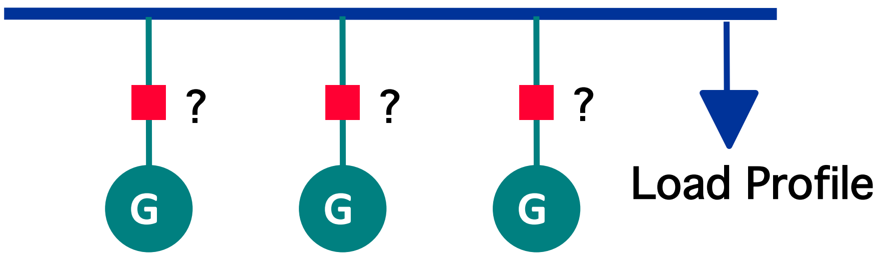
Incremental costs of 10, 12, 20 $/MWh, respectively

$$\min: \quad 10P_1 + 12P_2 + 20P_3$$

$$\begin{aligned} \text{st:} \quad & P_1 + P_2 + P_3 &= 180 \quad & \text{Power Balance} \\ & 0.66P_1 + 0.33P_2 &\le 100 \quad & \text{Line 1-3 Constraint} \\ & P_1, P_2, P_3 &\ge 0 & \end{aligned}$$

UNIT COMMITMENT PROBLEM

- Given load profile
(e.g. values of the load for each hour of a day)
- Given set of units available
- When should each unit be started, stopped and how much should it generate to meet the load at minimum cost?



UNIT COMMITMENT PROBLEM

- Unit 1:
  - $P_{Min} = 250$ MW, $P_{Max} = 600$ MW
  - $C_1 = 510.0 + 7.9\ P_1 + 0.00172\ P_1^2$ \$/h
- Unit 2:
  - $P_{Min} = 200$ MW, $P_{Max} = 400$ MW
  - $C_2 = 310.0 + 7.85\ P_2 + 0.00194\ P_2^2$ \$/h
- Unit 3:
  - $P_{Min} = 150$ MW, $P_{Max} = 500$ MW
  - $C_3 = 78.0 + 9.56\ P_3 + 0.00694\ P_3^2$ \$/h
- What *combination* of units 1, 2 and 3 will produce 550 MW at minimum cost?
- How much should each unit in that combination generate?

TRANSPORTATION PROBLEM

Suppose that there are m origins $R_1, R_2, \cdots, R_m$ (e.g., warehouses) and n destinations, $D_1, D_2, \cdots, D_n$ (e.g., factories). Let a_i be the amount of a commodity available at origin i $(i = 1, 2, \ldots, m)$ and b_j be the amount required at destination j $(j = 1, 2, \ldots, n)$. Let c_{ij} be the cost per unit of transporting the commodity from origin i to destination j. The objective is to determine the amount of commodity (x_{ij}) transported from origin i to destination j such that the total transportation costs are minimized. This problem can be formulated mathematically as

$$\text{Minimize } f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \tag{4.52}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \ldots, m \tag{4.53}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \ldots, n \tag{4.54}$$

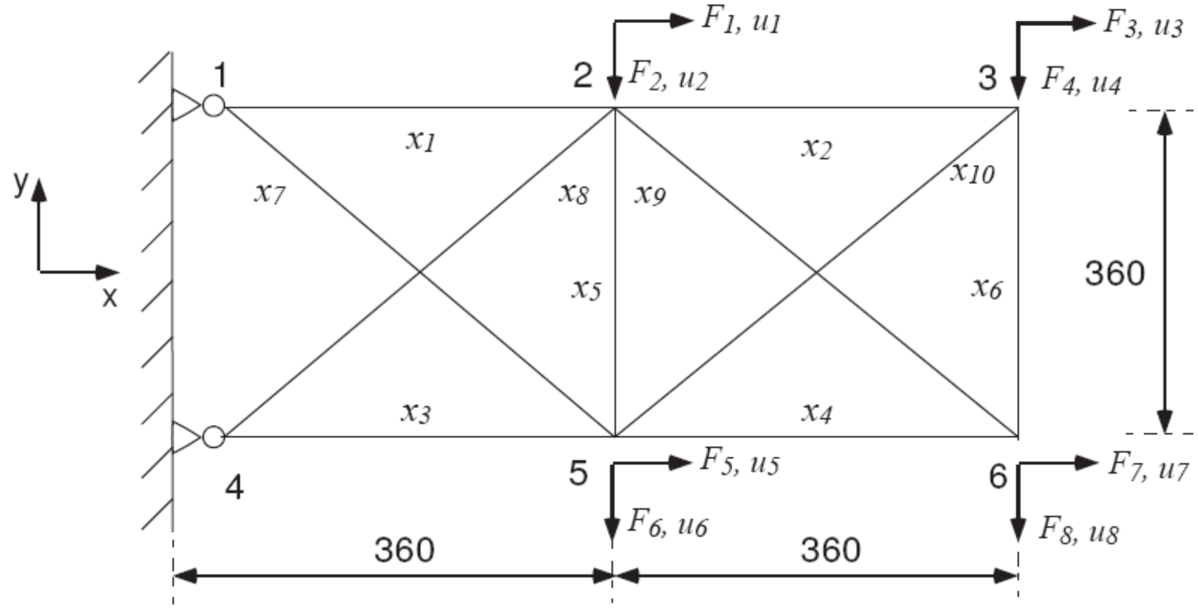
$$x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \tag{4.55}$$

Clearly, this is a LP problem in mn variables and $m + n$ equality constraints.

➢ Structural Example: The optimal design for the ten-bar truss

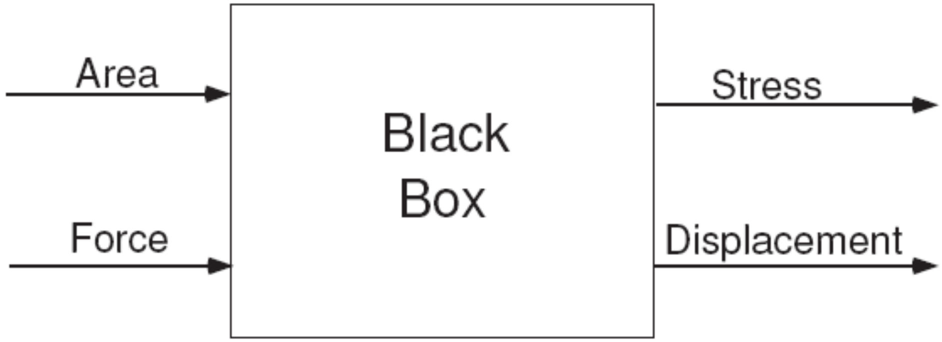
Objective:

- Minimizing the mass of the configuration and the maximum stress for a given set of conditions.
- Young's modulus: $E = 1\times10^6 N/mm^2$, the maximum allowable stress: $\sigma_{ult} = \pm100MPa$, the maximum allowable deflection is $\delta_{max} = \pm2mm$.



MECHANICAL ENGINEERING EXAMPLE

- ***blackbox–10bar.m***



- ***Loading conditions***

Forces at Nodes $\times 10^5$ N									
	i	1	2	3	4	5	6	7	8
Loading Conditions	1	0.6154	0.0579	0.0153	0.8381	0.1934	0.4966	0.7271	0.7948
	2	0.7919	0.3529	0.7468	0.0196	0.6822	0.8998	0.3093	0.9568
	3	0.9218	0.8132	0.4451	0.6813	0.3028	0.8216	0.8385	0.5226
	4	0.7382	0.0099	0.9318	0.3795	0.5417	0.6449	0.5681	0.8801
	5	0.1763	0.1389	0.4660	0.8318	0.1509	0.8180	0.3704	0.1730
	6	0.4057	0.2028	0.4186	0.5028	0.6979	0.6602	0.7027	0.9797
	7	0.9355	0.1987	0.8462	0.7095	0.3784	0.3420	0.5466	0.2714
	8	0.9169	0.6038	0.5252	0.4289	8600	0.2897	0.4449	0.2523
	9	0.4103	0.2722	0.2026	0.3046	0.8537	0.3412	0.6946	0.8757
	10	0.8936	0.1988	0.6721	0.1897	0.5936	0.5341	0.6213	0.7373

MECHANICAL ENGINEERING EXAMPLE

1. Generate the row vector F that contains all the 8 forces:

   F = [61540 5790 1530 83810 19340 49660 72710 79480]

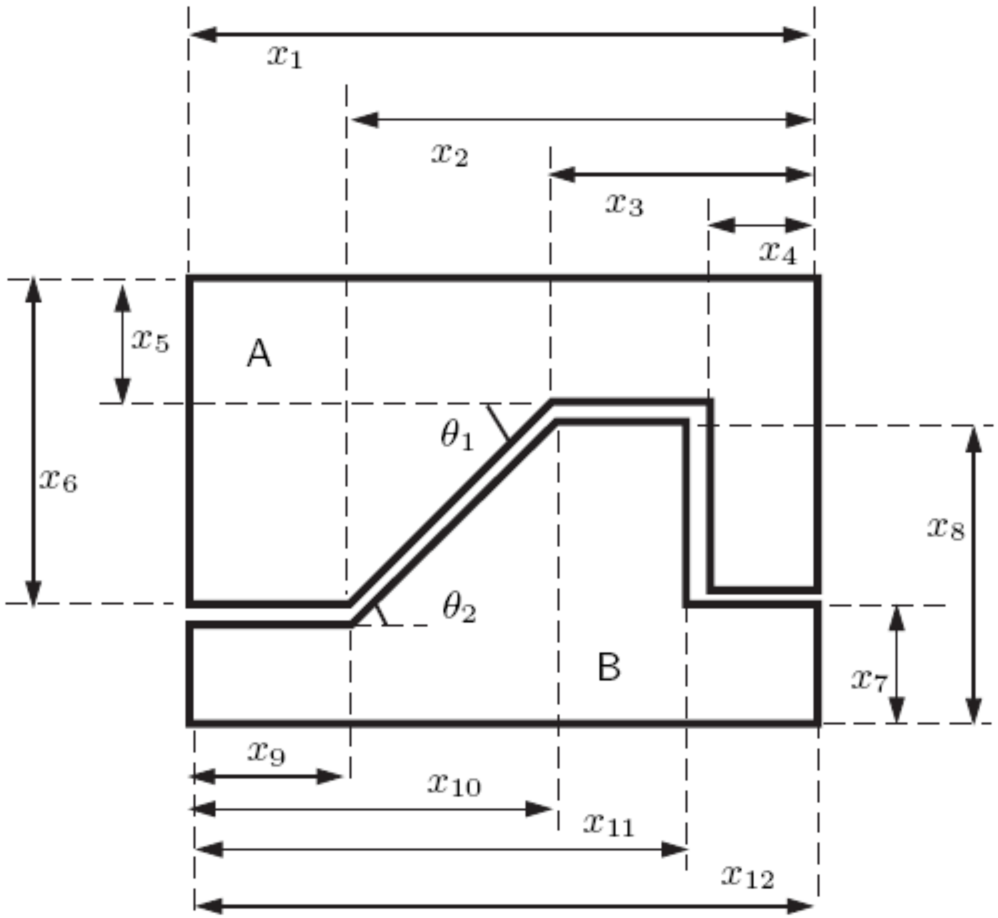
2. Generate the row vector x that contains all the 10 areas:

   x = [16.49 4.67 13.34 0.40 0.10 0.10 17.57 11.36 4.06 16.19]

3. Use these values in *blackbox−10bar.m to determine* (1) stresses and (2) deflections.
4. Double the value of the forces and use these new values in *blackbox−10bar.m to get the stresses and deflections.*
5. Halve the areas and repeat to get the corresponding stresses and deflections.
6. Minimize the total mass of the truss for the loading condition specified in Part 1 and the initial areas specified in Part 2.
7. Minimize the total mass for all ten loading conditions individually.
8. Minimize the maximum stress along with the total mass of the truss for the loading condition specified in Part 1 and the initial areas specified in Part 2. Plot the Pareto frontier for 11 solutions.

MECHANICAL ENGINEERING EXAMPLE

- **Tolerance Allocation Problem:** Tradeoff between the tolerances and the manufacturing cost.



MECHANICAL ENGINEERING EXAMPLE

- The tolerances of X_1 through X_{12} are assumed to be six times their respective standard deviations.
- The nominal or the mean values of X_1 through X_{12} are given as Xn = [50.0, 40.00125, 20.05, 9.9985, 9.9985, 30.0, 10.0, 30.0, 10.05, 30.0, 40.0, 50.0].
- The requirement that determines the quality of the assembly is θ_1 and θ_2 must be as close to each other as possible.

MECHANICAL ENGINEERING EXAMPLE

- Design variables: the standard deviations of the dimensions.
- The first objective, J_1: Minimize cost:

$$J_1 = 10 \sum_{i=1}^{12} \exp(-b_i * \sigma_{Xi})$$

b = [50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50]

- The second objective, J_2, Minimize the difference between the angles θ_1 and θ_2.

$$J_2 = \sqrt{V}$$

$$V = (X_n(2) - X_n(3))^2(\sigma_{X8}^2 + \sigma_{X7}^2) + (X_n(8) - X_n(7))^2(\sigma_{X2}^2 + \sigma_{X3}^2) + (X_n(6) - X_n(5))^2(\sigma_{X10}^2 + \sigma_{X9}^2) + (X_n(10) - X_n(9))^2(\sigma_{X6}^2 + \sigma_{X5}^2)$$

MECHANICAL ENGINEERING EXAMPLE

- Optimization problem statements

$$\min_{\sigma_X}\{J_1, J_2\}$$

such that

$$0.0001 \leq \sigma_X \leq 1$$

$$J_1 \leq 100$$

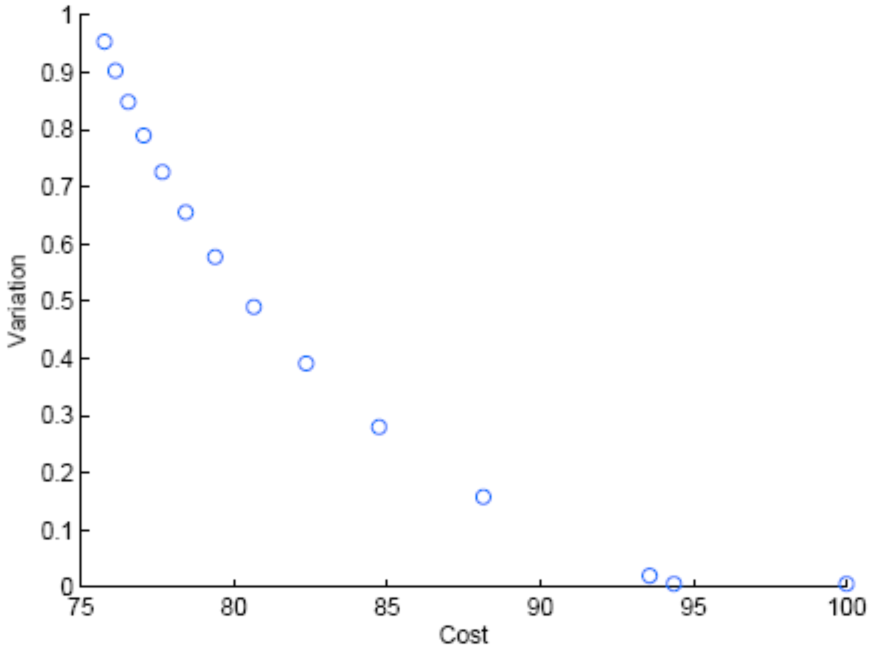
$$J_2 \leq 1$$

$$-(X_n(6) - X_n(5)) + (X_n(8) - X_n(7)) + 6\sqrt{\sigma_{X6}^2 + \sigma_{X5}^2 + \sigma_{X8}^2 + \sigma_{X7}^2} \leq 0$$

$$-(X_n(3) - X_n(4)) + (X_n(11) - X_n(10)) + 6\sqrt{\sigma_{X3}^2 + \sigma_{X4}^2 + \sigma_{X11}^2 + \sigma_{X10}^2} \leq 0$$

MECHANICAL ENGINEERING EXAMPLE

- Pareto Frontier



MATHEMATICAL EXAMPLE

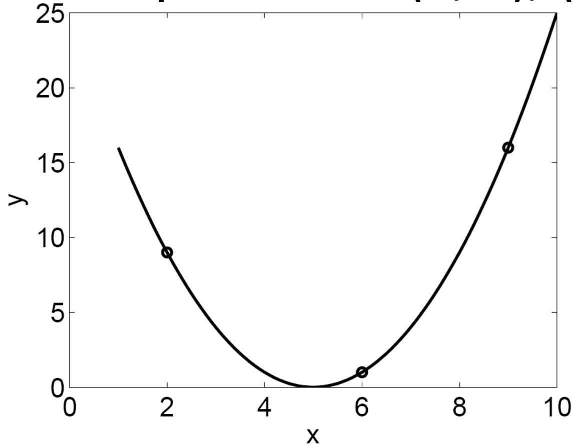
Data Fitting:

- **Data fitting** is the process of constructing **a mathematical function to fit to a series of data points**.
- Data points can be experiment results, simulation results, or observations of phenomena.
- There are many different methods for data fitting. The **least-squares approach**, which requires the use of optimization is introduced here.

MATHEMATICAL EXAMPLE

Data Fitting:

- Consider the three points marked as circles in Figure below. The coordinates of the three points are (2, 9), (6, 1), and (9, 16).



- To find **a quadratic function**, expressed by ***y = ax² + bx + c,*** to pass through these three points, the following system of equations must be solved;

$$2^2a + 2b + c = 9$$
$$6^2a + 6b + c = 1$$
$$9^2a + 9b + c = 16$$

MATHEMATICAL EXAMPLE

Data Fitting:

- The equations are satisfied by $a = 1$, $b = -10$, and $c = 25$.
- The method of **least-squares** is a popular and easy-to-implement approach to approximate the solution of over determined systems (sets of equations in which there are more equations than unknowns).
- The overall solution obtained by the least-squares data fitting minimizes the sum of the squares of the errors when solving every single equation.

MATHEMATICAL EXAMPLE

Least-Squares Data Fitting

- The least-squares data fitting minimizes the sum of squared residuals.
- The residuals are the differences between the recorded values and the fitted (or approximate) values provided by a model.
- Assume in the previous example, there is one more point (7.5, 6.5) in addition to the three points.
- A fitted quadratic function cannot pass through all the four points. Instead, the least-squares data fitting is used to fit a quadratic function that minimizes the sum of the errors between the fours points and the corresponding points on its curve.
- Let $\tilde{y}$ represent the fitted quadratic function. The sum of the errors is expressed as

Least-Squares Data Fitting

- Assume $\tilde{y}$ represent the fitted quadratic function. The sum of the errors is expressed as

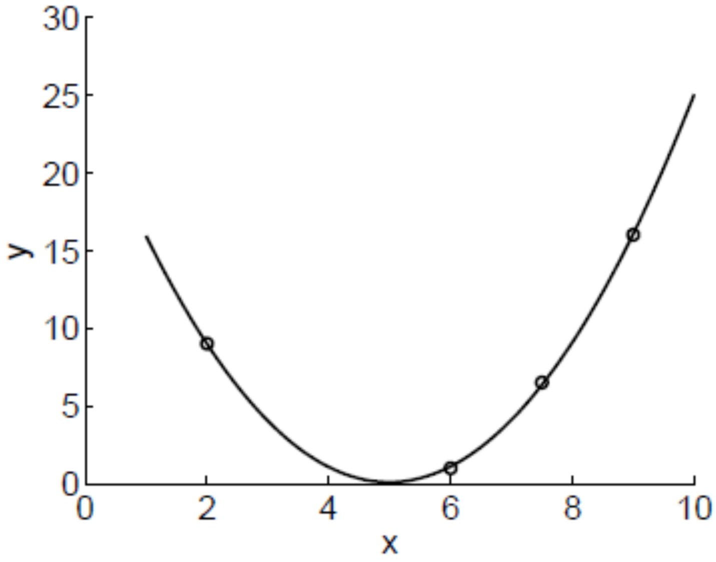
$$\sum_{n=1}^{4}(\tilde{y}-y)^2 = (2^2a+2b+c-9)^2+(6^2a+6b+c-1)^2$$
$$+(9^2a+9b+c-16)^2+(7.5^2a+7.5b+c-6.5)^2$$

- Perform an unconstrained minimization using Matlab. The corresponding values of the three parameters are a = 0.9937, b = −9.9198, and c = 24.8524, and the sum of the squared residuals is 0.0383

MATHEMATICAL EXAMPLE

Least-Squares Data Fitting

- The four points as well as the curve given by the fitted quadratic function is plotted below

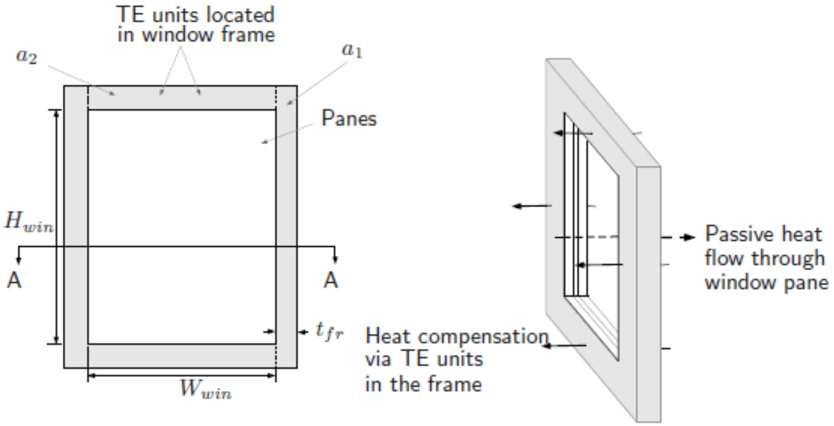


- It can be seen that the points are very close to, but not exactly on, the fitted curve.

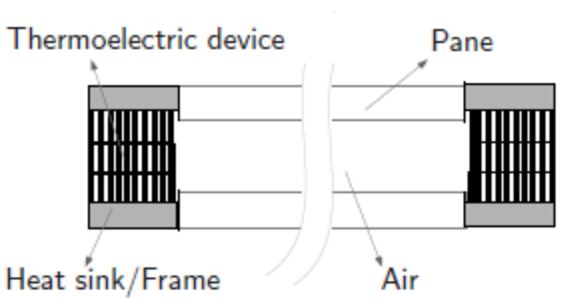
ELECTRICAL ENGINEERING EXAMPLE

Thermoelectric Window Design

- Thermoelectric (TE) window units are solid state devices that actively transfer heat in designated directions when supplied with electric power.
- They are small enough to be integrated into a window.
- The simplified schematic of the TE window shown in below



(a) Heat Transfer Process Taking Place through TE Window



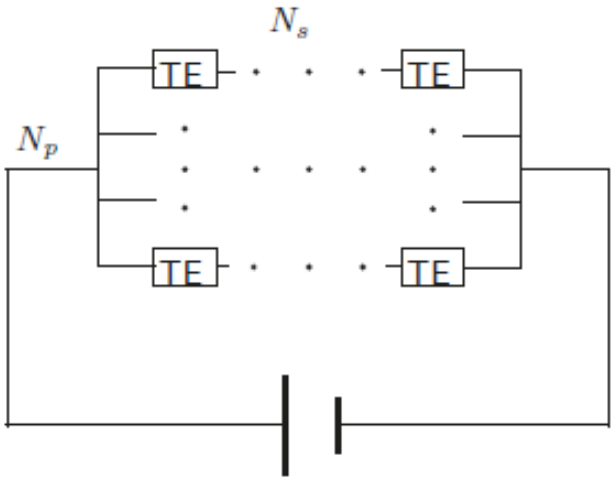
(b) Enlarged Broken View of Section A - A Showing TE Integration in Window Frame.

Thermoelectric Window Design

- TE units are installed within the frame of the window which, for practical purposes, also performs as a heat enhancer (heat sink).
- TE units in the window facilitate heat flow (solid lines) in the direction opposite to the passive heat flow through the window panes
- The frame of the TE window will be equipped with fins to maximize heat transfer.
- With this design, it is possible to integrate TE units in any glazing system once the frame is modified to accommodate the TE units.

Thermoelectric Window Design

- To achieve high heat transfer rate through the TE units installed on the window frame, the connection of the TE units in their electric network should be optimized.
- Suppose the TE units are connected in N_p parallel circuits. In each of the parallel circuits, there are N_s TE units connected in series.



Electric Network

ELECTRICAL ENGINEERING EXAMPLE

Thermoelectric Window Design

- The TE units are integrated into the window frames to transfer heat from inside to the outside in summer.
- Under this condition, the function of the TE units is to cool the air inside the room.
- The higher rate the heat is transferred through the TE units, the better the performance.
- The **objective of this optimization** is to maximize the total heat transfer rate through the TE units to the outside, expressed as $\dot{Q}_{cold}$.

Thermoelectric Window Design

- The numbers of TE units in series and in parallel, the total current, and the temperature difference across the TE units are the four parameters to be varied in the optimization.
- In this problem, the TE units are modeled based on their configuration and properties.

Modeling TE Units

- The ATI windows use TE units to control the heat transferred through the inner panes.
- Each TE unit consists of thermocouples which, when supplied with electric current, induce heat flow in the direction of the current. This is known as the **Peltier effect**.
- Because of the thermocouples' electrical resistance, heat is generated - the Joules effect.
- As the results of the two conflicting effects, heat is absorbed on the cold side and released from the hot side.

Modeling TE Units

- A temperature difference is created across the TE units. On the cold side of TE units, the heat rate is predicted as

$$\dot{Q}_{cold} = 2N_{te}N\left[\alpha I_{te}T_c - \frac{I_{te}^2\rho}{2G} - \kappa\Delta T_{te}G\right]$$

N_{te} number of TE units
N number of thermocouples in each TE unit;
α Seebeck coefficient;
I_{te} electric current;
T_c cold side temperature;
ρ resistivity;
G geometry factor, which represents the area to thickness ratio of the thermocouple;
κ thermal conductivity;
T_{te} temperature difference across the thermocouple.

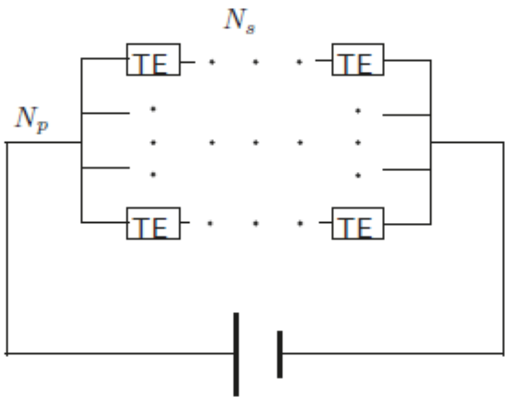
Modeling TE Units

- For a given TE unit, α, ρ, and κ are temperature **dependent properties**, N and G are **constants**, and are all provided by the manufacturer. The remaining four variables, I_{te}, T_{te}, T_c, and N_{te}, are **design variables**.
- TE units are connected in an electrical network, and the power is supplied to every TE unit.
- The TE units used for the TE window design are divided into several groups.
- The number of the groups is N_p . There are the same number of TE units in each group, which is N_s.
- Within each group, the TE units are connected in series. Then, all groups are connected in parallel.

ELECTRICAL ENGINEERING EXAMPLE

Modeling TE Units

- The electric voltage supplied to each group is the same.
- The connection of the TE units is shown below.



- The total number of TE units is

$$N_{te} = N_s N_p$$

ELECTRICAL ENGINEERING EXAMPLE

Modeling TE Units

- The voltage drop across the TE unit is given by

$$V_{te} = 2N\left[\frac{I_{te}\rho}{G} + \alpha\Delta T_{te}\right]$$

- The heat released from the hot side of the TE unit is the combination of the heat absorbed by the TE and the heat generated by electric current, which is given by

$$\dot{Q}_{hot} = N_{te}I_{te}V_{te} + \dot{Q}_{cold}$$

- The maximum allowable applied current

$$I_{max} = \frac{\kappa G}{\alpha}\left[\sqrt{(1+2ZT_h)} - 1\right]$$

Z figure-of-merit provided by the manufacturer
T_h hot side temperature

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

1. **Objective of Optimization**

    - The TE units are integrated into the window frames to transfer the heat from the inside of a room to the outside in summer.
    - In this condition, the function of the TE units is to cool the air inside the room.
    - The objective of optimization is to maximize the overall heat transfer rate through the TE units to the outside, which is $\dot{Q}_{cold}$.
    - The number of the TE units in series, N_s, the number of the TE units in parallel, N_p, the total electric current, I, and the temperature difference across the TE units, δ_T, are the **four variables for optimization**.

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

  2. **Physical Constraints**

     - The TE units are fixed on the frame of the window. The total area of all the TE units should not exceed the area of the frame, A_{frame}.
     - The total area of the TE units is expressed as

$$A_{all} = N_s N_p A_{te}$$

     - The electric current in each set of TE units should not exceed the maximum allowable current.

$$I_{te} \leq I_{max}$$

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

  2. **Physical Constraints**

     - According to energy conservation, the outside temperature is less than the addition of inside temperature and the temperature difference across the TE units

$$T_{out} \leq T_{cold} + N_s \Delta T_{te}$$

     - Bounds on the total current are used to avoid excessively large power requirement or unreasonable small power consumption.

$$0.01 \leq N_p I_{te} \leq 100$$

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

  3. **Modeling the Optimization Problem**

     - Considering the objective of the optimization and the constraints, the model of the optimization problem is as follows.

$$
\begin{aligned}
\min_{\{\Delta T, I, N_s, N_p\}} & \quad \dot{Q}_{cold}, \\
\text{subject to} & \\
& N_s N_p A_{te} \leq A_{frame}, \\
& I = N_p I_{te}, \\
& I_{te} \leq I_{max}, \\
& T_{out} \leq T_{cold} + N_s \Delta T_{te}, \\
& \Delta T_{te} \leq \Delta T_{max}, \\
& 0.01 \leq N_p I_{te} \leq 100.
\end{aligned}
$$

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

  4. **Optimization Procedure**

     - The trust region method is used for optimization. At the k iteration, the model of the TE units is approximated by a quadratic Taylor series expansion.

$$\dot{Q}_{cold_{approx}}(p) = \dot{Q}_{cold}(x_k) + G_k^T p + \frac{1}{2} p^T H_k p$$

$\dot{Q}_{cold_{approx}}(p)$	approximated objective function
$\dot{Q}_{cold}(x_k)$	optimal result from last iteration
p	step length
$G_k^T p$	gradient at the starting point
H_k	hessian at the starting point

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

4. **Optimization Procedure**

   - In each iteration, a ratio is defined as following to evaluate the agreement between the approximate objective function and the actual objective function.

$$\rho_k = \frac{\dot{Q}_{cold}(x_k) - \dot{Q}_{cold}(x_k + p)}{\dot{Q}_{cold_approx}(0) - \dot{Q}_{cold_approx}(p)}$$

   - The approximate objective is optimized using Matlab

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

4. **Optimization Procedure**
   - The approximate model is optimized as follows
     - Given $\hat{\Delta} > 0,\ \gamma = 0.3,\ \Delta_0 = \gamma\hat{\Delta},\ \alpha = 1,\ \text{and}\ \mu = \frac{1}{8}$
     - for k=1, 2, ...
     - Solve optimization problem for the approximate model
     - Check the improvement $\Delta\dot{Q}_{cold}$ on the actual objective function

if $\Delta\dot{Q}_{cold} \leq \alpha$

Stop optimization. The actual value of the objective function $\dot{Q}_{cold}$ is the final optimal value.

else continue.

Solving Optimization Problem

- The optimization of the connection of TE units can be solved in the following steps;

4. **Optimization Procedure**
   - The approximate model is optimized as follows

Evaluate ρ_k in Eq. 1.33

if $\rho_k \leq \frac{1}{4}$

$\Delta_{k+1} = \mu \Delta_k$

else

if $\rho_k \geq \frac{3}{4}$ and $\|p_k\| = \Delta_k$

$\Delta_{k+1} = min(2\Delta_k, \hat{\Delta})$

else $\Delta_{k+1} = \Delta_k$

if $\rho_k \ geq \mu$

$x_{k+1} = x_k + p_k$

else

$x_{k+1} = x_k$

end (for).

ELECTRICAL ENGINEERING EXAMPLE

Results

- The optimization of the problem is performed by Matlab.
- The maximum value of the objective function is $Q_{cold} = 1924$.
- The optimal values of the design variables obtained from the optimization are $T = 6$, $I = 23.36$, $N_s = 16.8$, and $N_p = 6.6$.
- Since the numbers of the TE units in series and in parallel are integers, the values of N_s and N_p are rounded to the closest integers.
- The final optimal results are $T = 6$, $I = 23.36$, $N_s = 17$, and $N_p = 7$.
- The optimization stops only after four iterations to evaluate the approximate model.

AEROSPACE ENGINEERING EXAMPLE

- In designing an aircraft landing gear, **the wheel track** (the length between the most left and the most right when looking at front-view) has several <u>requirements that must be considered</u>.
- These requirements include

(i) **Ground lateral control**,

The minimum allowable value for the wheel track must satisfy ground lateral control

(ii) **Ground lateral stability**,

lateral stability requirements. The maximum allowable value for the wheel

(iii) **Structural integrity**.

track must satisfy the structural integrity requirements.

➢Ground Controllability

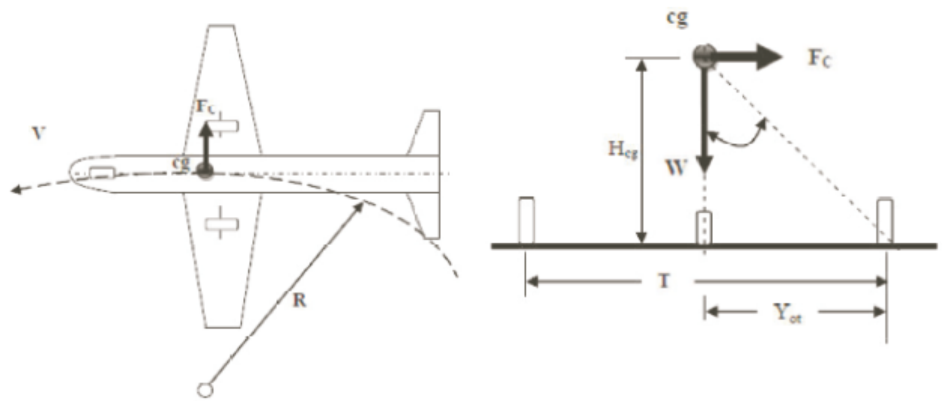
- The wheel track must be wide enough such that the aircraft does not roll over while taxiing on the ground.
- The centrifugal force (***F_c***) during a turn causes the aircraft to roll.

$$F_c = m\frac{V^2}{R}$$

aircraft ground speed

aircraft mass

radius of the turn



(a) Top View (b) Front View

An Aircraft in a Ground Turn

➢Ground Controllability

- The summation of the two contributing moments (the moment of the centrifugal force (Fc) and the moment of the aircraft weight, W) about the gear is given by:

$$\sum M_o = 0 \;\Rightarrow\; W \cdot T/2 + F_c \cdot H_{cg}$$

H_{cg}: distance of the aircraft center of gravity *cg*
T : length of the wheel track

- Therefore the wheel track (*T*) is given by:

$$T > 2\frac{F_c H_{cg}}{mg}$$

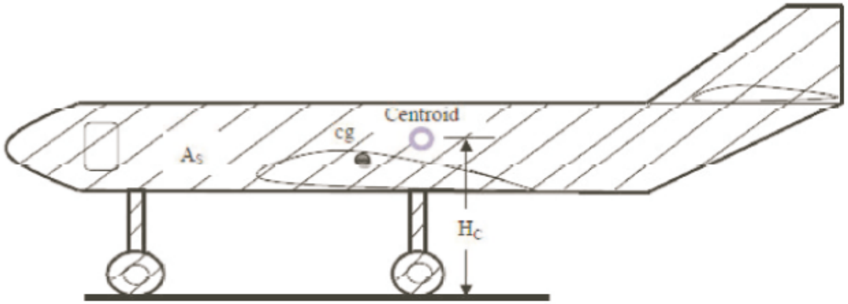
AEROSPACE ENGINEERING EXAMPLE

Ground Stability

- While taxiing on the ground, wind affects the stability of the aircraft and must be considered during the design process. A cross wind, i.e., perpendicular to the ground path or fuselage centerline of the aircraft, is the most important wind force when designing for ground stability.
- The cross wind force (***F_W***) on an aircraft can be modeled as a drag force given by

$$F_w = \frac{1}{2}\rho V_W^2 A_S C_{D_s}$$

ρ	**air density**
V_W	**wind speed**
C_{D_s}	**side drag coefficient**
A_S	**aircraft side area**



Aircraft Side Area and its Centroid

AEROSPACE ENGINEERING EXAMPLE

➢Ground Stability

- To prevent an aircraft from overturning due to the cross wind and to be stable on the ground, the wheel track (T) is given by:

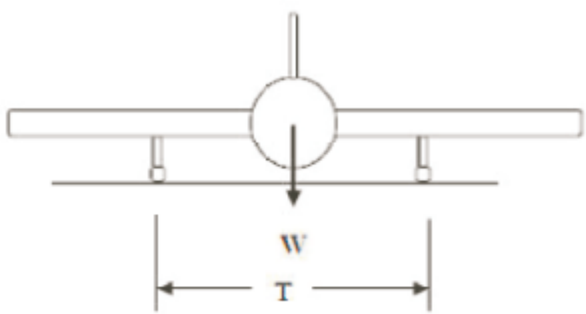
$$T > 2\frac{F_W H_c}{mg}$$

AEROSPACE ENGINEERING EXAMPLE

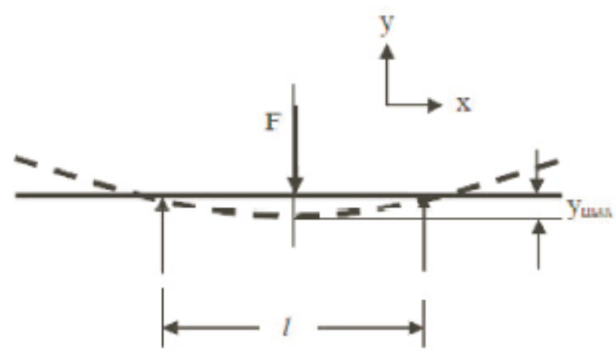
➢Structural Integrity

- When seen from the front, the aircraft structure can be modeled as a beam with a few simple supports.
- In an aircraft, at the main gear station, the wing is the beam and the two main wheels are the simple supports.

Front View of the Aircraft Structure Modeled as a Beam with Two Simple Supports:



(a) Aircraft Structure



(b) Beam with Two Simple Supports

➢Structural Integrity

- The maximum deflection (y_{max}) in a beam (wing) is determined as follows

$$y_{max} = -\frac{F_{m_{max}} T^3}{48EI}$$

$F_{m_{max}}$	maximum load on the main gear
E	modulus of elasticity
I	second moment of the area of the beam

Structural Integrity

- The maximum static load, which is carried by the main gear, can also be estimate by

$$F_{m_{max}} = \frac{B_{n_{max}}}{B} W$$

B wheel base (the distance between the nose gear and main)

$B_{n_{max}}$ maximum length between the aircraft *cg*, and the nose gear

- The wheel track in terms of maximum allowable deflection is given by:

$$T = \left[\frac{48EIBy_{max}}{WB_{n_{max}}}\right]^{\frac{1}{3}}$$

AEROSPACE ENGINEERING EXAMPLE

➢Structural Integrity

- For a twin engine jet transport aircraft, the maximum allowable wing deflection is **0.03 m** and the take-off mass is 50, 000 kg.
- From these requirements, we determine the necessary wheel track to satisfy all of the necessary requirements.

Constants and constraints are given as follows:

$$A_s = 150m^2 \qquad 40knot < V_w < 60knot \qquad (1knot = 0.5144m/s)$$
$$I = 0.003m^4 \qquad 25m < R < 35m$$
$$C_{D_s} = 0.8 \qquad 12m < B_{nmax} < 14m$$
$$E = 70Gpa \qquad 14m < B < 16m$$
$$H_{cg} = 3.5m \qquad 1.1455kgm^{-3} < \rho < 1.4224kgm^{-3}$$
$$H_c = 3.6m$$
$$V = 20knot$$

CIVIL ENGINEERING EXAMPLE

Design the exterior walls of a new building.

- The indoor temperature is T_i = 25°C. During the winter months, the outdoor temperature is consistently at T_o = 0°C.
- **The objective is to minimize the heat loss during the winter months**, while also being cost conscious.
- In the building and construction industry insulating materials are commonly described by their thermal resistance (R-value).
- The R-value being discussed is the unit thermal resistance. This is used for a unit value of any particular material. It is expressed as the thickness of the material divided by the thermal conductivity.
- For the thermal resistance of an entire section of material, instead of the unit resistance, divide the unit thermal resistance by the area of the material.
- The larger the R-value, the better the building insulation's effectiveness.

CIVIL ENGINEERING EXAMPLE

- The heat transfer in a simple system can be solved by using electrical resistance in series with a fixed potential.
- The only differences are: (i) the resistances are thermal resistances, and (ii) the potential is the difference in temperature from one side of the material to the other
- The resistance of each material to heat transfer depends on the specific thermal resistance [R-value]/[unit thickness].
- Assuming 1-D steady heat transfer, given by

$$\frac{d^2T}{dx^2} = 0$$

- The boundary conditions are given by

$$-k\frac{dT(0)}{dx} = h_0(T_0 - T(0)) \qquad -k\frac{dT(L)}{dx} = h_i(T(L) - T_i)$$

CIVIL ENGINEERING EXAMPLE

- The boundary conditions are given by

$$-k\frac{dT(0)}{dx} = h_0(T_0 - T(0)) \qquad -k\frac{dT(L)}{dx} = h_i(T(L) - T_i)$$

k : conduction coefficient
h : convection coefficient

- The convective heat transfer coefficients are h_i = 2 [W/mK] and h_0 = 22 [W/mK] for the inner and outer regions respectively.

- **The wall must cost no more than $100 and no thicker than 1m**

CIVIL ENGINEERING EXAMPLE

- Using the above equations along with the following material properties

Material	Thermal Conductivity, k (W/mK)	Cost ($/cm)
Concrete block	0.688	1
R-15 insulation board	0.04	15
Wood	0.212	3
Brick	1.24	0.50

- **Minimize the heat transfer rate and minimize the cost per unit wall area.**