



HW #06

Page 1 of 1

Fall 2021 | MECH 6318
Engineering Optimization – Prof. Jie Zhang
HOMEWORK #6
October 5, 2021

DUE: Tuesday, October 12, 2021 **11PM**
Submit the HW to eLearning

Points Distribution

30 points maximum

-5 to 0 points reserved for **Neatness and Professional Presentation**
(legible, stapled, show key Matlab commands, properly labeled plots, etc.)

Book Problems:

Problems 12.4 parts 1(b), 1(d), 2, 3, and 4.

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Book Problems:

Problems 12.4 parts 1(b), 1(d), 2, 3, and 4.

12.4 Consider the following function:

$$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{-x^2}, \quad \frac{1}{2} \leq x \leq 2$$

We wish to find the minimum of the above function in the given range of x .

1. Using the following listed methods, estimate the minimum of the above function using up to three iterations. Solve Parts (a) through (c) by hand, and please show your complete work.
 - (a) Interval Halving Method (*i.e.*, Bisection method)
 - (b) Newton-Raphson Method using $x_0 = 1$ as the starting point
 - (c) Golden Section Method
 - (d) Plot the function $f(x)$ using MATLAB, and identify the optimum solution in the range of x . For Parts (a) through (c), calculate the percentage error between the optimum x value and the value of x after three iterations. Present your results in a tabular form, as shown below, for methods in Parts (a), (b), and (c).
2. Write a MATLAB code for the Newton-Raphson method that evaluates the minimum of the function in Problem 1. Use an appropriate stopping criterion.
3. Verify your results using fmincon. Make sure your code gives you the same solution as that given by fmincon. You might have to change your stopping criterion.
4. How many function evaluations does your code need? How many function evaluations does fmincon need? Use the same starting point for your code and fmincon.

Table 12.6. Results Summary for Given Search Methods

Iteration	(a) x	(b) x	(c) x
	$f(x)$	$f(x)$	$f(x)$
1			
2			
3			

- Newton's method approximates the function $f(x)$ with a quadratic function at each iteration. The approximated quadratic function is minimized exactly, and a descent direction is evaluated.

$$\tilde{f}(x) \approx f(x_k) + \nabla f(x_k)^T(x - x_k) + \frac{1}{2}(x - x_k)^T \nabla^2 f(x_k)(x - x_k)$$

- The first derivative should satisfy $\nabla \tilde{f}(x) = 0$

$$\begin{aligned} \nabla \tilde{f}(x) &= \nabla f(x_k) + \nabla^2 f(x_k)(x - x_k) = 0 \\ \nabla^2 f(x_k)(x - x_k) &= -\nabla f(x_k). \\ x &= x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k) \end{aligned}$$

- The second term is used as the descent direction to calculate x_{k+1}

$$x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

- The descent direction

$$p_k = [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

- Look for the search direction p_k that satisfies

$$\nabla^2 f(x_k) p_k = -\nabla f(x_k)$$

- P_k can be solved by Gaussian Elimination, or other suitable methods.

12.4. 1b)

(b) Newton-Raphson Method using $x_0 = 1$ as the starting point

$$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{-x^2}, \quad \frac{1}{2} \leq x \leq 2$$

12.4.1c)

(b) Newton-Raphson Method using $x_0 = 1$ as the starting point

$$f(x) = -3x^3 + \frac{12}{x^2} + 2e^{x^2}, \quad \frac{1}{2} \leq x \leq 2$$

$$f'(x) = -9x^2 - 24x^{-3} + 4x e^{x^2} \quad \epsilon = 0.01$$

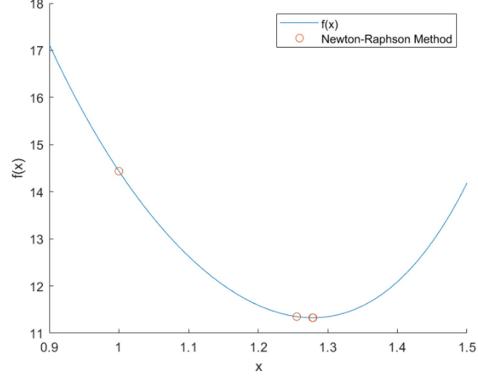
$$f''(x) = -18x + 72x^{-4} + 8x^2 e^{x^2} + 4e^{x^2}$$

K	x_k	$f(x_k)$	$f'(x_k)$	$f''(x_k)$	$P_k = \frac{f'(x_k)}{f''(x_k)}$	$x_{k+1} = x_k - P_k$	$ f'(x_k) < \epsilon ?$			
0	1.0	14.4	-22.12	8.662	-3.915	1.25527	N			
1	1.255	11.35	-2.027	6.71	-0.0234	1.27553	Y			
2	1.2788	11.32	0.065	9.1564	6e-4	1.2782	Y			
3										
4										

$x^* = 1.278$
 $f(x^*) = 11.327$

12.4.1d)

Problem 12.4 - Part 1d



See attached code for the rest of the problems...