MECH6318 HW03

Monday, September 20, 2021 1:13 PM

Fall 2021 | MECH 6318
Engineering Optimization – Prof. Jie Zhang
HOMEWORK #3
September 14, 2021

DUE: Tuesday, Sep 21, 2021 **5pm (central time)**
Submit the HW to eLearning

Points Distribution
30 points maximum
-5 to 0 points reserved for **Neatness and Professional Presentation**
(legible, stapled, show key Matlab commands, properly labeled plots, etc.)

Book Problems:

Problems 11.3 (Provide the Matlab plot), 11.4, 11.5, 11.7, 11.8 (Upload the M files to eLearning)

11.3 You are given the following optimization problem. Solve the following problem graphically.

$$\min_{x} \quad 8x_1 + 10x_2 + 4 \tag{11.118}$$

subject to

$$x_1 - x_2 \geq -4 \tag{11.119}$$
$$x_1 + x_2 \leq 6 \tag{11.120}$$
$$x_1, x_2 \geq 0 \tag{11.121}$$

Plot the objective function and the constraint equations. Identify the feasible design space and the optimal solution.

$$\min_{x} f(x) = 8x_1 + 10x_2 + 4$$

s.t.

$$x_1 - x_2 \geq -4 \longrightarrow -x_1 + x_2 \leq 4$$

s.t.

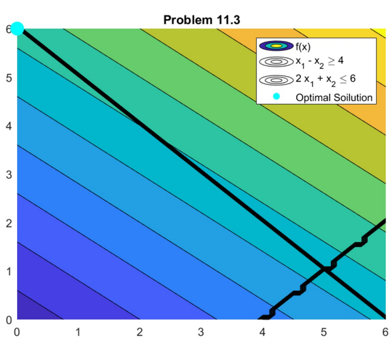
$$x_1 - x_2 \geq -4 \longrightarrow -x_1 + x_2 \leq 4$$

$$x_1 + x_2 \leq 6 \qquad A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$x_1, x_2 \geq 0$$

Problem 11.3

```
f = @(x) 8*x(1,:,:) + 10*x(2,:,:) + 4

A = [-1,-1;
     1, 1]
b = [4;
     6]

LB = [0;
     0]
UB = []

% Plot F(x)
lb = 0;
ub = 6;
[X1,X2] = meshgrid(lb:0.1:ub,lb:0.1:ub);
F = 8*X1 + 10*X2 + 4

figure()
hold on
contourf(X1, X2, F, 'DisplayName', 'f(x)')

contour(X1, X2, X1 - X2 >=  4, 'k',...
    'DisplayName', 'x_1 - x_2 \geq 4')
contour(X1, X2, X1 + X2 <=  6, 'k',...
    'DisplayName', '2 x_1 + x_2 \leq 6')
scatter(0,6, 150, 'filled', 'c',...
    'DisplayName', 'Optimal Soilution')
legend
title('Problem 11.3')
```



11.4 Transform the following problem into the standard LP formulation.

$$\min_{x} \; z = x_1 + 5x_2 + 3x_3 \tag{11.122}$$

subject to

$$x_1 - 5x_2 + x_3 \geq 1 \tag{11.123}$$
$$5x_1 + x_2 + 2x_3 \leq 5 \tag{11.124}$$
$$-2x_1 + 3x_2 + 3x_3 \leq 4 \tag{11.125}$$
$$3x_1 + 8x_2 + 5x_3 \leq 3 \tag{11.126}$$
$$x_1, x_2 \geq 0, x_3 \text{ unbounded} \tag{11.127}$$

$$\min_{x} \quad x_1 + 5x_2 + 3x_3 = F'x$$

$$F = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$$

s.t.

$$-x_1 + 5x_2 - x_3 \leq -1$$
$$5x_1 + x_2 + 2x_3 \leq 5$$
$$-2x_1 + 3x_2 + 3x_3 \leq 4$$
$$3x_1 + 8x_2 + 5x_3 \leq 3$$
$$x_1, x_2 \geq 0$$

$$Ax \leq b$$

$$A = \begin{bmatrix} -1 & 5 & -3 \\ 5 & 1 & 2 \\ -2 & 3 & 3 \\ 3 & 8 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} -1 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

11.5 Consider the following set of equations.

(1) How many basic solutions are possible for this set of constraints?

(2) Transform them into the reduced row echelon form with respect to the basic variables x_1, x_2, and x_3.

$$x_1 + 2x_2 + 2x_3 = 4 \tag{11.128}$$
$$x_1 + x_2 + x_3 - x_4 = 1 \tag{11.129}$$
$$3x_2 + x_3 + 2x_4 = 6 \tag{11.130}$$

Problem 11.5

```
A = [1, 2, 2, 0;
     1, 1, 1 -1;
     0, 3, 1, 2]
b = [4;
     1;
     6]

% Part a
n = size(A,1)
m = rank(A)
max_basic_sol = nchoosek(n,m)

% Part b
rref_11_5 = rref([A,b])
```

```
A = 3x4
       1     2     2     0
       1     1     1    -1
       0     3     1     2

b = 3x1
       4
       1
       6
```

```
n = 3
m = 3
max_basic_sol = 1
```

```
rref_11_5 = 3x5
      1.0000         0         0   -2.0000   -2.0000
           0    1.0000         0    0.5000    1.5000
           0         0    1.0000    0.5000    1.5000
```

11.7 Consider the following LP problem from Sec. 11.3.1. Solve it using the Simplex method.

$$\min_x \; x_1 - 2x_2 \tag{11.131}$$

subject to

$$-4x_1 + 6x_2 \le 9 \tag{11.132}$$

$$x_1 + x_2 \le 4 \tag{11.133}$$

$$x_1, x_2 \ge 0 \tag{11.134}$$

1) $c = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$A = \begin{bmatrix} -4 & 6 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$

$x_1, x_2 \ge 0$

2) simplex form:

$-4x_1 + 6x_2 + s_1 = 9$

$x_1 + x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2 \ge 0$

simplex table

$x_1 \quad x_2 \quad s_1 \quad s_2 \quad 1$

$$
\begin{array}{c|ccccc}
 & x_1 & x_2 & s_1 & s_2 & \\
\hline
R_1 & -4 & 6 & 1 & 0 & 9 \\
R_2 & 1 & 1 & 0 & 1 & 4 \\
R_3 & 1 & -2 & 0 & 0 & f
\end{array}
$$

3) pivital collumn

4) Pivital Row $\left(\frac{9}{6}\right) < \left(\frac{4}{1}\right)$

5) Pivital Element

Problem 11.7

```
c = [1;
    -2]

A = [-4, 6;
      1, 1]
b = [9;
     4]



syms f
T = [[A,eye(2),b];[c',zeros(1,2),f]]

R1 = T(1,:)
R2 = T(2,:)
R3 = T(3,:)

R1 = R1/6
T = [R1;R2;R3]

R2 = (3/5)*(R2 - R1)
T = [R1;R2;R3]

R1 = R1 + (2/3)*R2
T = [R1;R2;R3]

R3 = R3 + 2*R1
T = [R1;R2;R3]

R3 = R3 - R2
T = [R1;R2;R3]

final_simplex_tbl = T
final_simplex_soln = T(1:2,5)
final_simplex_value = c'*final_simplex_soln
```

T =

$$\begin{pmatrix} -\frac{2}{3} & 1 & \frac{1}{6} & 0 & \frac{3}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & -2 & 0 & 0 & f \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 1 & 0 & \frac{1}{5} & \frac{4}{5} & f+5 \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

T =

$$\begin{pmatrix} 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

```
final_simplex_tbl =
```

$$\begin{pmatrix} 1 & 0 & -\frac{1}{10} & \frac{3}{5} & \frac{3}{2} \\ 0 & 1 & \frac{1}{10} & \frac{2}{5} & \frac{5}{2} \\ 0 & 0 & \frac{3}{10} & \frac{1}{5} & f+\frac{7}{2} \end{pmatrix}$$

```
final_simplex_soln =
```

$$\begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$$

```
final_simplex_value =
```

$$-\frac{1}{2}$$

11.8 Solve the following problem using the Simplex method. Verify the correctness of your solution using `linprog`.

$$\min_{x} \; x_1 + 2x_2 - 7x_3 \tag{11.135}$$

subject to

$$2x_1 + x_2 + x_3 \leq 15 \tag{11.136}$$
$$-x_1 + 2x_2 - x_3 \leq 7 \tag{11.137}$$
$$x_1 + 5x_2 + 5x_3 \leq 25 \tag{11.138}$$
$$x_1, x_2, x_3 \geq 0 \tag{11.139}$$

$$c = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & 5 & 5 \end{bmatrix} \qquad b = \begin{bmatrix} 15 \\ 7 \\ 25 \end{bmatrix}$$

$$X_1, X_2, X_3 \geq 0$$

$$\begin{array}{c|cccccc|c|c} & X_1 & X_2 & X_3 & S_1 & S_2 & S_3 & & \\ \hline R_1 & 2 & 1 & 1 & 1 & 0 & 0 & 15 & \frac{15}{1} \\ R_2 & -1 & 2 & -1 & 0 & 1 & 0 & 7 & \cancel{\frac{7}{-1}} \\ R_3 & 1 & 5 & 5 & 0 & 0 & 1 & 25 & \frac{25}{5} \\ \hline R_4 & 1 & 2 & \underline{-7} & 0 & 0 & 0 & f & \end{array}$$

Problem 11.8

```
c = [1;
     2;
    -7]

A = [2, 1, 1;
    -1, 2,-1;
     1, 5, 5]
b = [15;
      7;
     25]


syms f
T = [[A,eye(3),b];[c',zeros(1,3),f]]

R1 = T(1,:)
R2 = T(2,:)
R3 = T(3,:)
R4 = T(4,:)
T = [R1; R2; R3; R4]

R3 = R3/5
T = [R1; R2; R3; R4]

R1 = R1 - R3
R2 = R2 + R3
R4 = R4 + 7*R3
T = [R1; R2; R3; R4]

R1 = (5/9) * R1
T = [R1;R2;R3;R4]

R2 = (1/3)*( R2 + (4/5)*R1)
T = [R1;R2;R3;R4]

R3 = R3 - R1/5 -R2
R4 = R4 - (12/5) * R1 - 9*R2
T = [R1;R2;R3;R4]

final_simplex_tbl = T
final_simplex_soln = T(1:3,7)
final_simplex_value = c'*final_simplex_soln
```

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ 1 & 5 & 5 & 0 & 0 & 1 & 25 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

R3 =

$$\begin{pmatrix} \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \end{pmatrix}$$

T =

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 & 15 \\ -1 & 2 & -1 & 0 & 1 & 0 & 7 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ 1 & 2 & -7 & 0 & 0 & 0 & f \end{pmatrix}$$

R1 =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \end{pmatrix}$$

R2 =

$$\begin{pmatrix} -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \end{pmatrix}$$

R4 =

$$\begin{pmatrix} \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

T =

$$\begin{pmatrix} \frac{9}{5} & 0 & 0 & 1 & 0 & -\frac{1}{5} & 10 \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ -\frac{4}{5} & 3 & 0 & 0 & 1 & \frac{1}{5} & 12 \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R2 =

$$\begin{pmatrix} 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ \frac{1}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 5 \\ \frac{12}{5} & 9 & 0 & 0 & 0 & \frac{7}{5} & f+35 \end{pmatrix}$$

R3 =

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

T =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

final_simplex_tbl =

$$\begin{pmatrix} 1 & 0 & 0 & \frac{5}{9} & 0 & -\frac{1}{9} & \frac{50}{9} \\ 0 & 1 & 0 & \frac{4}{27} & \frac{1}{3} & \frac{1}{27} & \frac{148}{27} \\ 0 & 0 & 1 & -\frac{7}{27} & -\frac{1}{3} & \frac{5}{27} & -\frac{43}{27} \\ 0 & 0 & 0 & -\frac{8}{3} & -3 & \frac{4}{3} & f-\frac{83}{3} \end{pmatrix}$$

Not Feasible….

final_simplex_soln =

$$\begin{pmatrix} \frac{50}{9} \\ \frac{148}{27} \\ -\frac{43}{27} \end{pmatrix}$$

final_simplex_value =

$$\frac{83}{3}$$